Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/190-7.2.4-f-x-^m-d+e-x^2-^p-a+b-arccosh-c-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [569]. This is test number [190].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (569)	0.00 (0)
Mathematica	97.54 (555)	2.46 (14)
Maple	82.78 (471)	17.22 (98)
Maxima	43.06 (245)	56.94 (324)
Fricas	41.65 (237)	58.35 (332)
Sympy	28.47 (162)	71.53 (407)
Mupad	25.31 (144)	74.69 (425)
Giac	18.63 (106)	81.37 (463)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

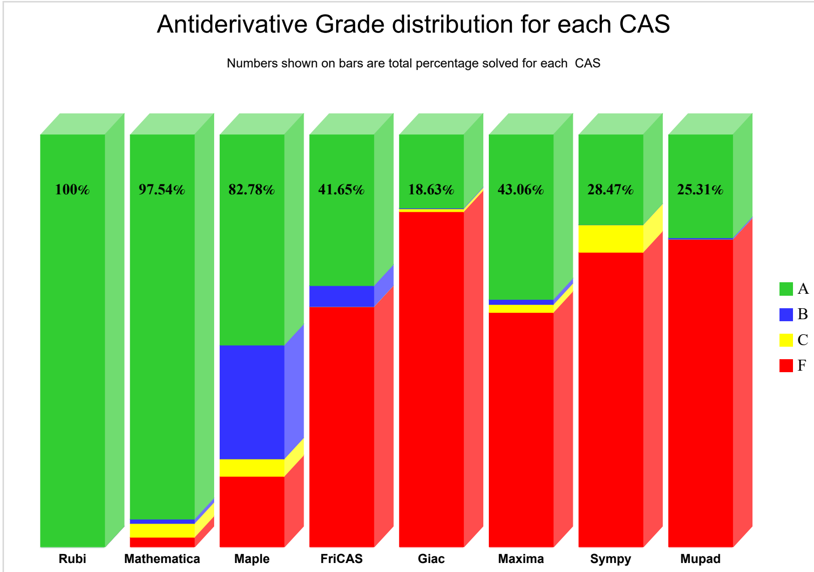
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

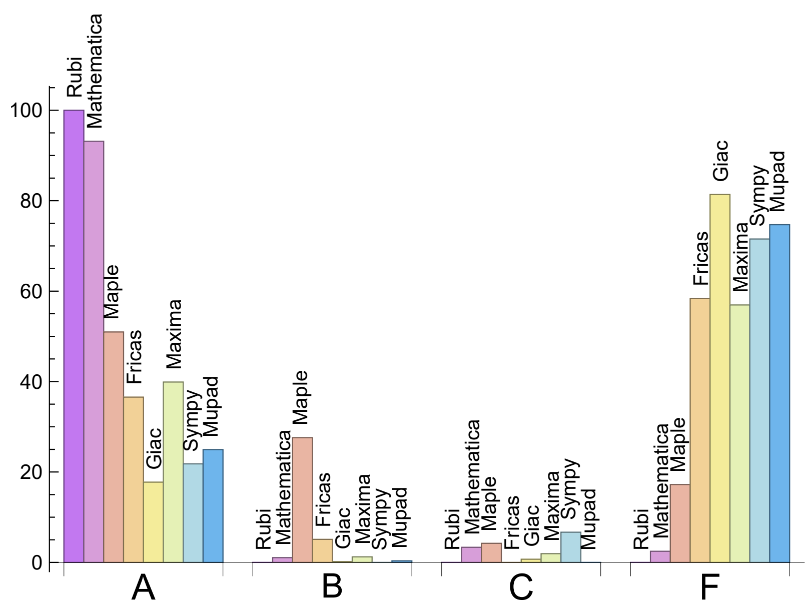
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.15	1.05	3.34	2.46
Maple	50.97	27.59	4.22	17.22
Maxima	39.89	1.23	1.93	56.94
Fricas	36.56	5.10	0.00	58.35
Mupad	N/A	0.35	0.00	74.69
Sympy	21.79	0.00	6.68	71.53
Giac	17.75	0.18	0.70	81.37

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	14	64.29 %	35.71 %	0.00 %
Maple	98	100.00 %	0.00 %	0.00 %
Fricas	332	80.12 %	0.00 %	19.88 %
Giac	463	52.48 %	0.65 %	46.87 %
Maxima	324	96.30 %	0.00 %	3.70 %
Sympy	407	84.28 %	9.34 %	6.39 %
Mupad	425	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

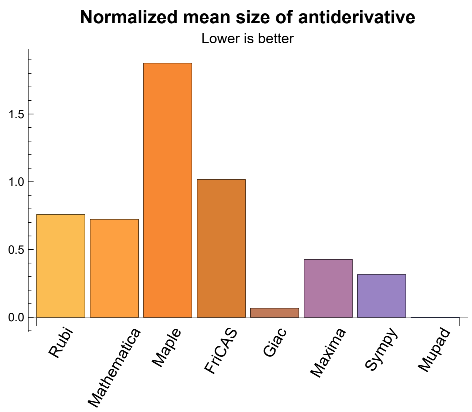
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

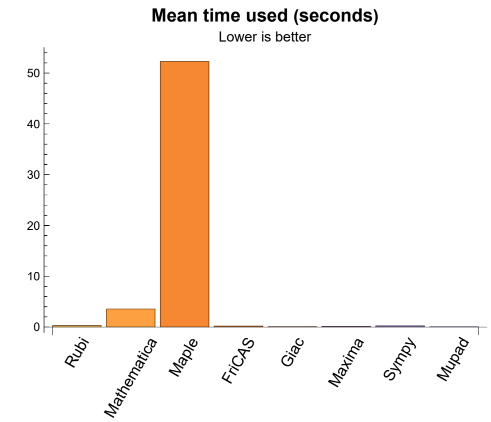
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.29	219.99	0.76	182.00	1.00
Mathematica	3.55	196.43	0.72	140.00	0.74
Maple	52.25	540.40	1.88	225.00	1.47
Maxima	0.15	91.63	0.43	0.00	0.00
Fricas	0.19	200.65	1.02	0.00	0.00
Sympy	0.23	75.30	0.31	0.00	0.00
Giac	0.03	5.21	0.07	0.00	0.00
Mupad	0.01	-0.24	-0.00	-1.00	-0.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{148, 149, 150, 233, 234, 235, 236, 237, 238, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {4, 6, 15, 17, 24, 26, 28, 29, 30, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 57, 58, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 112, 115, 117, 120, 122, 125, 130, 132, 142, 160, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 232, 245, 246, 247, 251, 252, 259, 260, 267, 268, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 315, 316, 317, 369, 370, 371, 372, 374, 375, 376, 377, 392, 397, 414, 415, 462, 464, 466, 475, 477, 484, 486, 489, 490, 491, 494, 495, 496, 497, 502, 506, 511, 512, 513, 543, 544, 545, 557, 558, 566, 567}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

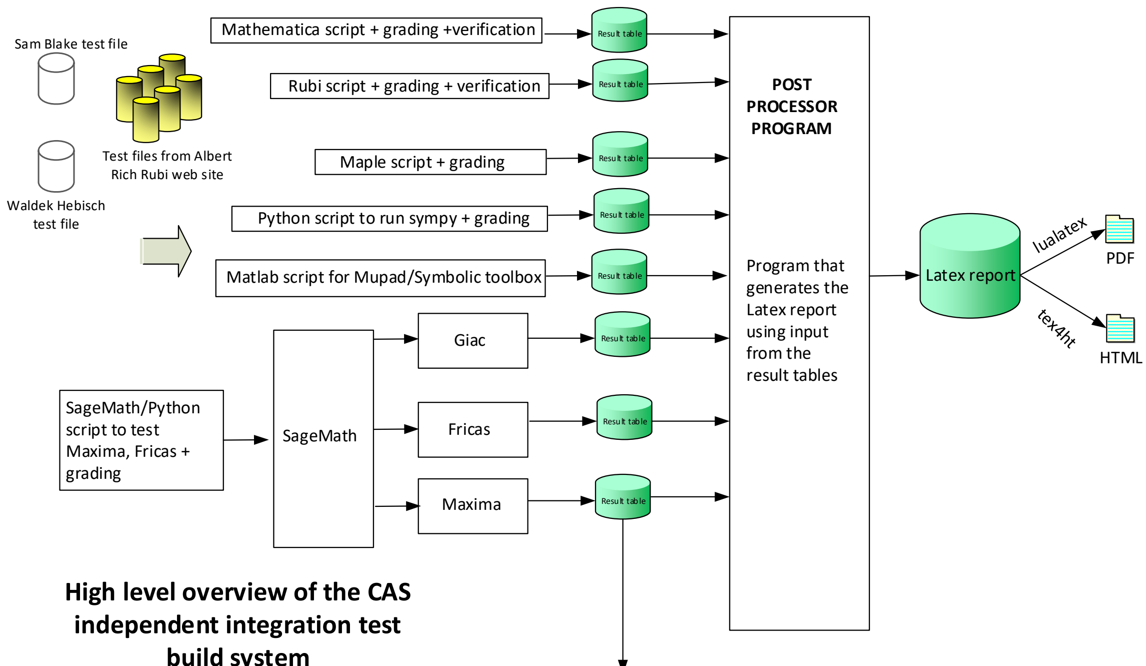
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 492, 493, 495, 497, 499, 507, 508, 514, 515, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { 44, 176, 260, 315, 316, 317 }

C grade: { 160, 251, 252, 489, 491, 494, 496, 498, 502, 503, 504, 505, 506, 511, 512, 513, 516, 517, 518 }

F grade: { 161, 162, 327, 334, 500, 501, 509, 510, 519, 520, 521, 546, 547, 551 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 68, 69, 70, 84, 85, 86, 100, 101, 102, 109, 110, 121, 130, 132, 134, 139, 142, 148, 149, 150, 166, 167, 168, 169, 227, 228, 229, 231, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 266, 269, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 288, 289, 290, 291, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 378, 381, 382, 383, 386, 387, 390, 391, 394, 395, 396, 399, 400, 405, 406, 407, 411, 412, 413, 416, 417, 418, 422, 423, 427, 428, 432, 433, 437, 438, 439, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 465, 466, 467, 468, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 514, 515, 522, 523, 524, 525, 526, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 42, 51, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 135, 136, 137, 138, 140, 141, 170, 171, 172, 173, 175, 177, 178, 179, 180, 181, 183, 185, 186, 187, 188, 189, 191, 193, 194, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 250, 251, 252, 253, 267, 268, 270, 276, 277, 278, 284, 285, 286, 287, 292, 293, 294, 295, 301, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 462, 464, 469, 499, 507, 508, 543 }

C grade: { 32, 55, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513 }

F grade: { 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 174, 176, 182, 184, 190, 192, 200, 202, 210, 212, 220, 222, 230, 232, 240, 241, 258, 260, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 384, 385, 388, 389, 392, 393, 397, 398, 401, 402, 403, 404, 408, 409, 410, 414, 415, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 516, 517, 518, 519, 520, 521, 527, 529, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 12, 14, 16, 18, 23, 25, 27, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 94, 95, 96, 97, 98, 99, 104, 106, 108, 113, 116, 119, 121, 126, 127, 129, 133, 134, 148, 149, 150, 164, 165, 166, 170, 172, 178, 180, 186, 188, 194, 196, 198, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 485, 487, 488, 514, 515, 522, 523, 524, 525, 526, 527, 528, 530, 531, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 11, 13, 19, 20, 21, 22, 40 }

C grade: { 62, 76, 93, 111, 136, 138, 141, 226, 228, 254, 256 }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 105, 107, 109, 110, 112, 114, 115, 117, 118, 120, 122, 123, 124, 125, 128, 130, 131, 132, 135, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 347, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 529, 532, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 63, 64, 65, 66, 67, 76, 77, 78, 79, 80, 81, 82, 83, 93, 94, 95, 96, 97, 98, 99, 104, 106, 108, 111, 113, 114, 116, 118, 124, 126, 128, 136, 138, 148, 149, 150, 164, 165, 166, 170, 172, 178, 180, 186, 188, 194, 196, 198, 226, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 254, 256, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 411, 416, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 470, 471, 472, 473, 474, 480, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551 }

B grade: { 62, 141, 296, 302, 347, 368, 437, 443, 467, 469, 476, 478, 479, 481, 482, 483, 485, 487, 488, 499, 507, 508, 516, 517, 518, 525, 526, 527, 528 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 105, 107, 109, 110, 112, 115, 117, 119, 120, 121, 122, 123, 125, 127, 129, 130, 131, 132, 133, 134, 135, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 299, 300, 301, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513, 519, 520, 521, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

2.1.6 Sympy

A grade: { 148, 149, 150, 164, 165, 166, 235, 236, 237, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 373, 378, 382, 383, 387, 395, 396, 400, 406, 407, 412, 422, 423, 427, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 453, 454, 455, 458, 459, 514, 515, 522, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 240, 241, 242, 461, 462, 463, 464, 465, 470, 471, 472, 473, 474, 479, 480, 481, 482, 483, 488, 525, 526, 527, 528 }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 341, 342, 343, 344, 345, 346, 347, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 452, 456, 457, 460, 466, 467, 468, 469, 475, 476, 477, 478, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 523, 524, 529, 534, 535, 536, 543, 544, 545, 547, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567, 569 }

2.1.7 Giac

A grade: { 134, 148, 149, 150, 236, 237, 238, 239, 261, 265, 266, 273, 275, 281, 283, 289, 291, 297, 298, 304, 305, 307, 309, 312, 313, 314, 318, 319, 325, 327, 332, 334, 339, 341, 349, 351, 353, 355, 356, 358, 360, 362, 365, 366, 367, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 528 }

C grade: { 138, 141, 228, 256 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 272, 274, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 290, 292, 293, 294, 295, 296, 299, 300, 301, 302, 303, 306, 308, 310, 311, 315, 316, 317, 320, 321, 322, 323, 324, 326, 328, 329, 330, 331, 333, 335, 336, 337, 338, 340, 342, 343, 344, 345, 346, 347, 348, 350, 352, 354, 357, 359, 361, 363, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 452, 453, 454, 457, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 525, 526, 527, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.8 Mupad

A grade: { 148, 149, 150, 233, 234, 235, 236, 237, 238, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 347, 368 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 525, 526, 527, 528, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	91	98	184	113	158	0	-1
N.S.	1	1.00	0.60	0.65	1.22	0.75	1.05	0.00	-0.01
time (sec)	N/A	0.103	0.112	2.726	0.294	0.369	0.723	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	166	186	202	108	144	0	-1
N.S.	1	1.00	1.23	1.38	1.50	0.80	1.07	0.00	-0.01
time (sec)	N/A	0.094	0.068	2.655	0.252	0.348	0.509	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	90	145	103	133	0	-1
N.S.	1	1.00	0.74	0.74	1.20	0.85	1.10	0.00	-0.01
time (sec)	N/A	0.091	0.079	2.670	0.261	0.351	0.363	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	146	162	98	124	0	-1
N.S.	1	1.00	1.02	1.49	1.65	1.00	1.27	0.00	-0.01
time (sec)	N/A	0.030	0.105	3.425	0.275	0.356	0.239	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	71	73	97	83	97	0	-1
N.S.	1	1.00	0.83	0.85	1.13	0.97	1.13	0.00	-0.01
time (sec)	N/A	0.050	0.063	2.109	0.253	0.344	0.139	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	116	131	0	0	0	0	-1
N.S.	1	1.00	0.99	1.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.166	5.021	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	110	101	66	127	0	0	-1
N.S.	1	1.00	1.45	1.33	0.87	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.127	1.947	0.467	0.391	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	106	137	0	0	0	0	-1
N.S.	1	1.00	0.79	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.132	7.592	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	127	119	89	146	0	0	-1
N.S.	1	1.00	1.41	1.32	0.99	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.169	1.987	0.474	0.392	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	264	124	128	319	165	236	0	-1
N.S.	1	1.28	0.60	0.62	1.55	0.80	1.15	0.00	-0.00
time (sec)	N/A	0.198	0.112	2.796	0.312	0.398	1.504	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	284	141	243	346	161	224	0	-1
N.S.	1	1.42	0.70	1.22	1.73	0.80	1.12	0.00	-0.00
time (sec)	N/A	0.184	0.165	2.770	0.277	0.362	1.084	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	223	116	120	261	153	209	0	-1
N.S.	1	1.26	0.66	0.68	1.47	0.86	1.18	0.00	-0.01
time (sec)	N/A	0.173	0.102	3.658	0.279	0.365	0.735	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	132	202	287	149	197	0	-1
N.S.	1	1.00	0.97	1.49	2.11	1.10	1.45	0.00	-0.01
time (sec)	N/A	0.051	0.122	2.753	0.266	0.386	0.527	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	177	99	102	194	133	172	0	-1
N.S.	1	1.24	0.69	0.71	1.36	0.93	1.20	0.00	-0.01
time (sec)	N/A	0.103	0.082	2.036	0.261	0.358	0.339	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	208	201	0	0	0	0	-1
N.S.	1	1.00	1.13	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.395	5.295	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	182	131	163	143	201	0	0	-1
N.S.	1	1.35	0.97	1.21	1.06	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.095	2.056	0.460	0.371	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	182	212	0	0	0	0	-1
N.S.	1	1.00	0.91	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.157	7.144	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	186	135	163	137	213	0	0	-1
N.S.	1	1.31	0.95	1.15	0.96	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.097	1.980	0.516	0.382	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	326	147	158	465	201	296	0	-1
N.S.	1	1.27	0.57	0.62	1.82	0.79	1.16	0.00	-0.00
time (sec)	N/A	0.289	0.126	3.540	0.269	0.348	2.906	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	328	168	286	501	197	287	0	-1
N.S.	1	1.43	0.73	1.24	2.18	0.86	1.25	0.00	-0.00
time (sec)	N/A	0.184	0.165	2.704	0.270	0.353	2.073	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	285	139	150	388	189	272	0	-1
N.S.	1	1.26	0.61	0.66	1.71	0.83	1.20	0.00	-0.00
time (sec)	N/A	0.266	0.150	2.686	0.274	0.351	1.473	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	156	244	423	185	260	0	-1
N.S.	1	1.00	0.94	1.47	2.55	1.11	1.57	0.00	-0.01
time (sec)	N/A	0.057	0.208	2.770	0.291	0.337	1.122	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	237	123	132	302	169	228	0	-1
N.S.	1	1.24	0.64	0.69	1.58	0.88	1.19	0.00	-0.01
time (sec)	N/A	0.191	0.130	1.991	0.278	0.349	0.739	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	305	255	0	0	0	0	-1
N.S.	1	1.00	1.28	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.377	6.141	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	239	136	212	231	249	0	0	-1
N.S.	1	1.33	0.76	1.18	1.28	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.165	2.010	0.471	0.434	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	303	267	0	0	0	0	-1
N.S.	1	1.00	1.13	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.374	10.750	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	252	142	216	208	253	0	0	-1
N.S.	1	1.29	0.73	1.11	1.07	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.178	2.023	0.487	0.400	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	242	0	0	0	0	-1
N.S.	1	1.00	1.44	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.267	5.237	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	151	222	0	0	0	0	-1
N.S.	1	1.00	1.08	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.218	4.572	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	155	187	0	0	0	0	-1
N.S.	1	1.00	1.52	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.108	4.254	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	85	162	0	0	0	0	-1
N.S.	1	1.00	1.15	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.065	2.657	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	312	0	0	0	0	-1
N.S.	1	1.00	1.08	5.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.048	11.767	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	93	88	0	0	0	0	-1
N.S.	1	1.00	1.52	1.44	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	0.107	5.273	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	132	163	0	0	0	0	-1
N.S.	1	1.00	1.39	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.201	4.876	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	144	283	0	0	0	0	-1
N.S.	1	1.00	1.22	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.371	4.490	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	223	219	0	0	0	0	-1
N.S.	1	1.00	1.42	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.240	5.822	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	244	268	0	0	0	0	-1
N.S.	1	1.00	1.38	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.660	7.879	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	209	278	0	0	0	0	-1
N.S.	1	1.00	1.17	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.374	7.829	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	206	230	0	0	0	0	-1
N.S.	1	1.00	1.66	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.457	6.573	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	64	134	65	0	0	-1
N.S.	1	1.00	0.87	1.05	2.20	1.07	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.063	2.864	0.282	0.351	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	189	230	0	0	0	0	-1
N.S.	1	1.00	1.58	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.875	4.252	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	149	339	0	0	0	0	-1
N.S.	1	1.00	1.28	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.468	4.545	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	283	256	0	0	0	0	-1
N.S.	1	1.00	1.66	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.429	5.842	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	319	347	0	0	0	0	-1
N.S.	1	1.00	2.10	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.329	5.913	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	377	335	0	0	0	0	-1
N.S.	1	1.00	1.52	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	1.032	6.296	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	287	352	0	0	0	0	-1
N.S.	1	1.00	1.15	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	1.061	10.216	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	83	136	0	101	0	0	-1
N.S.	1	1.00	0.61	1.00	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.101	2.031	0.000	0.332	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	287	352	0	0	0	0	-1
N.S.	1	1.00	1.54	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.986	7.188	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	64	86	0	98	0	0	-1
N.S.	1	1.00	0.70	0.95	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.084	1.964	0.000	0.344	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	316	352	0	0	0	0	-1
N.S.	1	1.00	1.76	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.679	5.118	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	210	508	0	0	0	0	-1
N.S.	1	1.00	1.23	2.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.882	7.316	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	362	387	0	0	0	0	-1
N.S.	1	1.00	1.57	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	1.120	5.760	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	273	606	0	0	0	0	-1
N.S.	1	1.00	1.09	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	1.794	9.605	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	471	481	0	0	0	0	-1
N.S.	1	1.00	1.52	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	1.126	6.750	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	169	0	0	0	0	-1
N.S.	1	1.00	1.45	3.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.037	11.244	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	161	0	0	0	0	-1
N.S.	1	1.00	1.10	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.581	3.951	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	223	205	0	0	0	0	-1
N.S.	1	1.00	1.36	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	1.532	4.391	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	198	878	0	0	0	0	-1
N.S.	1	1.00	0.71	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	0.788	5.357	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	151	367	0	0	0	0	-1
N.S.	1	1.00	0.75	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.709	4.123	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	144	278	0	0	0	0	-1
N.S.	1	1.00	1.16	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.386	4.065	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	137	286	0	0	0	0	-1
N.S.	1	1.00	1.16	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.288	4.631	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	1017	150	462	0	0	-1
N.S.	1	1.00	0.74	8.55	1.26	3.88	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.088	6.691	0.480	0.400	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	128	1742	146	549	0	0	-1
N.S.	1	1.00	0.64	8.75	0.73	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.141	7.102	0.487	0.426	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	146	2537	207	615	0	0	-1
N.S.	1	1.00	0.52	9.09	0.74	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.175	7.576	0.480	0.417	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	152	988	205	203	0	0	-1
N.S.	1	1.00	0.56	3.63	0.75	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.239	4.224	0.475	0.394	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	128	640	144	176	0	0	-1
N.S.	1	1.00	0.66	3.28	0.74	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.124	4.628	0.485	0.345	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	356	81	142	0	0	-1
N.S.	1	1.00	0.83	3.02	0.69	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.076	1.898	0.279	0.352	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	233	394	0	0	0	0	-1
N.S.	1	1.00	1.09	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.583	3.724	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	307	438	0	0	0	0	-1
N.S.	1	1.00	1.31	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.715	5.753	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	290	541	0	0	0	0	-1
N.S.	1	1.00	0.92	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.703	6.393	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	337	783	0	0	0	0	-1
N.S.	1	1.00	0.94	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	3.196	4.366	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	270	883	0	0	0	0	-1
N.S.	1	1.00	0.96	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	1.293	3.777	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	235	546	0	0	0	0	-1
N.S.	1	1.00	1.18	2.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.804	3.289	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	223	427	0	0	0	0	-1
N.S.	1	1.00	1.13	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.690	3.974	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	259	1181	0	0	0	0	-1
N.S.	1	1.00	1.28	5.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.516	5.967	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	94	2171	189	572	0	0	-1
N.S.	1	1.00	0.57	13.08	1.14	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.057	6.290	0.491	0.419	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	136	3145	163	648	0	0	-1
N.S.	1	1.00	0.55	12.73	0.66	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.099	6.934	0.491	0.429	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	154	4262	225	720	0	0	-1
N.S.	1	1.00	0.47	12.99	0.69	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.214	7.260	0.502	0.436	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	170	5523	287	792	0	0	-1
N.S.	1	1.00	0.42	13.50	0.70	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.292	9.740	0.530	0.426	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	182	1846	285	275	0	0	-1
N.S.	1	1.00	0.46	4.63	0.71	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.158	4.533	0.489	0.387	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	164	1376	223	245	0	0	-1
N.S.	1	1.00	0.51	4.29	0.69	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.145	0.121	3.266	0.485	0.345	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	136	966	161	215	0	0	-1
N.S.	1	1.00	0.56	3.98	0.66	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.131	2.815	0.500	0.354	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	107	620	102	185	0	0	-1
N.S.	1	1.00	0.65	3.76	0.62	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.126	1.405	0.262	0.350	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	336	499	0	0	0	0	-1
N.S.	1	1.00	1.15	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.803	3.475	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	500	542	0	0	0	0	-1
N.S.	1	1.00	1.61	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	1.032	7.225	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	574	570	0	0	0	0	-1
N.S.	1	1.00	1.79	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.818	5.770	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	500	1743	0	0	0	0	-1
N.S.	1	1.00	1.10	3.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	5.043	4.539	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	415	1289	0	0	0	0	-1
N.S.	1	1.00	1.12	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	3.143	3.809	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	347	885	0	0	0	0	-1
N.S.	1	1.00	1.18	3.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	1.621	2.527	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	305	550	0	0	0	0	-1
N.S.	1	1.00	1.07	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	1.154	4.014	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	319	1407	0	0	0	0	-1
N.S.	1	1.00	1.09	4.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.926	6.118	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	400	2429	0	0	0	0	-1
N.S.	1	1.00	1.37	8.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	2.200	8.072	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	105	3775	224	703	0	0	-1
N.S.	1	1.00	0.48	17.24	1.02	3.21	0.00	0.00	-0.00
time (sec)	N/A	0.093	0.068	7.069	0.506	0.428	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	147	5007	187	795	0	0	-1
N.S.	1	1.00	0.47	15.95	0.60	2.53	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.113	7.209	0.496	0.455	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	165	6382	251	879	0	0	-1
N.S.	1	1.00	0.43	16.58	0.65	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.141	8.098	0.499	0.421	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	193	2374	313	353	0	0	-1
N.S.	1	1.00	0.42	5.18	0.68	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.177	3.954	0.496	0.348	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	175	1840	249	317	0	0	-1
N.S.	1	1.00	0.46	4.87	0.66	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.133	3.380	0.484	0.372	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	145	1102	185	281	0	0	-1
N.S.	1	1.00	0.49	3.70	0.62	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.142	3.743	0.477	0.436	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	117	956	118	241	0	0	-1
N.S.	1	1.00	0.54	4.39	0.54	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.070	0.141	1.414	0.272	0.366	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	471	620	0	0	0	0	-1
N.S.	1	1.00	1.24	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	2.488	3.523	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	596	667	0	0	0	0	-1
N.S.	1	1.00	1.48	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	2.831	5.264	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	660	691	0	0	0	0	-1
N.S.	1	1.00	1.62	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.971	5.770	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	152	0	0	0	0	-1
N.S.	1	1.00	0.82	2.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.084	3.577	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	140	670	195	176	0	0	-1
N.S.	1	1.00	0.59	2.84	0.83	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.163	6.720	0.484	0.399	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	171	568	0	0	0	0	-1
N.S.	1	1.00	0.81	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.589	6.198	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	113	382	131	146	0	0	-1
N.S.	1	1.00	0.72	2.45	0.84	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.134	4.165	0.476	0.405	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	141	300	0	0	0	0	-1
N.S.	1	1.00	1.07	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.424	5.300	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	158	63	117	0	0	-1
N.S.	1	1.00	1.18	2.19	0.88	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.099	1.798	0.270	0.367	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	89	0	0	0	0	-1
N.S.	1	1.00	1.00	1.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.029	0.464	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	153	327	0	0	0	0	-1
N.S.	1	1.00	1.01	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.176	3.796	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	219	116	265	0	0	-1
N.S.	1	1.00	1.00	3.08	1.63	3.73	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.045	2.762	0.467	0.422	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	309	489	0	0	0	0	-1
N.S.	1	1.00	1.30	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.713	5.016	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	174	855	134	479	0	0	-1
N.S.	1	1.00	1.12	5.52	0.86	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.242	4.982	0.479	0.545	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	145	437	0	489	0	0	-1
N.S.	1	1.00	0.62	1.88	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.071	4.767	0.000	0.513	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	192	445	0	0	0	0	-1
N.S.	1	1.00	0.85	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.945	5.730	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	97	314	157	429	0	0	-1
N.S.	1	1.00	0.65	2.09	1.05	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.056	5.292	0.473	0.423	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	159	279	0	0	0	0	-1
N.S.	1	1.00	1.11	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.418	5.054	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	113	198	0	327	0	0	-1
N.S.	1	1.00	1.49	2.61	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.274	1.615	0.000	0.408	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	180	70	0	0	0	-1
N.S.	1	1.00	0.86	2.14	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.023	1.880	0.283	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	301	511	0	0	0	0	-1
N.S.	1	1.00	1.31	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.706	2.415	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	114	243	144	0	0	0	-1
N.S.	1	1.00	0.72	1.54	0.91	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.061	3.431	0.269	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	405	648	0	0	0	0	-1
N.S.	1	1.00	1.23	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	2.864	4.922	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	1053	0	0	0	0	-1
N.S.	1	1.00	0.64	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.083	4.908	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	167	471	0	529	0	0	-1
N.S.	1	1.00	0.69	1.94	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.116	4.944	0.000	0.407	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	236	225	1519	0	0	0	0	-1
N.S.	1	1.05	1.00	6.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.476	5.996	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	170	122	314	175	469	0	0	-1
N.S.	1	1.08	0.77	1.99	1.11	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.088	4.224	0.272	0.507	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	145	101	1251	169	0	0	0	-1
N.S.	1	1.09	0.76	9.41	1.27	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.141	7.244	0.276	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	139	140	249	0	421	0	0	-1
N.S.	1	1.09	1.10	1.96	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.169	2.082	0.000	0.409	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	174	132	1074	157	0	0	0	-1
N.S.	1	1.07	0.81	6.63	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.060	2.309	0.274	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	364	619	0	0	0	0	-1
N.S.	1	1.00	1.15	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	5.119	2.770	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	147	1350	0	0	0	0	-1
N.S.	1	1.00	0.59	5.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.271	2.954	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	468	801	0	0	0	0	-1
N.S.	1	1.00	0.98	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	6.200	4.615	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	218	1880	276	0	0	0	-1
N.S.	1	1.00	0.64	5.56	0.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.450	3.474	0.283	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	116	419	191	0	0	142	-1
N.S.	1	1.00	0.47	1.70	0.78	0.00	0.00	0.58	-0.00
time (sec)	N/A	0.097	0.067	3.648	0.280	0.000	0.000	0.456	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	93	456	0	0	0	0	-1
N.S.	1	1.00	0.64	3.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.193	7.062	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	311	62	101	0	0	-1
N.S.	1	1.00	0.67	2.83	0.56	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.096	4.526	0.505	0.369	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	75	223	0	0	0	0	-1
N.S.	1	1.00	0.85	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.118	5.947	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	55	123	28	72	0	40	-1
N.S.	1	1.00	1.12	2.51	0.57	1.47	0.00	0.82	-0.02
time (sec)	N/A	0.032	0.061	3.022	0.264	0.376	0.000	0.406	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	-1
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.016	1.257	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	113	270	0	0	0	0	-1
N.S.	1	1.00	1.10	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.106	3.917	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	168	73	89	0	80	-1
N.S.	1	1.00	1.19	3.50	1.52	1.85	0.00	1.67	-0.02
time (sec)	N/A	0.045	0.023	4.102	0.467	0.372	0.000	0.429	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	234	349	0	0	0	0	-1
N.S.	1	1.00	1.40	2.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.208	4.991	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.081	180.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	115	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.027	180.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	424	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.613	3.743	180.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	315	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.475	180.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	193	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.491	9.053	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.047	3.122	180.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	7.872	180.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	10.546	180.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	350	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	0.967	180.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	274	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.594	180.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	223	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.212	180.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.060	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	216	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.187	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	319	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.504	180.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	817	387	0	0	0	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	1.678	180.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	288	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.737	180.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	229	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.160	180.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	264	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	4.217	180.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	1.710	180.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.840	1.754	180.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	124	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.055	180.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	125	0	178	175	243	0	-1
N.S.	1	1.00	0.47	0.00	0.67	0.66	0.91	0.00	-0.00
time (sec)	N/A	0.448	0.124	180.000	0.272	0.392	1.082	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	101	0	134	142	182	0	-1
N.S.	1	1.00	0.52	0.00	0.69	0.73	0.93	0.00	-0.01
time (sec)	N/A	0.307	0.097	180.000	0.261	0.339	0.516	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	90	76	95	105	0	-1
N.S.	1	1.00	0.65	0.80	0.68	0.85	0.94	0.00	-0.01
time (sec)	N/A	0.179	0.103	1.176	0.254	0.356	0.199	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	187	0	0	0	0	-1
N.S.	1	1.00	0.97	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.059	2.309	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	191	255	0	0	0	0	-1
N.S.	1	1.00	1.17	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.579	3.148	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	319	320	0	0	0	0	-1
N.S.	1	1.00	1.24	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	3.045	3.487	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	237	1284	326	349	0	0	-1
N.S.	1	1.00	0.64	3.46	0.88	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.528	0.269	2.648	0.497	0.405	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	241	678	0	0	0	0	-1
N.S.	1	1.00	0.76	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	1.355	3.045	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	181	726	204	280	0	0	-1
N.S.	1	1.00	0.97	3.90	1.10	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.206	1.923	0.284	0.384	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	235	527	0	0	0	0	-1
N.S.	1	1.00	1.15	2.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.707	1.968	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	449	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	0.838	0.102	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	270	582	0	0	0	0	-1
N.S.	1	1.00	1.15	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	1.078	3.089	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	977	0	0	0	0	0	-1
N.S.	1	1.00	2.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	75.140	0.093	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	304	2633	0	0	0	0	-1
N.S.	1	1.00	0.90	7.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.696	4.480	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	262	1952	388	432	0	0	-1
N.S.	1	1.00	0.53	3.94	0.78	0.87	0.00	0.00	-0.00
time (sec)	N/A	0.784	0.343	2.392	0.492	0.433	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	485	1721	0	0	0	0	-1
N.S.	1	1.00	1.10	3.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.716	2.924	2.889	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	208	1270	278	367	0	0	-1
N.S.	1	1.00	0.60	3.65	0.80	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.298	1.144	0.287	0.392	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	374	1061	0	0	0	0	-1
N.S.	1	1.00	1.11	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	1.982	2.590	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	650	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	1.922	0.095	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	433	942	0	0	0	0	-1
N.S.	1	1.00	0.96	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	2.675	2.722	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1129	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	140.868	0.098	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	583	2879	0	0	0	0	-1
N.S.	1	1.00	1.37	6.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	1.374	5.644	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	880	880	288	2224	471	558	0	0	-1
N.S.	1	1.00	0.33	2.53	0.54	0.63	0.00	0.00	-0.00
time (sec)	N/A	1.233	0.418	2.394	0.495	0.416	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	841	841	910	2528	0	0	0	0	-1
N.S.	1	1.00	1.08	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.101	3.947	2.974	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	234	1958	337	477	0	0	-1
N.S.	1	1.00	0.50	4.17	0.72	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.363	1.150	0.283	0.411	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	740	1737	0	0	0	0	-1
N.S.	1	1.00	1.52	3.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	2.374	1.861	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	836	836	963	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	5.205	0.094	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	554	1227	0	0	0	0	-1
N.S.	1	1.00	0.91	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	3.849	2.802	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	890	890	1384	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	78.801	0.099	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	803	3431	0	0	0	0	-1
N.S.	1	1.00	1.26	5.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	2.170	4.304	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	255	1314	407	348	0	0	-1
N.S.	1	1.00	0.61	3.12	0.97	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.341	4.258	0.504	0.460	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	295	1092	0	0	0	0	-1
N.S.	1	1.00	0.83	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	1.045	4.561	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	201	752	279	282	0	0	-1
N.S.	1	1.00	0.69	2.58	0.96	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.294	3.295	0.485	0.430	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	228	563	0	0	0	0	-1
N.S.	1	1.00	1.01	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.558	4.037	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	149	314	145	218	0	0	-1
N.S.	1	1.00	0.96	2.03	0.94	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.221	1.923	0.283	0.384	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	149	0	0	0	0	-1
N.S.	1	1.00	1.00	2.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.034	0.398	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	315	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.377	180.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	237	513	0	0	0	0	-1
N.S.	1	1.00	1.27	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.516	2.037	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	697	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	75.109	0.112	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	370	2197	0	0	0	0	-1
N.S.	1	1.00	1.13	6.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.887	3.759	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	358	1101	0	0	0	0	-1
N.S.	1	1.00	0.64	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	2.456	3.869	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	343	1141	0	0	0	0	-1
N.S.	1	1.00	0.78	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	1.301	4.513	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	302	835	0	0	0	0	-1
N.S.	1	1.00	0.73	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	1.068	4.465	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	270	738	0	0	0	0	-1
N.S.	1	1.00	1.05	2.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	1.282	3.911	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	210	542	0	0	0	0	-1
N.S.	1	1.00	1.07	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.629	1.284	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	126	578	0	0	0	0	-1
N.S.	1	1.00	0.64	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.320	1.556	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	577	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	2.421	0.128	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	315	825	0	0	0	0	-1
N.S.	1	1.00	0.92	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	1.138	2.191	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	979	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.740	78.158	0.103	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	529	2867	0	0	0	0	-1
N.S.	1	1.00	1.07	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	1.698	3.638	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	437	1212	0	0	0	0	-1
N.S.	1	1.00	0.77	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.790	3.624	5.539	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	382	4074	0	0	0	0	-1
N.S.	1	1.00	0.79	8.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.653	1.707	4.526	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	341	834	0	0	0	0	-1
N.S.	1	1.00	1.01	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.466	3.002	3.503	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	264	3466	0	0	0	0	-1
N.S.	1	1.00	0.68	8.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	1.104	3.832	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	332	720	0	0	0	0	-1
N.S.	1	1.00	1.11	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	1.647	1.474	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	289	3049	0	0	0	0	-1
N.S.	1	1.00	0.87	9.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.993	2.162	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	806	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	9.296	0.107	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	457	3794	0	0	0	0	-1
N.S.	1	1.00	0.96	7.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	2.017	2.020	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	1181	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.006	86.494	0.105	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	534	5247	0	0	0	0	-1
N.S.	1	1.00	0.95	9.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.011	2.499	3.730	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	220	794	0	0	0	0	-1
N.S.	1	1.00	0.51	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	0.941	3.878	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	116	488	0	0	0	0	-1
N.S.	1	1.00	0.48	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.210	4.576	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	123	343	105	150	0	0	-1
N.S.	1	1.00	0.69	1.94	0.59	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.105	3.263	0.469	0.353	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	87	239	0	0	0	0	-1
N.S.	1	1.00	0.58	1.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.128	4.016	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	54	139	50	114	0	76	-1
N.S.	1	1.00	0.68	1.76	0.63	1.44	0.00	0.96	-0.01
time (sec)	N/A	0.053	0.070	2.330	0.275	0.351	0.000	0.424	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	-1
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.017	0.878	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.138	180.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	241	0	0	0	0	-1
N.S.	1	1.00	0.90	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.342	2.589	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	233	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.725	3.332	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1154	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.468	0.958	180.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	0.247	180.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.159	180.000	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.110	2.521	180.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.122	3.185	180.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.120	3.328	180.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.569	180.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	179	0	276	248	367	0	-1
N.S.	1	1.00	0.35	0.00	0.55	0.49	0.73	0.00	-0.00
time (sec)	N/A	1.002	0.158	180.000	0.266	0.351	1.482	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	147	0	210	204	274	0	-1
N.S.	1	1.00	0.38	0.00	0.54	0.53	0.71	0.00	-0.00
time (sec)	N/A	0.598	0.119	180.000	0.275	0.345	0.737	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	109	140	124	140	160	0	-1
N.S.	1	1.00	0.62	0.80	0.71	0.80	0.91	0.00	-0.01
time (sec)	N/A	0.301	0.114	1.175	0.262	0.427	0.323	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	253	0	0	0	0	-1
N.S.	1	1.00	0.90	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.075	1.516	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	276	416	0	0	0	0	-1
N.S.	1	1.00	1.06	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	1.429	2.909	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	455	527	0	0	0	0	-1
N.S.	1	1.00	1.18	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	7.311	4.280	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	189	887	0	0	0	0	-1
N.S.	1	1.00	0.31	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	0.839	2.959	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	148	536	0	0	0	0	-1
N.S.	1	1.00	0.37	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.330	2.553	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	98	256	0	0	0	0	-1
N.S.	1	1.00	0.42	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.151	2.767	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	55	0	0	0	0	-1
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.023	1.367	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	145	548	0	0	0	0	-1
N.S.	1	1.00	0.60	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.175	2.530	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	258	955	0	0	0	0	-1
N.S.	1	1.00	0.62	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.705	3.072	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	363	1319	0	0	0	0	-1
N.S.	1	1.00	0.60	2.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.622	1.330	2.853	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	136	520	0	0	0	0	-1
N.S.	1	1.00	0.43	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	0.324	5.080	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	140	375	131	205	0	0	-1
N.S.	1	1.00	0.58	1.54	0.54	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.116	3.417	0.478	0.403	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	98	255	0	0	0	0	-1
N.S.	1	1.00	0.52	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.166	4.382	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	101	155	65	159	0	103	-1
N.S.	1	1.00	0.92	1.41	0.59	1.45	0.00	0.94	-0.01
time (sec)	N/A	0.137	0.070	2.449	0.263	0.373	0.000	0.484	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	0	0	0	-1
N.S.	1	1.00	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.016	0.926	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	488	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.458	180.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	137	313	0	0	0	0	-1
N.S.	1	1.00	0.83	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.373	2.666	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	1051	0	0	0	0	0	-1
N.S.	1	1.00	2.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	4.029	3.374	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	2.962	180.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	42	0	0	0	0	-1
N.S.	1	1.00	0.67	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.209	2.433	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	35	0	0	0	0	-1
N.S.	1	1.00	0.68	0.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.124	2.768	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	0	0	0	0	-1
N.S.	1	1.00	0.86	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.105	1.738	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	1.111	180.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.020	4.401	2.334	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	188	597	0	0	0	0	-1
N.S.	1	1.00	0.55	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.377	4.585	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	171	547	0	0	0	0	-1
N.S.	1	1.00	0.58	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.322	3.352	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	103	229	0	0	0	0	-1
N.S.	1	1.00	0.74	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.224	5.304	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	127	363	0	0	0	0	-1
N.S.	1	1.00	0.64	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.234	2.287	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	105	229	0	0	0	0	-1
N.S.	1	1.00	0.76	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.157	3.243	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	0.910	180.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.228	0.812	180.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	1.036	180.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	0.627	180.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	215	731	0	0	0	0	-1
N.S.	1	1.00	0.54	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.678	3.295	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	188	597	0	0	0	0	-1
N.S.	1	1.00	0.55	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.540	4.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	172	547	0	0	0	0	-1
N.S.	1	1.00	0.58	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.505	2.923	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	147	413	0	0	0	0	-1
N.S.	1	1.00	0.62	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.338	3.235	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	0.930	180.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.403	1.070	180.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	1.086	180.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.653	180.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	216	731	0	0	0	0	-1
N.S.	1	1.00	0.54	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.951	4.286	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	233	781	0	0	0	0	-1
N.S.	1	1.00	0.53	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.875	3.920	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	216	731	0	0	0	0	-1
N.S.	1	1.00	0.54	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.789	2.145	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	191	597	0	0	0	0	-1
N.S.	1	1.00	0.56	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.547	4.404	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	0.943	180.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.659	0.986	180.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.094	1.073	180.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	0.734	180.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	69	249	0	0	0	0	-1
N.S.	1	1.00	0.70	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.088	4.812	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	200	0	0	0	0	-1
N.S.	1	1.00	0.92	3.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.114	0.068	5.560	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	149	0	0	0	0	-1
N.S.	1	1.00	0.92	2.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	0.077	4.308	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	100	0	0	0	0	-1
N.S.	1	1.00	1.79	3.57	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	0.061	3.006	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	48	0	55	0	0	-1
N.S.	1	1.00	1.68	1.71	0.00	1.96	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.044	3.639	0.000	0.355	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	0.409	180.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	0.519	2.992	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	130	347	0	0	0	0	-1
N.S.	1	1.00	0.66	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.219	4.224	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	99	232	0	0	0	0	-1
N.S.	1	1.00	0.71	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.208	5.127	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	171	0	0	0	0	-1
N.S.	1	1.00	0.88	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.158	3.550	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	54	55	0	65	0	0	-1
N.S.	1	1.00	1.54	1.57	0.00	1.86	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.081	2.572	0.000	0.379	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.545	180.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.090	0.957	180.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	3.387	180.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	5.133	180.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.114	180.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	2.619	180.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	1.656	180.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.094	0.790	180.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	0.141	180.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	0.465	180.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.879	180.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	1.271	180.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	257	109	0	0	0	0	-1
N.S.	1	1.00	2.62	1.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.272	4.332	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	194	85	0	0	0	0	-1
N.S.	1	1.00	2.37	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.206	2.479	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	217	61	0	0	0	0	-1
N.S.	1	1.00	3.74	1.05	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.154	1.290	2.118	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.161	2.273	180.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	9.143	2.007	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	322	1029	0	0	0	0	-1
N.S.	1	1.00	0.92	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	0.558	5.942	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	130	422	0	0	0	0	-1
N.S.	1	1.00	0.84	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.350	4.292	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	217	622	0	0	0	0	-1
N.S.	1	1.00	0.88	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.318	2.668	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	121	361	0	0	0	0	-1
N.S.	1	1.00	0.83	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.162	4.743	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	25.464	180.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	8.343	180.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	129.824	180.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	180.001	180.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	338	1176	0	0	0	0	-1
N.S.	1	1.00	0.95	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	0.748	4.971	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	327	1029	0	0	0	0	-1
N.S.	1	1.00	0.94	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	0.672	3.149	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	232	737	0	0	0	0	-1
N.S.	1	1.00	0.94	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.428	4.676	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	23.717	180.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	48.721	180.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.095	123.138	180.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	180.005	180.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	446	1676	0	0	0	0	-1
N.S.	1	1.00	0.98	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.834	1.200	5.314	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	436	1499	0	0	0	0	-1
N.S.	1	1.00	0.97	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.789	1.007	4.500	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	343	1176	0	0	0	0	-1
N.S.	1	1.00	0.98	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.719	4.028	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	5.946	180.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.430	11.892	180.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	15.707	180.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.095	15.423	180.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	190	1044	0	0	0	0	-1
N.S.	1	1.00	0.56	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.559	6.857	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	149	758	0	0	0	0	-1
N.S.	1	1.00	0.63	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.348	6.070	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	144	632	0	0	0	0	-1
N.S.	1	1.00	0.61	2.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.330	4.914	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	117	377	0	0	0	0	-1
N.S.	1	1.00	0.86	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.211	6.347	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	107	281	0	0	0	0	-1
N.S.	1	1.00	0.82	2.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.160	2.946	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	50	57	0	75	0	0	59
N.S.	1	1.00	1.35	1.54	0.00	2.03	0.00	0.00	1.59
time (sec)	N/A	0.033	0.024	1.375	0.000	0.349	0.000	0.000	0.451

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	3.366	180.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.122	180.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.100	21.746	180.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	4.708	180.000	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	15.543	180.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	1.649	180.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	16.290	180.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	14.557	180.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.295	4.090	180.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	35.324	180.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.100	5.247	180.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	31.668	180.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	2.539	180.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	26.680	180.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	10.230	180.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.107	0.894	180.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.105	0.167	180.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.545	180.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.100	1.023	180.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	1.380	180.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	51	0	56	0	0	48
N.S.	1	1.00	1.41	1.59	0.00	1.75	0.00	0.00	1.50
time (sec)	N/A	0.025	0.021	1.415	0.000	0.344	0.000	0.000	0.412

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	300	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.114	1.860	180.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	384	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.109	1.054	180.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	331	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.686	2.847	180.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	246	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	1.205	180.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.949	2.951	180.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	527	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.324	2.532	180.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	498	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.427	1.952	180.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	508	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.119	4.798	180.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	387	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.596	1.268	180.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.555	2.151	180.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	154	0	0	0	0	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.187	180.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	117	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.101	180.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.024	1.251	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	1.457	4.918	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	1.860	4.566	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	198	0	0	0	0	0	-1
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	0.351	180.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	136	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	0.251	180.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.024	1.444	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	1.516	4.339	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	213	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.937	0.363	180.000	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	148	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	0.344	180.000	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.024	1.420	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	1.159	4.098	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	165	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.203	180.000	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	121	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.106	180.000	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0	-1
N.S.	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.036	1.686	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.699	4.251	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	1.538	4.174	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	219	0	0	0	0	0	-1
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	0.334	180.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	144	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.271	180.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0	-1
N.S.	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.037	1.689	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.732	4.105	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.075	180.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	209	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.310	180.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	153	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.193	180.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	114	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.117	180.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	0	0	0	-1
N.S.	1	1.00	1.00	0.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.028	1.568	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	1.573	4.123	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.985	4.118	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	411	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.854	180.000	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	239	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.334	180.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	127	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.192	180.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	59	0	0	-1
N.S.	1	1.00	1.00	0.89	0.00	1.28	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.026	1.436	0.000	0.350	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	1.301	4.145	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.898	4.046	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	317	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.424	180.000	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	141	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.089	0.234	180.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	59	0	0	-1
N.S.	1	1.00	1.00	0.85	0.00	1.23	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.027	1.397	0.000	0.358	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.306	4.257	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.906	4.473	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	181	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.735	180.000	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	241	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.875	180.000	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	214	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.501	180.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.178	180.000	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.175	180.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	438	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	2.432	180.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	500	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	1.482	180.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	384	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	1.552	180.000	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	415	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.697	0.216	180.000	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	0.430	180.000	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	870	870	677	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	4.935	180.000	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	793	793	633	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	2.672	180.000	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	538	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	3.758	180.000	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	805	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.159	0.233	180.000	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.903	0.427	180.000	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	292	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.918	180.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	212	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.561	180.000	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.178	180.000	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	56	53	0	213	0	0	-1
N.S.	1	1.00	1.30	1.23	0.00	4.95	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.029	1.092	0.000	0.364	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	1.806	180.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	0.979	180.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	291	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.776	180.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.554	180.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	153	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.168	180.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	0	221	0	0	-1
N.S.	1	1.00	1.00	0.95	0.00	3.88	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.033	1.047	0.000	0.368	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.235	180.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.103	0.259	180.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.120	0.478	180.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	0.459	180.000	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.062	180.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.114	0.395	180.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.113	0.390	180.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	0.326	180.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	1.093	180.000	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	0.569	180.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.028	180.000	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.439	180.000	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.487	180.000	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.103	0.444	180.000	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.095	0.093	180.000	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.104	0.307	180.000	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.112	0.482	180.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	133	180	200	230	0	-1
N.S.	1	1.00	0.69	0.75	1.02	1.13	1.30	0.00	-0.01
time (sec)	N/A	0.096	0.080	2.719	0.266	0.347	0.700	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	140	332	198	192	212	0	-1
N.S.	1	1.00	0.87	2.06	1.23	1.19	1.32	0.00	-0.01
time (sec)	N/A	0.100	0.123	3.073	0.262	0.371	0.507	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	101	115	141	172	178	0	-1
N.S.	1	1.00	0.73	0.83	1.02	1.25	1.29	0.00	-0.01
time (sec)	N/A	0.087	0.074	2.577	0.259	0.382	0.336	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	285	158	163	160	0	-1
N.S.	1	1.00	0.98	2.34	1.30	1.34	1.31	0.00	-0.01
time (sec)	N/A	0.076	0.096	2.269	0.275	0.363	0.243	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	90	93	134	116	0	-1
N.S.	1	1.00	0.81	0.96	0.99	1.43	1.23	0.00	-0.01
time (sec)	N/A	0.053	0.065	1.914	0.264	0.354	0.151	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	119	130	0	0	0	0	-1
N.S.	1	1.00	0.45	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.159	4.985	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	105	108	65	174	0	0	-1
N.S.	1	1.00	1.40	1.44	0.87	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.094	1.951	0.476	0.409	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	101	147	0	0	0	0	-1
N.S.	1	1.00	0.40	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.092	6.224	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	128	167	87	192	0	0	-1
N.S.	1	1.00	1.36	1.78	0.93	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.184	1.862	0.461	0.423	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	192	227	305	444	422	0	-1
N.S.	1	1.00	0.60	0.71	0.96	1.39	1.32	0.00	-0.00
time (sec)	N/A	0.311	0.170	3.247	0.264	0.377	1.452	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	220	525	332	438	389	0	-1
N.S.	1	1.00	0.65	1.54	0.97	1.28	1.14	0.00	-0.00
time (sec)	N/A	0.248	0.203	2.870	0.283	0.392	1.077	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	163	195	247	385	340	0	-1
N.S.	1	1.00	0.63	0.75	0.95	1.48	1.31	0.00	-0.00
time (sec)	N/A	0.217	0.141	2.918	0.261	0.359	0.730	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	189	435	273	373	306	0	-1
N.S.	1	1.00	0.70	1.62	1.01	1.39	1.14	0.00	-0.00
time (sec)	N/A	0.168	0.170	2.800	0.265	0.364	0.526	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	130	157	180	321	246	0	-1
N.S.	1	1.00	0.66	0.80	0.92	1.64	1.26	0.00	-0.01
time (sec)	N/A	0.139	0.140	1.958	0.283	0.356	0.353	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	369	228	225	0	0	0	0	-1
N.S.	1	1.08	0.67	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	0.340	2.940	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	185	128	199	134	424	0	0	-1
N.S.	1	1.16	0.80	1.24	0.84	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.154	1.919	0.478	0.372	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	173	225	0	0	0	0	-1
N.S.	1	1.00	0.54	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	0.272	7.718	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	133	216	126	390	0	0	-1
N.S.	1	1.00	0.72	1.17	0.68	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.157	1.900	0.465	0.427	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	276	335	449	822	638	0	-1
N.S.	1	1.00	0.63	0.77	1.03	1.89	1.47	0.00	-0.00
time (sec)	N/A	0.437	0.253	3.507	0.311	0.390	2.808	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	310	745	485	815	604	0	-1
N.S.	1	1.00	0.63	1.51	0.98	1.65	1.22	0.00	-0.00
time (sec)	N/A	0.446	0.154	3.404	0.283	0.363	2.069	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	236	289	372	720	532	0	-1
N.S.	1	1.00	0.65	0.79	1.02	1.97	1.46	0.00	-0.00
time (sec)	N/A	0.362	0.208	2.754	0.278	0.371	1.474	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	262	614	407	705	490	0	-1
N.S.	1	1.00	0.73	1.72	1.14	1.97	1.37	0.00	-0.00
time (sec)	N/A	0.245	0.250	3.392	0.266	0.399	1.099	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	193	235	285	611	396	0	-1
N.S.	1	1.00	0.67	0.82	0.99	2.13	1.38	0.00	-0.00
time (sec)	N/A	0.252	0.200	1.849	0.271	0.361	0.741	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	344	351	0	0	0	0	-1
N.S.	1	1.00	0.68	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.703	0.717	1.212	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	182	309	220	771	0	0	-1
N.S.	1	1.00	0.69	1.17	0.83	2.91	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.210	2.107	0.469	0.411	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	278	296	0	0	0	0	-1
N.S.	1	1.00	0.58	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	0.495	2.488	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	184	309	195	767	0	0	-1
N.S.	1	1.00	0.71	1.19	0.75	2.95	0.00	0.00	-0.00
time (sec)	N/A	0.316	0.250	1.944	0.479	0.415	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	265	331	411	1039	600	0	-1
N.S.	1	1.00	0.67	0.84	1.04	2.63	1.52	0.00	-0.00
time (sec)	N/A	0.333	0.285	1.856	0.269	0.356	1.503	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	524	364	0	0	0	0	-1
N.S.	1	1.00	0.84	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	1.043	79.092	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	512	2970	0	0	0	0	-1
N.S.	1	1.00	0.98	5.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	0.404	9.769	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	457	301	0	0	0	0	-1
N.S.	1	1.00	0.84	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.562	15.787	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	447	2845	0	0	0	0	-1
N.S.	1	1.00	1.00	6.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.104	6.037	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	397	241	0	0	0	0	-1
N.S.	1	1.00	0.79	0.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	0.252	10.947	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	418	393	0	0	0	0	-1
N.S.	1	1.00	0.85	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	0.579	6.255	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	549	339	0	0	0	0	-1
N.S.	1	1.00	1.01	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	1.012	17.026	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	479	495	0	0	0	0	-1
N.S.	1	1.00	0.87	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.706	0.958	6.609	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	641	430	0	0	0	0	-1
N.S.	1	1.00	1.03	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.666	1.120	18.651	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	693	3019	0	0	0	0	-1
N.S.	1	1.00	1.23	5.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.684	1.332	10.666	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	646	0	978	0	0	-1
N.S.	1	1.00	1.09	5.72	0.00	8.65	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.212	7.670	0.000	0.411	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	0	529	0	0	0	0	-1
N.S.	1	1.00	0.00	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	2.883	14.827	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	0	744	0	0	0	0	-1
N.S.	1	1.00	0.00	1.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	3.485	8.529	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	777	1749	0	0	0	0	-1
N.S.	1	1.00	0.93	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.592	1.345	75.658	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	792	792	720	1689	0	0	0	0	-1
N.S.	1	1.00	0.91	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.407	1.060	31.326	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	734	1716	0	0	0	0	-1
N.S.	1	1.00	0.91	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.747	1.465	37.933	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	846	846	821	1821	0	0	0	0	-1
N.S.	1	1.00	0.97	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.474	1.714	89.476	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	1097	5257	0	0	0	0	-1
N.S.	1	1.00	1.49	7.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.845	6.546	30.056	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	241	192	2516	0	3548	0	0	-1
N.S.	1	1.04	0.83	10.89	0.00	15.36	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.528	10.163	0.000	0.512	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	183	2459	0	3391	0	0	-1
N.S.	1	1.00	1.03	13.89	0.00	19.16	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.628	10.171	0.000	0.478	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	772	772	0	1478	0	0	0	0	-1
N.S.	1	1.00	0.00	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	4.719	10.747	0.000	0.000	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	834	834	0	1938	0	0	0	0	-1
N.S.	1	1.00	0.00	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.925	7.248	12.177	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1224	1224	1185	3148	0	0	0	0	-1
N.S.	1	1.00	0.97	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.986	6.045	69.811	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1193	2292	0	0	0	0	-1
N.S.	1	1.00	0.97	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.269	6.042	60.570	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1162	3149	0	0	0	0	-1
N.S.	1	1.00	0.94	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.160	5.632	37.668	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	3.645	180.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	2.599	180.000	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	556	0	0	333	0	0	-1
N.S.	1	1.00	5.50	0.00	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.139	12.385	180.000	0.000	0.389	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	190	633	0	0	955	0	0	-1
N.S.	1	1.04	3.48	0.00	0.00	5.25	0.00	0.00	-0.01
time (sec)	N/A	0.138	1.883	180.000	0.000	0.462	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	685	0	0	2519	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	8.87	0.00	0.00	-0.00
time (sec)	N/A	0.566	2.946	180.000	0.000	0.494	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	558	529	0	0	0	0	0	0	-1
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.782	0.804	180.000	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	332	0	0	0	0	0	0	-1
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.372	180.000	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	187	0	0	0	0	0	0	-1
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.324	10.896	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	8.385	180.000	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	8.284	180.000	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	19.314	180.000	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	453	683	680	1528	996	0	-1
N.S.	1	1.00	0.74	1.12	1.12	2.51	1.64	0.00	-0.00
time (sec)	N/A	1.376	0.476	1.570	0.299	0.430	1.179	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	299	367	429	767	602	0	-1
N.S.	1	1.00	0.83	1.02	1.19	2.14	1.68	0.00	-0.00
time (sec)	N/A	0.839	0.319	1.549	0.284	0.380	0.582	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	174	0	222	304	286	0	-1
N.S.	1	1.00	1.04	0.00	1.32	1.81	1.70	0.00	-0.01
time (sec)	N/A	0.397	0.172	180.000	0.276	0.366	0.269	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	84	78	72	96	88	111	-1
N.S.	1	1.00	1.65	1.53	1.41	1.88	1.73	2.18	-0.02
time (sec)	N/A	0.109	0.054	1.731	0.265	0.358	0.095	0.473	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	763	763	623	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.988	0.400	180.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	9.147	180.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	7.805	180.000	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	12.751	180.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	60.602	180.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	254	380	0	0	0	0	-1
N.S.	1	1.00	0.65	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	0.389	9.415	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	180	125	178	0	0	0	0	-1
N.S.	1	1.29	0.90	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.168	7.378	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	56	0	0	0	0	-1
N.S.	1	1.00	0.85	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.050	3.664	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.427	180.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	2.171	180.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.719	180.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	0.795	180.000	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.075	180.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	2.715	180.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	663	1102	0	0	0	0	-1
N.S.	1	1.00	1.30	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	1.400	10.278	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	338	465	0	0	0	0	-1
N.S.	1	1.00	1.32	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.690	7.841	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	125	0	0	0	0	-1
N.S.	1	1.00	0.89	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.237	4.147	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	20.815	180.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	17.214	180.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	165.383	180.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	536	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.593	4.390	180.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	317	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.829	1.883	180.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	100	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.174	0.161	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	3.335	180.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	14.398	180.000	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	812	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.123	2.329	180.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	269	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.510	0.105	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	1.477	180.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	8.764	180.000	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	0.788	180.000	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	213	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.448	180.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	100	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.086	0.171	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.116	180.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.063	180.000	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	268	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	1.355	180.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	132	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.216	0.154	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.124	180.000	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.067	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [157] had the largest ratio of [35]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	23	0.261
2	A	7	7	1.00	23	0.304
3	A	6	6	1.00	23	0.261
4	A	4	3	1.00	21	0.143
5	A	4	4	1.00	20	0.200
6	A	8	8	1.00	23	0.348
7	A	5	6	1.00	23	0.261
8	A	9	9	1.00	23	0.391
9	A	5	6	1.00	23	0.261
10	A	7	7	1.28	25	0.280
11	A	9	10	1.42	25	0.400
12	A	6	6	1.26	25	0.240
13	A	5	3	1.00	23	0.130
14	A	6	6	1.24	22	0.273
15	A	12	8	1.00	25	0.320
16	A	8	8	1.35	25	0.320
17	A	13	11	1.00	25	0.440
18	A	8	9	1.31	25	0.360
19	A	6	6	1.27	25	0.240
20	A	11	10	1.43	25	0.400
21	A	6	6	1.26	25	0.240
22	A	6	3	1.00	23	0.130
23	A	6	6	1.24	22	0.273
24	A	17	8	1.00	25	0.320
25	A	8	8	1.33	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	18	11	1.00	25	0.440
27	A	9	9	1.29	25	0.360
28	A	12	8	1.00	25	0.320
29	A	8	8	1.00	25	0.320
30	A	8	6	1.00	25	0.240
31	A	5	5	1.00	23	0.217
32	A	6	4	1.00	22	0.182
33	A	7	5	1.00	25	0.200
34	A	9	7	1.00	25	0.280
35	A	9	7	1.00	25	0.280
36	A	14	9	1.00	25	0.360
37	A	12	9	1.00	25	0.360
38	A	10	10	1.00	25	0.400
39	A	8	6	1.00	25	0.240
40	A	2	2	1.00	23	0.087
41	A	8	6	1.00	22	0.273
42	A	9	7	1.00	25	0.280
43	A	13	11	1.00	25	0.440
44	A	13	10	1.00	25	0.400
45	A	20	13	1.00	25	0.520
46	A	13	10	1.00	25	0.400
47	A	7	7	1.00	25	0.280
48	A	10	7	1.00	25	0.280
49	A	3	3	1.00	23	0.130
50	A	10	6	1.00	22	0.273
51	A	12	8	1.00	25	0.320
52	A	17	11	1.00	25	0.440
53	A	17	11	1.00	25	0.440
54	A	26	13	1.00	25	0.520
55	A	6	4	1.00	18	0.222
56	A	8	6	1.00	18	0.333
57	A	10	6	1.00	18	0.333
58	A	7	4	1.00	27	0.148
59	A	5	4	1.00	27	0.148
60	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	27	0.111
62	A	4	3	1.00	27	0.111
63	A	4	5	1.00	27	0.185
64	A	4	5	1.00	27	0.185
65	A	3	4	1.00	27	0.148
66	A	3	4	1.00	27	0.148
67	A	3	2	1.00	25	0.080
68	A	8	6	1.00	27	0.222
69	A	8	6	1.00	27	0.222
70	A	10	7	1.00	27	0.259
71	A	11	7	1.00	27	0.259
72	A	9	7	1.00	27	0.259
73	A	7	6	1.00	24	0.250
74	A	7	6	1.00	27	0.222
75	A	7	6	1.00	27	0.222
76	A	5	4	1.00	27	0.148
77	A	5	6	1.00	27	0.222
78	A	5	6	1.00	27	0.222
79	A	5	6	1.00	27	0.222
80	A	4	5	1.00	27	0.185
81	A	4	5	1.00	27	0.185
82	A	4	5	1.00	27	0.185
83	A	4	3	1.00	25	0.120
84	A	11	8	1.00	27	0.296
85	A	12	9	1.00	27	0.333
86	A	12	9	1.00	27	0.333
87	A	16	9	1.00	27	0.333
88	A	14	9	1.00	27	0.333
89	A	10	7	1.00	24	0.292
90	A	12	9	1.00	27	0.333
91	A	12	8	1.00	27	0.296
92	A	12	8	1.00	27	0.296
93	A	5	4	1.00	27	0.148
94	A	6	7	1.00	27	0.259
95	A	5	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	5	1.00	27	0.185
97	A	4	5	1.00	27	0.185
98	A	4	5	1.00	27	0.185
99	A	4	3	1.00	25	0.120
100	A	15	9	1.00	27	0.333
101	A	15	11	1.00	27	0.407
102	A	16	10	1.00	27	0.370
103	A	3	3	1.00	14	0.214
104	A	6	4	1.00	27	0.148
105	A	5	3	1.00	27	0.111
106	A	4	4	1.00	27	0.148
107	A	3	3	1.00	27	0.111
108	A	2	2	1.00	25	0.080
109	A	1	1	1.00	24	0.042
110	A	6	4	1.00	27	0.148
111	A	2	2	1.00	27	0.074
112	A	8	6	1.00	27	0.222
113	A	4	4	1.00	27	0.148
114	A	5	6	1.00	27	0.222
115	A	7	6	1.00	27	0.222
116	A	4	6	1.00	27	0.222
117	A	4	4	1.00	27	0.148
118	A	3	3	1.00	25	0.120
119	A	2	2	1.00	24	0.083
120	A	9	7	1.00	27	0.259
121	A	5	6	1.00	27	0.222
122	A	13	9	1.00	27	0.333
123	A	5	6	1.00	27	0.222
124	A	5	7	1.00	27	0.259
125	A	9	6	1.05	27	0.222
126	A	4	6	1.08	27	0.222
127	A	5	4	1.09	27	0.148
128	A	4	4	1.09	25	0.160
129	A	5	5	1.07	24	0.208
130	A	13	9	1.00	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
131	A	5	7	1.00	27	0.259
132	A	18	13	1.00	27	0.482
133	A	5	7	1.00	27	0.259
134	A	8	5	1.00	20	0.250
135	A	5	3	1.00	22	0.136
136	A	4	4	1.00	22	0.182
137	A	3	3	1.00	22	0.136
138	A	2	2	1.00	20	0.100
139	A	1	1	1.00	19	0.053
140	A	6	4	1.00	22	0.182
141	A	2	2	1.00	22	0.091
142	A	8	6	1.00	22	0.273
143	A	1	1	1.00	30	0.033
144	A	1	1	1.00	31	0.032
145	A	8	9	1.00	27	0.333
146	A	7	8	1.00	27	0.296
147	A	6	7	1.00	25	0.280
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	11	7	1.00	29	0.241
152	A	7	6	1.00	29	0.207
153	A	3	3	1.00	29	0.103
154	A	1	1	1.00	29	0.034
155	A	4	4	1.00	29	0.138
156	A	7	4	1.00	29	0.138
157	A	11	7	1.00	35	0.200
158	A	7	6	1.00	35	0.171
159	A	3	3	1.00	35	0.086
160	A	1	1	1.00	35	0.029
161	A	4	4	1.00	35	0.114
162	A	7	4	1.00	35	0.114
163	A	1	1	1.00	24	0.042
164	A	17	6	1.00	20	0.300
165	A	12	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	7	5	1.00	18	0.278
167	A	8	5	1.00	20	0.250
168	A	12	9	1.00	20	0.450
169	A	17	11	1.00	20	0.550
170	A	16	8	1.00	29	0.276
171	A	11	8	1.00	29	0.276
172	A	6	6	1.00	27	0.222
173	A	5	5	1.00	26	0.192
174	A	12	8	1.00	29	0.276
175	A	7	7	1.00	29	0.241
176	A	12	9	1.00	29	0.310
177	A	11	11	1.00	29	0.379
178	A	26	13	1.00	29	0.448
179	A	20	13	1.00	29	0.448
180	A	8	8	1.00	27	0.296
181	A	11	9	1.00	26	0.346
182	A	18	13	1.00	29	0.448
183	A	15	14	1.00	29	0.483
184	A	18	15	1.00	29	0.517
185	A	18	13	1.00	29	0.448
186	A	35	18	1.00	29	0.621
187	A	31	21	1.00	29	0.724
188	A	8	8	1.00	27	0.296
189	A	18	9	1.00	26	0.346
190	A	26	17	1.00	29	0.586
191	A	25	16	1.00	29	0.552
192	A	28	22	1.00	29	0.759
193	A	30	17	1.00	29	0.586
194	A	16	7	1.00	29	0.241
195	A	11	7	1.00	29	0.241
196	A	9	7	1.00	29	0.241
197	A	5	5	1.00	29	0.172
198	A	4	3	1.00	27	0.111
199	A	1	1	1.00	26	0.038
200	A	8	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	6	1.00	29	0.207
202	A	12	9	1.00	29	0.310
203	A	9	9	1.00	29	0.310
204	A	23	13	1.00	29	0.448
205	A	15	12	1.00	29	0.414
206	A	14	10	1.00	29	0.345
207	A	8	8	1.00	29	0.276
208	A	8	6	1.00	27	0.222
209	A	6	6	1.00	26	0.231
210	A	16	11	1.00	29	0.379
211	A	15	11	1.00	29	0.379
212	A	27	15	1.00	29	0.517
213	A	26	12	1.00	29	0.414
214	A	28	12	1.00	29	0.414
215	A	20	12	1.00	29	0.414
216	A	18	9	1.00	29	0.310
217	A	12	12	1.00	29	0.414
218	A	10	8	1.00	27	0.296
219	A	10	10	1.00	26	0.385
220	A	26	13	1.00	29	0.448
221	A	21	15	1.00	29	0.517
222	A	41	19	1.00	29	0.655
223	A	36	17	1.00	29	0.586
224	A	15	11	1.00	22	0.500
225	A	11	7	1.00	24	0.292
226	A	8	7	1.00	24	0.292
227	A	5	5	1.00	24	0.208
228	A	3	3	1.00	22	0.136
229	A	1	1	1.00	21	0.048
230	A	8	5	1.00	24	0.208
231	A	6	6	1.00	24	0.250
232	A	12	9	1.00	24	0.375
233	A	0	0	0.00	0	0.000
234	A	0	0	0.00	0	0.000
235	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	0	0	0.00	0	0.000
237	A	0	0	0.00	0	0.000
238	A	0	0	0.00	0	0.000
239	A	0	0	0.00	0	0.000
240	A	29	15	1.00	20	0.750
241	A	20	12	1.00	20	0.600
242	A	11	8	1.00	18	0.444
243	A	10	6	1.00	20	0.300
244	A	19	11	1.00	20	0.550
245	A	30	12	1.00	20	0.600
246	A	29	13	1.00	22	0.591
247	A	16	12	1.00	22	0.546
248	A	6	5	1.00	22	0.227
249	A	1	1	1.00	22	0.045
250	A	7	7	1.00	22	0.318
251	A	12	12	1.00	22	0.546
252	A	20	15	1.00	22	0.682
253	A	13	6	1.00	24	0.250
254	A	10	8	1.00	24	0.333
255	A	6	6	1.00	24	0.250
256	A	4	4	1.00	22	0.182
257	A	1	1	1.00	21	0.048
258	A	10	6	1.00	24	0.250
259	A	7	7	1.00	24	0.292
260	A	18	11	1.00	24	0.458
261	A	0	0	0.00	0	0.000
262	A	7	3	1.00	20	0.150
263	A	6	3	1.00	20	0.150
264	A	5	3	1.00	18	0.167
265	A	0	0	0.00	0	0.000
266	A	0	0	0.00	0	0.000
267	A	12	5	1.00	28	0.179
268	A	12	5	1.00	28	0.179
269	A	6	5	1.00	28	0.179
270	A	9	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	5	1.00	25	0.200
272	A	0	0	0.00	0	0.000
273	A	0	0	0.00	0	0.000
274	A	0	0	0.00	0	0.000
275	A	0	0	0.00	0	0.000
276	A	15	5	1.00	28	0.179
277	A	12	5	1.00	28	0.179
278	A	12	5	1.00	26	0.192
279	A	9	5	1.00	25	0.200
280	A	0	0	0.00	0	0.000
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	15	5	1.00	28	0.179
285	A	15	5	1.00	28	0.179
286	A	15	5	1.00	26	0.192
287	A	12	5	1.00	25	0.200
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	0	0	0.00	0	0.000
292	A	5	3	1.00	24	0.125
293	A	5	3	1.00	24	0.125
294	A	4	3	1.00	24	0.125
295	A	2	2	1.00	22	0.091
296	A	1	1	1.00	21	0.048
297	A	0	0	0.00	0	0.000
298	A	0	0	0.00	0	0.000
299	A	9	5	1.00	28	0.179
300	A	6	5	1.00	28	0.179
301	A	4	4	1.00	26	0.154
302	A	1	1	1.00	25	0.040
303	A	0	0	0.00	0	0.000
304	A	0	0	0.00	0	0.000
305	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	0	0	0.00	0	0.000
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	0	0	0.00	0	0.000
311	A	0	0	0.00	0	0.000
312	A	0	0	0.00	0	0.000
313	A	0	0	0.00	0	0.000
314	A	0	0	0.00	0	0.000
315	A	8	4	1.00	20	0.200
316	A	7	4	1.00	20	0.200
317	A	6	4	1.00	18	0.222
318	A	0	0	0.00	0	0.000
319	A	0	0	0.00	0	0.000
320	A	22	6	1.00	28	0.214
321	A	16	7	1.00	28	0.250
322	A	14	7	1.00	26	0.269
323	A	7	7	1.00	25	0.280
324	A	0	0	0.00	0	0.000
325	A	0	0	0.00	0	0.000
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	21	7	1.00	28	0.250
329	A	24	10	1.00	26	0.385
330	A	11	7	1.00	25	0.280
331	A	0	0	0.00	0	0.000
332	A	0	0	0.00	0	0.000
333	A	0	0	0.00	0	0.000
334	A	0	0	0.00	0	0.000
335	A	30	7	1.00	28	0.250
336	A	30	10	1.00	26	0.385
337	A	14	7	1.00	25	0.280
338	A	0	0	0.00	0	0.000
339	A	0	0	0.00	0	0.000
340	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	0	0	0.00	0	0.000
342	A	13	6	1.00	28	0.214
343	A	10	6	1.00	28	0.214
344	A	10	6	1.00	28	0.214
345	A	7	7	1.00	28	0.250
346	A	5	5	1.00	26	0.192
347	A	1	1	1.00	25	0.040
348	A	0	0	0.00	0	0.000
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	0	0	0.00	0	0.000
352	A	0	0	0.00	0	0.000
353	A	0	0	0.00	0	0.000
354	A	0	0	0.00	0	0.000
355	A	0	0	0.00	0	0.000
356	A	0	0	0.00	0	0.000
357	A	0	0	0.00	0	0.000
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	0	0	0.00	0	0.000
365	A	0	0	0.00	0	0.000
366	A	0	0	0.00	0	0.000
367	A	0	0	0.00	0	0.000
368	A	1	1	1.00	21	0.048
369	A	27	7	1.00	27	0.259
370	A	32	7	1.00	27	0.259
371	A	17	9	1.00	25	0.360
372	A	14	7	1.00	24	0.292
373	A	0	0	0.00	0	0.000
374	A	32	7	1.00	29	0.241
375	A	42	7	1.00	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	32	9	1.00	27	0.333
377	A	19	7	1.00	26	0.269
378	A	0	0	0.00	0	0.000
379	A	25	12	1.00	24	0.500
380	A	10	9	1.00	24	0.375
381	A	1	1	1.00	24	0.042
382	A	0	0	0.00	0	0.000
383	A	0	0	0.00	0	0.000
384	A	27	13	1.00	24	0.542
385	A	11	9	1.00	24	0.375
386	A	1	1	1.00	24	0.042
387	A	0	0	0.00	0	0.000
388	A	41	17	1.00	24	0.708
389	A	13	11	1.00	24	0.458
390	A	1	1	1.00	24	0.042
391	A	0	0	0.00	0	0.000
392	A	25	12	1.00	24	0.500
393	A	10	9	1.00	24	0.375
394	A	1	1	1.00	24	0.042
395	A	0	0	0.00	0	0.000
396	A	0	0	0.00	0	0.000
397	A	27	13	1.00	24	0.542
398	A	11	9	1.00	24	0.375
399	A	1	1	1.00	24	0.042
400	A	0	0	0.00	0	0.000
401	A	6	5	1.00	19	0.263
402	A	18	6	1.00	24	0.250
403	A	13	6	1.00	24	0.250
404	A	8	6	1.00	24	0.250
405	A	1	1	1.00	24	0.042
406	A	0	0	0.00	0	0.000
407	A	0	0	0.00	0	0.000
408	A	20	8	1.00	24	0.333
409	A	15	8	1.00	24	0.333
410	A	9	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	1	1	1.00	24	0.042
412	A	0	0	0.00	0	0.000
413	A	0	0	0.00	0	0.000
414	A	19	11	1.00	24	0.458
415	A	7	6	1.00	24	0.250
416	A	1	1	1.00	24	0.042
417	A	0	0	0.00	0	0.000
418	A	0	0	0.00	0	0.000
419	A	6	4	1.00	29	0.138
420	A	9	4	1.00	27	0.148
421	A	6	4	1.00	26	0.154
422	A	0	0	0.00	0	0.000
423	A	0	0	0.00	0	0.000
424	A	12	4	1.00	29	0.138
425	A	12	4	1.00	27	0.148
426	A	9	4	1.00	26	0.154
427	A	0	0	0.00	0	0.000
428	A	0	0	0.00	0	0.000
429	A	15	4	1.00	29	0.138
430	A	15	4	1.00	27	0.148
431	A	12	4	1.00	26	0.154
432	A	0	0	0.00	0	0.000
433	A	0	0	0.00	0	0.000
434	A	9	4	1.00	28	0.143
435	A	6	4	1.00	28	0.143
436	A	4	3	1.00	26	0.115
437	A	1	1	1.00	25	0.040
438	A	0	0	0.00	0	0.000
439	A	0	0	0.00	0	0.000
440	A	9	4	1.00	29	0.138
441	A	6	4	1.00	29	0.138
442	A	4	3	1.00	27	0.111
443	A	1	1	1.00	26	0.038
444	A	0	0	0.00	0	0.000
445	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	0	0	0.00	0	0.000
447	A	0	0	0.00	0	0.000
448	A	0	0	0.00	0	0.000
449	A	0	0	0.00	0	0.000
450	A	0	0	0.00	0	0.000
451	A	0	0	0.00	0	0.000
452	A	0	0	0.00	0	0.000
453	A	0	0	0.00	0	0.000
454	A	0	0	0.00	0	0.000
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000
460	A	0	0	0.00	0	0.000
461	A	7	5	1.00	19	0.263
462	A	6	6	1.00	19	0.316
463	A	5	5	1.00	19	0.263
464	A	4	4	1.00	17	0.235
465	A	3	3	1.00	16	0.188
466	A	13	13	1.00	19	0.684
467	A	4	4	1.00	19	0.210
468	A	11	11	1.00	19	0.579
469	A	4	4	1.00	19	0.210
470	A	7	7	1.00	21	0.333
471	A	9	10	1.00	21	0.476
472	A	6	6	1.00	21	0.286
473	A	7	7	1.00	19	0.368
474	A	6	6	1.00	18	0.333
475	A	16	15	1.08	21	0.714
476	A	7	7	1.16	21	0.333
477	A	14	15	1.00	21	0.714
478	A	7	8	1.00	21	0.381
479	A	6	6	1.00	21	0.286
480	A	10	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	6	6	1.00	21	0.286
482	A	8	7	1.00	19	0.368
483	A	6	6	1.00	18	0.333
484	A	23	15	1.00	21	0.714
485	A	7	7	1.00	21	0.333
486	A	18	19	1.00	21	0.905
487	A	9	9	1.00	21	0.429
488	A	6	6	1.00	18	0.333
489	A	27	12	1.00	21	0.571
490	A	23	9	1.00	21	0.429
491	A	23	9	1.00	21	0.429
492	A	18	6	1.00	19	0.316
493	A	18	6	1.00	18	0.333
494	A	25	8	1.00	21	0.381
495	A	23	10	1.00	21	0.476
496	A	27	10	1.00	21	0.476
497	A	28	12	1.00	21	0.571
498	A	24	10	1.00	21	0.476
499	A	4	4	1.00	19	0.210
500	A	29	12	1.00	21	0.571
501	A	31	14	1.00	21	0.667
502	A	49	12	1.00	21	0.571
503	A	46	10	1.00	21	0.476
504	A	26	9	1.00	18	0.500
505	A	49	13	1.00	21	0.619
506	A	29	11	1.00	21	0.524
507	A	9	10	1.04	21	0.476
508	A	5	5	1.00	19	0.263
509	A	34	13	1.00	21	0.619
510	A	36	15	1.00	21	0.714
511	A	80	11	1.00	21	0.524
512	A	62	11	1.00	21	0.524
513	A	34	10	1.00	18	0.556
514	A	0	0	0.00	0	0.000
515	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	7	8	1.00	20	0.400
517	A	8	10	1.04	20	0.500
518	A	9	11	1.00	20	0.550
519	A	8	9	0.95	23	0.391
520	A	7	8	0.94	23	0.348
521	A	5	5	0.94	21	0.238
522	A	0	0	0.00	0	0.000
523	A	0	0	0.00	0	0.000
524	A	0	0	0.00	0	0.000
525	A	26	7	1.00	20	0.350
526	A	17	7	1.00	20	0.350
527	A	10	7	1.00	18	0.389
528	A	3	3	1.00	10	0.300
529	A	22	7	1.00	20	0.350
530	A	0	0	0.00	0	0.000
531	A	0	0	0.00	0	0.000
532	A	0	0	0.00	0	0.000
533	A	0	0	0.00	0	0.000
534	A	27	7	1.00	20	0.350
535	A	15	7	1.29	18	0.389
536	A	4	4	1.00	10	0.400
537	A	0	0	0.00	0	0.000
538	A	0	0	0.00	0	0.000
539	A	0	0	0.00	0	0.000
540	A	0	0	0.00	0	0.000
541	A	0	0	0.00	0	0.000
542	A	0	0	0.00	0	0.000
543	A	26	7	1.00	20	0.350
544	A	15	7	1.00	18	0.389
545	A	5	5	1.00	10	0.500
546	A	0	0	0.00	0	0.000
547	A	0	0	0.00	0	0.000
548	A	0	0	0.00	0	0.000
549	A	0	0	0.00	0	0.000
550	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	0	0	0.00	0	0.000
552	A	42	9	1.00	22	0.409
553	A	23	9	1.00	20	0.450
554	A	7	6	1.00	12	0.500
555	A	0	0	0.00	0	0.000
556	A	0	0	0.00	0	0.000
557	A	32	12	1.00	20	0.600
558	A	8	7	1.00	12	0.583
559	A	0	0	0.00	0	0.000
560	A	0	0	0.00	0	0.000
561	A	39	8	1.00	22	0.364
562	A	21	8	1.00	20	0.400
563	A	6	5	1.00	12	0.417
564	A	0	0	0.00	0	0.000
565	A	0	0	0.00	0	0.000
566	A	21	8	1.00	20	0.400
567	A	7	6	1.00	12	0.500
568	A	0	0	0.00	0	0.000
569	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=151

$$-\frac{152bd\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{76bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^3}-\frac{19bdx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c}+\frac{1}{49}bcdx^6\sqrt{-1+}$$

```
[Out] 1/5*d*x^5*(a+b*arccosh(c*x))-1/7*c^2*d*x^7*(a+b*arccosh(c*x))-152/3675*b*d*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-76/3675*b*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3-19/1225*b*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/49*b*c*d*x^6*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {14, 5921, 12, 471, 102, 75}

$$-\frac{1}{7}c^2dx^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{152bd\sqrt{cx-1}\,\sqrt{cx+1}}{3675c^5}-\frac{76bdx^2\sqrt{cx-1}\,\sqrt{cx+1}}{3675c^3}+\frac{1}{49}bcdx^6\sqrt{cx-1}\,\sqrt{cx+1}-\frac{19bdx^4\sqrt{cx-1}\,\sqrt{cx+1}}{1225c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-152*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (76*b*d*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(3675*c^3) - (19*b*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/
(1225*c) + (b*c*d*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/49 + (d*x^5*(a + b*ArcC
osh[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4(d-c^2dx^2)\,(a+b\cosh^{-1}(cx))\;dx = \frac{1}{5}dx^5(a+b\cosh^{-1}(cx)) - \frac{1}{7}c^2dx^7(a+b\cosh^{-1}(cx)) - (bc)\int \frac{}{35}$$

$$= \frac{1}{5}dx^5(a+b\cosh^{-1}(cx)) - \frac{1}{7}c^2dx^7(a+b\cosh^{-1}(cx)) - \frac{1}{35}(bcd)$$

$$= \frac{1}{49}bcdx^6\sqrt{-1+cx}\,\sqrt{1+cx} + \frac{1}{5}dx^5(a+b\cosh^{-1}(cx)) - \frac{1}{7}c^2dx$$

$$= -\frac{19bdx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c} + \frac{1}{49}bcdx^6\sqrt{-1+cx}\,\sqrt{1+cx} +$$

$$= -\frac{19bdx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c} + \frac{1}{49}bcdx^6\sqrt{-1+cx}\,\sqrt{1+cx} +$$

$$= -\frac{76bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c} +$$

$$= -\frac{76bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^3} - \frac{19bdx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c} +$$

$$= -\frac{152bd\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5} - \frac{76bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^3} - \frac{19}{}$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 0.60

$$\frac{d\left(-105ax^5(-7+5c^2x^2)+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(-152-76c^2x^2-57c^4x^4+75c^6x^6)}{c^5}-105bx^5(-7+5c^2x^2)\cosh^{-1}(cx)\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-152 - 7
6*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*ArcC
osh[c*x]))/3675
```

Maple [A]
time = 2.73, size = 98, normalized size = 0.65

method	result
derivativedivides	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-bd\left(\frac{\mathrm{arccosh}(cx)c^7x^7}{7}-\frac{\mathrm{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75x^6c^6-57c^4x^4-76c^2x^2-152)}{3675}\right)}{c^5}$

default	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-bd\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75x^6c^6-57c^4x^4-76c^2x^2-152\right)}{3675}\right)}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-d*a*(1/7*c^7*x^7-1/5*c^5*x^5)-b*d*(1/7*arccosh(c*x)*c^7*x^7-1/5*arc
cosh(c*x)*c^5*x^5-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-57*c^4*x^4
-76*c^2*x^2-152)))
```

Maxima [A]
time = 0.29, size = 184, normalized size = 1.22

$$-\frac{1}{7}ac^2dx^7+\frac{1}{5}adx^5-\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^2d+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x
^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6
 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
/c^6)*c)*b*d
```

Fricas [A]
time = 0.37, size = 113, normalized size = 0.75

$$-\frac{525\,ac^7dx^7-735\,ac^5dx^5+105\,(5\,bc^7dx^7-7\,bc^5dx^5)\log\left(cx+\sqrt{c^2x^2-1}\right)-(75\,bc^6dx^6-57\,bc^4dx^4-76\,bc^2dx^2-152\,bd)\sqrt{c^2x^2-1}}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*d
*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 76*
b*c^2*d*x^2 - 152*b*d)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.72, size = 158, normalized size = 1.05

$$\begin{cases} -\frac{ac^2dx^7}{7}+\frac{adx^5}{5}-\frac{bc^2dx^7\operatorname{acosh}(cx)}{7}+\frac{bcdx^6\sqrt{c^2x^2-1}}{49}+\frac{bdx^5\operatorname{acosh}(cx)}{5}-\frac{19bdx^4\sqrt{c^2x^2-1}}{1225c}-\frac{76bdx^2\sqrt{c^2x^2-1}}{3675c^3}-\frac{152bd\sqrt{c^2x^2-1}}{3675c^5} & \text{for}\ c\neq 0\\ \frac{dx^5\left(a+\frac{i\pi b}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**7/7 + a*d*x**5/5 - b*c**2*d*x**7*acosh(c*x)/7 + b*c
*d*x**6*sqrt(c**2*x**2 - 1)/49 + b*d*x**5*acosh(c*x)/5 - 19*b*d*x**4*sqrt(c
**2*x**2 - 1)/(1225*c) - 76*b*d*x**2*sqrt(c**2*x**2 - 1)/(3675*c**3) - 152*
b*d*sqrt(c**2*x**2 - 1)/(3675*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, T
rue))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(d-c^2\,d\,x^2\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

3.2 $\int x^3(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=135

$$-\frac{bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bd\cosh^{-1}(cx)}{24c^4}+\frac{1}{4}dx^4($$

```
[Out] -1/24*b*d*arccosh(c*x)/c^4+1/4*d*x^4*(a+b*arccosh(c*x))-1/6*c^2*d*x^6*(a+b*
arccosh(c*x))-1/24*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*d*x^3*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c+1/36*b*c*d*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {14, 5921, 12, 471, 102, 92, 54}

$$-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bd\cosh^{-1}(cx)}{24c^4}-\frac{bdx\sqrt{cx-1}\,\sqrt{cx+1}}{24c^3}+\frac{1}{36}bcdx^5\sqrt{cx-1}\,\sqrt{cx+1}-\frac{bdx^3\sqrt{cx-1}\,\sqrt{cx+1}}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/24*(b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (b*d*x^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/36 - (b*d*Ar
cCosh[c*x])/(24*c^4) + (d*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d*x^6*(a + b*A
rcCosh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
```

```
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{}{12\sqrt{}}$$

$$=\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int$$

$$=\frac{1}{36}bcdx^5\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6$$

$$=-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{4}dx$$

$$=-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{4}dx$$

$$=-\frac{bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5$$

$$=-\frac{bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5$$

Mathematica [A]
time = 0.07, size = 166, normalized size = 1.23

$$\frac{1}{4}adx^4-\frac{1}{6}ac^2dx^6-\frac{bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{4}bdx^4\cosh^{-1}(cx)-\frac{1}{6}bc^2dx^6\cosh^{-1}(cx)-\frac{bd\tanh^{-1}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*d*x^4)/4 - (a*c^2*d*x^6)/6 - (b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^
3) - (b*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/36 + (b*d*x^4*ArcCosh[c*x])/4 - (b*c^2*d*x^6*ArcCosh[c*x
])/6 - (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(12*c^4)
```

Maple [A]
time = 2.66, size = 186, normalized size = 1.38

method	result
derivativedivides	$\frac{-da\left(\frac{\left(c^2x^2-1\right)^3}{6}+\frac{\left(c^2x^2-1\right)^2}{4}\right)-\frac{bd\,\mathrm{arccosh}(cx)c^6x^6}{6}+\frac{bd\,\mathrm{arccosh}(cx)c^4x^4}{4}-\frac{bd\,\mathrm{arccosh}(cx)}{12}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}\,c^5x^5}{36}}{}$
default	$\frac{-da\left(\frac{\left(c^2x^2-1\right)^3}{6}+\frac{\left(c^2x^2-1\right)^2}{4}\right)-\frac{bd\,\mathrm{arccosh}(cx)c^6x^6}{6}+\frac{bd\,\mathrm{arccosh}(cx)c^4x^4}{4}-\frac{bd\,\mathrm{arccosh}(cx)}{12}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}\,c^5x^5}{36}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-d*a*(1/6*(c^2*x^2-1)^3+1/4*(c^2*x^2-1)^2)-1/6*b*d*arccosh(c*x)*c^6*
x^6+1/4*b*d*arccosh(c*x)*c^4*x^4-1/12*b*d*arccosh(c*x)+1/36*b*d*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*c^5*x^5-1/36*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*x^3-1/24*
b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/24*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c
^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.25, size = 202, normalized size = 1.50

$$-\frac{1}{6}\,ac^{2}dx^{6}+\frac{1}{4}\,adx^{4}-\frac{1}{288}\left(48\,x^{6}\operatorname{arcosh}\left(cx\right)-\left(\frac{8\,\sqrt{c^{2}x^{2}-1}\,x^{5}}{c^{2}}+\frac{10\,\sqrt{c^{2}x^{2}-1}\,x^{3}}{c^{4}}+\frac{15\,\sqrt{c^{2}x^{2}-1}\,x}{c^{6}}+\frac{15\,\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\,c\right)}{c^{7}}\right)c\right)bc^{2}d+\frac{1}{32}\left(8\,x^{4}\operatorname{arcosh}\left(cx\right)-\left(\frac{2\,\sqrt{c^{2}x^{2}-1}\,x^{3}}{c^{2}}+\frac{3\,\sqrt{c^{2}x^{2}-1}\,x}{c^{4}}+\frac{3\,\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\,c\right)}{c^{5}}\right)c\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x
^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6
 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*ar
ccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d
```

Fricas [A]
time = 0.35, size = 108, normalized size = 0.80

$$-\frac{12\,ac^{6}dx^{6}-18\,ac^{4}dx^{4}+3\,(4\,bc^{6}dx^{6}-6\,bc^{4}dx^{4}+bd)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-(2\,bc^{5}dx^{5}-2\,bc^{3}dx^{3}-3\,bcdx)\sqrt{c^{2}x^{2}-1}}{72\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4 +
 b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b*c
*d*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 144, normalized size = 1.07

$$\begin{cases} -\frac{ac^{2}dx^{6}}{6}+\frac{adx^{4}}{4}-\frac{bc^{2}dx^{6}\operatorname{acosh}(cx)}{6}+\frac{bcdx^{5}\sqrt{c^{2}x^{2}-1}}{36}+\frac{bdx^{4}\operatorname{acosh}(cx)}{4}-\frac{bdx^{3}\sqrt{c^{2}x^{2}-1}}{36c}-\frac{bdx\sqrt{c^{2}x^{2}-1}}{24c^{3}}-\frac{bd\operatorname{acosh}(cx)}{24c^{4}} & \text{for}\ c\neq 0 \\ \frac{dx^{4}\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**6/6 + a*d*x**4/4 - b*c**2*d*x**6*acosh(c*x)/6 + b*c
*d*x**5*sqrt(c**2*x**2 - 1)/36 + b*d*x**4*acosh(c*x)/4 - b*d*x**3*sqrt(c**2
*x**2 - 1)/(36*c) - b*d*x*sqrt(c**2*x**2 - 1)/(24*c**3) - b*d*acosh(c*x)/(2
4*c**4), Ne(c, 0)), (d*x**4*(a + I*pi*b/2)/4, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

3.3 $\int x^2(d - c^2dx^2)\left(a + b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=121

$$-\frac{26bd\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^3}-\frac{13bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}\right.$$

```
[Out] 1/3*d*x^3*(a+b*arccosh(c*x))-1/5*c^2*d*x^5*(a+b*arccosh(c*x))-26/225*b*d*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-13/225*b*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+
1/25*b*c*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {14, 5921, 12, 471, 102, 75}

$$-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{26bd\sqrt{cx-1}\,\sqrt{cx+1}}{225c^3}+\frac{1}{25}bcdx^4\sqrt{cx-1}\,\sqrt{cx+1}-\frac{13bdx^2\sqrt{cx-1}\,\sqrt{cx+1}}{225c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-26*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (13*b*d*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(225*c) + (b*c*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/25 + (
d*x^3*(a + b*ArcCosh[c*x]))/3 - (c^2*d*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{\ldots}{15\ldots}$$

$$=\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}(bcd)\int\ldots$$

$$=\frac{1}{25}bcdx^4\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\ldots$$

$$=-\frac{13bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{3}\ldots$$

$$=-\frac{13bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{3}\ldots$$

$$=-\frac{26bd\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^3}-\frac{13bdx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c}+\frac{1}{25}bc\ldots$$

Mathematica [A]

time = 0.08, size = 89, normalized size = 0.74

$$-\frac{d\left(15ac^3x^3(-5+3c^2x^2)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(26+13c^2x^2-9c^4x^4)+15bc^3x^3(-5+3c^2x^2)\cosh^{-1}(cx)\right)}{225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/225*(d*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
26 + 13*c^2*x^2 - 9*c^4*x^4) + 15*b*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCosh[c*x]))
/c^3
```

Maple [A]
time = 2.67, size = 90, normalized size = 0.74

method	result	size
derivativedivides	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-bd\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$	90
default	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-bd\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-d*a*(1/5*c^5*x^5-1/3*c^3*x^3)-b*d*(1/5*arccosh(c*x)*c^5*x^5-1/3*arc
cosh(c*x)*c^3*x^3-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*x^4-13*c^2*x^2-2
6)))
```

Maxima [A]
time = 0.26, size = 145, normalized size = 1.20

$$-\frac{1}{5}ac^2dx^5-\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d+\frac{1}{3}adx^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2
 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^2*d + 1/3*
a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c
^2*x^2 - 1)/c^4))*b*d
```

Fricas [A]
time = 0.35, size = 103, normalized size = 0.85

$$-\frac{45\,ac^5dx^5-75\,ac^3dx^3+15\,(3\,bc^5dx^5-5\,bc^3dx^3)\log\left(cx+\sqrt{c^2x^2-1}\right)-(9\,bc^4dx^4-13\,bc^2dx^2-26\,bd)\sqrt{c^2x^2-1}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^3
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d)*
sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 133, normalized size = 1.10

$$\begin{cases} -\frac{ac^2dx^5}{5} + \frac{adx^3}{3} - \frac{bc^2dx^5 \operatorname{acosh}(cx)}{5} + \frac{bcdx^4\sqrt{c^2x^2-1}}{25} + \frac{bdx^3 \operatorname{acosh}(cx)}{3} - \frac{13bdx^2\sqrt{c^2x^2-1}}{225c} - \frac{26bd\sqrt{c^2x^2-1}}{225c^3} & \text{for}\ c \neq 0 \\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**5/5 + a*d*x**3/3 - b*c**2*d*x**5*acosh(c*x)/5 + b*c
*d*x**4*sqrt(c**2*x**2 - 1)/25 + b*d*x**3*acosh(c*x)/3 - 13*b*d*x**2*sqrt(c
**2*x**2 - 1)/(225*c) - 26*b*d*sqrt(c**2*x**2 - 1)/(225*c**3), Ne(c, 0)), (
d*x**3*(a + I*pi*b/2)/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

3.4 $\int x(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=98

$$-\frac{3bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}+\frac{3bd\cosh^{-1}(cx)}{32c^2}-\frac{d(1-c^2x^2)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}$$

```
[Out] 1/16*b*d*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c+3/32*b*d*arccosh(c*x)/c^2-1/4*d*(-
c^2*x^2+1)^2*(a+b*arccosh(c*x))/c^2-3/32*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5914, 38, 54}

$$-\frac{d(1-c^2x^2)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{3bd\cosh^{-1}(cx)}{32c^2}+\frac{bdx(cx-1)^{3/2}(cx+1)^{3/2}}{16c}-\frac{3bdx\sqrt{cx-1}\,\sqrt{cx+1}}{32c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c) + (b*d*x*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(16*c) + (3*b*d*ArcCosh[c*x])/(32*c^2) - (d*(1 - c^2*x^2)^2*
(a + b*ArcCosh[c*x]))/(4*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{d(1-c^2x^2)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{(bd)\int(-1+cx)^{3/2}(1+cx)^{3/2}}{4c}$$
$$=\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d(1-c^2x^2)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}-\frac{(3}{}$$
$$=-\frac{3bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d(1-}{}$$
$$=-\frac{3bdx\sqrt{-1+cx}\,\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}+\frac{3bd\,\mathrm{co}}{3}$$

Mathematica [A]
time = 0.10, size = 100, normalized size = 1.02

$$-\frac{d\left(cx\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\,(5-2c^2x^2)+8acx(-2+c^2x^2)\right)+8bc^2x^2(-2+c^2x^2)\cosh^{-1}(cx)+10b\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{32c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
[Out] -1/32*(d*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5 - 2*c^2*x^2) + 8*a*c*x*(-2
 + c^2*x^2)) + 8*b*c^2*x^2*(-2 + c^2*x^2)*ArcCosh[c*x] + 10*b*ArcTanh[Sqrt[
(-1 + c*x)/(1 + c*x)]]))/c^2
```

Maple [A]
time = 3.42, size = 146, normalized size = 1.49

method	result
derivativedivides	$\frac{-\frac{d\left(c^2x^2-1\right)^2a}{4}-\frac{bd\,\mathrm{arccosh}(cx)c^4x^4}{4}+\frac{bd\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{bd\,\mathrm{arccosh}(cx)}{4}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}\,c^3x^3}{16}-\frac{5bcdx\sqrt{cx-}}{}}{c^2}$
default	$\frac{-\frac{d\left(c^2x^2-1\right)^2a}{4}-\frac{bd\,\mathrm{arccosh}(cx)c^4x^4}{4}+\frac{bd\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{bd\,\mathrm{arccosh}(cx)}{4}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}\,c^3x^3}{16}-\frac{5bcdx\sqrt{cx-}}{}}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*d*(c^2*x^2-1)^2*a-1/4*b*d*arccosh(c*x)*c^4*x^4+1/2*b*d*arccosh(
c*x)*c^2*x^2-1/4*b*d*arccosh(c*x)+1/16*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*
x^3-5/32*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3/32*b*d*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 162, normalized size = 1.65

$$-\frac{1}{4}ac^2dx^4-\frac{1}{32}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bc^2d+\frac{1}{2}adx^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2
+ 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c
)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/
c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d
```

Fricas [A]
time = 0.36, size = 98, normalized size = 1.00

$$-\frac{8\,ac^4dx^4-16\,ac^2dx^2+(8\,bc^4dx^4-16\,bc^2dx^2+5\,bd)\log\left(cx+\sqrt{c^2x^2-1}\right)-(2\,bc^3dx^3-5\,bcdx)\sqrt{c^2x^2-1}}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 + 5
*b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(c^2*x
^2 - 1))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.24, size = 124, normalized size = 1.27

$$\begin{cases}-\frac{ac^2dx^4}{4}+\frac{adx^2}{2}-\frac{bc^2dx^4\operatorname{acosh}(cx)}{4}+\frac{bcdx^3\sqrt{c^2x^2-1}}{16}+\frac{bdx^2\operatorname{acosh}(cx)}{2}-\frac{5bdx\sqrt{c^2x^2-1}}{32c}-\frac{5bd\operatorname{acosh}(cx)}{32c^2} & \text{for}\ c\neq 0\\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**4/4 + a*d*x**2/2 - b*c**2*d*x**4*acosh(c*x)/4 + b*c
*d*x**3*sqrt(c**2*x**2 - 1)/16 + b*d*x**2*acosh(c*x)/2 - 5*b*d*x*sqrt(c**2*
x**2 - 1)/(32*c) - 5*b*d*acosh(c*x)/(32*c**2), Ne(c, 0)), (d*x**2*(a + I*pi
*b)/2)/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

3.5 $\int (d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+\frac{1}{9}bcdx^2\sqrt{-1+cx}\,\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] d*x*(a+b*arccosh(c*x))-1/3*c^2*d*x^3*(a+b*arccosh(c*x))-7/9*b*d*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c+1/9*b*c*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5894, 12, 471, 75}

$$-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}bcdx^2\sqrt{cx-1}\,\sqrt{cx+1}-\frac{7bd\sqrt{cx-1}\,\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c) + (b*c*d*x^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/9 + d*x*(a + b*ArcCosh[c*x]) - (c^2*d*x^3*(a + b*ArcCosh[c*x]))
/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)\,dx = dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{dx\big(1}{\sqrt{-1+}}$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bcd)\int \frac{x\big(}{\sqrt{-1+}}$$

$$= \frac{1}{9}bcdx^2\sqrt{-1+cx}\,\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\big(a+b$$

$$= -\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+\frac{1}{9}bcdx^2\sqrt{-1+cx}\,\sqrt{1+cx}+dx\big(a+b$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 0.83

$$\frac{d\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\,(-7+c^2x^2)+a(9cx-3c^3x^3)-3bcx(-3+c^2x^2)\cosh^{-1}(cx)\right)}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3) -
 3*b*c*x*(-3 + c^2*x^2)*ArcCosh[c*x]))/(9*c)
```

Maple [A]
time = 2.11, size = 73, normalized size = 0.85

method	result	size
derivativedivides	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-bd\left(\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2x^2-7)}{9}\right)}{c}$	73
default	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-bd\left(\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2x^2-7)}{9}\right)}{c}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-d*a*(1/3*c^3*x^3-c*x)-b*d*(1/3*arccosh(c*x)*c^3*x^3-c*x*arccosh(c*x)-
1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*x^2-7)))
```

Maxima [A]
time = 0.25, size = 97, normalized size = 1.13

$$-\frac{1}{3}ac^2dx^3 - \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d + adx + \frac{\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x
^2 - 1))*b*d/c
```

Fricas [A]
time = 0.34, size = 83, normalized size = 0.97

$$-\frac{3ac^3dx^3 - 9acdx + 3\left(bc^3dx^3 - 3bcdx\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(bc^2dx^2 - 7bd\right)\sqrt{c^2x^2-1}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*log(c*x + sqr
t(c^2*x^2 - 1)) - (b*c^2*d*x^2 - 7*b*d)*sqrt(c^2*x^2 - 1))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.14, size = 97, normalized size = 1.13

$$\begin{cases} -\frac{ac^2dx^3}{3} + adx - \frac{bc^2dx^3\operatorname{acosh}(cx)}{3} + \frac{bcdx^2\sqrt{c^2x^2-1}}{9} + bdx\operatorname{acosh}(cx) - \frac{7bd\sqrt{c^2x^2-1}}{9c} & \text{for}\ c \neq 0 \\ dx\left(a + \frac{i\pi b}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*acosh(c*x)/3 + b*c*d*x*
*2*sqrt(c**2*x**2 - 1)/9 + b*d*x*acosh(c*x) - 7*b*d*sqrt(c**2*x**2 - 1)/(9*
c), Ne(c, 0)), (d*x*(a + I*pi*b/2), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

3.6 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=117

$$\frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d(1-c^2x^2)\,(a+b\cosh^{-1}(cx))+\frac{d(a+b\cosh^{-1}(cx))^2}{2b}+d(a$$

```
[Out] -1/4*b*d*arccosh(c*x)+1/2*d*(-c^2*x^2+1)*(a+b*arccosh(c*x))+1/2*d*(a+b*arcc
osh(c*x))^2/b+d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2)-1/2*b*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/4*b*c*d*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5919, 5882, 3799, 2221, 2317, 2438, 38, 54}

$$\frac{1}{2}d(1-c^2x^2)\,(a+b\cosh^{-1}(cx))+\frac{d(a+b\cosh^{-1}(cx))^2}{2b}+d\log\left(e^{-2\cosh^{-1}(cx)}+1\right)(a+b\cosh^{-1}(cx))-\frac{1}{2}bd\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+\frac{1}{4}bcdx\sqrt{cx-1}\,\sqrt{cx+1}-\frac{1}{4}bd\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*d*ArcCosh[c*x])/4 + (d*(1 - c
^2*x^2)*(a + b*ArcCosh[c*x]))/2 + (d*(a + b*ArcCosh[c*x])^2)/(2*b) + d*(a +
 b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] - (b*d*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx+\frac{1}{2}(bc$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+dS$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+$$
$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+$$

Mathematica [A]
time = 0.17, size = 116, normalized size = 0.99

$$-\frac{1}{4}d\left(2ac^2x^2-bcx\sqrt{-1+cx}\,\sqrt{1+cx}-2b\cosh^{-1}(cx)^2-2b\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)+2b\cosh^{-1}(cx)\left(c^2x^2-2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-4a\log(x)+2b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/4*(d*(2*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*ArcCosh[c*x
]^2 - 2*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(c^2*x^2 -
 2*Log[1 + E^(-2*ArcCosh[c*x])]) - 4*a*Log[x] + 2*b*PolyLog[2, -E^(-2*ArcCo
sh[c*x])]))
```

Maple [A]
time = 5.02, size = 131, normalized size = 1.12

method	result
derivativedivides	$-\frac{a\,c^2d\,x^2}{2}+ad\ln(cx)-\frac{bd\text{arccosh}(cx)^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}-\frac{bd\,\text{arccosh}(cx)c^2x^2}{2}+\frac{bd\,\text{arccc}}{4}$
default	$-\frac{a\,c^2d\,x^2}{2}+ad\ln(cx)-\frac{bd\text{arccosh}(cx)^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}-\frac{bd\,\text{arccosh}(cx)c^2x^2}{2}+\frac{bd\,\text{arccc}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*c^2*d*x^2+a*d*ln(c*x)-1/2*b*d*arccosh(c*x)^2+1/4*b*c*d*x*(c*x-1)^(1/
2)*(c*x+1)^(1/2)-1/2*b*d*arccosh(c*x)*c^2*x^2+1/4*b*d*arccosh(c*x)+b*d*arcc
osh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*d*x^2 + a*d*log(x) - integrate(b*c^2*d*x*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) - b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x}\right)dx+\int ac^{2}x\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x}\right)dx+\int bc^{2}x\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*acosh(c*x)/x, x
) + Integral(b*c**2*x*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x, x)
```

3.7 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=76

$$bcd\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{d(a+b\cosh^{-1}(cx))}{x}-c^2dx(a+b\cosh^{-1}(cx))+bcd\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)$$

```
[Out] -d*(a+b*arccosh(c*x))/x-c^2*d*x*(a+b*arccosh(c*x))+b*c*d*arctan((c*x-1)^(1/
2)*(c*x+1)^(1/2))+b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {14, 5921, 12, 471, 94, 211}

$$c^2(-d)x(a+b\cosh^{-1}(cx))-\frac{d(a+b\cosh^{-1}(cx))}{x}+bcd\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+bcd\sqrt{cx-1}\,\sqrt{cx+1}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (d*(a + b*ArcCosh[c*x]))/x - c^2*d*x*(
a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) - (bc)\int \frac{d(-}{x\sqrt{-1+}}$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) - (bcd)\int \frac{-}{x\sqrt{-1}}$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-}\right.$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-}\right.$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-}\right.$$

Mathematica [A]
time = 0.13, size = 110, normalized size = 1.45

$$-\frac{ad}{x} - ac^2dx + bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{bd\cosh^{-1}(cx)}{x} - bc^2dx\cosh^{-1}(cx) + \frac{bcd\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) - a*c^2*d*x + b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (b*d*ArcCosh[
c*x])/x - b*c^2*d*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A]
time = 1.95, size = 101, normalized size = 1.33

method	result
derivativedivides	$c\left(-ad\left(cx+\frac{1}{cx}\right)-bd\,\mathrm{arccosh}(cx)\,cx-\frac{bd\,\mathrm{arccosh}(cx)}{cx}+bd\sqrt{cx-1}\,\sqrt{cx+1}-\frac{bd\sqrt{cx-}}{}\right.$
default	$c\left(-ad\left(cx+\frac{1}{cx}\right)-bd\,\mathrm{arccosh}(cx)\,cx-\frac{bd\,\mathrm{arccosh}(cx)}{cx}+bd\sqrt{cx-1}\,\sqrt{cx+1}-\frac{bd\sqrt{cx-}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a*d*(c*x+1/c/x)-b*d*arccosh(c*x)*c*x-b*d*arccosh(c*x)/c/x+b*d*(c*x-1)^(
1/2)*(c*x+1)^(1/2)-b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan
(1/(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.47, size = 66, normalized size = 0.87

$$-ac^2dx-\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bcd-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -a*c^2*d*x - (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c*d - (c*arcsin(1/(c*
abs(x))) + arccosh(c*x)/x)*b*d - a*d/x
```

Fricas [A]
time = 0.39, size = 127, normalized size = 1.67

$$-\frac{ac^2dx^2-2\,bcdx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,bcdx-(bc^2+b)dx\log\left(-cx+\sqrt{c^2x^2-1}\right)+ad+(bc^2dx^2-(bc^2+b)dx+bd)\log\left(cx+\sqrt{c^2x^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d*x^2 - 2*b*c*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 -
 1)*b*c*d*x - (b*c^2 + b)*d*x*log(-c*x + sqrt(c^2*x^2 - 1)) + a*d + (b*c^2*
d*x^2 - (b*c^2 + b)*d*x + b*d)*log(c*x + sqrt(c^2*x^2 - 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int ac^2\,dx+\int\left(-\frac{a}{x^2}\right)\,dx+\int bc^2\operatorname{acosh}(cx)\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^2}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*acosh(c*x)
, x) + Integral(-b*acosh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2, x)
```

3.8 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=135

$$\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{1}{2}bc^2d\cosh^{-1}(cx)-\frac{d(1-c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d(a$$

```
[Out] -1/2*b*c^2*d*arccosh(c*x)-1/2*d*(-c^2*x^2+1)*(a+b*arccosh(c*x))/x^2-1/2*c^2
*d*(a+b*arccosh(c*x))^2/b-c^2*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)+1/2*b*c^2*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)+1/2*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5920, 99, 12, 54, 5882, 3799, 2221, 2317, 2438}

$$-\frac{d(1-c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}bc^2d\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-\frac{1}{2}bc^2d\cosh^{-1}(cx)+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*c^2*d*ArcCosh[c*x])/2 - (d*
(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/(2*x^2) - (c^2*d*(a + b*ArcCosh[c*x])^2
)/(2*b) - c^2*d*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] + (b*c^2*
d*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d(1-c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2}\,d$$
$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{d(1-c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)$$
$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{d(1-c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2d(a+}{}$$
$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)\left(a+b\cos\right.}{2x^2}$$
$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)\left(a+b\cos\right.}{2x^2}$$
$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d(1-c^2x^2)\left(a+b\cos\right.}{2x^2}$$

Mathematica [A]
time = 0.13, size = 106, normalized size = 0.79

$$-\frac{d\left(a-bcx\sqrt{-1+cx}\,\sqrt{1+cx}+bc^2x^2\cosh^{-1}(cx)^2+b\cosh^{-1}(cx)\left(1+2c^2x^2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+2ac^2x^2\log(x)-bc^2x^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(d*(a - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*c^2*x^2*ArcCosh[c*x]^2
+ b*ArcCosh[c*x]*(1 + 2*c^2*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*c^2*x^2
*Log[x] - b*c^2*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/x^2
```

Maple [A]
time = 7.59, size = 137, normalized size = 1.01

method	result
derivativedivides	$c^2\left(-ad\ln(cx)-\frac{ad}{2c^2x^2}+\frac{bd\operatorname{arccosh}(cx)^2}{2}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}}{2cx}-\frac{bd}{2}-\frac{bd\operatorname{arccosh}(cx)}{2c^2x^2}-bd\,\mathrm{ar}\right.$
default	$c^2\left(-ad\ln(cx)-\frac{ad}{2c^2x^2}+\frac{bd\operatorname{arccosh}(cx)^2}{2}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}}{2cx}-\frac{bd}{2}-\frac{bd\operatorname{arccosh}(cx)}{2c^2x^2}-bd\,\mathrm{ar}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a*d*ln(c*x)-1/2*a*d/c^2/x^2+1/2*b*d*arccosh(c*x)^2+1/2*b*d*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c/x-1/2*b*d-1/2*b*d*arccosh(c*x)/c^2/x^2-b*d*arccosh(c*x)
*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -b*c^2*d*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) - a*c^2*d*l
og(x) + 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x^3}\right)dx+\int\frac{ac^2}{x}\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^3}\right)dx+\int\frac{bc^2\operatorname{acosh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*acosh(c*x)/x
**3, x) + Integral(b*c**2*acosh(c*x)/x, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3, x)
```

3.9 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d(a+b\cosh^{-1}(cx))}{3x^3}+\frac{c^2d(a+b\cosh^{-1}(cx))}{x}-\frac{5}{6}bc^3d\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)$$

[Out] -1/3*d*(a+b*arccosh(c*x))/x^3+c^2*d*(a+b*arccosh(c*x))/x-5/6*b*c^3*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2

Rubi [A]

time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {14, 5921, 12, 465, 94, 211}

$$\frac{c^2d(a+b\cosh^{-1}(cx))}{x}-\frac{d(a+b\cosh^{-1}(cx))}{3x^3}-\frac{5}{6}bc^3d\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]

[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*x^3) + (c^2*d*(a + b*ArcCosh[c*x]))/x - (5*b*c^3*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/6

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d-c^2dx^2)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-(bc)\int \frac{d(-1}{3x^3\sqrt{-1+}}\\ &= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{3}(bcd)\int \frac{-}{x^3\sqrt{-1}}\\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(\right.}{x}\\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(\right.}{x}\\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(\right.}{x}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 127, normalized size = 1.41

$$-\frac{ad}{3x^3}+\frac{ac^2d}{x}+\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{bd\cosh^{-1}(cx)}{3x^3}+\frac{bc^2d\cosh^{-1}(cx)}{x}-\frac{5bc^3d\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{6\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

[Out] -1/3*(a*d)/x^3 + (a*c^2*d)/x + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (b*d*ArcCosh[c*x])/(3*x^3) + (b*c^2*d*ArcCosh[c*x])/x - (5*b*c^3*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Maple [A]
time = 1.99, size = 119, normalized size = 1.32

method	result
derivativedivides	$c^3\left(-ad\left(-\frac{1}{cx}+\frac{1}{3c^3x^3}\right)+\frac{bd\,\mathrm{arccosh}(cx)}{cx}-\frac{bd\,\mathrm{arccosh}(cx)}{3c^3x^3}+\frac{5bd\sqrt{cx-1}\,\sqrt{cx+1}\,\arctan\left(\frac{}{\sqrt{c^2x}}\right.}{6\sqrt{c^2x^2-1}}\right.$
default	$c^3\left(-ad\left(-\frac{1}{cx}+\frac{1}{3c^3x^3}\right)+\frac{bd\,\mathrm{arccosh}(cx)}{cx}-\frac{bd\,\mathrm{arccosh}(cx)}{3c^3x^3}+\frac{5bd\sqrt{cx-1}\,\sqrt{cx+1}\,\arctan\left(\frac{}{\sqrt{c^2x}}\right.}{6\sqrt{c^2x^2-1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)

[Out] c^3*(-a*d*(-1/c/x+1/3/c^3/x^3)+b*d*arccosh(c*x)/c/x-1/3*b*d*arccosh(c*x)/c^3/x^3+5/6*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/x^2)

Maxima [A]
time = 0.47, size = 89, normalized size = 0.99

$$\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bc^2d-\frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)bd+\frac{ac^2d}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")

[Out] (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d + a*c^2*d/x - 1/3*a*d/x^3

Fricas [A]
time = 0.39, size = 146, normalized size = 1.62

$$-\frac{10\,bc^3dx^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-6\,ac^2dx^2+2\,(3\,bc^2-b)dx^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,bcdx+2\,ad-2\,(3\,bc^2dx^2-(3\,bc^2-b)dx^3-bd)\log\left(cx+\sqrt{c^2x^2-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")

[Out] -1/6*(10*b*c^3*d*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 6*a*c^2*d*x^2 + 2*(3*b*c^2 - b)*d*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*b*c*d*

```
x + 2*a*d - 2*(3*b*c^2*d*x^2 - (3*b*c^2 - b)*d*x^3 - b*d)*log(c*x + sqrt(c^
2*x^2 - 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x^4}\right)dx+\int\frac{ac^2}{x^2}\,dx+\int\left(-\frac{b\,\mathrm{acosh}\,(cx)}{x^4}\right)dx+\int\frac{bc^2\,\mathrm{acosh}\,(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d*(Integral(-a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(-b*acosh(c*x
)/x**4, x) + Integral(b*c**2*acosh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4, x)
```

3.10 $\int x^4(d-c^2dx^2)^2 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=206

$$-\frac{8bd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{315c^5}+\frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^5}-\frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^5}-\frac{10bd^2(-1+cx)^{7/2}}{441c^5}$$

```
[Out] 4/945*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^5-1/525*b*d^2*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^5-10/441*b*d^2*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^5-1/81*b*d^2*(c*x-1
)^(9/2)*(c*x+1)^(9/2)/c^5+1/5*d^2*x^5*(a+b*arccosh(c*x))-2/7*c^2*d^2*x^7*(a
+b*arccosh(c*x))+1/9*c^4*d^2*x^9*(a+b*arccosh(c*x))-8/315*b*d^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c^5
```

Rubi [A]
time = 0.20, antiderivative size = 264, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {276, 5921, 12, 534, 1265, 911, 1167}

$$\frac{1}{9}c^4d^2x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^2(1-c^2x^2)^5}{81c^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{10bd^2(1-c^2x^2)^4}{441c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2(1-c^2x^2)^3}{525c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bd^2(1-c^2x^2)^2}{945c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^2(1-c^2x^2)}{315c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(945*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(525*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*d^2*(1 - c^2*x^2)^4)/(
441*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^5)/(81*c^5*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b*ArcCosh[c*x]))/5 - (2*c^2*d^2*
x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
```

```
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x$$

$$= \frac{8bd^2(1-c^2x^2)}{315c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bd^2(1-c^2x^2)^2}{945c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{525c}$$

Mathematica [A]
time = 0.11, size = 124, normalized size = 0.60

$$\frac{d^2\left(315ac^5x^5(63-90c^2x^2+35c^4x^4)-b\sqrt{-1+cx}\,\sqrt{1+cx}\,(2104+1052c^2x^2+789c^4x^4-2650c^6x^6+1225c^8x^8)+315bc^5x^5(63-90c^2x^2+35c^4x^4)\cosh^{-1}(cx)\right)}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8)
+ 315*b*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcCosh[c*x]))/(99225*c^5)
```

Maple [A]
time = 2.80, size = 128, normalized size = 0.62

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\mathrm{arccosh}(cx)c^9x^9}{9}-\frac{2\,\mathrm{arccosh}(cx)c^7x^7}{7}+\frac{\mathrm{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(12}{}\right.}{c^5}$

default	$\frac{d^2 a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2 b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{2\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1225c}{}\right.}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9-2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arccosh(c*x)*
c^9*x^9-2/7*arccosh(c*x)*c^7*x^7+1/5*arccosh(c*x)*c^5*x^5-1/99225*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-2650*c^6*x^6+789*c^4*x^4+1052*c^2*x^2+2104
)))
```

Maxima [A]
time = 0.31, size = 319, normalized size = 1.55

$$\frac{1}{9}ac^4d^2x^9-\frac{2}{7}ac^2d^2x^7+\frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)bc^4d^2+\frac{1}{5}ad^2x^5-\frac{2}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^2d^2+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arccosh(c*x) - (35*
sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
- 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2
- 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 +
16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqr
t(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/
c^6)*c)*b*d^2
```

Fricas [A]
time = 0.40, size = 165, normalized size = 0.80

$$\frac{11025\,ac^9d^2x^9-28350\,ac^7d^2x^7+19845\,ac^5d^2x^5+315\,(35\,bc^9d^2x^9-90\,bc^7d^2x^7+63\,bc^5d^2x^5)\log\left(cx+\sqrt{c^2x^2-1}\right)-(1225\,bc^8d^2x^8-2650\,bc^6d^2x^6+789\,bc^4d^2x^4+1052\,bc^2d^2x^2+2104\,bd^2)\sqrt{c^2x^2-1}}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 - 1)) - (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 + 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [C] Result contains complex when optimal does not.
time = 1.50, size = 236, normalized size = 1.15

$$\begin{cases}\frac{ac^4d^2x^9}{9}-\frac{2ac^2d^2x^7}{7}+\frac{ad^2x^5}{5}+\frac{bc^4d^2x^9\operatorname{acosh}(cx)}{9}-\frac{bc^3d^2x^8\sqrt{c^2x^2-1}}{81}-\frac{2bc^2d^2x^7\operatorname{acosh}(cx)}{7}+\frac{106bcd^2x^6\sqrt{c^2x^2-1}}{3969}+\frac{bd^2x^5\operatorname{acosh}(cx)}{5}-\frac{263bd^2x^4\sqrt{c^2x^2-1}}{33075c}-\frac{1052bd^2x^2\sqrt{c^2x^2-1}}{99225c^3}-\frac{2104bd^2\sqrt{c^2x^2-1}}{99225c^5} & \text{for } c\neq 0\\ \frac{d^2x^5\left(a+\frac{i\pi b}{2}\right)}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 - 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*acosh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 - 1)/81 - 2*b*c
**2*d**2*x**7*acosh(c*x)/7 + 106*b*c*d**2*x**6*sqrt(c**2*x**2 - 1)/3969 + b
*d**2*x**5*acosh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 - 1)/(33075*c) - 1
052*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 - 1)/(99225*c**5), Ne(c, 0)), (d**2*x**5*(a + I*pi*b/2)/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

3.11 $\int x^3(d-c^2dx^2)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=200

$$-\frac{73bd^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{3072c^3}-\frac{73bd^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4608c}+\frac{43bcd^2x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{-}$$

```
[Out] -73/3072*b*d^2*arccosh(c*x)/c^4+1/4*d^2*x^4*(a+b*arccosh(c*x))-1/3*c^2*d^2*
x^6*(a+b*arccosh(c*x))+1/8*c^4*d^2*x^8*(a+b*arccosh(c*x))-73/3072*b*d^2*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c+43/1152*b*c*d^2*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c*x
-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 284, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 45, 5921, 12, 534, 1281, 470, 327, 223, 212}

$$\frac{1}{8}c^4d^2x^8\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{43bcd^2x^5(1-c^2x^2)}{1152\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{73bd^2x^3(1-c^2x^2)}{4608c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{73bd^2\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{3072c^4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{73bd^2x(1-c^2x^2)}{3072c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d^2x^7(1-c^2x^2)}{64\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (73*b*d^2*x*(1 - c^2*x^2))/(3072*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (73*b*
d^2*x^3*(1 - c^2*x^2))/(4608*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (43*b*c*d^2*
x^5*(1 - c^2*x^2))/(1152*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^7*(1
- c^2*x^2))/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c*x
]))/4 - (c^2*d^2*x^6*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^8*(a + b*ArcCosh[
c*x]))/8 - (73*b*d^2*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/
(3072*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
```

```
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{bc^3d^2x^7(1-c^2x^2)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a\right.$$

$$= -\frac{43bcd^2x^5(1-c^2x^2)}{1152\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^2x^7(1-c^2x^2)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a\right.$$

$$= \frac{73bd^2x^3(1-c^2x^2)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{43bcd^2x^5(1-c^2x^2)}{1152\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d}{64\sqrt{-}}$$

$$= \frac{73bd^2x(1-c^2x^2)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3(1-c^2x^2)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4}{1152}$$

$$= \frac{73bd^2x(1-c^2x^2)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3(1-c^2x^2)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4}{1152}$$

$$= \frac{73bd^2x(1-c^2x^2)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3(1-c^2x^2)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4}{1152}$$

Mathematica [A]

time = 0.17, size = 141, normalized size = 0.70

$$\frac{d^2\left(2304ax^4-3072ac^2x^6+1152ac^4x^8-\frac{bx\sqrt{-1+cx}\sqrt{1+cx}\,(219+146c^2x^2-344c^4x^4+144c^6x^6)}{c^3}+384bx^4(6-8c^2x^2+3c^4x^4)\cosh^{-1}(cx)-\frac{219b\log\left(cx+\sqrt{-1+cx}\sqrt{1+cx}\right)}{c^4}\right)}{9216}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(2304*a*x^4 - 3072*a*c^2*x^6 + 1152*a*c^4*x^8 - (b*x*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(219 + 146*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6))/c^3 + 384*b*x^
4*(6 - 8*c^2*x^2 + 3*c^4*x^4)*ArcCosh[c*x] - (219*b*Log[c*x + Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/c^4))/9216
```

Maple [A]
time = 2.77, size = 243, normalized size = 1.22

method	result
derivativedivides	$\frac{d^2 a\left(\frac{\left(c^2x^2-1\right)^4}{8}+\frac{\left(c^2x^2-1\right)^3}{6}\right)+\frac{d^2 b\,\mathrm{arccosh}(cx)c^8x^8}{8}-\frac{d^2 b\,\mathrm{arccosh}(cx)c^6x^6}{3}+\frac{d^2 b\,\mathrm{arccosh}(cx)c^4x^4}{4}-\frac{b\,d^2\mathrm{arccosh}(cx)}{24}-\frac{d^2 b\sqrt{c\ldots}}{\ldots}}{\ldots}$
default	$\frac{d^2 a\left(\frac{\left(c^2x^2-1\right)^4}{8}+\frac{\left(c^2x^2-1\right)^3}{6}\right)+\frac{d^2 b\,\mathrm{arccosh}(cx)c^8x^8}{8}-\frac{d^2 b\,\mathrm{arccosh}(cx)c^6x^6}{3}+\frac{d^2 b\,\mathrm{arccosh}(cx)c^4x^4}{4}-\frac{b\,d^2\mathrm{arccosh}(cx)}{24}-\frac{d^2 b\sqrt{c\ldots}}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^2*a*(1/8*(c^2*x^2-1)^4+1/6*(c^2*x^2-1)^3)+1/8*d^2*b*arccosh(c*x)*c
^8*x^8-1/3*d^2*b*arccosh(c*x)*c^6*x^6+1/4*d^2*b*arccosh(c*x)*c^4*x^4-1/24*b
*d^2*arccosh(c*x)-1/64*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^7*x^7+43/1152*d^
2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5-73/4608*d^2*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*c^3*x^3-73/3072*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+55/3072*d^2*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(168) = 336$.
time = 0.28, size = 346, normalized size = 1.73

$$\frac{1}{8}ac^4d^2x^8-\frac{1}{3}ac^2d^2x^6+\frac{1}{3072}\left(384x^8\,\mathrm{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)bc^4d^2+\frac{1}{4}ad^2x^4-\frac{1}{144}\left(48x^6\,\mathrm{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bc^2d^2+\frac{1}{32}\left(8x^4\,\mathrm{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arccosh(c*x) - (48*
sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
- 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x
^2 - 1)*c)/c^9)*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*arccosh(c*x) -
 (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d^2
```

```
+ 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2
- 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2
```

Fricas [A]
time = 0.36, size = 161, normalized size = 0.80

$$\frac{1152\,ac^8d^2x^8 - 3072\,ac^6d^2x^6 + 2304\,ac^4d^2x^4 + 3\,(384\,bc^8d^2x^8 - 1024\,bc^6d^2x^6 + 768\,bc^4d^2x^4 - 73\,bd^2)\log\left(cx + \sqrt{c^2x^2 - 1}\right) - (144\,bc^7d^2x^7 - 344\,bc^5d^2x^5 + 146\,bc^3d^2x^3 + 219\,bcd^2x)\sqrt{c^2x^2 - 1}}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 + 219*b*c*d^2*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 224, normalized size = 1.12

$$\begin{cases} \frac{ac^4d^2x^8}{8} - \frac{ac^2d^2x^6}{3} + \frac{ad^2x^4}{4} + \frac{bc^4d^2x^8\operatorname{acosh}(cx)}{8} - \frac{bc^3d^2x^7\sqrt{c^2x^2-1}}{64} - \frac{bc^2d^2x^6\operatorname{acosh}(cx)}{3} + \frac{43bcd^2x^5\sqrt{c^2x^2-1}}{1152} + \frac{bd^2x^4\operatorname{acosh}(cx)}{4} - \frac{73bd^2x^3\sqrt{c^2x^2-1}}{4608c} - \frac{73bd^2x\sqrt{c^2x^2-1}}{3072c^3} - \frac{73bd^2\operatorname{acosh}(cx)}{3072c^4} & \text{for}\ c \neq 0 \\ \frac{d^2x^4\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 - a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*acosh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 - 1)/64 - b*c**2*
d**2*x**6*acosh(c*x)/3 + 43*b*c*d**2*x**5*sqrt(c**2*x**2 - 1)/1152 + b*d**2
*x**4*acosh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 - 1)/(4608*c) - 73*b*d**
2*x*sqrt(c**2*x**2 - 1)/(3072*c**3) - 73*b*d**2*acosh(c*x)/(3072*c**4), Ne(
c, 0)), (d**2*x**4*(a + I*pi*b/2)/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

3.12 $\int x^2(d-c^2dx^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=177

$$-\frac{8bd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{105c^3}+\frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{315c^3}-\frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{175c^3}-\frac{bd^2(-1+cx)^{7/2}(1+}{49c^3}$$

```
[Out] 4/315*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-1/175*b*d^2*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^3-1/49*b*d^2*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*ar
ccosh(c*x))-2/5*c^2*d^2*x^5*(a+b*arccosh(c*x))+1/7*c^4*d^2*x^7*(a+b*arccosh
(c*x))-8/105*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {276, 5921, 12, 534, 1265, 785}

$$\frac{1}{7}c^4d^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(1-c^2x^2)^4}{49c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2(1-c^2x^2)^3}{175c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bd^2(1-c^2x^2)^2}{315c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^2(1-c^2x^2)}{105c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(105*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(175*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^2*(1 - c^2*x^2)^4)/(49*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 - (2*c
^2*d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
```

```
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2$$

$$=\frac{8bd^2(1-c^2x^2)}{105c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bd^2(1-c^2x^2)^2}{315c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{175c}$$

Mathematica [A]
time = 0.10, size = 116, normalized size = 0.66

$$\frac{d^2\left(105ac^3x^3(35-42c^2x^2+15c^4x^4)-b\sqrt{-1+cx}\,\sqrt{1+cx}\,(818+409c^2x^2-612c^4x^4+225c^6x^6)+105bc^3x^3(35-42c^2x^2+15c^4x^4)\cosh^{-1}(cx)\right)}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(3
5 - 42*c^2*x^2 + 15*c^4*x^4)*ArcCosh[c*x]))/(11025*c^3)
```

Maple [A]
time = 3.66, size = 120, normalized size = 0.68

method	result
derivativedivides	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(225x}{11025}\right.}{c^3}$
default	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(225x}{11025}\right.}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7-2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arccosh(c*x)*
c^7*x^7-2/5*arccosh(c*x)*c^5*x^5+1/3*arccosh(c*x)*c^3*x^3-1/11025*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(225*c^6*x^6-612*c^4*x^4+409*c^2*x^2+818)))
```

Maxima [A]
time = 0.28, size = 261, normalized size = 1.47

$$\frac{1}{7}ac^4d^2x^7-\frac{2}{5}ac^2d^2x^5+\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^4d^2-\frac{2}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d^2+\frac{1}{3}ad^2x^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arccosh(c*x) - (5*sqr
t(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*
x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^2 - 2/75*(15*x^5*arccosh(c*x
) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 - 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(
sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2
```

Fricas [A]
time = 0.36, size = 153, normalized size = 0.86

$$\frac{1575\,ac^{7}d^{2}x^{7} - 4410\,ac^{5}d^{2}x^{5} + 3675\,ac^{3}d^{2}x^{3} + 105\,(15\,bc^{7}d^{2}x^{7} - 42\,bc^{5}d^{2}x^{5} + 35\,bc^{3}d^{2}x^{3})\log\left(cx + \sqrt{c^{2}x^{2} - 1}\right) - (225\,bc^{6}d^{2}x^{6} - 612\,bc^{4}d^{2}x^{4} + 409\,bc^{2}d^{2}x^{2} + 818\,bd^{2})\sqrt{c^{2}x^{2} - 1}}{11025\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 - 1)) - (225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 +
818*b*d^2)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.73, size = 209, normalized size = 1.18

$$\begin{cases} \frac{ac^{4}d^{2}x^{7}}{7} - \frac{2ac^{2}d^{2}x^{5}}{5} + \frac{ad^{2}x^{3}}{3} + \frac{bc^{4}d^{2}x^{7}\operatorname{acosh}(cx)}{7} - \frac{bc^{3}d^{2}x^{6}\sqrt{c^{2}x^{2}-1}}{49} - \frac{2bc^{2}d^{2}x^{5}\operatorname{acosh}(cx)}{5} + \frac{68bcd^{2}x^{4}\sqrt{c^{2}x^{2}-1}}{1225} + \frac{bd^{2}x^{3}\operatorname{acosh}(cx)}{3} - \frac{409bd^{2}x^{2}\sqrt{c^{2}x^{2}-1}}{11025c} - \frac{818bd^{2}\sqrt{c^{2}x^{2}-1}}{11025c^{3}} & \text{for}\: c \neq 0 \\ \frac{d^{2}x^{3}\left(a + \frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 - 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*acosh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 - 1)/49 - 2*b*c
**2*d**2*x**5*acosh(c*x)/5 + 68*b*c*d**2*x**4*sqrt(c**2*x**2 - 1)/1225 + b*
d**2*x**3*acosh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(11025*c) - 81
8*b*d**2*sqrt(c**2*x**2 - 1)/(11025*c**3), Ne(c, 0)), (d**2*x**3*(a + I*pi*
b/2)/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

3.13 $\int x(d-c^2dx^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=136

$$-\frac{5bd^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}$$

```
[Out] 5/144*b*d^2*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-1/36*b*d^2*x*(c*x-1)^(5/2)*(c*x
+1)^(5/2)/c+5/96*b*d^2*arccosh(c*x)/c^2-1/6*d^2*(-c^2*x^2+1)^3*(a+b*arccosh
(c*x))/c^2-5/96*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5914, 38, 54}

$$-\frac{d^2(1-c^2x^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}-\frac{bd^2x(cx-1)^{5/2}(cx+1)^{5/2}}{36c}+\frac{5bd^2x(cx-1)^{3/2}(cx+1)^{3/2}}{144c}-\frac{5bd^2x\sqrt{cx-1}\,\sqrt{cx+1}}{96c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c) + (5*b*d^2*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/(144*c) - (b*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(
36*c) + (5*b*d^2*ArcCosh[c*x])/(96*c^2) - (d^2*(1 - c^2*x^2)^3*(a + b*ArcCo
sh[c*x]))/(6*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{d^2(1-c^2x^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}-\frac{(bd^2)\int(-1+cx)^{5/2}(1+cx}{6c}$$
$$=-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-\frac{d^2(1-c^2x^2)^3\left(a+b\cosh^{-1}(cx\right.}{6c^2}$$
$$=\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-$$
$$=-\frac{5bd^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-$$
$$=-\frac{5bd^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-$$

Mathematica [A]
time = 0.12, size = 132, normalized size = 0.97

$$\frac{d^2\left(cx\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\,(-33+26c^2x^2-8c^4x^4)+48acx(3-3c^2x^2+c^4x^4)\right)+48bc^2x^2(3-3c^2x^2+c^4x^4)\cosh^{-1}(cx)-33b\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)\right)}{288c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) +
48*a*c*x*(3 - 3*c^2*x^2 + c^4*x^4)) + 48*b*c^2*x^2*(3 - 3*c^2*x^2 + c^4*x^4
)*ArcCosh[c*x] - 33*b*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/(288*c^2)
```

Maple [A]
time = 2.75, size = 202, normalized size = 1.49

method	result
derivativedivides	$\frac{\frac{d^2\left(c^2x^2-1\right)^3a}{6}+\frac{d^2b\,\mathrm{arccosh}(cx)c^6x^6}{6}-\frac{d^2b\,\mathrm{arccosh}(cx)c^4x^4}{2}+\frac{d^2b\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{b\,d^2\mathrm{arccosh}(cx)}{6}-\frac{d^2b\sqrt{cx-1}\,\sqrt{cx-}}{36}}{}$
default	$\frac{\frac{d^2\left(c^2x^2-1\right)^3a}{6}+\frac{d^2b\,\mathrm{arccosh}(cx)c^6x^6}{6}-\frac{d^2b\,\mathrm{arccosh}(cx)c^4x^4}{2}+\frac{d^2b\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{b\,d^2\mathrm{arccosh}(cx)}{6}-\frac{d^2b\sqrt{cx-1}\,\sqrt{cx-}}{36}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*d^2*(c^2*x^2-1)^3*a+1/6*d^2*b*arccosh(c*x)*c^6*x^6-1/2*d^2*b*arc
cosh(c*x)*c^4*x^4+1/2*d^2*b*arccosh(c*x)*c^2*x^2-1/6*b*d^2*arccosh(c*x)-1/3
6*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5+13/144*d^2*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*c^3*x^3-11/96*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+5/96*d^2*b*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(113) = 226$.
time = 0.27, size = 287, normalized size = 2.11

$$\frac{1}{6}\,ac^4d^2x^6 - \frac{1}{2}\,ac^2d^2x^4 + \frac{1}{288}\left(48\,x^6\,\mathrm{arcosh}\left(cx\right) - \left(\frac{8\,\sqrt{c^2x^2-1}\,x^5}{c^2} + \frac{10\,\sqrt{c^2x^2-1}\,x^3}{c^4} + \frac{15\,\sqrt{c^2x^2-1}\,x}{c^6} + \frac{15\,\log\left(2\,c^2x + 2\,\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bc^4d^2 - \frac{1}{16}\left(8\,x^4\,\mathrm{arcosh}\left(cx\right) - \left(\frac{2\,\sqrt{c^2x^2-1}\,x^3}{c^2} + \frac{3\,\sqrt{c^2x^2-1}\,x}{c^4} + \frac{3\,\log\left(2\,c^2x + 2\,\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bc^2d^2 + \frac{1}{2}\,ad^2x^2 + \frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2} + \frac{\log\left(2\,c^2x + 2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arccosh(c*x) - (8*sqr
t(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1
)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^4*d^2 - 1/16*
(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/
c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c^2*d^2 + 1/2*a*d^2*
x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^2
```

Fricas [A]
time = 0.39, size = 149, normalized size = 1.10

$$\frac{48\,ac^6d^2x^6 - 144\,ac^4d^2x^4 + 144\,ac^2d^2x^2 + 3\,(16\,bc^6d^2x^6 - 48\,bc^4d^2x^4 + 48\,bc^2d^2x^2 - 11\,bd^2)\log\left(cx + \sqrt{c^2x^2-1}\right) - (8\,bc^5d^2x^5 - 26\,bc^3d^2x^3 + 33\,bcd^2x)\sqrt{c^2x^2-1}}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 - 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 - 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 - 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 - 1)) - (8*b*c^5*d^2*x^5 - 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 - 1))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 197, normalized size = 1.45

$$\begin{cases} \frac{ac^4d^2x^6}{6} - \frac{ac^2d^2x^4}{2} + \frac{ad^2x^2}{2} + \frac{bc^4d^2x^6\,\mathrm{acosh}\,(cx)}{6} - \frac{bc^3d^2x^5\sqrt{c^2x^2-1}}{36} - \frac{bc^2d^2x^4\,\mathrm{acosh}\,(cx)}{2} + \frac{13bcd^2x^3\sqrt{c^2x^2-1}}{144} + \frac{bd^2x^2\,\mathrm{acosh}\,(cx)}{2} - \frac{11bd^2x\sqrt{c^2x^2-1}}{96c} - \frac{11bd^2\,\mathrm{acosh}\,(cx)}{96c^2} & \text{for}\: c \neq 0 \\ \frac{d^2x^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 - a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*acosh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 - 1)/36 - b*c**2*
d**2*x**4*acosh(c*x)/2 + 13*b*c*d**2*x**3*sqrt(c**2*x**2 - 1)/144 + b*d**2*
x**2*acosh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 - 1)/(96*c) - 11*b*d**2*acos
h(c*x)/(96*c**2), Ne(c, 0)), (d**2*x**2*(a + I*pi*b/2)/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

3.14 $\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=143

$$-\frac{8bd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c}+\frac{4bd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{45c}-\frac{bd^2(-1+cx)^{5/2}(1+cx)^{5/2}}{25c}+d^2x\left(a+b\cosh^{-1}(cx\right.$$

```
[Out] 4/45*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-1/25*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(
5/2)/c+d^2*x*(a+b*arccosh(c*x))-2/3*c^2*d^2*x^3*(a+b*arccosh(c*x))+1/5*c^4*
d^2*x^5*(a+b*arccosh(c*x))-8/15*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {200, 5894, 12, 534, 1261, 712}

$$\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^2\left(1-c^2x^2\right)^3}{25c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{45c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^2\left(1-c^2x^2\right)}{15c\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(15*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(1 -
 c^2*x^2)^2)/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^3)/
(25*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) - (2*c^2*d
^2*x^3*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\, dx = d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a\right.$$

$$= \frac{8bd^2(1-c^2x^2)}{15c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bd^2(1-c^2x^2)^2}{45c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2(1}{25c\sqrt{-1+}}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 0.69

$$\frac{d^2\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\,(-149+38c^2x^2-9c^4x^4)+15acx(15-10c^2x^2+3c^4x^4)+15bcx(15-10c^2x^2+3c^4x^4)\cosh^{-1}(cx)\right)}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*a
*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)
*ArcCosh[c*x]))/(225*c)
```

Maple [A]
time = 2.04, size = 102, normalized size = 0.71

method	result
derivativedivides	$\frac{d^2 a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2 b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2\operatorname{arccosh}(cx)c^3x^3}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4x^4-38\right.}{225}\right.}{c}$
default	$\frac{d^2 a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2 b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2\operatorname{arccosh}(cx)c^3x^3}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4x^4-38\right.}{225}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(d^2*a*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b*(1/5*arccosh(c*x)*c^5*x^5-2/
3*arccosh(c*x)*c^3*x^3+c*x*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(
9*c^4*x^4-38*c^2*x^2+149)))
```

Maxima [A]
time = 0.26, size = 194, normalized size = 1.36

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^4d^2-\frac{2}{3}\,ac^2d^2x^3-\frac{2}{9}\left(3\,x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d^2+ad^2x+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^
2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^4*d^2 - 2
/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c
^2*x^2 - 1))*b*d^2/c
```

Fricas [A]
time = 0.36, size = 133, normalized size = 0.93

$$\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4d^2x^4-38\,bc^2d^2x^2+149\,bd^2\right)\sqrt{c^2x^2-1}}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (9
*b*c^4*d^2*x^4 - 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 - 1))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 172, normalized size = 1.20

$$\begin{cases} \frac{ac^4d^2x^5}{5} - \frac{2ac^2d^2x^3}{3} + ad^2x + \frac{bc^4d^2x^5 \operatorname{acosh}(cx)}{5} - \frac{bc^3d^2x^4\sqrt{c^2x^2-1}}{25} - \frac{2bc^2d^2x^3 \operatorname{acosh}(cx)}{3} + \frac{38bcd^2x^2\sqrt{c^2x^2-1}}{225} + bd^2x \operatorname{acosh}(cx) - \frac{149bd^2\sqrt{c^2x^2-1}}{225c} & \text{for } c \neq 0 \\ d^2x\left(a + \frac{i\pi b}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*acosh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 - 1)/25 - 2*b*c**2*d
**2*x**3*acosh(c*x)/3 + 38*b*c*d**2*x**2*sqrt(c**2*x**2 - 1)/225 + b*d**2*x
*acosh(c*x) - 149*b*d**2*sqrt(c**2*x**2 - 1)/(225*c), Ne(c, 0)), (d**2*x*(a
 + I*pi*b/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

3.15 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=184

$$\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2\cosh^{-1}(cx)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}\right.$$

```
[Out] -1/16*b*c*d^2*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)-11/32*b*d^2*arccosh(c*x)+1/2*d^
2*(-c^2*x^2+1)*(a+b*arccosh(c*x))+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))
+1/2*d^2*(a+b*arccosh(c*x))^2/b+d^2*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)+11/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5919, 5882, 3799, 2221, 2317, 2438, 38, 54}

$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d^2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{2}bd^2\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-\frac{1}{16}bcd^2x(cx-1)^{3/2}(cx+1)^{3/2}+\frac{11}{32}bcd^2x\sqrt{cx-1}\,\sqrt{cx+1}-\frac{11}{32}bd^2\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (11*b*c*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (b*c*d^2*x*(-1 + c*x)^(3/2
)*(1 + c*x)^(3/2))/16 - (11*b*d^2*ArcCosh[c*x])/32 + (d^2*(1 - c^2*x^2)*(a
+ b*ArcCosh[c*x]))/2 + (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^2*
(a + b*ArcCosh[c*x])^2)/(2*b) + d^2*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcC
osh[c*x])] - (b*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^2\,(a+b\cosh^{-1}(cx))}{x}\,dx = \frac{1}{4}d^2(1-c^2x^2)^2\,(a+b\cosh^{-1}(cx)) + d\int \frac{(d-c^2dx^2)\,(a+b\cosh^{-1}(}{x}$$

$$= -\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{1}{2}d^2(1-c^2x^2)\,(a+b\cosh^{-1}(cx$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} +$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} -$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} -$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} -$$

$$= \frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2} -$$

Mathematica [A]
time = 0.39, size = 208, normalized size = 1.13

$$\frac{1}{4}d^2\left(-4ac^2x^2+ac^4x^4-4bc^2x^2\cosh^{-1}(cx)+bc^4x^4\cosh^{-1}(cx)+2b\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}+2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)-\frac{1}{8}b\left(cx\sqrt{\frac{-1+cx}{1+cx}}\,(3+3cx+2c^2x^2+2c^3x^3)+6\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)+2b\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+4a\log(x)-2b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (d^2*(-4*a*c^2*x^2 + a*c^4*x^4 - 4*b*c^2*x^2*ArcCosh[c*x] + b*c^4*x^4*ArcCo
sh[c*x] + 2*b*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)
/(1 + c*x)]]) - (b*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3 + 3*c*x + 2*c^2*x^2 +
 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/8 + 2*b*ArcCosh[c*x]*
(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 4*a*Log[x] - 2*b*PolyLog[
2, -E^(-2*ArcCosh[c*x])]))/4
```

Maple [A]
time = 5.30, size = 201, normalized size = 1.09

method	result
derivativedivides	$\frac{a\,d^2c^4x^4}{4} - a\,d^2c^2x^2 + a\,d^2\ln(cx) + d^2b\,\mathrm{arccosh}(cx)\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)$

default	$\frac{a\,d^2c^4x^4}{4}-a\,d^2c^2x^2+a\,d^2\ln\left(cx\right)+d^2b\,\text{arccosh}(cx)\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*d^2*c^4*x^4-a*d^2*c^2*x^2+a*d^2*ln(c*x)+d^2*b*arccosh(c*x)*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/4*d^2*b*arccosh(c*x)*c^4*x^4-d^2*b*arccos
h(c*x)*c^2*x^2-1/2*d^2*b*arccosh(c*x)^2-1/16*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*c^3*x^3+13/32*b*d^2*arccosh(c*x)+1/2*d^2*b*polylog(2,-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)+13/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*x*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a}{x}\,dx+\int\left(-2ac^2x\right)\,dx+\int ac^4x^3\,dx+\int\frac{b\,\text{acosh}\left(cx\right)}{x}\,dx+\int\left(-2bc^2x\,\text{acosh}\left(cx\right)\right)\,dx+\int bc^4x^3\,\text{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3, x
) + Integral(b*acosh(c*x)/x, x) + Integral(-2*b*c**2*x*acosh(c*x), x) + Int
egral(b*c**4*x**3*acosh(c*x), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x, x)
```

3.16 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=135

$$\frac{5}{3}bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{9}bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+$$

```
[Out] -1/9*b*c*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)-d^2*(a+b*arccosh(c*x))/x-2*c^2*d^2
*x*(a+b*arccosh(c*x))+1/3*c^4*d^2*x^3*(a+b*arccosh(c*x))+b*c*d^2*arctan((c*
x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 182, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {276, 5921, 12, 534, 1265, 911, 1167, 211}

$$\frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(cx)\right)-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bcd^2\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2(1-c^2x^2)^2}{9\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5bcd^2(1-c^2x^2)}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 -
 c^2*x^2)^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/
x - 2*c^2*d^2*x*(a + b*ArcCosh[c*x]) + (c^4*d^2*x^3*(a + b*ArcCosh[c*x]))/3
 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
```

```
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^2\,(a+b\cosh^{-1}(cx))}{x^2}\,dx = -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} - 2c^2d^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}c^4d^2x^3(a+$$

$$= -\frac{5bcd^2(1-c^2x^2)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2(1-c^2x^2)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d^2(a+b\cosh}{x}$$

$$= -\frac{5bcd^2(1-c^2x^2)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2(1-c^2x^2)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d^2(a+b\cosh}{x}$$

Mathematica [A]
time = 0.09, size = 131, normalized size = 0.97

$$\frac{d^2\left(-9a-18ac^2x^2+3ac^4x^4+16bcx\sqrt{-1+cx}\,\sqrt{1+cx}-bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}+3b(-3-6c^2x^2+c^4x^4)\cosh^{-1}(cx)-9bcx\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)\right)}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 + 16*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 +
c*x] - b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 3*b*(-3 - 6*c^2*x^2 + c^4*x
^4)*ArcCosh[c*x] - 9*b*c*x*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))/(9*x)
```

Maple [A]
time = 2.06, size = 163, normalized size = 1.21

method	result

derivativedivides	$c\left(a\,d^2\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+\frac{d^2b\,\mathrm{arccosh}(cx)c^3x^3}{3}-2d^2b\,\mathrm{arccosh}(cx)\,cx-\frac{d^2b\,\mathrm{arccosh}(cx)}{cx}-\frac{d^2b\sqrt{}}{}\right.$
default	$c\left(a\,d^2\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+\frac{d^2b\,\mathrm{arccosh}(cx)c^3x^3}{3}-2d^2b\,\mathrm{arccosh}(cx)\,cx-\frac{d^2b\,\mathrm{arccosh}(cx)}{cx}-\frac{d^2b\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a*d^2*(1/3*c^3*x^3-2*c*x-1/c/x)+1/3*d^2*b*arccosh(c*x)*c^3*x^3-2*d^2*b*a
rccosh(c*x)*c*x-d^2*b*arccosh(c*x)/c/x-1/9*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*c^2*x^2-d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2
*x^2-1)^(1/2))+16/9*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [A]
time = 0.46, size = 143, normalized size = 1.06

$$\frac{1}{3}\,ac^4d^2x^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^2-2\,ac^2d^2x-2\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bcd^2-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^2-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
+ 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*c*d^2 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b
*d^2 - a*d^2/x
```

Fricas [A]
time = 0.37, size = 201, normalized size = 1.49

$$\frac{3\,ac^4d^2x^4-18\,ac^2d^2x^2+18\,bcd^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-3\,(bc^4-6\,bc^2-3\,b)d^2x\log\left(-cx+\sqrt{c^2x^2-1}\right)-9\,ad^2+3\,(bc^4d^2x^4-6\,bc^2d^2x^2-(bc^4-6\,bc^2-3\,b)d^2x-3\,bd^2)\log\left(cx+\sqrt{c^2x^2-1}\right)-(bc^3d^2x^3-16\,bcd^2x)\sqrt{c^2x^2-1}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 - 18*a*c^2*d^2*x^2 + 18*b*c*d^2*x*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - 3*(b*c^4 - 6*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 - 1)
) - 9*a*d^2 + 3*(b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (b*c^4 - 6*b*c^2 - 3*b)*
d^2*x - 3*b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^3*d^2*x^3 - 16*b*c*d^2
*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \left(-2ac^{2}\right)\,dx+\int \frac{a}{x^{2}}\,dx+\int ac^{4}x^{2}\,dx+\int \left(-2bc^{2}\operatorname{acosh}\left(cx\right)\right)\,dx+\int \frac{b\operatorname{acosh}\left(cx\right)}{x^{2}}\,dx+\int bc^{4}x^{2}\operatorname{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(-2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
x) + Integral(-2*b*c**2*acosh(c*x), x) + Integral(b*acosh(c*x)/x**2, x) + I
ntegral(b*c**4*x**2*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2, x)
```

3.17 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=200

$$\frac{1}{4}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2d^2\cosh^{-1}(cx)-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-}\right.$$

```
[Out] -1/2*b*c*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/x-1/4*b*c^2*d^2*arccosh(c*x)-c^2*d
^2*(-c^2*x^2+1)*(a+b*arccosh(c*x))-1/2*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x)
)/x^2-c^2*d^2*(a+b*arccosh(c*x))^2/b-2*c^2*d^2*(a+b*arccosh(c*x))*ln(1+1/(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+b*c^2*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)+1/4*b*c^3*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5920, 99, 12, 38, 54, 5919, 5882, 3799, 2221, 2317, 2438}

$$-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d^2\left(a+b\cosh^{-1}(cx)\right)^2}{b}-2c^2d^2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}bc^3d^2x\sqrt{cx-1}\,\sqrt{cx+1}+bc^2d^2\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-\frac{1}{4}bc^2d^2\cosh^{-1}(cx)-\frac{bcd^2(cx-1)^{3/2}(cx+1)^{3/2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c*d^2*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(2*x) - (b*c^2*d^2*ArcCosh[c*x])/4 - c^2*d^2*(1 - c^2*x^2)*(
a + b*ArcCosh[c*x]) - (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/(2*x^2) -
(c^2*d^2*(a + b*ArcCosh[c*x])^2)/b - 2*c^2*d^2*(a + b*ArcCosh[c*x])*Log[1 +
 E^(-2*ArcCosh[c*x])] + b*c^2*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
```

```
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx &= -\frac{d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\left(2c^2d\right)\int\frac{\left(d-c^2dx^2\right)\left(a+b\,\mathrm{co}\right.}{x} \\
&= -\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-c^2d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right) \\
&= -\frac{1}{2}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-c^2d \\
&= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}+\frac{1}{2}bc^2 \\
&= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2 \\
&= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2 \\
&= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x}-\frac{1}{4}bc^2
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 182, normalized size = 0.91

$$\frac{d^2\left(-2a+2ac^4x^4+2bcx\sqrt{-1+cx}\,\sqrt{1+cx}-bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}-4bc^2x^2\cosh^{-1}(cx)^2-2bc^2x^2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)+2b\cosh^{-1}(cx)\left(-1+c^4x^4-4c^2x^2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-8ac^2x^2\log(x)+4bc^2x^2\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a + 2*a*c^4*x^4 + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - b*c^3*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^2*x^2*ArcCosh[c*x]^2 - 2*b*c^2*x^2*Ar
cTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(-1 + c^4*x^4 - 4*c^2*
x^2*Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*c^2*x^2*Log[x] + 4*b*c^2*x^2*PolyLo
g[2, -E^(-2*ArcCosh[c*x])]))/(4*x^2)
```

Maple [A]
time = 7.14, size = 212, normalized size = 1.06

method	result
derivativedivides	$c^2\left(\frac{a\,d^2c^2x^2}{2}-\frac{a\,d^2}{2c^2x^2}-2a\,d^2\ln(cx)+d^2b\mathrm{arccosh}(cx)^2+\frac{d^2b\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{bc\,d^2x\sqrt{cx-1}}{4}\right.$
default	$c^2\left(\frac{a\,d^2c^2x^2}{2}-\frac{a\,d^2}{2c^2x^2}-2a\,d^2\ln(cx)+d^2b\mathrm{arccosh}(cx)^2+\frac{d^2b\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{bc\,d^2x\sqrt{cx-1}}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a*d^2*c^2*x^2-1/2*a*d^2/c^2/x^2-2*a*d^2*ln(c*x)+d^2*b*arccosh(c*x)
^2+1/2*d^2*b*arccosh(c*x)*c^2*x^2-1/4*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
-1/4*b*d^2*arccosh(c*x)-1/2*d^2*b+1/2*d^2*b/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)
-1/2*d^2*b*arccosh(c*x)/c^2/x^2-2*d^2*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)-d^2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x -
 arccosh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a}{x^3}\,dx + \int \left(-\frac{2ac^2}{x}\right)dx + \int ac^4x\,dx + \int \frac{b\operatorname{acosh}(cx)}{x^3}\,dx + \int \left(-\frac{2bc^2\operatorname{acosh}(cx)}{x}\right)dx + \int bc^4x\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x, x
) + Integral(b*acosh(c*x)/x**3, x) + Integral(-2*b*c**2*acosh(c*x)/x, x) +
Integral(b*c**4*x*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3, x)
```

3.18 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=142

$$-bc^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c$$

```
[Out] -1/3*d^2*(a+b*arccosh(c*x))/x^3+2*c^2*d^2*(a+b*arccosh(c*x))/x+c^4*d^2*x*(a
+b*arccosh(c*x))-11/6*b*c^3*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))-b*c^3*d
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {276, 5921, 12, 534, 1265, 911, 1171, 396, 211}

$$c^4d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{11bc^3d^2\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d^2(1-c^2x^2)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c^3*d^2*(1 - c^2*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^
2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(
3*x^3) + (2*c^2*d^2*(a + b*ArcCosh[c*x]))/x + c^4*d^2*x*(a + b*ArcCosh[c*x]
) - (11*b*c^3*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
```

```
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\big(a+b\,\text{c}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\big(a+b\,\text{c}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\big(a+b\,\text{c}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\big(a+b\,\text{c}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\big(a+b\,\text{c}$$

$$= -\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\big(a+b\,\text{c}}{x}$$

$$= \frac{bc^3d^2(1-c^2x^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\big(a+b\cosh^{-}}{3x^3}$$

$$= \frac{bc^3d^2(1-c^2x^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\big(a+b\cosh^{-}}{3x^3}$$

Mathematica [A]
time = 0.10, size = 135, normalized size = 0.95

$$\frac{d^2\left(-2a+12ac^2x^2+6ac^4x^4+bcx\sqrt{-1+cx}\,\sqrt{1+cx}-6bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}+2b(-1+6c^2x^2+3c^4x^4)\cosh^{-1}(cx)+11bc^3x^3\,\text{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x
] - 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(-1 + 6*c^2*x^2 + 3*c^4*
x^4)*ArcCosh[c*x] + 11*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))
/(6*x^3)
```

Maple [A]
time = 1.98, size = 163, normalized size = 1.15

method	result
derivativedivides	$c^3\left(a\,d^2\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)+d^2b\,\text{arccosh}(cx)\,cx-\frac{d^2b\,\text{arccosh}(cx)}{3c^3x^3}+\frac{2d^2b\,\text{arccosh}(cx)}{cx}+\frac{11d^2b\sqrt{cx}}{}\right.$
default	$c^3\left(a\,d^2\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)+d^2b\,\text{arccosh}(cx)\,cx-\frac{d^2b\,\text{arccosh}(cx)}{3c^3x^3}+\frac{2d^2b\,\text{arccosh}(cx)}{cx}+\frac{11d^2b\sqrt{cx}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*d^2*(c*x-1/3/c^3/x^3+2/c/x)+d^2*b*arccosh(c*x)*c*x-1/3*d^2*b*arccosh
(c*x)/c^3/x^3+2*d^2*b*arccosh(c*x)/c/x+11/6*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))-d^2*b*(c*x-1)^(1/2)*(c*x+1
)^(1/2)+1/6*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/x^2)
```

Maxima [A]
time = 0.52, size = 137, normalized size = 0.96

$$ac^4d^2x+\left(cx\,\text{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bc^3d^2+2\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\text{arcosh}\,(cx)}{x}\right)bc^2d^2-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\text{arcosh}\,(cx)}{x^3}\right)bd^2+\frac{2\,ac^2d^2}{x}-\frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c^3*d^2 + 2*(c*arcsi
n(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d^2 - 1/6*((c^2*arcsin(1/(c*abs(x))
) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d^2 + 2*a*c^2*d^2/x -
1/3*a*d^2/x^3
```

Fricas [A]
time = 0.38, size = 213, normalized size = 1.50

$$\frac{6\,ac^4d^2x^4-22\,bc^3d^2x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+12\,ac^2d^2x^2-2\,(3\,bc^4+6\,bc^2-b)d^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-2\,ad^2+2\,(3\,bc^4d^2x^4+6\,bc^2d^2x^2-(3\,bc^4+6\,bc^2-b)d^2x^3-bd^2)\log\left(cx+\sqrt{c^2x^2-1}\right)-(6\,bc^3d^2x^3-bcd^2x)\sqrt{c^2x^2-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 22*b*c^3*d^2*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
12*a*c^2*d^2*x^2 - 2*(3*b*c^4 + 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^
2 - 1)) - 2*a*d^2 + 2*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - (3*b*c^4 + 6*b*c
```

```
^2 - b)*d^2*x^3 - b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*d^2*x^3 -
b*c*d^2*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \left(-\frac{2ac^2}{x^2}\right)dx + \int bc^4\operatorname{acosh}(cx)\,dx + \int \frac{b\operatorname{acosh}(cx)}{x^4}\,dx + \int \left(-\frac{2bc^2\operatorname{acosh}(cx)}{x^2}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(-2*a*c**2/x**2,
x) + Integral(b*c**4*acosh(c*x), x) + Integral(b*acosh(c*x)/x**4, x) + Inte
gral(-2*b*c**2*acosh(c*x)/x**2, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4, x)
```

3.19 $\int x^4(d-c^2dx^2)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=256

$$-\frac{16bd^3\sqrt{-1+cx}\,\sqrt{1+cx}}{1155c^5}+\frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{3465c^5}-\frac{2bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{1925c^5}+\frac{bd^3(-1+cx)^{7/2}(}{1617c^5}$$

```
[Out] 8/3465*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^5-2/1925*b*d^3*(c*x-1)^(5/2)*(c*
x+1)^(5/2)/c^5+1/1617*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^5+4/297*b*d^3*(c*
x-1)^(9/2)*(c*x+1)^(9/2)/c^5+1/121*b*d^3*(c*x-1)^(11/2)*(c*x+1)^(11/2)/c^5+
1/5*d^3*x^5*(a+b*arccosh(c*x))-3/7*c^2*d^3*x^7*(a+b*arccosh(c*x))+1/3*c^4*d
^3*x^9*(a+b*arccosh(c*x))-1/11*c^6*d^3*x^11*(a+b*arccosh(c*x))-16/1155*b*d^
3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5
```

Rubi [A]
time = 0.29, antiderivative size = 326, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {276, 5921, 12, 1624, 1813, 1634}

$$-\frac{1}{11}c^6d^3x^{11}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^3(1-c^2x^2)^6}{121c^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4bd^3(1-c^2x^2)^5}{297c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^3(1-c^2x^2)^4}{1617c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bd^3(1-c^2x^2)^3}{1925c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^3(1-c^2x^2)^2}{3465c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bd^3(1-c^2x^2)}{1155c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(1155*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3
*(1 - c^2*x^2)^2)/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c
^2*x^2)^3)/(1925*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4
)/(1617*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*d^3*(1 - c^2*x^2)^5)/(297*
c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^6)/(121*c^5*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (3*c^2*d^3*x^7
*(a + b*ArcCosh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcCosh[c*x]))/3 - (c^6*d^3*
x^11*(a + b*ArcCosh[c*x]))/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{16bd^3(1-c^2x^2)}{1155c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bd^3(1-c^2x^2)^2}{3465c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{192}$$

Mathematica [A]
time = 0.13, size = 147, normalized size = 0.57

$$-\frac{d^3\left(3465ac^5x^5(-231+495c^2x^2-385c^4x^4+105c^6x^6)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(50488+25244c^2x^2+18933c^4x^4-117625c^6x^6+111475c^8x^8-33075c^{10}x^{10})+3465bc^5x^5(-231+495c^2x^2-385c^4x^4+105c^6x^6)\cosh^{-1}(cx)\right)}{4002075c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/4002075*(d^3*(3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6
*x^6) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(50488 + 25244*c^2*x^2 + 18933*c^4*x
^4 - 117625*c^6*x^6 + 111475*c^8*x^8 - 33075*c^10*x^10) + 3465*b*c^5*x^5*(-
231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*ArcCosh[c*x]))/c^5
```

Maple [A]
time = 3.54, size = 158, normalized size = 0.62

method	result
derivativedivides	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\mathrm{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\mathrm{arccosh}(cx)c^9x^9}{3}+\frac{3\,\mathrm{arccosh}(cx)c^7x^7}{7}-\frac{\mathrm{arccosh}(cx)c^5x^5}{5}-\right.}{c^5}$
default	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\mathrm{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\mathrm{arccosh}(cx)c^9x^9}{3}+\frac{3\,\mathrm{arccosh}(cx)c^7x^7}{7}-\frac{\mathrm{arccosh}(cx)c^5x^5}{5}-\right.}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-d^3*a*(1/11*c^11*x^11-1/3*c^9*x^9+3/7*c^7*x^7-1/5*c^5*x^5)-d^3*b*(1
/11*arccosh(c*x)*c^11*x^11-1/3*arccosh(c*x)*c^9*x^9+3/7*arccosh(c*x)*c^7*x^
7-1/5*arccosh(c*x)*c^5*x^5-1/4002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^1
0*x^10-111475*c^8*x^8+117625*c^6*x^6-18933*c^4*x^4-25244*c^2*x^2-50488)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(212) = 424$.
time = 0.27, size = 465, normalized size = 1.82

$$-\frac{1}{11}\,ac^{6}d^{3}x^{11}+\frac{1}{3}\,ac^{4}d^{3}x^{9}-\frac{3}{7}\,ac^{2}d^{3}x^{7}-\frac{1}{7623}\left(693\,x^{11}\operatorname{arcosh}\left(cx\right)-\left(\frac{63\,\sqrt{c^{2}x^{2}-1}\,x^{10}}{c^{2}}+\frac{70\,\sqrt{c^{2}x^{2}-1}\,x^{8}}{c^{4}}+\frac{80\,\sqrt{c^{2}x^{2}-1}\,x^{6}}{c^{6}}+\frac{96\,\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{8}}+\frac{128\,\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{10}}+\frac{256\,\sqrt{c^{2}x^{2}-1}}{c^{12}}\right)c\right)bc^{6}d^{3}+\frac{1}{945}\left(315\,x^{9}\operatorname{arcosh}\left(cx\right)-\left(\frac{35\,\sqrt{c^{2}x^{2}-1}\,x^{8}}{c^{2}}+\frac{40\,\sqrt{c^{2}x^{2}-1}\,x^{6}}{c^{4}}+\frac{48\,\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{6}}+\frac{64\,\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{8}}+\frac{128\,\sqrt{c^{2}x^{2}-1}}{c^{10}}\right)c\right)bc^{4}d^{3}+\frac{1}{5}\,ad^{3}x^{5}-\frac{3}{245}\left(35\,x^{7}\operatorname{arcosh}\left(cx\right)-\left(\frac{5\,\sqrt{c^{2}x^{2}-1}\,x^{6}}{c^{2}}+\frac{6\,\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{6}}+\frac{16\,\sqrt{c^{2}x^{2}-1}}{c^{8}}\right)c\right)bc^{2}d^{3}+\frac{1}{75}\left(15\,x^{5}\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{2}}+\frac{4\,\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}-1}}{c^{6}}\right)c\right)bd^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693*
x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 - 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x
^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6 + 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*
sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(c^2*x^2 - 1)/c^12)*c)*b*c^6*d^3 + 1/9
45*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2
- 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8
+ 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 - 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^3 + 1/7
5*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)
*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3
```

Fricas [**A**]
time = 0.35, size = 201, normalized size = 0.79

$$-\frac{363825\,ac^{11}d^{3}x^{11}-1334025\,ac^{9}d^{3}x^{9}+1715175\,ac^{7}d^{3}x^{7}-800415\,ac^{5}d^{3}x^{5}+3465\,\left(105\,bc^{11}d^{3}x^{11}-385\,bc^{9}d^{3}x^{9}+495\,bc^{7}d^{3}x^{7}-231\,bc^{5}d^{3}x^{5}\right)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(33075\,bc^{10}d^{3}x^{10}-111475\,bc^{8}d^{3}x^{8}+117625\,bc^{6}d^{3}x^{6}-18933\,bc^{4}d^{3}x^{4}-25244\,bc^{2}d^{3}x^{2}-50488\,bd^{3}\right)\sqrt{c^{2}x^{2}-1}}{4002075\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*
d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^3*
x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) -
 (33075*b*c^10*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 - 189
33*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*sqrt(c^2*x^2 - 1))/c^
5
```

Sympy [C] Result contains complex when optimal does not.
time = 2.91, size = 296, normalized size = 1.16

$$\begin{cases} -\frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} - \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} - \frac{bc^6d^3x^{11}\operatorname{acosh}(cx)}{11} + \frac{bc^5d^3x^{10}\sqrt{c^2x^2-1}}{121} + \frac{bc^4d^3x^9\operatorname{acosh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2-1}}{3267} - \frac{3bc^2d^3x^7\operatorname{acosh}(cx)}{7} + \frac{4705bcd^3x^6\sqrt{c^2x^2-1}}{160083} + \frac{bd^3x^5\operatorname{acosh}(cx)}{5} - \frac{6311bd^3x^4\sqrt{c^2x^2-1}}{1334025c} - \frac{25244bd^3x^2\sqrt{c^2x^2-1}}{4002075c^3} - \frac{50488bd^3\sqrt{c^2x^2-1}}{4002075c^5} & \text{for } c \neq 0 \\ \frac{d^3x^5\left(a+\frac{i\pi b}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 - 3*a*c**2*d**3*x**7/
7 + a*d**3*x**5/5 - b*c**6*d**3*x**11*acosh(c*x)/11 + b*c**5*d**3*x**10*sqr
t(c**2*x**2 - 1)/121 + b*c**4*d**3*x**9*acosh(c*x)/3 - 91*b*c**3*d**3*x**8*
sqrt(c**2*x**2 - 1)/3267 - 3*b*c**2*d**3*x**7*acosh(c*x)/7 + 4705*b*c*d**3*
x**6*sqrt(c**2*x**2 - 1)/160083 + b*d**3*x**5*acosh(c*x)/5 - 6311*b*d**3*x*
*4*sqrt(c**2*x**2 - 1)/(1334025*c) - 25244*b*d**3*x**2*sqrt(c**2*x**2 - 1)/
(4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 - 1)/(4002075*c**5), Ne(c, 0))
, (d**3*x**5*(a + I*pi*b/2)/5, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

3.20 $\int x^3(d-c^2dx^2)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=230

$$-\frac{49bd^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{5120c^3}+\frac{49bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{7680c^3}-\frac{49bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{9600c^3}+\frac{7bd^3x(-1$$

```
[Out] 49/7680*b*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-49/9600*b*d^3*x*(c*x-1)^(5/
2)*(c*x+1)^(5/2)/c^3+7/1600*b*d^3*x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/100*b
*d^3*x*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^3+49/5120*b*d^3*arccosh(c*x)/c^4-1/8*d
^3*(c*x-1)^4*(c*x+1)^4*(a+b*arccosh(c*x))/c^4-1/10*d^3*(c*x-1)^5*(c*x+1)^5*
(a+b*arccosh(c*x))/c^4-49/5120*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 328, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {272, 45, 5921, 12, 580, 21, 396, 201, 223, 212}

$$\frac{d^3(1-c^2x^2)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{49bd^3\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{5120c^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd^3x(1-c^2x^2)^5}{100c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{7bd^3x(1-c^2x^2)^4}{1600c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{49bd^3x(1-c^2x^2)^3}{9600c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{49bd^3x(1-c^2x^2)^2}{7680c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{49bd^3x(1-c^2x^2)}{5120c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (49*b*d^3*x*(1 - c^2*x^2))/(5120*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*
d^3*x*(1 - c^2*x^2)^2)/(7680*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*d^3*
x*(1 - c^2*x^2)^3)/(9600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*d^3*x*(1
- c^2*x^2)^4)/(1600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*x*(1 - c^2*x
^2)^5)/(100*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(1 - c^2*x^2)^4*(a + b
*ArcCosh[c*x]))/(8*c^4) + (d^3*(1 - c^2*x^2)^5*(a + b*ArcCosh[c*x]))/(10*c^
4) + (49*b*d^3*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*
c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 580

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Dist[(e1 + f1*x^(n/2))^FracPart[r]*((e2 + f2*x^(n/2))^FracPart[r]/(e1*e2 +
f1*f2*x^n)^FracPart[r]), Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1-c^2x^2)^5\left(a+b\cosh^{-1}\right.}{10c^4}$$

$$=-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1-c^2x^2)^5\left(a+b\cosh^{-1}\right.}{10c^4}$$

$$=-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1-c^2x^2)^5\left(a+b\cosh^{-1}\right.}{10c^4}$$

$$=-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1-c^2x^2)^5\left(a+b\cosh^{-1}\right.}{10c^4}$$

$$=-\frac{bd^3x(1-c^2x^2)^5}{100c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(1-c^2x^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}$$

$$=\frac{7bd^3x(1-c^2x^2)^4}{1600c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bd^3x(1-c^2x^2)^5}{100c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(}{}$$

$$=\frac{49bd^3x(1-c^2x^2)^3}{9600c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{7bd^3x(1-c^2x^2)^4}{1600c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{}{10}$$

$$=\frac{49bd^3x(1-c^2x^2)^2}{7680c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{49bd^3x(1-c^2x^2)^3}{9600c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{16}$$

$$=\frac{49bd^3x(1-c^2x^2)}{5120c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{49bd^3x(1-c^2x^2)^2}{7680c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{96}$$

$$=\frac{49bd^3x(1-c^2x^2)}{5120c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{49bd^3x(1-c^2x^2)^2}{7680c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{96}$$

$$=\frac{49bd^3x(1-c^2x^2)}{5120c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{49bd^3x(1-c^2x^2)^2}{7680c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{96}$$

Mathematica [A]
time = 0.16, size = 168, normalized size = 0.73

$$-\frac{d^3\left(1920ac^4x^4(-10+20c^2x^2-15c^4x^4+4c^6x^6)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1185+790c^2x^2-3208c^4x^4+2736c^6x^6-768c^8x^8)+1920bc^4x^4(-10+20c^2x^2-15c^4x^4+4c^6x^6)\cosh^{-1}(cx)+1185b\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)\right)}{76800c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/76800*(d^3*(1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) +
 b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 + 27
36*c^6*x^6 - 768*c^8*x^8) + 1920*b*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 +
 4*c^6*x^6)*ArcCosh[c*x] + 1185*b*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))
/c^4
```

Maple [A]
time = 2.70, size = 286, normalized size = 1.24

method	result
derivativedivides	$-d^3a\left(\frac{\left(c^2x^2-1\right)^5}{10}+\frac{\left(c^2x^2-1\right)^4}{8}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^{10}x^{10}}{10}+\frac{3d^3b\,\mathrm{arccosh}(cx)c^8x^8}{8}-\frac{d^3b\,\mathrm{arccosh}(cx)c^6x^6}{2}+\frac{d^3b\,\mathrm{arccosh}(cx)c^4x^4}{4}-$
default	$-d^3a\left(\frac{\left(c^2x^2-1\right)^5}{10}+\frac{\left(c^2x^2-1\right)^4}{8}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^{10}x^{10}}{10}+\frac{3d^3b\,\mathrm{arccosh}(cx)c^8x^8}{8}-\frac{d^3b\,\mathrm{arccosh}(cx)c^6x^6}{2}+\frac{d^3b\,\mathrm{arccosh}(cx)c^4x^4}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-d^3*a*(1/10*(c^2*x^2-1)^5+1/8*(c^2*x^2-1)^4)-1/10*d^3*b*arccosh(c*x
)*c^10*x^10+3/8*d^3*b*arccosh(c*x)*c^8*x^8-1/2*d^3*b*arccosh(c*x)*c^6*x^6+1
/4*d^3*b*arccosh(c*x)*c^4*x^4-1/40*b*d^3*arccosh(c*x)+1/100*d^3*b*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c^9*x^9-57/1600*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^7*x^
7+401/9600*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5-79/7680*d^3*b*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*c^3*x^3-79/5120*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+4
9/5120*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-
1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(194) = 388.
time = 0.27, size = 501, normalized size = 2.18

$$-\frac{1}{10}ac^6d^3x^{10}+\frac{3}{8}ac^4d^3x^8-\frac{1}{2}ac^2d^3x^6-\frac{1}{12800}\left(1280x^{10}\,\mathrm{arcosh}\left(cx\right)-\left(\frac{128\sqrt{c^2x^2-1}\,x^9}{c^2}+\frac{144\sqrt{c^2x^2-1}\,x^7}{c^4}+\frac{168\sqrt{c^2x^2-1}\,x^5}{c^6}+\frac{210\sqrt{c^2x^2-1}\,x^3}{c^8}+\frac{315\sqrt{c^2x^2-1}\,x}{c^{10}}+\frac{315\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^{11}}\right)c\right)bc^6d^3+\frac{1}{1024}\left(384x^8\,\mathrm{arcosh}\left(cx\right)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)bc^4d^3+\frac{1}{4}ad^3x^4-\frac{1}{96}\left(48x^6\,\mathrm{arcosh}\left(cx\right)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bc^2d^3+\frac{1}{32}\left(8x^4\,\mathrm{arcosh}\left(cx\right)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(128
0*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sqrt(c^2*x^2 - 1
)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2 - 1)*x^3/c^8 +
 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^
11)*c)*b*c^6*d^3 + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7
/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sq
rt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*
c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*
x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*lo
g(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arccosh(
c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c
^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3
```

Fricas [A]
time = 0.35, size = 197, normalized size = 0.86

$$-\frac{7680\,ac^{10}d^3x^{10}-28800\,ac^8d^3x^8+38400\,ac^6d^3x^6-19200\,ac^4d^3x^4+15\,(512\,bc^{10}d^3x^{10}-1920\,bc^8d^3x^8+2560\,bc^6d^3x^6-1280\,bc^4d^3x^4+79\,bd^3)\log\left(cx+\sqrt{c^2x^2-1}\right)-(768\,bc^9d^3x^9-2736\,bc^7d^3x^7+3208\,bc^5d^3x^5-790\,bc^3d^3x^3-1185\,bcd^3x)\sqrt{c^2x^2-1}}{76800\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
- 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 2560
*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)
) - (768*b*c^9*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*b*c^
3*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 287, normalized size = 1.25

$$\begin{cases} -\frac{ac^6d^3x^{10}}{10}+\frac{3ac^4d^3x^8}{8}-\frac{ac^2d^3x^6}{2}+\frac{ad^3x^4}{4}-\frac{bc^6d^3x^{10}\operatorname{acosh}(cx)}{10}+\frac{bc^5d^3x^9\sqrt{c^2x^2-1}}{100}+\frac{3bc^4d^3x^8\operatorname{acosh}(cx)}{8}-\frac{57bc^3d^3x^7\sqrt{c^2x^2-1}}{1600}-\frac{bc^2d^3x^6\operatorname{acosh}(cx)}{2}+\frac{401bcd^3x^5\sqrt{c^2x^2-1}}{9600}+\frac{bd^3x^4\operatorname{acosh}(cx)}{4}-\frac{79bd^3x^3\sqrt{c^2x^2-1}}{7680c}-\frac{79bd^3x\sqrt{c^2x^2-1}}{5120c^3}-\frac{79bd^3\operatorname{acosh}(cx)}{5120c^4} & \text{for}\ c\neq 0\\ \frac{d^3x^4\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 - a*c**2*d**3*x**6/
2 + a*d**3*x**4/4 - b*c**6*d**3*x**10*acosh(c*x)/10 + b*c**5*d**3*x**9*sqrt
(c**2*x**2 - 1)/100 + 3*b*c**4*d**3*x**8*acosh(c*x)/8 - 57*b*c**3*d**3*x**7
*sqrt(c**2*x**2 - 1)/1600 - b*c**2*d**3*x**6*acosh(c*x)/2 + 401*b*c*d**3*x*
*5*sqrt(c**2*x**2 - 1)/9600 + b*d**3*x**4*acosh(c*x)/4 - 79*b*d**3*x**3*sqr
t(c**2*x**2 - 1)/(7680*c) - 79*b*d**3*x*sqrt(c**2*x**2 - 1)/(5120*c**3) - 7
9*b*d**3*acosh(c*x)/(5120*c**4), Ne(c, 0)), (d**3*x**4*(a + I*pi*b/2)/4, Tr
ue))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

3.21 $\int x^2(d-c^2dx^2)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=227

$$-\frac{16bd^3\sqrt{-1+cx}\,\sqrt{1+cx}}{315c^3}+\frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{945c^3}-\frac{2bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{525c^3}+\frac{bd^3(-1+cx)^{7/2}}{441c^3}$$

```
[Out] 8/945*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-2/525*b*d^3*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^3+1/441*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/81*b*d^3*(c*x-1)
^(9/2)*(c*x+1)^(9/2)/c^3+1/3*d^3*x^3*(a+b*arccosh(c*x))-3/5*c^2*d^3*x^5*(a+
b*arccosh(c*x))+3/7*c^4*d^3*x^7*(a+b*arccosh(c*x))-1/9*c^6*d^3*x^9*(a+b*arc
cosh(c*x))-16/315*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.27, antiderivative size = 285, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {276, 5921, 12, 1624, 1813, 1634}

$$-\frac{1}{9}c^6d^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^3(1-c^2x^2)^5}{81c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^3(1-c^2x^2)^4}{441c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bd^3(1-c^2x^2)^3}{525c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^3(1-c^2x^2)^2}{945c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bd^3(1-c^2x^2)}{315c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*
(1 - c^2*x^2)^2)/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c^2
*x^2)^3)/(525*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(
441*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*(1 - c^2*x^2)^5)/(81*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (3*c^2*d^3*
x^5*(a + b*ArcCosh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCosh[c*x]))/7 - (c^6
*d^3*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
```

```
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7$$

$$=\frac{16bd^3(1-c^2x^2)}{315c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{945c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2}{525c^3}$$

Mathematica [A]
time = 0.15, size = 139, normalized size = 0.61

$$-\frac{d^3\left(315ac^3x^3(-105+189c^2x^2-135c^4x^4+35c^6x^6)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(5258+2629c^2x^2-6297c^4x^4+4675c^6x^6-1225c^8x^8)+315bc^3x^3(-105+189c^2x^2-135c^4x^4+35c^6x^6)\cosh^{-1}(cx)\right)}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/99225*(d^3*(315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6
) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 + 46
75*c^6*x^6 - 1225*c^8*x^8) + 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^
4 + 35*c^6*x^6)*ArcCosh[c*x]))/c^3
```

Maple [A]
time = 2.69, size = 150, normalized size = 0.66

method	result
derivativedivides	$\frac{-d^3a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\right.}{c^3}$
default	$\frac{-d^3a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{\operatorname{arccosh}(cx)c^3x^3}{3}-\right.}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-d^3*a*(1/9*c^9*x^9-3/7*c^7*x^7+3/5*c^5*x^5-1/3*c^3*x^3)-d^3*b*(1/9*
arccosh(c*x)*c^9*x^9-3/7*arccosh(c*x)*c^7*x^7+3/5*arccosh(c*x)*c^5*x^5-1/3*
arccosh(c*x)*c^3*x^3-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-4675
*c^6*x^6+6297*c^4*x^4-2629*c^2*x^2-5258)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(189) = 378.
time = 0.27, size = 388, normalized size = 1.71

$$-\frac{1}{9}ac^6d^3x^9+\frac{3}{7}ac^4d^3x^7-\frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)bc^6d^3-\frac{3}{5}ac^2d^3x^5+\frac{3}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^4d^3-\frac{1}{25}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d^3+\frac{1}{3}ad^3x^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arccosh(c*x) - (35
*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*
c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2
```

```
*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c
^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arccosh(c*x) - (
3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
- 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^3
```

Fricas [A]
time = 0.35, size = 189, normalized size = 0.83

$$-\frac{11025\,ac^{9}d^{3}x^{9} - 42525\,ac^{7}d^{3}x^{7} + 59535\,ac^{5}d^{3}x^{5} - 33075\,ac^{3}d^{3}x^{3} + 315\,(35\,bc^{9}d^{3}x^{9} - 135\,bc^{7}d^{3}x^{7} + 189\,bc^{5}d^{3}x^{5} - 105\,bc^{3}d^{3}x^{3})\log\left(cx + \sqrt{c^{2}x^{2} - 1}\right) - (1225\,bc^{8}d^{3}x^{8} - 4675\,bc^{6}d^{3}x^{6} + 6297\,bc^{4}d^{3}x^{4} - 2629\,bc^{2}d^{3}x^{2} - 5258\,bd^{3})\sqrt{c^{2}x^{2} - 1}}{99225\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 -
 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*c^
5*d^3*x^5 - 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^8*d
^3*x^8 - 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 - 525
8*b*d^3)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 1.47, size = 272, normalized size = 1.20

$$\begin{cases} -\frac{ac^{6}d^{3}x^{9}}{9} + \frac{3ac^{4}d^{3}x^{7}}{7} - \frac{3ac^{2}d^{3}x^{5}}{5} + \frac{ad^{3}x^{3}}{3} - \frac{bc^{6}d^{3}x^{9}\operatorname{acosh}{(cx)}}{9} + \frac{bc^{5}d^{3}x^{8}\sqrt{c^{2}x^{2} - 1}}{81} + \frac{3bc^{4}d^{3}x^{7}\operatorname{acosh}{(cx)}}{7} - \frac{187bc^{3}d^{3}x^{6}\sqrt{c^{2}x^{2} - 1}}{3969} - \frac{3bc^{2}d^{3}x^{5}\operatorname{acosh}{(cx)}}{5} + \frac{2099bcd^{3}x^{4}\sqrt{c^{2}x^{2} - 1}}{33075} + \frac{bd^{3}x^{3}\operatorname{acosh}{(cx)}}{3} - \frac{2629bd^{3}x^{2}\sqrt{c^{2}x^{2} - 1}}{99225c} - \frac{5258bd^{3}\sqrt{c^{2}x^{2} - 1}}{99225c^{3}} & \text{for}\: c \neq 0 \\ \frac{d^{3}x^{3}\left(a + \frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 - 3*a*c**2*d**3*x**5/
5 + a*d**3*x**3/3 - b*c**6*d**3*x**9*acosh(c*x)/9 + b*c**5*d**3*x**8*sqrt(c
**2*x**2 - 1)/81 + 3*b*c**4*d**3*x**7*acosh(c*x)/7 - 187*b*c**3*d**3*x**6*s
qrt(c**2*x**2 - 1)/3969 - 3*b*c**2*d**3*x**5*acosh(c*x)/5 + 2099*b*c*d**3*x
**4*sqrt(c**2*x**2 - 1)/33075 + b*d**3*x**3*acosh(c*x)/3 - 2629*b*d**3*x**2
*sqrt(c**2*x**2 - 1)/(99225*c) - 5258*b*d**3*sqrt(c**2*x**2 - 1)/(99225*c**
3), Ne(c, 0)), (d**3*x**3*(a + I*pi*b/2)/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

3.22 $\int x(d - c^2dx^2)^3 \left(a + b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=166

$$-\frac{35bd^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-1+c}{}$$

```
[Out] 35/1536*b*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-7/384*b*d^3*x*(c*x-1)^(5/2)*(
c*x+1)^(5/2)/c+1/64*b*d^3*x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c+35/1024*b*d^3*arc
cosh(c*x)/c^2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arccosh(c*x))/c^2-35/1024*b*d^3*x
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5914, 38, 54}

$$-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{35bd^3\cosh^{-1}(cx)}{1024c^2}+\frac{bd^3x(cx-1)^{7/2}(cx+1)^{7/2}}{64c}-\frac{7bd^3x(cx-1)^{5/2}(cx+1)^{5/2}}{384c}+\frac{35bd^3x(cx-1)^{3/2}(cx+1)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{cx-1}\,\sqrt{cx+1}}{1024c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1024*c) + (35*b*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/(1536*c) - (7*b*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(
5/2))/(384*c) + (b*d^3*x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(64*c) + (35*b*d
^3*ArcCosh[c*x])/(1024*c^2) - (d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/(8
*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
```

```
c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx &= -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{\left(bd^3\right)\int(-1+cx)^{7/2}(1+ca\\
&= \frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}\\
&= -\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\\
&= \frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}\\
&= -\frac{35bd^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}\\
&= -\frac{35bd^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 156, normalized size = 0.94

$$-\frac{d^3\left(cx\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\,(279-326c^2x^2+200c^4x^4-48c^6x^6)+384acx(-4+6c^2x^2-4c^4x^4+c^6x^6)\right)+384bc^2x^2(-4+6c^2x^2-4c^4x^4+c^6x^6)\cosh^{-1}(cx)+279b\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)\right)}{3072c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/3072*(d^3*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(279 - 326*c^2*x^2 + 200*
c^4*x^4 - 48*c^6*x^6) + 384*a*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)) +
 384*b*c^2*x^2*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)*ArcCosh[c*x] + 279*b*
Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/c^2
```

Maple [A]

time = 2.77, size = 244, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{d^3\left(c^2x^2-1\right)^4a}{8}-\frac{d^3b\,\mathrm{arccosh}(cx)c^8x^8}{8}+\frac{d^3b\,\mathrm{arccosh}(cx)c^6x^6}{2}-\frac{3d^3b\,\mathrm{arccosh}(cx)c^4x^4}{4}+\frac{d^3b\,\mathrm{arccosh}(cx)c^2x^2}{2}-\frac{b\,d^3\mathrm{arccosh}(cx)}{8}+}{}$

default	$\frac{-\frac{d^3\left(c^2x^2-1\right)^4 a}{8}-\frac{d^3 b \operatorname{arccosh}(cx) c^8 x^8}{8}+\frac{d^3 b \operatorname{arccosh}(cx) c^6 x^6}{2}-\frac{3d^3 b \operatorname{arccosh}(cx) c^4 x^4}{4}+\frac{d^3 b \operatorname{arccosh}(cx) c^2 x^2}{2}-\frac{b\, d^3 \operatorname{arccosh}(cx)}{8}+\frac{d^3 \cdots}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/8*d^3*(c^2*x^2-1)^4*a-1/8*d^3*b*arccosh(c*x)*c^8*x^8+1/2*d^3*b*ar
ccosh(c*x)*c^6*x^6-3/4*d^3*b*arccosh(c*x)*c^4*x^4+1/2*d^3*b*arccosh(c*x)*c^
2*x^2-1/8*b*d^3*arccosh(c*x)+1/64*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^7*x^7
-25/384*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5+163/1536*d^3*b*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*c^3*x^3-93/1024*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+35/
1024*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)
^(1/2)))
```

<u>Maxima</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(137) = 274$.
time = 0.29, size = 423, normalized size = 2.55

$$-\frac{1}{8}ac^6d^3x^8+\frac{1}{2}ac^4d^3x^6-\frac{1}{3072}\left(384x^8\operatorname{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)bc^6d^3-\frac{3}{4}ac^2d^3x^4+\frac{1}{96}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bc^4d^3-\frac{3}{32}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bc^2d^3+\frac{1}{2}ad^3x^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arccosh(c*x) - (48
*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*c)/c^9)*c)*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arccosh(c*
x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(
c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^4*
d^3 - 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*
x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c^2*d^3 +
 1/2*a*d^3*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^3
```

<u>Fricas</u> [A]
time = 0.34, size = 185, normalized size = 1.11

$$-\frac{384\,ac^8d^3x^8-1536\,ac^6d^3x^6+2304\,ac^4d^3x^4-1536\,ac^2d^3x^2+3\left(128\,bc^8d^3x^8-512\,bc^6d^3x^6+768\,bc^4d^3x^4-512\,bc^2d^3x^2+93\,bd^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(48\,bc^7d^3x^7-200\,bc^5d^3x^5+326\,bc^3d^3x^3-279\,bcd^3x\right)\sqrt{c^2x^2-1}}{3072\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 1536
*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x
```

```
^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7
*d^3*x^7 - 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(c^2*
x^2 - 1))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.12, size = 260, normalized size = 1.57

$$\begin{cases} -\frac{ac^6d^3x^8}{8} + \frac{ac^4d^3x^6}{2} - \frac{3ac^2d^3x^4}{4} + \frac{ad^3x^2}{2} - \frac{bc^6d^3x^8\operatorname{acosh}(cx)}{8} + \frac{bc^5d^3x^7\sqrt{c^2x^2-1}}{64} + \frac{bc^4d^3x^6\operatorname{acosh}(cx)}{2} - \frac{25bc^3d^3x^5\sqrt{c^2x^2-1}}{384} - \frac{3bc^2d^3x^4\operatorname{acosh}(cx)}{4} + \frac{163bcd^3x^3\sqrt{c^2x^2-1}}{1536} + \frac{bd^3x^2\operatorname{acosh}(cx)}{2} - \frac{93bd^3x\sqrt{c^2x^2-1}}{1024c} - \frac{93bd^3\operatorname{acosh}(cx)}{1024c^2} & \text{for}\ c \neq 0 \\ \frac{d^3x^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 - 3*a*c**2*d**3*x**4/4
+ a*d**3*x**2/2 - b*c**6*d**3*x**8*acosh(c*x)/8 + b*c**5*d**3*x**7*sqrt(c**
2*x**2 - 1)/64 + b*c**4*d**3*x**6*acosh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c
**2*x**2 - 1)/384 - 3*b*c**2*d**3*x**4*acosh(c*x)/4 + 163*b*c*d**3*x**3*sqr
t(c**2*x**2 - 1)/1536 + b*d**3*x**2*acosh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x*
*2 - 1)/(1024*c) - 93*b*d**3*acosh(c*x)/(1024*c**2), Ne(c, 0)), (d**3*x**2*
(a + I*pi*b/2)/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

3.23 $\int \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=191

$$-\frac{16bd^3\sqrt{-1+cx}\,\sqrt{1+cx}}{35c}+\frac{8bd^3(-1+cx)^{3/2}(1+cx)^{3/2}}{105c}-\frac{6bd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{175c}+\frac{bd^3(-1+cx)^{7/2}(}{49c}$$

```
[Out] 8/105*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-6/175*b*d^3*(c*x-1)^(5/2)*(c*x+1)
^(5/2)/c+1/49*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c+d^3*x*(a+b*arccosh(c*x))-
c^2*d^3*x^3*(a+b*arccosh(c*x))+3/5*c^4*d^3*x^5*(a+b*arccosh(c*x))-1/7*c^6*d
^3*x^7*(a+b*arccosh(c*x))-16/35*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]

time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {200, 5894, 12, 1624, 1813, 1864}

$$-\frac{1}{7}c^6d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^3(1-c^2x^2)^4}{49c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{6bd^3(1-c^2x^2)^3}{175c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^3(1-c^2x^2)^2}{105c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bd^3(1-c^2x^2)}{35c\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(35*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*(1
- c^2*x^2)^2)/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*d^3*(1 - c^2*x^2)
^3)/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(49*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[c*x]) - c^2*d^3*x^3*(a +
 b*ArcCosh[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (c^6*d^3*x^7*(a
 + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx = d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+\right.$$

$$= \frac{16bd^3(1-c^2x^2)}{35c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bd^3(1-c^2x^2)^2}{105c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6bd^3(}{175c\sqrt{-1}}$$

Mathematica [A]
time = 0.13, size = 123, normalized size = 0.64

$$-\frac{d^3\left(b\sqrt{-1+cx}\sqrt{1+cx}\,(2161-757c^2x^2+351c^4x^4-75c^6x^6)+105acx(-35+35c^2x^2-21c^4x^4+5c^6x^6)+105bcx(-35+35c^2x^2-21c^4x^4+5c^6x^6)\cosh^{-1}(cx)\right)}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/3675*(d^3*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2161 - 757*c^2*x^2 + 351*c^4*
x^4 - 75*c^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) +
 105*b*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCosh[c*x]))/c
```

Maple [A]
time = 1.99, size = 132, normalized size = 0.69

method	result
derivativedivides	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+\operatorname{arccosh}(cx)c^3x^3-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-}}{}\right.}{c}$
default	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+\operatorname{arccosh}(cx)c^3x^3-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-}}{}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-d^3*a*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b*(1/7*arccosh(c*x)*c
^7*x^7-3/5*arccosh(c*x)*c^5*x^5+arccosh(c*x)*c^3*x^3-c*x*arccosh(c*x)-1/367
5*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-351*c^4*x^4+757*c^2*x^2-2161)))
```

Maxima [A]
time = 0.28, size = 302, normalized size = 1.58

$$-\frac{1}{7}ac^6d^3x^7+\frac{3}{5}ac^4d^3x^5-\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bc^6d^3+\frac{1}{25}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^4d^3-ac^2d^3x^3-\frac{1}{3}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d^3+ad^3x+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sq
rt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)
*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arccosh(c*
x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 - 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccosh(c*x) - c*
(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + a*d^3*x
+ (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Fricas [A]
time = 0.35, size = 169, normalized size = 0.88

$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\,(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3-35\,bcd^3x)\log\left(cx+\sqrt{c^2x^2-1}\right)-(75\,bc^6d^3x^6-351\,bc^4d^3x^4+757\,bc^2d^3x^2-2161\,bd^3)\sqrt{c^2x^2-1}}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 3675
*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 - 3
5*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d^3*x^6 - 351*b*c^4*d
^3*x^4 + 757*b*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(c^2*x^2 - 1))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 228, normalized size = 1.19

$$\begin{cases} -\frac{ac^6d^3x^7}{7} + \frac{3ac^4d^3x^5}{5} - ac^2d^3x^3 + ad^3x - \frac{bc^6d^3x^7\operatorname{acosh}(cx)}{7} + \frac{bc^5d^3x^6\sqrt{c^2x^2-1}}{49} + \frac{3bc^4d^3x^5\operatorname{acosh}(cx)}{5} - \frac{117bc^3d^3x^4\sqrt{c^2x^2-1}}{1225} - bc^2d^3x^3\operatorname{acosh}(cx) + \frac{757bcd^3x^2\sqrt{c^2x^2-1}}{3675} + bd^3x\operatorname{acosh}(cx) - \frac{2161bd^3\sqrt{c^2x^2-1}}{3675c} & \text{for}\ c \neq 0 \\ d^3x\left(a + \frac{i\pi b}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 +
a*d**3*x - b*c**6*d**3*x**7*acosh(c*x)/7 + b*c**5*d**3*x**6*sqrt(c**2*x**2
- 1)/49 + 3*b*c**4*d**3*x**5*acosh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*
x**2 - 1)/1225 - b*c**2*d**3*x**3*acosh(c*x) + 757*b*c*d**3*x**2*sqrt(c**2*
x**2 - 1)/3675 + b*d**3*x*acosh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 - 1)/(367
5*c), Ne(c, 0)), (d**3*x*(a + I*pi*b/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

3.24 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=239

$$\frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}-\frac{19}{48}bd^3\cosh^{-1}($$

```
[Out] -7/72*b*c*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/36*b*c*d^3*x*(c*x-1)^(5/2)*(c
*x+1)^(5/2)-19/48*b*d^3*arccosh(c*x)+1/2*d^3*(-c^2*x^2+1)*(a+b*arccosh(c*x)
)+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))+1/6*d^3*(-c^2*x^2+1)^3*(a+b*arc
cosh(c*x))+1/2*d^3*(a+b*arccosh(c*x))^2/b+d^3*(a+b*arccosh(c*x))*ln(1+1/(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)+19/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5919, 5882, 3799, 2221, 2317, 2438, 38, 54}

$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^3\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d^3\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{2}bd^3\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+\frac{1}{36}bcd^3x(cx-1)^{5/2}(cx+1)^{5/2}-\frac{7}{72}bcd^3x(cx-1)^{3/2}(cx+1)^{3/2}+\frac{19}{48}bcd^3x\sqrt{cx-1}\,\sqrt{cx+1}-\frac{19}{48}bd^3\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (19*b*c*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/48 - (7*b*c*d^3*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/72 + (b*c*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/36 -
 (19*b*d^3*ArcCosh[c*x])/48 + (d^3*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 +
(d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^3*(1 - c^2*x^2)^3*(a + b*
ArcCosh[c*x]))/6 + (d^3*(a + b*ArcCosh[c*x])^2)/(2*b) + d^3*(a + b*ArcCosh[
c*x])*Log[1 + E^(-2*ArcCosh[c*x])] - (b*d^3*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d-c^2dx^2)^3\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx &= \frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{(d-c^2dx^2)^2\left(a+b\cosh^{-1}\right.}{x} \\
&= \frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx\right. \\
&= -\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/} \\
&= \frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+ \\
&= \frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+ \\
&= \frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+ \\
&= \frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+ \\
&= \frac{19}{48}bcd^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 305, normalized size = 1.28

$$-\frac{1}{144}d^3\left(216ac^2x^2-108ac^4x^4+24ac^6x^6+33bcx\sqrt{\frac{-1+cx}{1+cx}}+33bc^2x^2\sqrt{\frac{-1+cx}{1+cx}}+22bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}+22bc^4x^4\sqrt{\frac{-1+cx}{1+cx}}-4bc^5x^5\sqrt{\frac{-1+cx}{1+cx}}-4bc^6x^6\sqrt{\frac{-1+cx}{1+cx}}-108bcx\sqrt{-1+cx}\,\sqrt{1+cx}-72b\cosh^{-1}(cx)^2-150b\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)+12b\cosh^{-1}(cx)\left(18c^2x^2-9c^4x^4+2c^6x^6-12\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-144a\log(x)+72b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/144*(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 + 33*b*c*x*Sqrt[(
-1 + c*x)/(1 + c*x)] + 33*b*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 22*b*c^3*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 22*b*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 4
*b*c^5*x^5*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*
x)] - 108*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 72*b*ArcCosh[c*x]^2 - 150*b*
ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 12*b*ArcCosh[c*x]*(18*c^2*x^2 - 9*c^4
*x^4 + 2*c^6*x^6 - 12*Log[1 + E^(-2*ArcCosh[c*x])]) - 144*a*Log[x] + 72*b*P
olyLog[2, -E^(-2*ArcCosh[c*x])]))
```

Maple [A]
time = 6.14, size = 255, normalized size = 1.07

method	result

derivativedivides	$-\frac{a\,d^3c^6x^6}{6}+\frac{3a\,d^3c^4x^4}{4}-\frac{3a\,d^3c^2x^2}{2}+a\,d^3\ln\left(cx\right)+\frac{d^3b\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}$
default	$-\frac{a\,d^3c^6x^6}{6}+\frac{3a\,d^3c^4x^4}{4}-\frac{3a\,d^3c^2x^2}{2}+a\,d^3\ln\left(cx\right)+\frac{d^3b\,\mathrm{polylog}\left(2,-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*d^3*c^6*x^6+3/4*a*d^3*c^4*x^4-3/2*a*d^3*c^2*x^2+a*d^3*ln(c*x)+1/2*d^
3*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+25/48*b*d^3*arccosh(c*x
)-1/2*d^3*b*arccosh(c*x)^2+1/36*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5-1
1/72*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*x^3+25/48*b*c*d^3*x*(c*x-1)^(1/2
)*(c*x+1)^(1/2)-1/6*d^3*b*arccosh(c*x)*c^6*x^6+3/4*d^3*b*arccosh(c*x)*c^4*x
^4+d^3*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/2*d^3*b*a
rccosh(c*x)*c^2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) -
 integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) - b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-\frac{a}{x}\right)dx+\int 3ac^2x\,dx+\int\left(-3ac^4x^3\right)dx+\int ac^6x^5\,dx+\int\left(-\frac{b\,\mathrm{acosh}\left(cx\right)}{x}\right)dx+\int 3bc^2x\,\mathrm{acosh}\left(cx\right)dx+\int\left(-3bc^4x^3\,\mathrm{acosh}\left(cx\right)\right)dx+\int bc^6x^5\,\mathrm{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x**
3, x) + Integral(a*c**6*x**5, x) + Integral(-b*acosh(c*x)/x, x) + Integral(
3*b*c**2*x*acosh(c*x), x) + Integral(-3*b*c**4*x**3*acosh(c*x), x) + Integr
al(b*c**6*x**5*acosh(c*x), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x, x)
```

3.25 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=180

$$\frac{11}{5}bcd^3\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{5}bcd^3(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{25}bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}-\frac{d^3(a+b\cosh^{-1}(c}{x}$$

```
[Out] -1/5*b*c*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/25*b*c*d^3*(c*x-1)^(5/2)*(c*x+1)
^(5/2)-d^3*(a+b*arccosh(c*x))/x-3*c^2*d^3*x*(a+b*arccosh(c*x))+c^4*d^3*x^3*
(a+b*arccosh(c*x))-1/5*c^6*d^3*x^5*(a+b*arccosh(c*x))+b*c*d^3*arctan((c*x-1
)^(1/2)*(c*x+1)^(1/2))+11/5*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 239, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {276, 5921, 12, 1624, 1813, 1634, 65, 211}

$$-\frac{1}{5}c^6d^3x^5(a+b\cosh^{-1}(cx))+c^4d^3x^3(a+b\cosh^{-1}(cx))-3c^2d^3x(a+b\cosh^{-1}(cx))-\frac{d^3(a+b\cosh^{-1}(cx))}{x}+\frac{bcd^3\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^3(1-c^2x^2)^3}{25\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^3(1-c^2x^2)^2}{5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{11bcd^3(1-c^2x^2)}{5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*(1 - c^2*x^2))/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
- c^2*x^2)^2)/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1 - c^2*x^2)^3)/
(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x - 3*c^2*d^
3*x*(a + b*ArcCosh[c*x]) + c^4*d^3*x^3*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^5*
(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^3\,(a+b\cosh^{-1}(cx))}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+\right.$$

$$= -\frac{11bcd^3(1-c^2x^2)}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)^2}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-}{25\sqrt{-1+cx}}$$

$$= -\frac{11bcd^3(1-c^2x^2)}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)^2}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-}{25\sqrt{-1+cx}}$$

$$= -\frac{11bcd^3(1-c^2x^2)}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)^2}{5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-}{25\sqrt{-1+cx}}$$

Mathematica [A]
time = 0.16, size = 136, normalized size = 0.76

$$\frac{1}{25}d^3\left(-\frac{25a}{x} - 75ac^2x + 25ac^4x^3 - 5ac^6x^5 + bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(61-7c^2x^2+c^4x^4) - \frac{5b(5+15c^2x^2-5c^4x^4+c^6x^6)\cosh^{-1}(cx)}{x} - 25bc\text{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^3*((-25*a)/x - 75*a*c^2*x + 25*a*c^4*x^3 - 5*a*c^6*x^5 + b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(61 - 7*c^2*x^2 + c^4*x^4) - (5*b*(5 + 15*c^2*x^2 - 5*c^4
*x^4 + c^6*x^6)*ArcCosh[c*x])/x - 25*b*c*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])]))/25
```

Maple [A]
time = 2.01, size = 212, normalized size = 1.18

method	result

derivativedivides	$c\left(-a\,d^3\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^5x^5}{5}+d^3b\,\mathrm{arccosh}(cx)\,c^3x^3-3d^3b\,\mathrm{arccc}\right.$
default	$c\left(-a\,d^3\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^5x^5}{5}+d^3b\,\mathrm{arccosh}(cx)\,c^3x^3-3d^3b\,\mathrm{arccc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a*d^3*(1/5*c^5*x^5-c^3*x^3+3*c*x+1/c/x)-1/5*d^3*b*arccosh(c*x)*c^5*x^5+
d^3*b*arccosh(c*x)*c^3*x^3-3*d^3*b*arccosh(c*x)*c*x-d^3*b*arccosh(c*x)/c/x+
1/25*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^4*x^4-7/25*d^3*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*c^2*x^2-d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arct
an(1/(c^2*x^2-1)^(1/2))+61/25*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [A]
time = 0.47, size = 231, normalized size = 1.28

$$-\frac{1}{5}\,ac^6d^3x^5-\frac{1}{75}\left(15\,x^5\,\mathrm{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^6d^3+ac^4d^3x^3+\frac{1}{3}\left(3\,x^3\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^3-3\,ac^2d^3x-3\left(cx\,\mathrm{arcosh}\left(cx\right)-\sqrt{c^2x^2-1}\right)bcd^3-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\left(cx\right)}{x}\right)bd^3-\frac{ad^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c
^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^6*d^3 +
a*c^4*d^3*x^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*
sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arccosh(c*x) - s
qrt(c^2*x^2 - 1))*b*c*d^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d^3
 - a*d^3/x
```

Fricas [A]
time = 0.43, size = 249, normalized size = 1.38

$$-\frac{5\,ac^6d^3x^6-25\,ac^4d^3x^4+75\,ac^2d^3x^2-50\,bcd^3x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-5\,(bc^6-5\,bc^4+15\,bc^2+5\,b)d^3x\log\left(-cx+\sqrt{c^2x^2-1}\right)+25\,ad^3+5\,(bc^6d^3x^6-5\,bc^4d^3x^4+15\,bc^2d^3x^2-(bc^6-5\,bc^4+15\,bc^2+5\,b)d^3x+5\,bd^3)\log\left(cx+\sqrt{c^2x^2-1}\right)-(bc^5d^3x^5-7\,bc^3d^3x^3+61\,bcd^3x)\sqrt{c^2x^2-1}}{25\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/25*(5*a*c^6*d^3*x^6 - 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 50*b*c*d^3*x
*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^
3*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 25*a*d^3 + 5*(b*c^6*d^3*x^6 - 5*b*c^4*d
```

```
^3*x^4 + 15*b*c^2*d^3*x^2 - (b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x + 5*b*
d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 + 61*b
*c*d^3*x)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^{3}\left(\int 3ac^{2}\,dx + \int\left(-\frac{a}{x^{2}}\right)dx + \int\left(-3ac^{4}x^{2}\right)dx + \int ac^{6}x^{4}\,dx + \int 3bc^{2}\operatorname{acosh}{\left(cx\right)}\,dx + \int\left(-\frac{b\operatorname{acosh}{\left(cx\right)}}{x^{2}}\right)dx + \int\left(-3bc^{4}x^{2}\operatorname{acosh}{\left(cx\right)}\right)dx + \int bc^{6}x^{4}\operatorname{acosh}{\left(cx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d**3*(Integral(3*a*c**2, x) + Integral(-a/x**2, x) + Integral(-3*a*c**4*x*
*2, x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*acosh(c*x), x) + Inte
gral(-b*acosh(c*x)/x**2, x) + Integral(-3*b*c**4*x**2*acosh(c*x), x) + Inte
gral(b*c**6*x**4*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2, x)
```

3.26 $\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=267

$$-\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x}+\frac{3}{32}bc^2d^3\cosh$$

```
[Out] -7/16*b*c^3*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/2*b*c*d^3*(c*x-1)^(5/2)*(c*
x+1)^(5/2)/x+3/32*b*c^2*d^3*arccosh(c*x)-3/2*c^2*d^3*(-c^2*x^2+1)*(a+b*arcc
osh(c*x))-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))-1/2*d^3*(-c^2*x^2+1
)^3*(a+b*arccosh(c*x))/x^2-3/2*c^2*d^3*(a+b*arccosh(c*x))^2/b-3*c^2*d^3*(a+
b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3/2*b*c^2*d^3*p
olylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/32*b*c^3*d^3*x*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5920, 99, 12, 38, 54, 5919, 5882, 3799, 2221, 2317, 2438}

$$-\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{2}c^2d^3\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-3c^2d^3\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{7}{16}bc^3d^3x(cx-1)^{3/2}(cx+1)^{3/2}-\frac{3}{32}bc^3d^3x\sqrt{cx-1}\,\sqrt{cx+1}+\frac{3}{2}bc^2d^3\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+\frac{3}{32}bc^2d^3\cosh^{-1}(cx)+\frac{bcd^3(cx-1)^{5/2}(cx+1)^{5/2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (7*b*c^3*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/16 + (b*c*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(2
*x) + (3*b*c^2*d^3*ArcCosh[c*x])/32 - (3*c^2*d^3*(1 - c^2*x^2)*(a + b*ArcCo
sh[c*x]))/2 - (3*c^2*d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^3*(1
- c^2*x^2)^3*(a + b*ArcCosh[c*x]))/(2*x^2) - (3*c^2*d^3*(a + b*ArcCosh[c*x]
)^2)/(2*b) - 3*c^2*d^3*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] +
(3*b*c^2*d^3*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^3\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx &= -\frac{d^3(1-c^2x^2)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\left(3c^2d\right)\int \frac{(d-c^2dx^2)^2\left(a+b\right.}{x}\\
&= \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x}-\frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\\
&= \frac{3}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x}-\\
&= -\frac{33}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}\\
&= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}\\
&= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}\\
&= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}\\
&= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 303, normalized size = 1.13

$$\frac{d^3\left(-16a+48ac^4x^4-8ac^6x^6+3bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}+3bc^4x^4\sqrt{\frac{-1+cx}{1+cx}}+2bc^5x^5\sqrt{\frac{-1+cx}{1+cx}}+2bc^6x^6\sqrt{\frac{-1+cx}{1+cx}}+16bcx\sqrt{-1+cx}\,\sqrt{1+cx}-24bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}-48bc^2x^2\cosh^{-1}(cx)^2-42bc^2x^2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)-8b\cosh^{-1}(cx)\left(2-6c^4x^4+c^6x^6+12c^2x^2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-96ac^2x^2\log(x)+48bc^2x^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)}{32x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-16*a + 48*a*c^4*x^4 - 8*a*c^6*x^6 + 3*b*c^3*x^3*Sqrt[(-1 + c*x)/(1 +
 c*x)] + 3*b*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c^5*x^5*Sqrt[(-1 + c*
x)/(1 + c*x)] + 2*b*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x] - 24*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 48*b*c^2*
x^2*ArcCosh[c*x]^2 - 42*b*c^2*x^2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] - 8*b
*ArcCosh[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 + 12*c^2*x^2*Log[1 + E^(-2*ArcCosh[c
*x])]) - 96*a*c^2*x^2*Log[x] + 48*b*c^2*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])
]))/(32*x^2)
```

Maple [**A**]
time = 10.75, size = 267, normalized size = 1.00

method	result
derivativedivides	$c^2 \left(-\frac{a\,d^3 c^4 x^4}{4} + \frac{3a\,d^3 c^2 x^2}{2} - \frac{a\,d^3}{2c^2 x^2} - 3a\,d^3 \ln{(cx)} - 3d^3 b \operatorname{arccosh}(cx) \ln\left(1 + (cx + \sqrt{cx -}\right.\right.$
default	$c^2 \left(-\frac{a\,d^3 c^4 x^4}{4} + \frac{3a\,d^3 c^2 x^2}{2} - \frac{a\,d^3}{2c^2 x^2} - 3a\,d^3 \ln{(cx)} - 3d^3 b \operatorname{arccosh}(cx) \ln\left(1 + (cx + \sqrt{cx -}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/4*a*d^3*c^4*x^4+3/2*a*d^3*c^2*x^2-1/2*a*d^3/c^2/x^2-3*a*d^3*ln(c*x)
-3*d^3*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/4*d^3*b*a
rccosh(c*x)*c^4*x^4+3/2*d^3*b*arccosh(c*x)*c^2*x^2-1/2*d^3*b-21/32*b*d^3*ar
ccosh(c*x)+1/16*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*x^3-21/32*b*c*d^3*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)-3/2*d^3*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)+3/2*d^3*b*arccosh(c*x)^2+1/2*d^3*b/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2
)-1/2*d^3*b*arccosh(c*x)/c^2/x^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) + 1/2*b*d^3*(sq
rt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate(b*c^6*d
```

```
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x^3,
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-\frac{a}{x^3}\right)dx+\int\frac{3ac^2}{x}\,dx+\int\left(-3ac^4x\right)dx+\int ac^6x^3\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^3}\right)dx+\int\frac{3bc^2\operatorname{acosh}(cx)}{x}\,dx+\int\left(-3bc^4x\operatorname{acosh}(cx)\right)dx+\int bc^6x^3\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4*
x, x) + Integral(a*c**6*x**3, x) + Integral(-b*acosh(c*x)/x**3, x) + Integr
al(3*b*c**2*acosh(c*x)/x, x) + Integral(-3*b*c**4*x*acosh(c*x), x) + Integr
al(b*c**6*x**3*acosh(c*x), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\operatorname{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3, x)
```

3.27 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=195

$$-\frac{8}{3}bc^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{bcd^3\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}+\frac{1}{9}bc^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{d^3(a+b\cosh^{-1}(cx}{3x^3}$$

```
[Out] 1/9*b*c^3*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)-1/3*d^3*(a+b*arccosh(c*x))/x^3+3*
c^2*d^3*(a+b*arccosh(c*x))/x+3*c^4*d^3*x*(a+b*arccosh(c*x))-1/3*c^6*d^3*x^3
*(a+b*arccosh(c*x))-17/6*b*c^3*d^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))-8/3*
b*c^3*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/x^2
```

Rubi [A]
time = 0.26, antiderivative size = 252, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {276, 5921, 12, 1624, 1813, 1635, 911, 1167, 211}

$$-\frac{1}{3}c^6d^3x^3(a+b\cosh^{-1}(cx))+3c^4d^3x(a+b\cosh^{-1}(cx))+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}-\frac{17bc^3d^3\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d^3(1-c^2x^2)^2}{9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bc^3d^3(1-c^2x^2)}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (8*b*c^3*d^3*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
- c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^3*(1 - c^2*x^2)
^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/(3*x^3) +
 (3*c^2*d^3*(a + b*ArcCosh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcCosh[c*x]) - (c
^6*d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (17*b*c^3*d^3*Sqrt[-1 + c^2*x^2]*ArcTa
n[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^3\,(a+b\cosh^{-1}(cx))}{x^4}\,dx = -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b\cosh^{-1}(cx))}{x}+3c^4d^3x(a+b$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3}+\frac{3c^2d^3(a+b$$

$$= \frac{8bc^3d^3(1-c^2x^2)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^3(1-}{9\sqrt{-1+cx}}$$

$$= \frac{8bc^3d^3(1-c^2x^2)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^3(1-}{9\sqrt{-1+cx}}$$

Mathematica [A]
time = 0.18, size = 142, normalized size = 0.73

$$\frac{d^3\left(-6a+54ac^2x^2+54ac^4x^4-6ac^6x^6+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(3-50c^2x^2+2c^4x^4)-6b(1-9c^2x^2-9c^4x^4+c^6x^6)\cosh^{-1}(cx)+51bc^3x^3\text{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)\right)}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 + b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(3 - 50*c^2*x^2 + 2*c^4*x^4) - 6*b*(1 - 9*c^2*x^2 - 9*c^4*
x^4 + c^6*x^6)*ArcCosh[c*x] + 51*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x])]))/(18*x^3)
```

Maple [A]
time = 2.02, size = 216, normalized size = 1.11

method	result
derivativedivides	$c^3\left(-a\,d^3\left(\frac{c^3x^3}{3}-3cx-\frac{3}{cx}+\frac{1}{3c^3x^3}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^3x^3}{3}+3d^3b\,\mathrm{arccosh}(cx)\,cx+\frac{3d^3b\,\mathrm{arccosh}}{cx}\right.$
default	$c^3\left(-a\,d^3\left(\frac{c^3x^3}{3}-3cx-\frac{3}{cx}+\frac{1}{3c^3x^3}\right)-\frac{d^3b\,\mathrm{arccosh}(cx)c^3x^3}{3}+3d^3b\,\mathrm{arccosh}(cx)\,cx+\frac{3d^3b\,\mathrm{arccosh}}{cx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-a*d^3*(1/3*c^3*x^3-3*c*x-3/c/x+1/3/c^3/x^3)-1/3*d^3*b*arccosh(c*x)*c^
3*x^3+3*d^3*b*arccosh(c*x)*c*x+3*d^3*b*arccosh(c*x)/c/x-1/3*d^3*b*arccosh(c
*x)/c^3/x^3+1/9*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+17/6*d^3*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))-25/9*d^
3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/x
^2)
```

Maxima [A]
time = 0.49, size = 208, normalized size = 1.07

$$-\frac{1}{3}\,ac^6d^3x^3-\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\,\mathrm{arcosh}\left(cx\right)-\sqrt{c^2x^2-1}\right)bc^3d^3+3\left(c\,\mathrm{arcsin}\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\left(cx\right)}{x}\right)bc^2d^3-\frac{1}{6}\left(\left(c^2\,\mathrm{arcsin}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\left(cx\right)}{x^3}\right)bd^3+\frac{3\,ac^2d^3}{x}-\frac{ad^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arccosh(c*x
) - sqrt(c^2*x^2 - 1))*b*c^3*d^3 + 3*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)
/x)*b*c^2*d^3 - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c +
 2*arccosh(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Fricas [A]
time = 0.40, size = 253, normalized size = 1.30

$$-\frac{6\,ac^6d^3x^6-54\,ac^4d^3x^4+102\,bc^3d^3x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-54\,ac^2d^3x^2-6\,(bc^6-9\,bc^4-9\,bc^2+b)d^3x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)+6\,ad^3+6\,(bc^6d^3x^6-9\,bc^4d^3x^4-9\,bc^2d^3x^2-(bc^6-9\,bc^4-9\,bc^2+b)d^3x^3+bd^3)\log\left(cx+\sqrt{c^2x^2-1}\right)-(2\,bc^5d^3x^5-50\,bc^3d^3x^3+3\,bcd^3x)\sqrt{c^2x^2-1}}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*a*c^6*d^3*x^6 - 54*a*c^4*d^3*x^4 + 102*b*c^3*d^3*x^3*arctan(-c*x +
 sqrt(c^2*x^2 - 1)) - 54*a*c^2*d^3*x^2 - 6*(b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*
d^3*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + 6*a*d^3 + 6*(b*c^6*d^3*x^6 - 9*b*c^
4*d^3*x^4 - 9*b*c^2*d^3*x^2 - (b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3 + b*d
^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3 + 3*
b*c*d^3*x)*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int(-3ac^4)\,dx+\int\left(-\frac{a}{x^4}\right)dx+\int\frac{3ac^2}{x^2}\,dx+\int ac^6x^2\,dx+\int(-3bc^4\operatorname{acosh}(cx))\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^4}\right)dx+\int\frac{3bc^2\operatorname{acosh}(cx)}{x^2}\,dx+\int bc^6x^2\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d**3*(Integral(-3*a*c**4, x) + Integral(-a/x**4, x) + Integral(3*a*c**2/x*
*2, x) + Integral(a*c**6*x**2, x) + Integral(-3*b*c**4*acosh(c*x), x) + Int
egral(-b*acosh(c*x)/x**4, x) + Integral(3*b*c**2*acosh(c*x)/x**2, x) + Inte
gral(b*c**6*x**2*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4, x)
```

3.28 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx$

Optimal. Leaf size=158

$$\frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d}+\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d}-\frac{x(a+b\cosh^{-1}(cx))}{c^4d}-\frac{x^3(a+b\cosh^{-1}(cx))}{3c^2d}+\frac{2(a+b\cosh$$

```
[Out] -x*(a+b*arccosh(c*x))/c^4/d-1/3*x^3*(a+b*arccosh(c*x))/c^2/d+2*(a+b*arccosh
(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d+b*polylog(2,-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/c^5/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/c^5/d+11/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d+1/9*b*x^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5938, 5903, 4267, 2317, 2438, 75, 102, 12}

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{c^5d}-\frac{x(a+b\cosh^{-1}(cx))}{c^4d}-\frac{x^3(a+b\cosh^{-1}(cx))}{3c^2d}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^5d}+\frac{11b\sqrt{cx-1}\,\sqrt{cx+1}}{9c^5d}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{9c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^5*d) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(9*c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^4*d) - (x^3*(a + b*ArcCos
h[c*x]))/(3*c^2*d) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^5*
d) + (b*PolyLog[2, -E^ArcCosh[c*x]])/(c^5*d) - (b*PolyLog[2, E^ArcCosh[c*x]
])/(c^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d} + \frac{\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2} + \frac{b\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3cd}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d} + \frac{\int}{}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3}{}$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} -$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} -$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} -$$

Mathematica [A]

time = 0.27, size = 227, normalized size = 1.44

$$-\frac{18acx+6ac^3x^3-18b\sqrt{\frac{-1+cx}{1+cx}}-18bcx\sqrt{\frac{-1+cx}{1+cx}}-4b\sqrt{-1+cx}\,\sqrt{1+cx}-2bc^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}+18bcx\cosh^{-1}(cx)+6bc^3x^3\cosh^{-1}(cx)-9b\cosh^{-1}(cx)^2-18b\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)+18b\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)+9a\log(1-cx)-9a\log(1+cx)+18b\,\mathrm{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+18b\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{18c^5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -1/18*(18*a*c*x + 6*a*c^3*x^3 - 18*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 18*b*c*x*
Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*x^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 18*b*c*x*ArcCosh[c*x] + 6*b*c^3*x^3*ArcCosh
[c*x] - 9*b*ArcCosh[c*x]^2 - 18*b*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
 18*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 9*a*Log[1 - c*x] - 9*a*Log[1 +
 c*x] + 18*b*PolyLog[2, -E^(-ArcCosh[c*x])] + 18*b*PolyLog[2, E^ArcCosh[c*x
]])/(c^5*d)
```

Maple [A]

time = 5.24, size = 242, normalized size = 1.53

method	result
derivativedivides	$-\frac{a\,c^3x^3}{3d}-\frac{acx}{d}-\frac{a\ln(cx-1)}{2d}+\frac{a\ln(cx+1)}{2d}-\frac{b\,\mathrm{polylog}\left(2,cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{9d}+$
default	$-\frac{a\,c^3x^3}{3d}-\frac{acx}{d}-\frac{a\ln(cx-1)}{2d}+\frac{a\ln(cx+1)}{2d}-\frac{b\,\mathrm{polylog}\left(2,cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{9d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-1/3*a/d*c^3*x^3-a/d*c*x-1/2*a/d*ln(c*x-1)+1/2*a/d*ln(c*x+1)-b/d*pol
ylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/9*b/d*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*c^2*x^2+b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*l
n(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+11/9*b/d*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1
/3*b/d*arccosh(c*x)*c^3*x^3+b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))-b/d*arccosh(c*x)*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/72*(4*c^4*(2*(c^2*x^3 + 3*x)/(c^8*d) - 3*log(c*x + 1)/(c^9*d) + 3*log(c*x
 - 1)/(c^9*d)) + 36*c^2*(2*x/(c^6*d) - log(c*x + 1)/(c^7*d) + log(c*x - 1)/
(c^7*d)) + 648*c*integrate(1/12*x*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x) - 3*
(4*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 3*log(c*x + 1)^2 + 6*log(c*x + 1)*log(c*x - 1))/(c^5
*d) + 72*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1
))/(c^7*d*x^3 - c^5*d*x + (c^6*d*x^2 - c^4*d)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x) - 216*integrate(1/12*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x))*b - 1/6*a*(2
*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1)/(c^5*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^4}{c^2x^2-1}\,dx + \int \frac{bx^4\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

3.29 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx$

Optimal. Leaf size=140

$$\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d}+\frac{b\cosh^{-1}(cx)}{4c^4d}-\frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d}+\frac{(a+b\cosh^{-1}(cx))^2}{2bc^4d}-\frac{(a+b\cosh^{-1}(cx))\log}{c^4d}$$

```
[Out] 1/4*b*arccosh(c*x)/c^4/d-1/2*x^2*(a+b*arccosh(c*x))/c^2/d+1/2*(a+b*arccosh(
c*x))^2/b/c^4/d-(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)/c^4/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d+1/4*b*x*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5938, 5913, 3797, 2221, 2317, 2438, 92, 54}

$$\frac{(a+b\cosh^{-1}(cx))^2}{2bc^4d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{c^4d}-\frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^4d}+\frac{b\cosh^{-1}(cx)}{4c^4d}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d) + (b*ArcCosh[c*x])/(4*c^4*d) -
 (x^2*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (a + b*ArcCosh[c*x])^2/(2*b*c^4*d)
- ((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d) - (b*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*c^4*d)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx = -\frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} + \frac{\int \frac{x(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx}{c^2} + \frac{b\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2cd}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d} - \frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} - \frac{\text{Subst}(\int (a+bx)\coth(x)\,}{c^4d}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} + \frac{(a+b\cos}{2bc}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} + \frac{(a+b\cos}{2bc}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} + \frac{(a+b\cos}{2bc}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2(a+b\cosh^{-1}(cx))}{2c^2d} + \frac{(a+b\cos}{2bc}$$

Mathematica [A]
time = 0.22, size = 151, normalized size = 1.08

$$-\frac{2c^2x^2(a+b\cosh^{-1}(cx)) - \frac{2(a+b\cosh^{-1}(cx))^2}{b} - b\left(cx\sqrt{-1+cx}\sqrt{1+cx} + 2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right) + 4(a+b\cosh^{-1}(cx))\log\left(1-e^{\cosh^{-1}(cx)}\right) + 4(a+b\cosh^{-1}(cx))\log\left(1+e^{\cosh^{-1}(cx)}\right) + 4b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) + 4b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -1/4*(2*c^2*x^2*(a + b*ArcCosh[c*x]) - (2*(a + b*ArcCosh[c*x])^2)/b - b*(c*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]) + 4
*(a + b*ArcCosh[c*x])*Log[1 - E^ArcCosh[c*x]] + 4*(a + b*ArcCosh[c*x])*Log[
1 + E^ArcCosh[c*x]] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*PolyLog[2, E^Ar
cCosh[c*x]])/(c^4*d)
```

Maple [A]
time = 4.57, size = 222, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{a\,c^2x^2}{2d} - \frac{a\ln(cx-1)}{2d} - \frac{a\ln(cx+1)}{2d} + \frac{b\text{arccosh}(cx)^2}{2d} - \frac{b\,\text{arccosh}(cx)c^2x^2}{2d} + \frac{b\sqrt{cx+1}\sqrt{cx-1}\,cx}{4d} + \frac{b\,\text{arccosh}(cx)}{4d} - \frac{b\,\text{arcc}}{}}{}$
default	$\frac{-\frac{a\,c^2x^2}{2d} - \frac{a\ln(cx-1)}{2d} - \frac{a\ln(cx+1)}{2d} + \frac{b\text{arccosh}(cx)^2}{2d} - \frac{b\,\text{arccosh}(cx)c^2x^2}{2d} + \frac{b\sqrt{cx+1}\sqrt{cx-1}\,cx}{4d} + \frac{b\,\text{arccosh}(cx)}{4d} - \frac{b\,\text{arcc}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*a/d*c^2*x^2-1/2*a/d*ln(c*x-1)-1/2*a/d*ln(c*x+1)+1/2*b/d*arccosh
(c*x)^2-1/2*b/d*arccosh(c*x)*c^2*x^2+1/4*b/d*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*
x+1/4*b/d*arccosh(c*x)-b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))-b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*ln(1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/8*b*((2*c^2*x^2 - 4*(c^
2*x^2 + log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
 + 2*(log(c*x - 1) + 1)*log(c*x + 1) + log(c*x + 1)^2 + log(c*x - 1)^2 + 2*
log(c*x - 1))/(c^4*d) - 8*integrate(1/2*(c^2*x^2 + log(c*x + 1) + log(c*x -
 1))/(c^6*d*x^3 - c^4*d*x + (c^5*d*x^2 - c^3*d)*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3*arccosh(c*x) + a*x^3)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^3}{c^2x^2-1}\,dx + \int \frac{bx^3\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a+b\,\mathrm{acosh}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

3.30 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx$

Optimal. Leaf size=102

$$\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x(a+b\cosh^{-1}(cx))}{c^2d}+\frac{2(a+b\cosh^{-1}(cx))\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^3d}+\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^3d}$$

```
[Out] -x*(a+b*arccosh(c*x))/c^2/d+2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/c^3/d+b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d-b*
polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d+b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5938, 5903, 4267, 2317, 2438, 75}

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{c^3d}-\frac{x(a+b\cosh^{-1}(cx))}{c^2d}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^3d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^2*d)
 + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3*d) + (b*PolyLog[2,
 -E^ArcCosh[c*x]])/(c^3*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c^3*d)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{c^2}+\frac{b\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{cd}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,\right.}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}}{c^3d}$$

Mathematica [A]
time = 0.11, size = 155, normalized size = 1.52

$$\frac{-2acx+2b\sqrt{\frac{-1+cx}{1+cx}}+2bcx\sqrt{\frac{-1+cx}{1+cx}}-2bcx\cosh^{-1}(cx)+b\cosh^{-1}(cx)^2+2b\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)-2b\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)-a\log(1-cx)+a\log(1+cx)-2b\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-2b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (-2*a*c*x + 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c*x*Sqrt[(-1 + c*x)/(1 + c
*x)] - 2*b*c*x*ArcCosh[c*x] + b*ArcCosh[c*x]^2 + 2*b*ArcCosh[c*x]*Log[1 + E
^(-ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] - a*Log[1 - c*
x] + a*Log[1 + c*x] - 2*b*PolyLog[2, -E^(-ArcCosh[c*x])] - 2*b*PolyLog[2, E
^ArcCosh[c*x]])/(2*c^3*d)
```

Maple [A]
time = 4.25, size = 187, normalized size = 1.83

method	result
derivativedivides	$\frac{-\frac{acx}{d}-\frac{a\ln(cx-1)}{2d}+\frac{a\ln(cx+1)}{2d}+\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d}-\frac{b\,\mathrm{arccosh}(}{}}{}$
default	$\frac{-\frac{acx}{d}-\frac{a\ln(cx-1)}{2d}+\frac{a\ln(cx+1)}{2d}+\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d}-\frac{b\,\mathrm{arccosh}(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a/d*c*x-1/2*a/d*ln(c*x-1)+1/2*a/d*ln(c*x+1)+b/d*arccosh(c*x)*ln(1+c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*(c*x-1)^(1/2)*(c*x+1)^(1/2)-b/d*arccosh
(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*c*x+b/d*polylo
g(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(4*c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^5*d) + log(c*x - 1)/(c^5*d)) + 24
*c*integrate(1/4*x*log(c*x - 1)/(c^4*d*x^2 - c^2*d), x) - (4*(2*c*x - log(c
*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + log(c*x +
1)^2 + 2*log(c*x + 1)*log(c*x - 1))/(c^3*d) + 8*integrate(-1/2*(2*c*x - log
(c*x + 1) + log(c*x - 1))/(c^5*d*x^3 - c^3*d*x + (c^4*d*x^2 - c^2*d)*sqrt(c
*x + 1)*sqrt(c*x - 1)), x) - 8*integrate(1/4*log(c*x - 1)/(c^4*d*x^2 - c^2*
d), x))*b - 1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d
))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^2}{c^2x^2-1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
[Out] -(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

3.31 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=74

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^2d}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^2/b/c^2/d-(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/c^2/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2)/c^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5913, 3797, 2221, 2317, 2438}

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(2*b*c^2*d) - ((a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*c^2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\text{Subst}(\int \text{lo}}{}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\text{Subst}\left(\int \frac{\text{l}}{}\right.}{}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} - \frac{b\text{Li}_2\left(e^{2\cosh}\right.}{2c^2d}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 1.15

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)-2b^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-2b^2\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2bc^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2
*b*Log[1 + E^ArcCosh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*Pol
yLog[2, E^ArcCosh[c*x]])/(2*b*c^2*d)
```

Maple [A]
time = 2.66, size = 162, normalized size = 2.19

method	result
derivativedivides	$\frac{-\frac{a\ln(cx-1)}{2d}-\frac{a\ln(cx+1)}{2d}+\frac{b\operatorname{arccosh}(cx)^2}{2d}-\frac{b\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}-\frac{b\operatorname{polylog}\left(2,-cx-\sqrt{cx-1}\right.}{d}}{c^2}$
default	$\frac{-\frac{a\ln(cx-1)}{2d}-\frac{a\ln(cx+1)}{2d}+\frac{b\operatorname{arccosh}(cx)^2}{2d}-\frac{b\operatorname{arccosh}(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}-\frac{b\operatorname{polylog}\left(2,-cx-\sqrt{cx-1}\right.}{d}}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a/d*ln(c*x-1)-1/2*a/d*ln(c*x+1)+1/2*b/d*arccosh(c*x)^2-b/d*arcc
osh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))-b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-
b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*b*((4*(log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) - log(c*x - 1)^2)/(c^2*
d) + 8*integrate(1/2*(log(c*x + 1) + log(c*x - 1))/(c^4*d*x^3 - c^2*d*x + (
c^3*d*x^2 - c*d)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) - 1/2*a*log(
c^2*d*x^2 - d)/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x*arccosh(c*x) + a*x)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{c^2x^2-1}\,dx+\int \frac{bx\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*acosh(c*x)/(c**2*x**2 - 1
), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

3.32 $\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=59

$$\frac{2(a+b\cosh^{-1}(cx))\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd}+\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d+b*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/c/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5903, 4267, 2317, 2438}

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d) + (b*PolyLog[2, -E^A
rcCosh[c*x]])/(c*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \cosh^{-1}(cx\right.}{cd}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(cx)}\right)}{cd} -$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{cd} - \frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 1.08

$$\frac{-\left(\left(a+b\cosh^{-1}(cx)\right)\left(\log\left(1-e^{\cosh^{-1}(cx)}\right)-\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)\right)+b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*(Log[1 - E^ArcCosh[c*x]] - Log[1 + E^ArcCosh[c*x]])
) + b*PolyLog[2, -E^ArcCosh[c*x]] - b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Maple [C] Result contains complex when optimal does not.
time = 11.77, size = 312, normalized size = 5.29

method	result
derivativedivides	$\frac{\frac{a\,\text{arctanh}(cx)}{d}+\frac{b\,\text{arctanh}(cx)\text{arccosh}(cx)}{d}+\frac{2ib\sqrt{-c^2x^2+1}\,\sqrt{\frac{cx}{2}+\frac{1}{2}}\,\sqrt{\frac{cx}{2}-\frac{1}{2}}\,\text{arctanh}(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+}}\right.}{d\left(c^2x^2-1\right)}}{}$
default	$\frac{\frac{a\,\text{arctanh}(cx)}{d}+\frac{b\,\text{arctanh}(cx)\text{arccosh}(cx)}{d}+\frac{2ib\sqrt{-c^2x^2+1}\,\sqrt{\frac{cx}{2}+\frac{1}{2}}\,\sqrt{\frac{cx}{2}-\frac{1}{2}}\,\text{arctanh}(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+}}\right.}{d\left(c^2x^2-1\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/d*arctanh(c*x)+b/d*arctanh(c*x)*arccosh(c*x)+2*I*b/d*(-c^2*x^2+1)^(1
/2)*(1/2*c*x+1/2)^(1/2)*(1/2*c*x-1/2)^(1/2)/(c^2*x^2-1)*arctanh(c*x)*ln(1+I
*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I*b/d*(-c^2*x^2+1)^(1/2)*(1/2*c*x+1/2)^(1/2)
*(1/2*c*x-1/2)^(1/2)/(c^2*x^2-1)*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(
1/2))+2*I*b/d*(-c^2*x^2+1)^(1/2)*(1/2*c*x+1/2)^(1/2)*(1/2*c*x-1/2)^(1/2)/(c
^2*x^2-1)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I*b/d*(-c^2*x^2+1)^(1/2)*
(1/2*c*x+1/2)^(1/2)*(1/2*c*x-1/2)^(1/2)/(c^2*x^2-1)*dilog(1-I*(c*x+1)/(-c^2
*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*b*((4*(log(c*x + 1) - log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1))/(c*d) + 8*integrate(1/4
*(3*c*x - 1)*log(c*x - 1)/(c^2*d*x^2 - d), x) + 8*integrate(1/2*(log(c*x +
1) - log(c*x - 1))/(c^3*d*x^3 - c*d*x + (c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(
c*x - 1)), x)) + 1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^2-1}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acosh(c*x)/(c**2*x**2 - 1), x
))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2),x)
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2), x)
```

3.33 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*p
olylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5916, 5569, 4267, 2317, 2438}

$$\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*PolyLog[2, -E^(
2*ArcCosh[c*x])])/(2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx &= -\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{d}\\
&= -\frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \cosh^{-1}(cx)\right)}{d}\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \cosh^{-1}(\right.}{d}\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\cosh^{-1}(cx)}\right)}{2d}\\
&= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d} - \frac{b\text{Li}_2\left(e^{2\cosh^{-1}(\right.}{2d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 1.52

$$\frac{a\log(x)}{d}-\frac{a\log\left(1-c^2x^2\right)}{2d}-\frac{b\left(2\cosh^{-1}(cx)\left(\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (a*Log[x])/d - (a*Log[1 - c^2*x^2])/(2*d) - (b*(2*ArcCosh[c*x]*(Log[1 - E^(
-2*ArcCosh[c*x])] - Log[1 + E^(-2*ArcCosh[c*x])]) + PolyLog[2, -E^(-2*ArcCo
sh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d)
```

Maple [A]
time = 5.27, size = 88, normalized size = 1.44

method	result
derivativedivides	$-\frac{a\ln(cx-1)}{2d}-\frac{a\ln(cx+1)}{2d}+\frac{a\ln(cx)}{d}-\frac{b\left(\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2}\right)-\frac{\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{\right.}\right.}\right.}{4}\right.}{d}$
default	$-\frac{a\ln(cx-1)}{2d}-\frac{a\ln(cx+1)}{2d}+\frac{a\ln(cx)}{d}-\frac{b\left(\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2}\right)-\frac{\operatorname{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{\right.}\right.}\right.}{4}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*ln(c*x-1)-1/2*a/d*ln(c*x+1)+a/d*ln(c*x)-b/d*(dilog(1/(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)-1/4*dilog(1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^3 - d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^3-x}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^2x^3-x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**3 - x), x) + Integral(b*acosh(c*x)/(c**2*x**3 - x), x
))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)), x)
```

3.34 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)}\,dx$

Optimal. Leaf size=95

$$-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d}+\frac{bc\text{PolyLo}}{}$$

```
[Out] (-a-b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*c*(a+b*
arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+b*c*polylog(2,-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/d
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5932, 5903, 4267, 2317, 2438, 94, 211}

$$-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{2c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}+\frac{bc\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (2*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d + (b*c*PolyLog[2,
-E^ArcCosh[c*x]])/d - (b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{dx}+c^2\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx+\frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}-\frac{c\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\cosh^{-1}(cx)\right)}{d}+\frac{\left(bc^2\right)\text{Subst}\big(}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tan}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tan}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tan}{d}$$

Mathematica [A]
time = 0.20, size = 132, normalized size = 1.39

$$\frac{-\frac{a+b\cosh^{-1}(cx)}{x}+\frac{bc\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-c\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{\cosh^{-1}(cx)}\right)+c\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{\cosh^{-1}(cx)}\right)+bc\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-bc\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])/x) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x
^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c*(a + b*ArcCosh[c*x])*Log[1 - E^Arc
Cosh[c*x]] + c*(a + b*ArcCosh[c*x])*Log[1 + E^ArcCosh[c*x]] + b*c*PolyLog[2
, -E^ArcCosh[c*x]] - b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Maple [A]
time = 4.88, size = 163, normalized size = 1.72

method	result
derivativedivides	$c\left(-\frac{a}{dcx}+\frac{a\ln(cx+1)}{2d}-\frac{a\ln(cx-1)}{2d}-\frac{b\,\mathrm{arccosh}(cx)}{dcx}+\frac{2b\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\,\mathrm{dilog}\left(1\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}+\frac{a\ln(cx+1)}{2d}-\frac{a\ln(cx-1)}{2d}-\frac{b\,\mathrm{arccosh}(cx)}{dcx}+\frac{2b\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}+\frac{b\,\mathrm{dilog}\left(1\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/d/c/x+1/2*a/d*ln(c*x+1)-1/2*a/d*ln(c*x-1)-b/d*arccosh(c*x)/c/x+2*b/d*
arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))+b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*dilog(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(24*c^3*integrate(1/4*x*log(c*x - 1)/(c^2*d*x^2 - d), x) - 4*c^2*(log(c
*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 8*c^2*integrate(1/4*log(c*x - 1)/(c^2
*d*x^2 - d), x) - (c*x*log(c*x + 1)^2 + 2*c*x*log(c*x + 1)*log(c*x - 1) - 4
*(c*x*log(c*x + 1) - c*x*log(c*x - 1) - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(d*x) + 8*integrate(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(c*x - 1) -
```

```
2*c)/(c^3*d*x^4 - c*d*x^2 + (c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x))*b + 1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^4-x^2}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^2x^4-x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d),x)
[Out] -(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*acosh(c*x)/(c**2*x**4 - x*
*2), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x^2\,\left(d-c^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)), x)
```

3.35 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)}\,dx$

Optimal. Leaf size=118

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}+\frac{bc^2\mathrm{PolyLog}\left(2,-e^2\right.}{2d}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d/x^2+2*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)/d-1/2*b*c^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*
b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5932, 5916, 5569, 4267, 2317, 2438, 97}

$$\frac{2c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{bc^2\mathrm{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{bc^2\mathrm{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (2*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*c^2*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])
/(2*d)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2dx^2} + c^2\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)}\,dx + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} - \frac{c^2\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\mathrm{sech}(x)\,dx\right.}{d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} - \frac{(2c^2)\,\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(2x)\,dx, x\right.}{d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh}\right.}{d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh}\right.}{d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh}\right.}{d}$$

Mathematica [A]

time = 0.37, size = 144, normalized size = 1.22

$$-\frac{\frac{a}{x^2} - 2ac^2\log(x) + ac^2\log(1-c^2x^2) + bc^2\left(-\frac{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}{cx} + \frac{\cosh^{-1}(cx)}{c^2x^2} + 2\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right) - 2\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right) + \mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right) - \mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] -1/2*(a/x^2 - 2*a*c^2*Log[x] + a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + ArcCosh[c*x]/(c^2*x^2) + 2*ArcCosh[c*
x]*Log[1 - E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x]
)] + PolyLog[2, -E^(-2*ArcCosh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/d
```

Maple [A]

time = 4.49, size = 283, normalized size = 2.40

method	result
derivativedivides	$c^2\left(-\frac{a}{2d\,c^2x^2} + \frac{a\ln(cx)}{d} - \frac{a\ln(cx-1)}{2d} - \frac{a\ln(cx+1)}{2d} + \frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2dcx} - \frac{b}{2d} - \frac{b\,\mathrm{arccosh}(cx)}{2d\,c^2x^2} - \right.$
default	$c^2\left(-\frac{a}{2d\,c^2x^2} + \frac{a\ln(cx)}{d} - \frac{a\ln(cx-1)}{2d} - \frac{a\ln(cx+1)}{2d} + \frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2dcx} - \frac{b}{2d} - \frac{b\,\mathrm{arccosh}(cx)}{2d\,c^2x^2} - \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/d/c^2/x^2+a/d*ln(c*x)-1/2*a/d*ln(c*x-1)-1/2*a/d*ln(c*x+1)+1/2*b
/d/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2*b/d-1/2*b/d*arccosh(c*x)/c^2/x^2-b/d
*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))+b/d*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-b/d*arccos
h(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d*polylog(2,c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2))
*a - b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^5 - d*x^3)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^5-x^3}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^2x^5-x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*acosh(c*x)/(c**2*x**5 - x*
*3), x))/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)), x)
```

3.36 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)}\,dx$

Optimal. Leaf size=157

$$\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2(a+b\cosh^{-1}(cx))}{dx}+\frac{7bc^3\text{ArcTan}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{6d}+\frac{2c}{}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3-c^2*(a+b*arccosh(c*x))/d/x+7/6*b*c^3*arctan((
c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/d+b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-
b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+1/6*b*c*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d/x^2
```

Rubi [A]
time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5932, 5903, 4267, 2317, 2438, 94, 211, 105, 12}

$$\frac{2c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{d}-\frac{c^2(a+b\cosh^{-1}(cx))}{dx}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{7bc^3\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{6d}+\frac{bc^3\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc^3\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{d}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) - (c^2*(a + b*ArcCosh[c*x]))/(d*x) + (7*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(6*d) + (2*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d
+ (b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/d - (b*c^3*PolyLog[2, E^ArcCosh[c*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\,(d-c^2dx^2)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3dx^3} + c^2\int \frac{a+b\cosh^{-1}(cx)}{x^2\,(d-c^2dx^2)}\,dx + \frac{(bc)\int \frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} - \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx} + c^4\int \frac{a+b}{d-}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} - \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx} - \frac{c^3\text{Subst}\left(\int(\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} - \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx} + \frac{bc^3\tan^{-1}\left(\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} - \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx} + \frac{7bc^3\tan^{-1}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} - \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx} + \frac{7bc^3\tan^{-1}}{}$$

Mathematica [A]
time = 0.24, size = 223, normalized size = 1.42

$$\frac{-\frac{2a}{x^3}-\frac{6ac^2}{x}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2}-\frac{2b\cosh^{-1}(cx)}{x^3}-\frac{6bc^2\cosh^{-1}(cx)}{x}+\frac{7bc^3\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-6ac^3\log\left(1-e^{\cosh^{-1}(cx)}\right)-6bc^3\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)+6ac^3\log\left(1+e^{\cosh^{-1}(cx)}\right)+6bc^3\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right)+6bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-6bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] ((-2*a)/x^3 - (6*a*c^2)/x + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x^2 - (2*b*A
rcCosh[c*x])/x^3 - (6*b*c^2*ArcCosh[c*x])/x + (7*b*c^3*Sqrt[-1 + c^2*x^2]*A
rcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*a*c^3*Log[1 -
 E^ArcCosh[c*x]] - 6*b*c^3*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 6*a*c^3*L
og[1 + E^ArcCosh[c*x]] + 6*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 6*b
*c^3*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(6*d
)
```

Maple [A]
time = 5.82, size = 219, normalized size = 1.39

method	result
derivativedivides	$c^3\left(-\frac{a\ln(cx-1)}{2d}-\frac{a}{3d\,c^3x^3}-\frac{a}{dcx}+\frac{a\ln(cx+1)}{2d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,c^2x^2}-\frac{b\,\text{arccosh}(cx)}{dcx}-\frac{b\,\text{arccosh}(}{3d\,c^3x^3}\right.$

default	$c^3\left(-\frac{a\ln(cx-1)}{2d}-\frac{a}{3d\,c^3x^3}-\frac{a}{dcx}+\frac{a\ln(cx+1)}{2d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,c^2x^2}-\frac{b\,\mathrm{arccosh}(cx)}{dcx}-\frac{b\,\mathrm{arccosh}(cx)}{3d\,c^3x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/2*a/d*ln(c*x-1)-1/3*a/d/c^3/x^3-a/d/c/x+1/2*a/d*ln(c*x+1)+1/6*b/d/c
^2/x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-b/d*arccosh(c*x)/c/x-1/3*b/d*arccosh(c*x
)/c^3/x^3+7/3*b/d*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*dilog(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))+b/d*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^3
))*a + 1/24*(216*c^5*integrate(1/12*x^3*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x
) - 12*c^4*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 72*c^4*integrate(1/1
2*x^2*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x) - 4*c^2*(c*log(c*x + 1)/d - c*lo
g(c*x - 1)/d - 2/(d*x)) - (3*c^3*x^3*log(c*x + 1)^2 + 6*c^3*x^3*log(c*x + 1
)*log(c*x - 1) - 4*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2
*x^2 - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(d*x^3) + 24*integrate(1/
6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log(c*x - 1) - 6*c^3*x^2 - 2*c)/(c^3*
d*x^6 - c*d*x^4 + (c^2*d*x^5 - d*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^6-x^4}\,dx+\int \frac{b\,\mathrm{acosh}\,(cx)}{c^2x^6-x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*acosh(c*x)/(c**2*x**6 - x*
*4), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)), x)
```

3.37 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{3\left(a+b\right.}{}$$

```
[Out] 3/2*x*(a+b*arccosh(c*x))/c^4/d^2+1/2*x^3*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x
^2+1)-3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2
-3/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2+3/2*b*polylog(2,
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2-1/2*b*x^2/c^3/d^2/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5934, 5938, 5903, 4267, 2317, 2438, 75, 100, 21}

$$-\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{3b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{3b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2c^5d^2}-\frac{bx^2}{2c^3d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(b*x^2)/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/(2*c^5*d^2) + (3*x*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) + (x^3*(a
+ b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/(c^5*d^2) - (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^5*d
^2) + (3*b*PolyLog[2, E^ArcCosh[c*x]])/(2*c^5*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\int\frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}-\frac{3\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2c^2d}\\
&= -\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}\\
&= -\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}\\
&= -\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}\\
&= -\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}\\
&= -\frac{bx^2}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 244, normalized size = 1.38

$$\frac{4acx-3b\sqrt{\frac{-1+cx}{1+cx}}-4bcx\sqrt{\frac{-1+cx}{1+cx}}+\frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2acx}{-1+c^2x^2}+4bcx\cosh^{-1}(cx)+\frac{b\cosh^{-1}(cx)}{1-cx}-\frac{b\cosh^{-1}(cx)}{1+cx}+6b\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)-6b\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right)+3a\log(1-cx)-3a\log(1+cx)-6b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+6b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4c^5d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (4*a*c*x - 3*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c*x*Sqrt[(-1 + c*x)/(1 + c*
x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + (b*c*x*Sqrt[(-1 + c*x)/(1
+ c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + 4*b*c*x*ArcCosh[c*x] + (b*A
rcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 6*b*ArcCosh[c*x]*Log[
1 - E^ArcCosh[c*x]] - 6*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 3*a*Log[1
```

```
- c*x] - 3*a*Log[1 + c*x] - 6*b*PolyLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2
, E^ArcCosh[c*x]])/(4*c^5*d^2)
```

Maple [A]
time = 7.88, size = 268, normalized size = 1.51

method	result
derivativedivides	$\frac{\frac{acx}{d^2}-\frac{a}{4d^2(cx-1)}+\frac{3a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}-\frac{3a\ln(cx+1)}{4d^2}+\frac{b\,\mathrm{arccosh}(cx)cx}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d^2}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-b}{}$
default	$\frac{\frac{acx}{d^2}-\frac{a}{4d^2(cx-1)}+\frac{3a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}-\frac{3a\ln(cx+1)}{4d^2}+\frac{b\,\mathrm{arccosh}(cx)cx}{d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{d^2}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/d^2*c*x-1/4*a/d^2/(c*x-1)+3/4*a/d^2*ln(c*x-1)-1/4*a/d^2/(c*x+1)-3/
4*a/d^2*ln(c*x+1)+b/d^2*arccosh(c*x)*c*x-b/d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-
1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c*x-1/2*b/d^2/(c^2*x^2-1)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)-3/2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-
3/2*b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*arccosh(c*x
)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*polylog(2,c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(16*c^4*(2*x/(c^10*d^2*x^2 - c^8*d^2) - 4*x/(c^8*d^2) + 3*log(c*x + 1)
/(c^9*d^2) - 3*log(c*x - 1)/(c^9*d^2)) - 576*c^3*integrate(1/8*x^3*log(c*x
- 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*c^2*(2*x/(c^8*d^2*x^2
 - c^6*d^2) + log(c*x + 1)/(c^7*d^2) - log(c*x - 1)/(c^7*d^2)) + 192*c^2*in
tegrate(1/8*x^2*log(c*x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) -
9*(c*(2/(c^8*d^2*x - c^7*d^2) - log(c*x + 1)/(c^7*d^2) + log(c*x - 1)/(c^7*
d^2)) + 4*log(c*x - 1)/(c^8*d^2*x^2 - c^6*d^2))*c + 4*(3*(c^2*x^2 - 1)*log(
c*x + 1)^2 + 6*(c^2*x^2 - 1)*log(c*x + 1)*log(c*x - 1) + 4*(4*c^3*x^3 - 6*c
*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x - 1))*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^2*x^2 - c^5*d^2) - 64*integrate(-1/4*
(4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x
 - 1))/(c^9*d^2*x^5 - 2*c^7*d^2*x^3 + c^5*d^2*x + (c^8*d^2*x^4 - 2*c^6*d^2*
```

```
x^2 + c^4*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 192*integrate(1/8*log(c*x
 - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b - 1/4*a*(2*x/(c^6*d^2*
x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*d^2) - 3*log(c*x - 1)/
(c^5*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^4 \operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

3.38 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^2} dx$

Optimal. Leaf size=179

$$-\frac{b}{2c^4d^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}}+\frac{b\cosh^{-1}(cx)}{2c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+$$

```
[Out] 1/2*b*arccosh(c*x)/c^4/d^2+1/2*x^2*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-
1/2*(a+b*arccosh(c*x))^2/b/c^4/d^2+(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)/c^4/d^2+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)/c^4/d^2-1/2*b/c^4/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*(c*x-1)^(1/2
)/c^4/d^2/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5934, 5913, 3797, 2221, 2317, 2438, 91, 12, 79, 54}

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^4d^2}-\frac{b\sqrt{cx-1}}{2c^4d^2\sqrt{cx+1}}-\frac{b}{2c^4d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{b\cosh^{-1}(cx)}{2c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -1/2*b/(c^4*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x])/(2*c^4*d
^2*Sqrt[1 + c*x]) + (b*ArcCosh[c*x])/(2*c^4*d^2) + (x^2*(a + b*ArcCosh[c*x]
))/(2*c^2*d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*c^4*d^2) + ((a +
 b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d^2) + (b*PolyLog[2, E^(
2*ArcCosh[c*x])])/(2*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2} - \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\,\text{cot}\ldots\right.}{c\ldots}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2} \ldots$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{(a\ldots}{\ldots}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cos\ldots\right.}{2c^2d^2\left(1\ldots\right.}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cos\ldots\right.}{2c^2d^2\left(1\ldots\right.}$$

Mathematica [A]
time = 0.37, size = 209, normalized size = 1.17

$$\frac{-b\sqrt{\frac{-1+cx}{1+cx}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} + \frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} - \frac{2a}{-1+c^2x^2} + \frac{b\cosh^{-1}(cx)}{1-cx} + \frac{b\cosh^{-1}(cx)}{1+cx} - 2b\cosh^{-1}(cx)^2 + 4b\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right) + 4b\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right) + 2a\log\left(1-c^2x^2\right) + 4b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right) + 4b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4c^4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (-(b*Sqrt[(-1 + c*x)/(1 + c*x)]) + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x)
 + (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a)/(-1 + c^2*x^2) + (b
```

```
*ArcCosh[c*x])/(1 - c*x) + (b*ArcCosh[c*x])/(1 + c*x) - 2*b*ArcCosh[c*x]^2
+ 4*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Log[1 + E^Arc
Cosh[c*x]] + 2*a*Log[1 - c^2*x^2] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*P
olyLog[2, E^ArcCosh[c*x]])/(4*c^4*d^2)
```

Maple [A]
time = 7.83, size = 278, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{a}{4d^2(cx-1)}+\frac{a\ln(cx-1)}{2d^2}+\frac{a}{4d^2(cx+1)}+\frac{a\ln(cx+1)}{2d^2}-\frac{b\,\mathrm{arccosh}(cx)^2}{2d^2}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{2d^2(c^2x^2-1)}+\frac{b\,c^2x^2}{2d^2(c^2x^2-1)}-\frac{b\,\mathrm{arccosh}(\ldots}{2d^2(c^2x^2\ldots}}{\ldots}$
default	$\frac{-\frac{a}{4d^2(cx-1)}+\frac{a\ln(cx-1)}{2d^2}+\frac{a}{4d^2(cx+1)}+\frac{a\ln(cx+1)}{2d^2}-\frac{b\,\mathrm{arccosh}(cx)^2}{2d^2}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{2d^2(c^2x^2-1)}+\frac{b\,c^2x^2}{2d^2(c^2x^2-1)}-\frac{b\,\mathrm{arccosh}(\ldots}{2d^2(c^2x^2\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/4*a/d^2/(c*x-1)+1/2*a/d^2*ln(c*x-1)+1/4*a/d^2/(c*x+1)+1/2*a/d^2*l
n(c*x+1)-1/2*b/d^2*arccosh(c*x)^2-1/2*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*c*x+1/2*b/d^2/(c^2*x^2-1)*c^2*x^2-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*
x)-1/2*b/d^2/(c^2*x^2-1)+b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))+b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d^2*arccosh(c*x)
*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c*
x - 1) + (c^2*x^2 - 1)*log(c*x - 1)^2 - 4*((c^2*x^2 - 1)*log(c*x + 1) + (c^
2*x^2 - 1)*log(c*x - 1) - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2)/(c
^6*d^2*x^2 - c^4*d^2) - 8*integrate(1/2*((c^2*x^2 - 1)*log(c*x + 1) + (c^2*
x^2 - 1)*log(c*x - 1) - 1)/(c^8*d^2*x^5 - 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*
d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))
, x)) - 1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

3.39 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x(a+b\cosh^{-1}(cx))}{2c^2d^2\,(1-c^2x^2)}-\frac{(a+b\cosh^{-1}(cx))\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^3d^2}-\frac{b\mathrm{PolyLog}\left(2,-\right.}{2c^3d}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-(a+b*arccosh(c*x))*arctanh(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2-1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/c^3/d^2+1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/
d^2-1/2*b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5934, 5903, 4267, 2317, 2438, 75}

$$-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{c^3d^2}+\frac{x(a+b\cosh^{-1}(cx))}{2c^2d^2\,(1-c^2x^2)}-\frac{b\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{b\mathrm{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}-\frac{b}{2c^3d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -1/2*b/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2
*c^2*d^2*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c
^3*d^2) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^3*d^2) + (b*PolyLog[2, E^Arc
Cosh[c*x]])/(2*c^3*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{\mathrm{Subst}\left(\int(a+bx)\mathrm{csch}(\right.}{2c^3}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tan}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tan}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)\tan}{c^3d^2}$$

Mathematica [A]

time = 0.46, size = 206, normalized size = 1.66

$$\frac{b\sqrt{\frac{-1+cx}{1+cx}}+\frac{b\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bcx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2acx}{-1+c^2x^2}+\frac{b\cosh^{-1}(cx)}{1-cx}-\frac{b\cosh^{-1}(cx)}{1+cx}+2b\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right)+a\log(1-cx)-a\log(1+cx)-2b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+2b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4c^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (b*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) +
(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + (
b*ArcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 2*b*ArcCosh[c*x]*L
og[1 - E^ArcCosh[c*x]] - 2*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + a*Log[1
 - c*x] - a*Log[1 + c*x] - 2*b*PolyLog[2, -E^ArcCosh[c*x]] + 2*b*PolyLog[2,
 E^ArcCosh[c*x]])/(4*c^3*d^2)
```

Maple [A]
time = 6.57, size = 230, normalized size = 1.85

method	result
derivativedivides	$-\frac{a}{4d^2(cx-1)}+\frac{a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}-\frac{a\ln(cx+1)}{4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{c\right.}}{2d}$
default	$-\frac{a}{4d^2(cx-1)}+\frac{a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}-\frac{a\ln(cx+1)}{4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2\left(c^2x^2-1\right)}-\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{c\right.}}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/4*a/d^2/(c*x-1)+1/4*a/d^2*ln(c*x-1)-1/4*a/d^2/(c*x+1)-1/4*a/d^2*l
n(c*x+1)-1/2*b/d^2/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*b/d^2/(c^2*x
^2-1)*arccosh(c*x)*c*x-1/2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))-1/2*b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*b/d^2*ar
ccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*b/d^2*polylog(2,c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2
+ c^2*d^2), x) + 8*c^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) + log(c*x + 1)/(c^5*d^2
```

```
) - log(c*x - 1)/(c^5*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^6*d^
2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x) + 3*(c*(2/(c^6*d^2*x - c^5*d^2) - log(
c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 4*log(c*x - 1)/(c^6*d^2*x^2
- c^4*d^2))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x +
 1)*log(c*x - 1) - 4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*lo
g(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^5*d^2*x^2 - c^3*d^2)
 + 64*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log
(c*x - 1))/(c^7*d^2*x^5 - 2*c^5*d^2*x^3 + c^3*d^2*x + (c^6*d^2*x^4 - 2*c^4*
d^2*x^2 + c^2*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(
c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))*b - 1/4*a*(2*x/(c^4*d
^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

3.40 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{bx}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\,(1-c^2x^2)}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-1/2*b*x/c/d^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5914, 39}

$$\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\,(1-c^2x^2)}-\frac{bx}{2cd^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(b*x)/(c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*c
^2*d^2*(1 - c^2*x^2))
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{a+b\cosh^{-1}(cx)}{2c^2d^2\,(1-c^2x^2)}+\frac{b\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}\\ &= -\frac{bx}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\,(1-c^2x^2)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.87

$$\frac{a + bcx\sqrt{-1+cx}\sqrt{1+cx} + b\cosh^{-1}(cx)}{2c^2d^2 - 2c^4d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*ArcCosh[c*x])/(2*c^2*d^2 - 2*c^
4*d^2*x^2)
```

Maple [A]
time = 2.86, size = 64, normalized size = 1.05

method	result	size
derivativedivides	$\frac{-\frac{a}{2d^2(c^2x^2-1)}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2(c^2x^2-1)}-\frac{cx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}}{c^2}$	64
default	$\frac{-\frac{a}{2d^2(c^2x^2-1)}+\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2(c^2x^2-1)}-\frac{cx}{2\sqrt{cx-1}\sqrt{cx+1}}\right)}{d^2}}{c^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2-1)+b/d^2*(-1/2/(c^2*x^2-1)*arccosh(c*x)-1/2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*c*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(52) = 104$.
time = 0.28, size = 134, normalized size = 2.20

$$-\frac{1}{4}\left(\left(\frac{\sqrt{c^2x^2-1}\,c^2d^2}{c^7d^4x+c^6d^4}+\frac{\sqrt{c^2x^2-1}\,c^2d^2}{c^7d^4x-c^6d^4}\right)c^2+\frac{2\,\operatorname{arcosh}(cx)}{c^4d^2x^2-c^2d^2}\right)b-\frac{a}{2\,(c^4d^2x^2-c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((sqrt(c^2*x^2 - 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(c^2*x^2 - 1)*
c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 + 2*arccosh(c*x)/(c^4*d^2*x^2 - c^2*d^2)
)*b - 1/2*a/(c^4*d^2*x^2 - c^2*d^2)
```

Fricas [A]
time = 0.35, size = 65, normalized size = 1.07

$$-\frac{ac^2x^2 + \sqrt{c^2x^2-1}\,bcx + b\log\left(cx + \sqrt{c^2x^2-1}\right)}{2\left(c^4d^2x^2 - c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] -1/2*(a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b*c*x + b*log(c*x + sqrt(c^2*x^2 - 1)))
/(c^4*d^2*x^2 - c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx\,\mathrm{acosh}\,(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*acosh(c*x)/(
c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

3.41 $\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=120

$$-\frac{b}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x(a+b\cosh^{-1}(cx))}{2d^2(1-c^2x^2)}+\frac{(a+b\cosh^{-1}(cx))\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd^2}+\frac{b\text{PolyLog}\left(2,-e\right.}{2cd^2}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+(a+b*arccosh(c*x))*arctanh(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2+1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/c/d^2-1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2-1/2*b/
c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5901, 5903, 4267, 2317, 2438, 75}

$$\frac{x(a+b\cosh^{-1}(cx))}{2d^2(1-c^2x^2)}+\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{cd^2}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b}{2cd^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] -1/2*b/(c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*d
^2*(1 - c^2*x^2)) + ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d^2)
+ (b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c*d^2) - (b*PolyLog[2, E^ArcCosh[c*x]]
)/(2*c*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{(bc)\int\frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}+\frac{\int\frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2d}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\,dx\right.}{2cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{cd^2}
\end{aligned}$$

Mathematica [A]
time = 0.87, size = 189, normalized size = 1.58

$$\frac{\frac{-2acx-2b\sqrt{\frac{-1+cx}{1+cx}}-2bcx\sqrt{\frac{-1+cx}{1+cx}}-2b\cosh^{-1}(cx)\left(cx+\left(-1+c^2x^2\right)\log\left(1-e^{\cosh^{-1}(cx)}\right)+\left(1-c^2x^2\right)\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)+\left(a-ac^2x^2\right)\log(1-cx)-a\log(1+cx)+ac^2x^2\log(1+cx)}{-1+c^2x^2}+2b\operatorname{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-2b\operatorname{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4cd^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((-2*a*c*x - 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*b*c*x*Sqrt[(-1 + c*x)/(1 +
c*x)] - 2*b*ArcCosh[c*x]*(c*x + (-1 + c^2*x^2)*Log[1 - E^ArcCosh[c*x]] + (1
 - c^2*x^2)*Log[1 + E^ArcCosh[c*x]]) + (a - a*c^2*x^2)*Log[1 - c*x] - a*Log
[1 + c*x] + a*c^2*x^2*Log[1 + c*x])/(-1 + c^2*x^2) + 2*b*PolyLog[2, -E^ArcC
osh[c*x]] - 2*b*PolyLog[2, E^ArcCosh[c*x]])/(4*c*d^2)
```

Maple [A]
time = 4.25, size = 230, normalized size = 1.92

method	result
derivativedivides	$\frac{-\frac{a}{4d^2(cx+1)}+\frac{a\ln(cx+1)}{4d^2}-\frac{a}{4d^2(cx-1)}-\frac{a\ln(cx-1)}{4d^2}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2(c^2x^2-1)}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2(c^2x^2-1)}+\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{c\right.}}{2d}}{}$
default	$\frac{-\frac{a}{4d^2(cx+1)}+\frac{a\ln(cx+1)}{4d^2}-\frac{a}{4d^2(cx-1)}-\frac{a\ln(cx-1)}{4d^2}-\frac{b\,\mathrm{arccosh}(cx)cx}{2d^2(c^2x^2-1)}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2(c^2x^2-1)}+\frac{b\,\mathrm{arccosh}(cx)\ln\left(1+cx+\sqrt{c\right.}}{2d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4*a/d^2/(c*x+1)+1/4*a/d^2*ln(c*x+1)-1/4*a/d^2/(c*x-1)-1/4*a/d^2*ln(
c*x-1)-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c*x-1/2*b/d^2/(c^2*x^2-1)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)+1/2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))+1/2*b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b/d^2*arcc
osh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b/d^2*polylog(2,c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 +
 d^2), x) - 8*c^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - l
og(c*x - 1)/(c^3*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x^4
 - 2*c^2*d^2*x^2 + d^2), x) + 3*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/
(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2
))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c
*x - 1) + 4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1
```

```
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^3*d^2*x^2 - c*d^2) + 64*integ
rate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))
/(c^5*d^2*x^5 - 2*c^3*d^2*x^3 + c*d^2*x + (c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^
2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(c*x - 1)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x))*b - 1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(
c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acosh(c*x)/(c**4
*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2,x)
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2, x)
```

3.42 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{bcx}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2(1-c^2x^2)}+\frac{2(a+b\cosh^{-1}(cx))\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d^2}+\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+2*(a+b*arccosh(c*x))*arctanh((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)/d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-
1/2*b*c*x/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5936, 5916, 5569, 4267, 2317, 2438, 39}

$$\frac{a+b\cosh^{-1}(cx)}{2d^2(1-c^2x^2)}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{d^2}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{bcx}{2d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c*x)/(d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*d
^2*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^
2 + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, E^(2*ArcCos
h[c*x])])/(2*d^2)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx = \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} - \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,d\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} - \frac{2\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x,\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cos}\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cos}\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)} + \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cos}\right.}{d^2}$$

Mathematica [A]
time = 0.47, size = 149, normalized size = 1.28

$$\frac{\frac{a}{1-c^2x^2} + 2a\log(x) - a\log\left(1-c^2x^2\right) + b\left(\frac{cx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx} + \frac{\cosh^{-1}(cx)}{1-c^2x^2} - 2\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right) + 2\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right) - \text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right) + \text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a/(1 - c^2*x^2) + 2*a*Log[x] - a*Log[1 - c^2*x^2] + b*((c*x*Sqrt[(-1 + c*x
)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 2*ArcCosh[c*x]*Log[1
 - E^(-2*ArcCosh[c*x])] + 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(143) = 286$.
time = 4.54, size = 339, normalized size = 2.92

method	result
derivativedivides	$\frac{a\ln(cx)}{d^2} + \frac{a}{4d^2(cx+1)} - \frac{a\ln(cx+1)}{2d^2} - \frac{a}{4d^2(cx-1)} - \frac{a\ln(cx-1)}{2d^2} - \frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{2d^2(c^2x^2-1)} + \frac{b\,c^2x^2}{2d^2(c^2x^2-1}$
default	$\frac{a\ln(cx)}{d^2} + \frac{a}{4d^2(cx+1)} - \frac{a\ln(cx+1)}{2d^2} - \frac{a}{4d^2(cx-1)} - \frac{a\ln(cx-1)}{2d^2} - \frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{2d^2(c^2x^2-1)} + \frac{b\,c^2x^2}{2d^2(c^2x^2-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^2*ln(c*x)+1/4*a/d^2/(c*x+1)-1/2*a/d^2*ln(c*x+1)-1/4*a/d^2/(c*x-1)-1/2*a
/d^2*ln(c*x-1)-1/2*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x+1/2*b/
d^2/(c^2*x^2-1)*c^2*x^2-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)-1/2*b/d^2/(c^2*x
^2-1)-b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*polylo
g(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d^2*arccosh(c*x)*ln(1+(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2)/d^2-b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*polyl
og(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*log
(x)/d^2) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^4*d^2*x^5
- 2*c^2*d^2*x^3 + d^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^5-2c^2x^3+x}\,dx+\int \frac{b\operatorname{acosh}\left(cx\right)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*acosh(c*x)/(c**4
*x**5 - 2*c**2*x**3 + x), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2), x)
```

3.43 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{bc}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^2x\,(1-c^2x^2)}+\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\,(1-c^2x^2)}+\frac{bc\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d^2}-$$

```
[Out] (-a-b*arccosh(c*x))/d^2/x/(-c^2*x^2+1)+3/2*c^2*x*(a+b*arccosh(c*x))/d^2/(-c
^2*x^2+1)+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+3*c*(a+b*arccosh(c*x)
)*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+3/2*b*c*polylog(2,-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/d^2-3/2*b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/d^2-1/2*b*c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5932, 5901, 5903, 4267, 2317, 2438, 75, 106, 21, 94, 211}

$$\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\,(1-c^2x^2)}-\frac{a+b\cosh^{-1}(cx)}{d^2x\,(1-c^2x^2)}+\frac{3c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{bc\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d^2}+\frac{3bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{3bc\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{bc}{2d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^2*x
*(1 - c^2*x^2)) + (3*c^2*x*(a + b*ArcCosh[c*x]))/(2*d^2*(1 - c^2*x^2)) + (b
*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (3*c*(a + b*ArcCosh[c*x])*Ar
cTanh[E^ArcCosh[c*x]])/d^2 + (3*b*c*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) -
(3*b*c*PolyLog[2, E^ArcCosh[c*x]])/(2*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
```

```
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx - \frac{(bc)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2}$$

$$= \frac{bc}{d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{\cdots}{x\sqrt{-\cdots}}}{\cdots}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} - \frac{(3c)\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{3c(a\cdots}{\cdots}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan\cdots}{\cdots}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan\cdots}{\cdots}$$

Mathematica [A]
time = 0.43, size = 283, normalized size = 1.66

$$\frac{-\frac{4a}{x}+bc\sqrt{\frac{-1+cx}{1+cx}}+\frac{bc\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{bc^2x\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}-\frac{2ac^2x}{-1+c^2x^2}-\frac{4b\cosh^{-1}(cx)}{x}+\frac{bc\cosh^{-1}(cx)}{1-cx}-\frac{bc\cosh^{-1}(cx)}{1+cx}+\frac{4bc\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-6bc\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)+6bc\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right)-3ac\log(1-cx)+3ac\log(1+cx)+6bc\,\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-6bc\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{4d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]

[Out] ((-4*a)/x + b*c*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + (b*c^2*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c^2*x)/(-1 + c^2*x^2) - (4*b*ArcCosh[c*x])/x + (b*c*ArcCosh[c*x])/(1 - c*x) - (b*c*ArcCosh[c*x])/(1 + c*x) + (4*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*b*c*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 6*b*c*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] - 3*a*c*Log[1 - c*x] + 3*a*c*Log[1 + c*x] + 6*b*c*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c*PolyLog[2, E^ArcCosh[c*x]])/(4*d^2)

Maple [A]
time = 5.84, size = 256, normalized size = 1.51

method	result
derivativedivides	$c\left(-\frac{a}{4d^2(cx+1)}+\frac{3a\ln(cx+1)}{4d^2}-\frac{a}{d^2cx}-\frac{a}{4d^2(cx-1)}-\frac{3a\ln(cx-1)}{4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2(c^2x^2-1)}-\frac{3b\operatorname{arccos}}{2d^2(c^2}\right.$
default	$c\left(-\frac{a}{4d^2(cx+1)}+\frac{3a\ln(cx+1)}{4d^2}-\frac{a}{d^2cx}-\frac{a}{4d^2(cx-1)}-\frac{3a\ln(cx-1)}{4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{2d^2(c^2x^2-1)}-\frac{3b\operatorname{arccos}}{2d^2(c^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] c*(-1/4*a/d^2/(c*x+1)+3/4*a/d^2*ln(c*x+1)-a/d^2/c/x-1/4*a/d^2/(c*x-1)-3/4*a/d^2*ln(c*x-1)-1/2*b/d^2/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-3/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c*x+b/d^2/(c^2*x^2-1)/c/x*arccosh(c*x)+2*b/d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*b/d^2*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")

[Out] 1/64*(576*c^5*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x) - 24*c^4*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2)) - 192*c^4*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x) + 9*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1

```
)/(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d
^2))*c^3 + 16*c^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x
 - 1)/(c*d^2)) + 192*c^2*integrate(1/8*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^
2*x^2 + d^2), x) - 4*(3*(c^3*x^3 - c*x)*log(c*x + 1)^2 + 6*(c^3*x^3 - c*x)*
log(c*x + 1)*log(c*x - 1) + 4*(6*c^2*x^2 - 3*(c^3*x^3 - c*x)*log(c*x + 1) +
 3*(c^3*x^3 - c*x)*log(c*x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
)/(c^2*d^2*x^3 - d^2*x) + 64*integrate(-1/4*(6*c^3*x^2 - 3*(c^4*x^3 - c^2*x
)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(c*x - 1) - 4*c)/(c^5*d^2*x^6 - 2*c
^3*d^2*x^4 + c*d^2*x^2 + (c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(c*x + 1
)*sqrt(c*x - 1)), x))*b - 1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) -
3*c*log(c*x + 1)/d^2 + 3*c*log(c*x - 1)/d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^6-2c^2x^4+x^2}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^4x^6-2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*acosh(c*x)/(c
**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^2\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

3.44 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=152

$$-\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cos}\right.}{d^2}$$

```
[Out] c^2*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+1/2*(-a-b*arccosh(c*x))/d^2/x^2/(-c
^2*x^2+1)+4*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)/d^2+b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-b*c^2*po
lylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5932, 5936, 5916, 5569, 4267, 2317, 2438, 39, 105, 12}

$$\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{bc^2\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc}{2d^2x\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*(a + b*ArcCosh[c*x])
)/(d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (4
*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2 + (b*c^2*PolyLog
[2, -E^(2*ArcCosh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\left(2c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx-\frac{(bc)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{(bc)\int}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3x}{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{\left(4c^2\right)\mathrm{S}}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2(a}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2(a}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2(a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(152) = 304.
time = 0.33, size = 319, normalized size = 2.10

$$\frac{a-2ac^2x^2-bcx\sqrt{\frac{-1+cx}{1+cx}}-bc^2x^2\sqrt{\frac{-1+cx}{1+cx}}+b\cosh^{-1}(cx)-2bc^2x^2\cosh^{-1}(cx)+4bc^2x^2\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-4bc^4x^4\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-4bc^2x^2\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)+4bc^4x^4\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)-4ac^2x^2\log(x)+4ac^4x^4\log(x)+2ac^2x^2\log\left(1-c^2x^2\right)-2ac^4x^4\log\left(1-c^2x^2\right)-2bc^2x^2(-1+c^2x^2)\,\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+2bc^2x^2(-1+c^2x^2)\,\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)}{2d^2x^2\left(-1+c^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a - 2*a*c^2*x^2 - b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)] + b*ArcCosh[c*x] - 2*b*c^2*x^2*ArcCosh[c*x] + 4*b*c^2*x^2*A
rcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 -
 E^(-2*ArcCosh[c*x])] - 4*b*c^2*x^2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x]
)] + 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 4*a*c^2*x^2*Lo
g[x] + 4*a*c^4*x^4*Log[x] + 2*a*c^2*x^2*Log[1 - c^2*x^2] - 2*a*c^4*x^4*Log[
1 - c^2*x^2] - 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, -E^(-2*ArcCosh[c*x])]
+ 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, E^(-2*ArcCosh[c*x])])/(2*d^2*x^2*(-
1 + c^2*x^2))
```

Maple [A]
time = 5.91, size = 347, normalized size = 2.28

method	result
derivativedivides	$c^2\left(-\frac{a}{4d^2(cx-1)}-\frac{a\ln(cx-1)}{d^2}+\frac{a}{4d^2(cx+1)}-\frac{a\ln(cx+1)}{d^2}-\frac{a}{2d^2c^2x^2}+\frac{2a\ln(cx)}{d^2}-\frac{b\sqrt{cx+1}\,\sqrt{cx-}}{2d^2(c^2x^2-1)cx}\right.$
default	$c^2\left(-\frac{a}{4d^2(cx-1)}-\frac{a\ln(cx-1)}{d^2}+\frac{a}{4d^2(cx+1)}-\frac{a\ln(cx+1)}{d^2}-\frac{a}{2d^2c^2x^2}+\frac{2a\ln(cx)}{d^2}-\frac{b\sqrt{cx+1}\,\sqrt{cx-}}{2d^2(c^2x^2-1)cx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/4*a/d^2/(c*x-1)-a/d^2*ln(c*x-1)+1/4*a/d^2/(c*x+1)-a/d^2*ln(c*x+1)-1
/2*a/d^2/c^2/x^2+2*a/d^2*ln(c*x)-1/2*b/d^2/(c^2*x^2-1)/c/x*(c*x+1)^(1/2)*(c
*x-1)^(1/2)-b/d^2/(c^2*x^2-1)*arccosh(c*x)+1/2*b/d^2/(c^2*x^2-1)/c^2/x^2*ar
ccosh(c*x)-2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b/d
^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b/d^2*arccosh(c*x)*ln(1+(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)/d^2-2*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*
b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2
+ (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^7-2c^2x^5+x^3}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*acosh(c*x)/(c
**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2),x)
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2), x)
```

3.45 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}+\frac{5c^4x\left(a+\right.}{2d^2}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d^2/x^3/(-c^2*x^2+1)-5/3*c^2*(a+b*arccosh(c*x))/d^2
/x/(-c^2*x^2+1)+5/2*c^4*x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+13/6*b*c^3*ar
ctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+5*c^3*(a+b*arccosh(c*x))*arctanh(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+5/2*b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/d^2-5/2*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-1/
3*b*c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c/d^2/x^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5932, 5901, 5903, 4267, 2317, 2438, 75, 106, 21, 94, 211, 105, 12}

$$\frac{5c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}+\frac{5c^4x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{13bc^3\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{6d^2}+\frac{5bc^3\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{5bc^3\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{bc^3}{3d^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc}{6d^2x^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/3*(b*c^3)/(d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c)/(6*d^2*x^2*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(3*d^2*x^3*(1 - c^2*x^2)) - (5
*c^2*(a + b*ArcCosh[c*x]))/(3*d^2*x*(1 - c^2*x^2)) + (5*c^4*x*(a + b*ArcCos
h[c*x]))/(2*d^2*(1 - c^2*x^2)) + (13*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c
*x]])/(6*d^2) + (5*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d^2 +
(5*b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (5*b*c^3*PolyLog[2, E^ArcCo
sh[c*x]])/(2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}+\frac{1}{3}\left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx-\frac{(bc)\int \frac{1}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}}{3d^2}$$

$$= -\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}+$$

$$= \frac{5bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}-$$

Mathematica [A]
time = 1.03, size = 377, normalized size = 1.52

$$-\frac{\frac{4a}{x^3}+\frac{24ac^2}{x}-3bc^3\sqrt{\frac{-1+cx}{1+cx}}+\frac{3bc^3\sqrt{\frac{-1+cx}{1+cx}}}{-1+cx}+\frac{3bc^4x\sqrt{\frac{-1+cx}{1+cx}}}{-1+cx}-\frac{2bc^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{6ac^4x}{-1+c^2x^2}+\frac{4b\cosh^{-1}(cx)}{x^3}+\frac{24bc^2\cosh^{-1}(cx)}{x}+\frac{3bc^3\cosh^{-1}(cx)}{-1+cx}+\frac{3bc^3\cosh^{-1}(cx)}{1+cx}-\frac{26bc^3\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}+30bc^3\cosh^{-1}(cx)\log\left(1-e^{\cosh^{-1}(cx)}\right)-30bc^3\cosh^{-1}(cx)\log\left(1+e^{\cosh^{-1}(cx)}\right)+15ac^3\log(1-cx)-15ac^3\log(1+cx)-30bc^3\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+30bc^3\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{12d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/12*((4*a)/x^3 + (24*a*c^2)/x - 3*b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)] + (3*b
*c^3*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) + (3*b*c^4*x*Sqrt[(-1 + c*x)/(1
 + c*x)])/(-1 + c*x) - (2*b*c^3)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c)/(
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*a*c^4*x)/(-1 + c^2*x^2) + (4*b*ArcCo
sh[c*x])/x^3 + (24*b*c^2*ArcCosh[c*x])/x + (3*b*c^3*ArcCosh[c*x])/(-1 + c*x
) + (3*b*c^3*ArcCosh[c*x])/(1 + c*x) - (26*b*c^3*Sqrt[-1 + c^2*x^2]*ArcTan[
Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 30*b*c^3*ArcCosh[c*x]
*Log[1 - E^ArcCosh[c*x]] - 30*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] +
15*a*c^3*Log[1 - c*x] - 15*a*c^3*Log[1 + c*x] - 30*b*c^3*PolyLog[2, -E^ArcC
osh[c*x]] + 30*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/d^2
```

Maple [A]
time = 6.30, size = 335, normalized size = 1.35

method	result
derivativedivides	$c^3\left(-\frac{a}{4d^2(cx-1)}-\frac{5a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}+\frac{5a\ln(cx+1)}{4d^2}-\frac{a}{3d^2c^3x^3}-\frac{2a}{d^2cx}-\frac{5b\,\mathrm{arccosh}(cx)cx}{2d^2(c^2x^2-1)}-\frac{b\sqrt{c}}{}\right.$
default	$c^3\left(-\frac{a}{4d^2(cx-1)}-\frac{5a\ln(cx-1)}{4d^2}-\frac{a}{4d^2(cx+1)}+\frac{5a\ln(cx+1)}{4d^2}-\frac{a}{3d^2c^3x^3}-\frac{2a}{d^2cx}-\frac{5b\,\mathrm{arccosh}(cx)cx}{2d^2(c^2x^2-1)}-\frac{b\sqrt{c}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*a/d^2/(c*x-1)-5/4*a/d^2*ln(c*x-1)-1/4*a/d^2/(c*x+1)+5/4*a/d^2*ln(
c*x+1)-1/3*a/d^2/c^3/x^3-2*a/d^2/c/x-5/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c*x
-1/3*b/d^2/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+5/3*b/d^2/(c^2*x^2-1)/c/
x*arccosh(c*x)-1/6*b/d^2/(c^2*x^2-1)/c^2/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/
3*b/d^2/(c^2*x^2-1)/c^3/x^3*arccosh(c*x)+13/3*b/d^2*arctan(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))+5/2*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*b/d
^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*b/d^2*dilog(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 - 1
0*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a + 1/192*(8640*c^7*integrate(1/24*
x^5*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x) - 120*c^6*(2*x
/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
 - 2880*c^6*integrate(1/24*x^4*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 +
d^2*x^2), x) + 45*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/(c^3*d^2) + lo
g(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2))*c^5 + 80*c^
4*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2)) +
 2880*c^4*integrate(1/24*x^2*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^
2*x^2), x) + 16*c^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x
+ 1)/d^2 + 3*c*log(c*x - 1)/d^2) - 4*(15*(c^5*x^5 - c^3*x^3)*log(c*x + 1)^2
 + 30*(c^5*x^5 - c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 20*c^
2*x^2 - 15*(c^5*x^5 - c^3*x^3)*log(c*x + 1) + 15*(c^5*x^5 - c^3*x^3)*log(c*
x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*d^2*x^5 - d^2*x^3)
 + 192*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^5 - c^4*x^3)*lo
g(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(c*x - 1) - 4*c)/(c^5*d^2*x^8 - 2*c^
```

```
3*d^2*x^6 + c*d^2*x^4 + (c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3)*sqrt(c*x +
1)*sqrt(c*x - 1)), x))*b
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8-2c^2x^6+x^4}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^4x^8-2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*acosh(c*x)/(c
**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2),x)
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2), x)
```

3.46 $\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=249

$$\frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}}-\frac{b(-1+cx)^{3/2}}{12c^5d^3(1+cx)^{3/2}}+\frac{3b}{8c^5d^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/12*b*x^3/c^2/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-1/12*b*(c*x-1)^(3/2)/c^5/d^3
/(c*x+1)^(3/2)+1/4*x^3*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b
*arccosh(c*x))/c^4/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3+3/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/c^5/d^3-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3+1/
4*b/c^5/d^3/(c*x+1)^(3/2)/(c*x-1)^(1/2)+3/8*b/c^5/d^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5934, 5903, 4267, 2317, 2438, 75, 96, 91, 21, 37}

$$\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4c^5d^3}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}+\frac{3b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}-\frac{3b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3b}{8c^5d^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b(cx-1)^{3/2}}{12c^5d^3(cx+1)^{3/2}}+\frac{b}{4c^5d^3\sqrt{cx-1}\,(cx+1)^{3/2}}+\frac{bx^3}{12c^2d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c^2*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^5*d^3*Sqrt[-
1 + c*x]*(1 + c*x)^(3/2)) - (b*(-1 + c*x)^(3/2))/(12*c^5*d^3*(1 + c*x)^(3/2
)) + (3*b)/(8*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(a + b*ArcCosh[c
*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*x*(a + b*ArcCosh[c*x]))/(8*c^4*d^3*(
1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*c^5*d^3
) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c^5*d^3) - (3*b*PolyLog[2, E^ArcCo
sh[c*x]])/(8*c^5*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx &= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\int\frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3}-\frac{3\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx}{4c^2d} \\
&= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)} \\
&= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}}+\frac{3b}{8c^5d^3\sqrt{-1+}} \\
&= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}}+\frac{3b}{8c^5d^3\sqrt{-1+}} \\
&= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}}-\frac{b(-1+cx)^3}{12c^5d^3(1+cx} \\
&= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}}-\frac{b(-1+cx)^3}{12c^5d^3(1+cx}
\end{aligned}$$

Mathematica [A]

time = 1.06, size = 287, normalized size = 1.15

$$\frac{-\frac{b(-2+cx)\sqrt{1+cx}}{(-1+cx)^{3/2}}+\frac{b\sqrt{-1+cx}\,(2+cx)}{(1+cx)^{3/2}}+\frac{12acx}{\left(-1+c^2x^2\right)^2}+\frac{30acx}{-1+c^2x^2}+\frac{3b\cosh^{-1}(cx)}{(-1+cx)^2}-\frac{3b\cosh^{-1}(cx)}{(1+cx)^2}-15b\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}}+\frac{\cosh^{-1}(cx)}{1-cx}\right)-15b\left(\sqrt{\frac{-1+cx}{1+cx}}-\frac{\cosh^{-1}(cx)}{1+cx}\right)+\frac{9}{2}b\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1-e^{\cosh^{-1}(cx)}\right)\right)-\frac{9}{2}b\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)-9a\log(1-cx)+9a\log(1+cx)+18b\,\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-18b\,\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{48c^5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (30*a*c*x)/(-1 + c^2*
x^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 1
5*b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 15*b*(Sqrt
[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) + (9*b*ArcCosh[c*x]*(ArcCo
sh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 - (9*b*ArcCosh[c*x]*(ArcCosh[c*x] -
 4*Log[1 + E^ArcCosh[c*x]]))/2 - 9*a*Log[1 - c*x] + 9*a*Log[1 + c*x] + 18*b
*PolyLog[2, -E^ArcCosh[c*x]] - 18*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^5*d^3
)
```

Maple [A]
time = 10.22, size = 352, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{a}{16d^3(cx-1)^2}+\frac{5a}{16d^3(cx-1)}-\frac{3a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}+\frac{5a}{16d^3(cx+1)}+\frac{3a\ln(cx+1)}{16d^3}+\frac{5b\,\mathrm{arccosh}(cx)c^3x^3}{8d^3\left(c^4x^4-2c^2x^2+1\right)}+\frac{5b\sqrt{cx+1}\,\cdots}{8d^3\left(c^4x^4-\cdots\right)}\cdots}{\cdots}$
default	$\frac{\frac{a}{16d^3(cx-1)^2}+\frac{5a}{16d^3(cx-1)}-\frac{3a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}+\frac{5a}{16d^3(cx+1)}+\frac{3a\ln(cx+1)}{16d^3}+\frac{5b\,\mathrm{arccosh}(cx)c^3x^3}{8d^3\left(c^4x^4-2c^2x^2+1\right)}+\frac{5b\sqrt{cx+1}\,\cdots}{8d^3\left(c^4x^4-\cdots\right)}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/16*a/d^3/(c*x-1)^2+5/16*a/d^3/(c*x-1)-3/16*a/d^3*ln(c*x-1)-1/16*a/
d^3/(c*x+1)^2+5/16*a/d^3/(c*x+1)+3/16*a/d^3*ln(c*x+1)+5/8*b/d^3/(c^4*x^4-2*
c^2*x^2+1)*arccosh(c*x)*c^3*x^3+5/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*c^2*x^2-3/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c*x-1
3/24*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3/8*b/d^3*arcc
osh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*b/d^3*polylog(2,-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))-3/8*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))-3/8*b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3
*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 80*c^4*(2*(5*c^2*x^3 - 3*x)/(c^12*d^3
```

```
*x^4 - 2*c^10*d^3*x^2 + c^8*d^3) + 3*log(c*x + 1)/(c^9*d^3) - 3*log(c*x - 1
)/(c^9*d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^10*d^3*x^6 - 3*c
^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6
)/(c^12*d^3*x^3 - c^11*d^3*x^2 - c^10*d^3*x + c^9*d^3) - 5*log(c*x + 1)/(c^
9*d^3) + 5*log(c*x - 1)/(c^9*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^12*
d^3*x^4 - 2*c^10*d^3*x^2 + c^8*d^3))*c^3 - 48*c^2*(2*(c^2*x^3 + x)/(c^10*d^
3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3) - log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c
^7*d^3)) + 12288*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*
d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 3*log(c*x + 1)/(c^7*d^3)
 + 3*log(c*x - 1)/(c^7*d^3)) - 16*log(c*x - 1)/(c^10*d^3*x^4 - 2*c^8*d^3*x^
2 + c^6*d^3))*c - 32*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x
^4 - 2*c^2*x^2 + 1)*log(c*x + 1)*log(c*x - 1) - 4*(10*c^3*x^3 - 6*c*x + 3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x
 - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^9*d^3*x^4 - 2*c^7*d^3*x^2
 + c^5*d^3) + 2048*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3*(c^4*x^4 - 2*c^2*
x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^11*d^3
*x^7 - 3*c^9*d^3*x^5 + 3*c^7*d^3*x^3 - c^5*d^3*x + (c^10*d^3*x^6 - 3*c^8*d^
3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*in
tegrate(1/32*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c
^4*d^3), x))*b + 1/16*a*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
 c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^4}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx^4\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**4*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

3.47 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^3} dx$

Optimal. Leaf size=136

$$\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^4d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{x^4(a+b\cosh^{-1}(\ldots}{4d^3(1-c^2x^2)\ldots}$$

```
[Out] 1/12*b*x^3/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-1/4*b*arccosh(c*x)/c^4/d^3+1/4
*x^4*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+1/4*b/c^4/d^3/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+1/4*b*(c*x-1)^(1/2)/c^4/d^3/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5917, 100, 21, 91, 12, 79, 54}

$$\frac{x^4(a+b\cosh^{-1}(cx))}{4d^3(1-c^2x^2)^2}+\frac{b\sqrt{cx-1}}{4c^4d^3\sqrt{cx+1}}+\frac{b}{4c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^4*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b*Sqrt[-1 + c*x])/(4*c^4*d^3*Sqrt[1 + c*x]) - (b*A
rcCosh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{b\int \frac{x^2(-3-3cx)}{(-1+cx)^{3/2}(1+cx)^{5/2}}}{12cd^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}}{4cd^3}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^4\left(a+b\cosh^{-1}\right.}{4d^3\left(1-c^2x^2\right.}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^4\left(a+b\cosh^{-1}\right.}{4d^3\left(1-c^2x^2\right.}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}}$$

$$= \frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}}$$

Mathematica [A]
time = 0.10, size = 83, normalized size = 0.61

$$\frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(-3+4c^2x^2)+a(-3+6c^2x^2)+3b(-1+2c^2x^2)\cosh^{-1}(cx)}{12c^4d^3\left(-1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + 4*c^2*x^2) + a*(-3 + 6*c^2*x^2) +
 3*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

Maple [A]
time = 2.03, size = 136, normalized size = 1.00

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}\right)}{d^3}-\frac{b\left(-\frac{\mathrm{arccosh}(cx)}{16(cx+1)^2}+\frac{3\,\mathrm{arccosh}(cx)}{16(cx+1)}-\frac{\mathrm{arccosh}(cx)}{16(cx-1)^2}-\frac{3\,\mathrm{arccosh}(cx)}{16(cx-1)}-\frac{cx\left(4c^2x\right.}{12(cx-1)^{\frac{3}{2}}}\right.}{d^3}}{c^4}$

default	$-\frac{-\frac{a\left(-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}\right)}{d^3}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{16(cx+1)^2}+\frac{3\operatorname{arccosh}(cx)}{16(cx+1)}-\frac{\operatorname{arccosh}(cx)}{16(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16(cx-1)}-\frac{cx(4c}{12(cx-1)}\right.}{d^3}}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(-a/d^3*(-1/16/(c*x+1)^2+3/16/(c*x+1)-1/16/(c*x-1)^2-3/16/(c*x-1))-b/d^3*(-1/16*arccosh(c*x)/(c*x+1)^2+3/16*arccosh(c*x)/(c*x+1)-1/16*arccosh(c*x)/(c*x-1)^2-3/16*arccosh(c*x)/(c*x-1)-1/12*c*x*(4*c^2*x^2-3)/(c*x-1)^(3/2)/(c*x+1)^(3/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] 1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 16*integrate(1/4*(2*c^2*x^2 - 1)/(c^10*d^3*x^7 - 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 - c^4*d^3*x + (c^9*d^3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) + 1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)

Fricas [A]
time = 0.33, size = 101, normalized size = 0.74

$$\frac{3\,ac^4x^4+3\,(2\,bc^2x^2-b)\log\left(cx+\sqrt{c^2x^2-1}\right)+(4\,bc^3x^3-3\,bcx)\sqrt{c^2x^2-1}}{12\,(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] 1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx^3\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**3*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

3.48 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=186

$$\frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{8c^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}-$$

```
[Out] 1/12*b/c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*x*(a+b*arccosh(c*x))/c^2/d^3
/(-c^2*x^2+1)^2-1/8*x*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)-1/4*(a+b*arcc
osh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3-1/8*b*polylog(2,
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3+1/8*b*polylog(2,c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/c^3/d^3+1/8*b/c^3/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5934, 5901, 5903, 4267, 2317, 2438, 75}

$$-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4c^3d^3}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b}{8c^3d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b}{12c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(8*c^3*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) -
(x*(a + b*ArcCosh[c*x]))/(8*c^2*d^3*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*
ArcTanh[E^ArcCosh[c*x]])/(4*c^3*d^3) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c
^3*d^3) + (b*PolyLog[2, E^ArcCosh[c*x]])/(8*c^3*d^3)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^2}\,dx}{4c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}\right.}{4c^2d^3\left(1-c\right.}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}\right.}{4c^2d^3\left(1-c\right.}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}\right.}{4c^2d^3\left(1-c\right.}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}\right.}{4c^2d^3\left(1-c\right.}$$

Mathematica [A]
time = 0.99, size = 287, normalized size = 1.54

$$\frac{-\frac{b(-2+cx)\sqrt{1+cx}}{(-1+cx)^{3/2}}+\frac{b\sqrt{-1+cx}\,(2+cx)}{(1+cx)^{3/2}}+\frac{12acx}{(-1+c^2x^2)^2}+\frac{6acx}{-1+c^2x^2}+\frac{3b\cosh^{-1}(cx)}{(-1+cx)^2}-\frac{3b\cosh^{-1}(cx)}{(1+cx)^2}-3b\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}}+\frac{\cosh^{-1}(cx)}{1-cx}\right)-3b\left(\sqrt{\frac{-1+cx}{1+cx}}-\frac{\cosh^{-1}(cx)}{1+cx}\right)-\frac{3}{2}b\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1-e^{\cosh^{-1}(cx)}\right)\right)+\frac{3}{2}b\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)+3a\log(1-cx)-3a\log(1+cx)-6b\,\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)+6b\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{48c^3d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (6*a*c*x)/(-1 + c^2*x
^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 3*
b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) - (3*b*ArcCosh[c*x]*(ArcCosh[
c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 + (3*b*ArcCosh[c*x]*(ArcCosh[c*x] - 4*
Log[1 + E^ArcCosh[c*x]]))/2 + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] - 6*b*Pol
yLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^3*d^3)
```

Maple [A]
time = 7.19, size = 352, normalized size = 1.89

method	result
derivativedivides	$-\frac{a}{16d^3(cx+1)^2}+\frac{a}{16d^3(cx+1)}-\frac{a\ln(cx+1)}{16d^3}+\frac{a}{16d^3(cx-1)^2}+\frac{a}{16d^3(cx-1)}+\frac{a\ln(cx-1)}{16d^3}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{8d^3\left(c^4x^4-2c^2x^2+1\right)}+\frac{b\,a}{8d^3(}$

default	$-\frac{a}{16d^3(cx+1)^2}+\frac{a}{16d^3(cx+1)}-\frac{a\ln(cx+1)}{16d^3}+\frac{a}{16d^3(cx-1)^2}+\frac{a}{16d^3(cx-1)}+\frac{a\ln(cx-1)}{16d^3}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{8d^3(c^4x^4-2c^2x^2+1)}+\frac{b\,\mathrm{arcc}\cdots}{8d^3(c^4\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/16*a/d^3/(c*x+1)^2+1/16*a/d^3/(c*x+1)-1/16*a/d^3*ln(c*x+1)+1/16*a
/d^3/(c*x-1)^2+1/16*a/d^3/(c*x-1)+1/16*a/d^3*ln(c*x-1)+1/8*b/d^3/(c^4*x^4-2
*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+1/8*b/d^3/(c^4*x^4-2*c^2*x^
2+1)*arccosh(c*x)*c^3*x^3-1/24*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)+1/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c*x-1/8*b/d^3*arcc
osh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*b/d^3*polylog(2,-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))+1/8*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))+1/8*b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(6144*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*
x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) - 16*c^4*(2*(5*c^2*x^3 - 3*x)/(c^10*d^3*
x^4 - 2*c^8*d^3*x^2 + c^6*d^3) + 3*log(c*x + 1)/(c^7*d^3) - 3*log(c*x - 1)/
(c^7*d^3)) - 2048*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*
d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) + 6*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 5*log(c*x + 1)/(c^7*d^3)
 + 5*log(c*x - 1)/(c^7*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^10*d^3*x^
4 - 2*c^8*d^3*x^2 + c^6*d^3))*c^3 - 16*c^2*(2*(c^2*x^3 + x)/(c^8*d^3*x^4 -
2*c^6*d^3*x^2 + c^4*d^3) - log(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3))
 + 4096*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 +
3*c^4*d^3*x^2 - c^2*d^3), x) + 3*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^8*d^3*x^3
 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 3*log(c*x + 1)/(c^5*d^3) + 3*log(c*
x - 1)/(c^5*d^3)) - 16*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
)*c - 32*((c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 2*(c^4*x^4 - 2*c^2*x^2
 + 1)*log(c*x + 1)*log(c*x - 1) + 4*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x
^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3) + 2048*
integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)
 + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^9*d^3*x^7 - 3*c^7*d^3*x^5 + 3
*c^5*d^3*x^3 - c^3*d^3*x + (c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c
^2*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 2048*integrate(1/32*log(c*x - 1)
```

```
/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x))*b + 1/16*a*(2
*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*
d^3) + log(c*x - 1)/(c^3*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx^2\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
[Out] -(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**2*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

3.49 $\int \frac{x(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=91

$$\frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{bx}{6cd^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\,(1-c^2x^2)^2}$$

```
[Out] 1/12*b*x/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*(a+b*arccosh(c*x))/c^2/d^3/(
-c^2*x^2+1)^2-1/6*b*x/c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5914, 40, 39}

$$\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\,(1-c^2x^2)^2}-\frac{bx}{6cd^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (b*x)/(6*c*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx=\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3}$$

$$=\frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{6cd^3}$$

$$=\frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{bx}{6cd^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.70

$$\frac{3a+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(3-2c^2x^2\right)+3b\cosh^{-1}(cx)}{12c^2d^3\left(-1+c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - 2*c^2*x^2) + 3*b*ArcCosh[c*x
])/(12*c^2*d^3*(-1 + c^2*x^2)^2)
```

Maple [A]
time = 1.96, size = 86, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3}}{c^2}$	86
default	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3}}{c^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arccosh(c*x)+1/12/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*c*x*(2*c^2*x^2-3)/(c^2*x^2-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b*((4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 1)/(c^6*d^3*x^4 - 2*c^4
*d^3*x^2 + c^2*d^3) + 16*integrate(1/4/(c^8*d^3*x^7 - 3*c^6*d^3*x^5 + 3*c^4
*d^3*x^3 - c^2*d^3*x + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3
)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) + 1/4*a/(c^6*d^3*x^4 - 2*c^
4*d^3*x^2 + c^2*d^3)
```

Fricas [A]
time = 0.34, size = 98, normalized size = 1.08

$$-\frac{3\,ac^4x^4-6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(2\,bc^3x^3-3\,bcx\right)\sqrt{c^2x^2-1}}{12\,\left(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 - 1)) + (2*b*
c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{ax}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int\frac{bx\operatorname{acosh}\left(cx\right)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b
*x*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

3.50 $\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=180

$$\frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{3b}{8cd^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{3(a}{}$$

```
[Out] 1/12*b/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*x*(a+b*arccosh(c*x))/d^3/(-c^2
*x^2+1)^2+3/8*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*
arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3+3/8*b*polylog(2,-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))/c/d^3-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/c/d^3-3/8*b/c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5901, 5903, 4267, 2317, 2438, 75}

$$\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4cd^3}+\frac{3b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\frac{3b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\frac{3b}{8cd^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (3*b)/(8*c*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (3*x
*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/(4*c*d^3) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c*d^3
) - (3*b*PolyLog[2, E^ArcCosh[c*x]])/(8*c*d^3)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)} + \frac{(3$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

Mathematica [A]
time = 0.68, size = 316, normalized size = 1.76

$$\frac{\frac{4ax}{(-1+c^2x^2)^2} - \frac{6ax}{-1+c^2x^2} + \frac{b\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,(2+cx)-3\cosh^{-1}(cx)\right)}{3c(1+cx)^2} + \frac{b\left((2-cx)\sqrt{-1+cx}\,\sqrt{1+cx}+3\cosh^{-1}(cx)\right)}{3c(-1+cx)^2} + \frac{3b\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}}+\frac{\cosh^{-1}(cx)}{1-cx}\right)}{c} + \frac{3b\left(\sqrt{\frac{-1+cx}{1+cx}}-\frac{\cosh^{-1}(cx)}{1+cx}\right)}{c} - \frac{3a\log(1-cx)}{c} + \frac{3a\log(1+cx)}{c} - \frac{3b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)\right)}{2c} + \frac{3b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1-e^{\cosh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)\right)}{2c}}{16d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(3*c*(1 + c*x)^2) + (b*((2 - c*x)*S
qrt[-1 + c*x]*Sqrt[1 + c*x] + 3*ArcCosh[c*x]))/(3*c*(-1 + c*x)^2) + (3*b*(-
(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))/c + (3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/c - (3*a*Log[1 - c*x])/c + (
3*a*Log[1 + c*x])/c - (3*b*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCos
h[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/(2*c) + (3*b*(ArcCosh[c*x]*(ArcC
osh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(2*c
))/(16*d^3)
```

Maple [A]
time = 5.12, size = 352, normalized size = 1.96

method	result

derivativedivides	$\frac{\frac{a}{16d^3(cx-1)^2}-\frac{3a}{16d^3(cx-1)}-\frac{3a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}-\frac{3a}{16d^3(cx+1)}+\frac{3a\ln(cx+1)}{16d^3}-\frac{3b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{8d^3(c^4x^4-2c^2x^2+1)}-\frac{3b}{8d^3}}{}$
default	$\frac{\frac{a}{16d^3(cx-1)^2}-\frac{3a}{16d^3(cx-1)}-\frac{3a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}-\frac{3a}{16d^3(cx+1)}+\frac{3a\ln(cx+1)}{16d^3}-\frac{3b\sqrt{cx+1}\,\sqrt{cx-1}\,c^2x^2}{8d^3(c^4x^4-2c^2x^2+1)}-\frac{3b}{8d^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/16*a/d^3/(c*x-1)^2-3/16*a/d^3/(c*x-1)-3/16*a/d^3*ln(c*x-1)-1/16*a/d^
3/(c*x+1)^2-3/16*a/d^3/(c*x+1)+3/16*a/d^3*ln(c*x+1)-3/8*b/d^3/(c^4*x^4-2*c^
2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2-3/8*b/d^3/(c^4*x^4-2*c^2*x^2+1
)*arccosh(c*x)*c^3*x^3+11/24*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+5/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c*x+3/8*b/d^3*arccos
h(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*b/d^3*polylog(2,-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))-3/8*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))-3/8*b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 48*c^4*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 -
 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*
d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x
^4 + 3*c^2*d^3*x^2 - d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3*x
^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*log(
c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*
d^3*x^2 + c^4*d^3))*c^3 + 80*c^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*
x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 12288*c
^2*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3
*x^2 - d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^3*x^
2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d
^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c - 32*(3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c
*x + 1)*log(c*x - 1) + 4*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*
log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3) + 2048*integrat
```

```
e(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*
(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^7*d^3*x^7 - 3*c^5*d^3*x^5 + 3*c^
3*d^3*x^3 - c*d^3*x + (c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)*s
qrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*integrate(1/32*log(c*x - 1)/(c^6*d^3
*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x))*b - 1/16*a*(2*(3*c^2*x^3 -
 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(
c*x - 1)/(c*d^3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
[Out] -(Integral(a/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*a
cosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3,x)
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3, x)
```

3.51 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)}+\frac{2\left(a+b\cosh\right.}{}$$

```
[Out] 1/12*b*c*x/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*(a+b*arccosh(c*x))/d^3/(-c^2
*x^2+1)^2+1/2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+2*(a+b*arccosh(c*x))*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+1/2*b*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)/d^3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2)/d^3-2/3*b*c*x/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5936, 5916, 5569, 4267, 2317, 2438, 39, 40}

$$\frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)}+\frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^3}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{2bcx}{3d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bcx}{12d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c*x)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*d^3*(1 - c^2*x^2)^2) + (a
 + b*ArcCosh[c*x])/(2*d^3*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[
E^(2*ArcCosh[c*x])])/d^3 + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (b
*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx}{d}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{}{(-1\cdots}}{\cdots}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a\cdots}{2\cdots}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a\cdots}{2\cdots}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a\cdots}{2\cdots}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a\cdots}{2\cdots}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a\cdots}{2\cdots}$$

Mathematica [A]
time = 0.88, size = 210, normalized size = 1.23

$$-\frac{-\frac{3a}{(-1+c^2x^2)^2}+\frac{6a}{-1+c^2x^2}-12a\log(x)+6a\log\left(1-c^2x^2\right)+b\left(\frac{8cx\sqrt{\frac{-1+cx}{1+cx}}}{-1+cx}-\frac{cx\left(\frac{-1+cx}{1+cx}\right)^{3/2}}{(-1+cx)^3}-\frac{3\cosh^{-1}(cx)}{(-1+c^2x^2)^2}+\frac{6\cosh^{-1}(cx)}{-1+c^2x^2}+12\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-12\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)+6\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-6\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{12d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

```
[Out] -1/12*((-3*a)/(-1 + c^2*x^2)^2 + (6*a)/(-1 + c^2*x^2) - 12*a*Log[x] + 6*a*L
og[1 - c^2*x^2] + b*((8*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) - (c*x*(
(-1 + c*x)/(1 + c*x))^(3/2))/(-1 + c*x)^3 - (3*ArcCosh[c*x])/(-1 + c^2*x^2)
^2 + (6*ArcCosh[c*x])/(-1 + c^2*x^2) + 12*ArcCosh[c*x]*Log[1 - E^(-2*ArcCos
h[c*x])] - 12*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 6*PolyLog[2, -E^(
-2*ArcCosh[c*x])] - 6*PolyLog[2, E^(-2*ArcCosh[c*x])]))/d^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(190) = 380.
time = 7.32, size = 508, normalized size = 2.97

method	result

derivativedivides	$\frac{a\ln(cx)}{d^3}+\frac{a}{16d^3(cx-1)^2}-\frac{5a}{16d^3(cx-1)}-\frac{a\ln(cx-1)}{2d^3}+\frac{a}{16d^3(cx+1)^2}+\frac{5a}{16d^3(cx+1)}-\frac{a\ln(cx+1)}{2d^3}-\frac{2b\sqrt{cx}}{3d}$
default	$\frac{a\ln(cx)}{d^3}+\frac{a}{16d^3(cx-1)^2}-\frac{5a}{16d^3(cx-1)}-\frac{a\ln(cx-1)}{2d^3}+\frac{a}{16d^3(cx+1)^2}+\frac{5a}{16d^3(cx+1)}-\frac{a\ln(cx+1)}{2d^3}-\frac{2b\sqrt{cx}}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^3*ln(c*x)+1/16*a/d^3/(c*x-1)^2-5/16*a/d^3/(c*x-1)-1/2*a/d^3*ln(c*x-1)+1
/16*a/d^3/(c*x+1)^2+5/16*a/d^3/(c*x+1)-1/2*a/d^3*ln(c*x+1)-2/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3+2/3*b/d^3/(c^4*x^4-2*c^
2*x^2+1)*c^4*x^4-1/2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^2*x^2+3/4*b
/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-4/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2/3*b/
d^3/(c^4*x^4-2*c^2*x^2+1)-b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))-b/d^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+b/d^3*arccosh(c*x
)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)/d^3-b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x + 1
)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
 - d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx+\int \frac{b\operatorname{acosh}{(cx)}}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(b*a
cosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x\,{\left(d-c^2\,d\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3), x)
```

3.52 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{7bc}{8d^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\,(1-c^2x^2)^2}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\,(1-c^2x^2)^2}+\frac{15c^2x(\cdots}{8\cdots}$$

```
[Out] 1/12*b*c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+(-a-b*arccosh(c*x))/d^3/x/(-c^2*x^
2+1)^2+5/4*c^2*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+15/8*c^2*x*(a+b*arcc
osh(c*x))/d^3/(-c^2*x^2+1)+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+15/4
*c*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+15/8*b*c
*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3-15/8*b*c*polylog(2,c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))/d^3-7/8*b*c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5932, 5901, 5903, 4267, 2317, 2438, 75, 106, 21, 94, 211}

$$\frac{15c^2x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\,(1-c^2x^2)}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\,(1-c^2x^2)^2}-\frac{a+b\cosh^{-1}(cx)}{d^3x\,(1-c^2x^2)^2}+\frac{15c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4d^3}+\frac{bc\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d^3}+\frac{15bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{15bc\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{7bc}{8d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc}{12d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^3*x*(1 - c^2*x^2)^2) + (5*c^2
*x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (15*c^2*x*(a + b*ArcCosh
[c*x]))/(8*d^3*(1 - c^2*x^2)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d^3 + (15*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (15*b*c
*PolyLog[2, -E^ArcCosh[c*x]])/(8*d^3) - (15*b*c*PolyLog[2, E^ArcCosh[c*x]])
/(8*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
```

```
1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^3}$$

$$= -\frac{bc}{3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{b\int}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{15c}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2} + \frac{5c}{}$$

Mathematica [A]
time = 1.12, size = 362, normalized size = 1.57

$$\frac{-\frac{96a}{x}+\frac{24ac^2x}{(-1+c^2x^2)^2}-\frac{84ac^2x}{-1+c^2x^2}-\frac{2bc\left((-2+cx)\sqrt{-1+cx}\sqrt{1+cx}-3\cosh^{-1}(cx)\right)}{(-1+cx)^2}+\frac{2bc\left(\sqrt{-1+cx}\sqrt{1+cx}(2+cx)-3\cosh^{-1}(cx)\right)}{(1+cx)^2}-\frac{96b\cosh^{-1}(cx)}{x}+42bc\left(-\frac{1}{\sqrt{\frac{-1+cx}{1+cx}}}+\frac{\cosh^{-1}(cx)}{1-cx}\right)+42bc\left(\sqrt{\frac{-1+cx}{1+cx}}-\frac{\cosh^{-1}(cx)}{1+cx}\right)+\frac{96bc\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-90ac\log(1-cx)+90ac\log(1+cx)-45bc\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)-4\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)\right)+45bc\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(1-e^{\cosh^{-1}(cx)}\right)\right)-4\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)\right)}{96d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-96*a)/x + (24*a*c^2*x)/(-1 + c^2*x^2)^2 - (84*a*c^2*x)/(-1 + c^2*x^2) -
(2*b*c*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCosh[c*x]))/(-1 + c*
x)^2 + (2*b*c*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(1
 + c*x)^2 - (96*b*ArcCosh[c*x])/x + 42*b*c*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)])
 + ArcCosh[c*x]/(1 - c*x)) + 42*b*c*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c
*x]/(1 + c*x)) + (96*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - 90*a*c*Log[1 - c*x] + 90*a*c*Log[1 + c*x] - 4
5*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*PolyLog[
2, -E^ArcCosh[c*x]]) + 45*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 - E^Arc
Cosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(96*d^3)
```

Maple [A]
time = 5.76, size = 387, normalized size = 1.68

method	result
derivativedivides	$c\left(\frac{a}{16d^3(cx-1)^2}-\frac{7a}{16d^3(cx-1)}-\frac{15a\ln(cx-1)}{16d^3}-\frac{a}{d^3cx}-\frac{a}{16d^3(cx+1)^2}-\frac{7a}{16d^3(cx+1)}+\frac{15a\ln(cx+1)}{16d^3}-\frac{15}{8d^3}\right.$
default	$c\left(\frac{a}{16d^3(cx-1)^2}-\frac{7a}{16d^3(cx-1)}-\frac{15a\ln(cx-1)}{16d^3}-\frac{a}{d^3cx}-\frac{a}{16d^3(cx+1)^2}-\frac{7a}{16d^3(cx+1)}+\frac{15a\ln(cx+1)}{16d^3}-\frac{15}{8d^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/16*a/d^3/(c*x-1)^2-7/16*a/d^3/(c*x-1)-15/16*a/d^3*ln(c*x-1)-a/d^3/c/x-
1/16*a/d^3/(c*x+1)^2-7/16*a/d^3/(c*x+1)+15/16*a/d^3*ln(c*x+1)-15/8*b/d^3/(c
^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^3*x^3-7/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+25/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(
c*x)*c*x+23/24*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-b/d^
3/c/x/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2*b/d^3*arctan(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))+15/8*b/d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+15/8*b/d^
3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+15/8*b/d^3*dilog(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(92160*c^7*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 240*c^6*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4
- 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5
*d^3)) - 30720*c^6*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 90*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3
*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*lo
g(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^
6*d^3*x^2 + c^4*d^3))*c^5 + 400*c^4*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d
^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 6144
0*c^4*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 - d^3), x) + 45*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^
3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c
^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c^3 + 1
28*c^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x
 + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 30720*c^2*integrate(1/32*log(c*x
```

```
- 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) - 32*(15*(c^5*
x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1)^2 + 30*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(
c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 50*c^2*x^2 - 15*(c^5*x^5 - 2*c^3*x^
3 + c*x)*log(c*x + 1) + 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x - 1) + 16)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x
) + 2048*integrate(-1/16*(30*c^5*x^4 - 50*c^3*x^2 - 15*(c^6*x^5 - 2*c^4*x^3
 + c^2*x)*log(c*x + 1) + 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(c*x - 1) + 16
*c)/(c^7*d^3*x^8 - 3*c^5*d^3*x^6 + 3*c^3*d^3*x^4 - c*d^3*x^2 + (c^6*d^3*x^7
 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))
*b - 1/16*a*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 +
 d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^{6}x^{8}-3c^{4}x^{6}+3c^{2}x^{4}-x^{2}}\,dx+\int \frac{b\operatorname{acosh}{(cx)}}{c^{6}x^{8}-3c^{4}x^{6}+3c^{2}x^{4}-x^{2}}\,dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(
b*acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x^2\,(d - c^2\,d\,x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3), x)
```

3.53 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=250

$$\frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{3d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3c^2\left(a+b\cosh^{-1}(cx\right.}{4d^3\left(1-c^2x^2\right)^2}$$

```
[Out] 1/2*b*c/d^3/x/(c*x-1)^(3/2)/(c*x+1)^(3/2)-5/12*b*c^3*x/d^3/(c*x-1)^(3/2)/(c
*x+1)^(3/2)+3/4*c^2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+1/2*(-a-b*arccosh
(c*x))/d^3/x^2/(-c^2*x^2+1)^2+3/2*c^2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+6
*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+3/
2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-3/2*b*c^2*polyl
og(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-2/3*b*c^3*x/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5932, 5936, 5916, 5569, 4267, 2317, 2438, 39, 40, 105, 12}

$$\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^3\left(1-c^2x^2\right)}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{6c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^3}-\frac{2bc^3x}{3d^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5bc^3x}{12d^3(cx-1)^{3/2}(cx+1)^{3/2}}+\frac{3bc^2\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{bc}{2d^3x(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c)/(2*d^3*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (5*b*c^3*x)/(12*d^3*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c^3*x)/(3*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*c^2*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) - (a + b*ArcCo
sh[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(a + b*ArcCosh[c*x]))/(2*d^3*
(1 - c^2*x^2)) + (6*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d
^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2} + \left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2} + \frac{(bc)\ \cdots}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{bc^3x}{4d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\cdots\right.}{4d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{d^3\sqrt{-1+cx}\,\sqrt{1+\cdots}}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\,\sqrt{1\cdots}}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\,\sqrt{1\cdots}}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\,\sqrt{1\cdots}}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{-1+cx}\,\sqrt{1\cdots}}$$

Mathematica [**A**]
time = 1.79, size = 273, normalized size = 1.09

$$-\frac{\frac{6a}{x^2}-\frac{3ac^2}{(-1+c^2x^2)^2}+\frac{12ac^2}{-1+c^2x^2}-36ac^2\log(x)+18ac^2\log\left(1-c^2x^2\right)+bc^2\left(-\frac{cx}{\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3}+\frac{14cx}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}-\frac{6\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}{cx}+\frac{6\cosh^{-1}(cx)}{c^2x^2}-\frac{3\cosh^{-1}(cx)}{(-1+c^2x^2)^2}+\frac{12\cosh^{-1}(cx)}{-1+c^2x^2}+36\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-36\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)+18\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-18\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{12d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

```
[Out] -1/12*((6*a)/x^2 - (3*a*c^2)/(-1 + c^2*x^2)^2 + (12*a*c^2)/(-1 + c^2*x^2) -
 36*a*c^2*Log[x] + 18*a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((c*x)/(((-1 + c*x)/
(1 + c*x))^(3/2)*(1 + c*x)^3)) + (14*c*x)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)) - (6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*ArcCosh[c*x])/(
c^2*x^2) - (3*ArcCosh[c*x])/(-1 + c^2*x^2)^2 + (12*ArcCosh[c*x])/(-1 + c^2*
x^2) + 36*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 36*ArcCosh[c*x]*Log[1
 + E^(-2*ArcCosh[c*x])] + 18*PolyLog[2, -E^(-2*ArcCosh[c*x])] - 18*PolyLog[
2, E^(-2*ArcCosh[c*x])]))/d^3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(264) = 528.
time = 9.60, size = 606, normalized size = 2.42

method	result
derivativedivides	$c^2\left(\frac{a}{16d^3(cx+1)^2}+\frac{9a}{16d^3(cx+1)}-\frac{3a\ln(cx+1)}{2d^3}-\frac{a}{2d^3c^2x^2}+\frac{3a\ln(cx)}{d^3}+\frac{a}{16d^3(cx-1)^2}-\frac{9a}{16d^3(cx-1)}-\right.$
default	$c^2\left(\frac{a}{16d^3(cx+1)^2}+\frac{9a}{16d^3(cx+1)}-\frac{3a\ln(cx+1)}{2d^3}-\frac{a}{2d^3c^2x^2}+\frac{3a\ln(cx)}{d^3}+\frac{a}{16d^3(cx-1)^2}-\frac{9a}{16d^3(cx-1)}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/16*a/d^3/(c*x+1)^2+9/16*a/d^3/(c*x+1)-3/2*a/d^3*ln(c*x+1)-1/2*a/d^3/
c^2/x^2+3*a/d^3*ln(c*x)+1/16*a/d^3/(c*x-1)^2-9/16*a/d^3/(c*x-1)-3/2*a/d^3*l
n(c*x-1)-2/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^
3+2/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*c^4*x^4+1/4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-3/2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)
*c^2*x^2-4/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*c^2*x^2+1/2*b/d^3/(c^4*x^4-2*c^2*x
^2+1)/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)+9/4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arcco
sh(c*x)+2/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)-1/2*b/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2
/x^2*arccosh(c*x)-3*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)-3*b/d^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3*b/d^3*arccosh(c*x)*
ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3/2*b*polylog(2,-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)/d^3-3*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-3*b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2)
 + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) - b
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^6*d^3*x^9 - 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
 - d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(
b*acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3), x)
```

3.54 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=310

$$-\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right.}$$

```
[Out] -1/12*b*c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/6*b*c/d^3/x^2/(c*x-1)^(3/2)/(
c*x+1)^(3/2)+1/3*(-a-b*arccosh(c*x))/d^3/x^3/(-c^2*x^2+1)^2-7/3*c^2*(a+b*ar
ccosh(c*x))/d^3/x/(-c^2*x^2+1)^2+35/12*c^4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x
^2+1)^2+35/8*c^4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+19/6*b*c^3*arctan((c
*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/4*c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/8*b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/d^3-35/8*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3-29/2
4*b*c^3/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5932, 5901, 5903, 4267, 2317, 2438, 75, 106, 21, 94, 211, 105, 12}

$$\frac{35c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4d^3}-\frac{7c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^3x\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{12d^3\left(1-c^2x^2\right)^2}+\frac{19bc^3\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{6d^3}+\frac{35bc^3\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{35bc^3\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{29bc^3}{24d^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^3}{12d^3(cx-1)^{3/2}(cx+1)^{3/2}}+\frac{bc}{6d^3x^2(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

```
[Out] -1/12*(b*c^3)/(d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (b*c)/(6*d^3*x^2*(-1
 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (29*b*c^3)/(24*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (a + b*ArcCosh[c*x])/(3*d^3*x^3*(1 - c^2*x^2)^2) - (7*c^2*(a + b*A
rcCosh[c*x]))/(3*d^3*x*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(
12*d^3*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x
^2)) + (19*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(6*d^3) + (35*c^3*(a
 + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (35*b*c^3*PolyLog[2,
-E^ArcCosh[c*x]])/(8*d^3) - (35*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(8*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{1}{3}\left(7c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{3d^3}$$

$$= \frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^3x\left(1-c^2x^2\right)^2}+\frac{1}{3}(3$$

$$= -\frac{7bc^3}{9d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{49bc^3}{24d^3\sqrt{-1+cx}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}}$$

Mathematica [A]
time = 1.13, size = 471, normalized size = 1.52

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-16*a)/x^3 - (144*a*c^2)/x + (12*a*c^4*x)/(-1 + c^2*x^2)^2 - (66*a*c^4*x)
/(-1 + c^2*x^2) - (b*c^3*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCo
sh[c*x]))/(-1 + c*x)^2 + (b*c^3*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3
*ArcCosh[c*x]))/(1 + c*x)^2 + 33*b*c^3*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + A
rcCosh[c*x]/(1 - c*x)) + 33*b*c^3*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x
]/(1 + c*x)) + 144*b*c^2*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]*ArcTan[
Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (8*b*(-2*ArcCosh[c*x
```

```
] + (c*x*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^
2]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x^3 - 105*a*c^3*Log[1 - c*x] + 105*a
*c^3*Log[1 + c*x] - (105*b*c^3*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^Ar
cCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/2 + (105*b*c^3*(ArcCosh[c*x]
*(ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]])
)/2)/(48*d^3)
```

Maple [A]
time = 6.75, size = 481, normalized size = 1.55

method	result
derivativedivides	$c^3\left(\frac{a}{16d^3(cx-1)^2}-\frac{11a}{16d^3(cx-1)}-\frac{35a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}-\frac{11a}{16d^3(cx+1)}+\frac{35a\ln(cx+1)}{16d^3}-\frac{a}{3d^3c^3x^3}\right.$
default	$c^3\left(\frac{a}{16d^3(cx-1)^2}-\frac{11a}{16d^3(cx-1)}-\frac{35a\ln(cx-1)}{16d^3}-\frac{a}{16d^3(cx+1)^2}-\frac{11a}{16d^3(cx+1)}+\frac{35a\ln(cx+1)}{16d^3}-\frac{a}{3d^3c^3x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(1/16*a/d^3/(c*x-1)^2-11/16*a/d^3/(c*x-1)-35/16*a/d^3*ln(c*x-1)-1/16*a/
d^3/(c*x+1)^2-11/16*a/d^3/(c*x+1)+35/16*a/d^3*ln(c*x+1)-1/3*a/d^3/c^3/x^3-3
*a/d^3/c/x-35/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^3*x^3-29/24*b/d^
3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^2*x^2+175/24*b/d^3/(c
^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c*x+9/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)-7/3*b/d^3/c/x/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+1/6*
b/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/3*b/d^3/(
c^4*x^4-2*c^2*x^2+1)/c^3/x^3*arccosh(c*x)+19/3*b/d^3*arctan(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))+35/8*b/d^3*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+35/8*b/
d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+35/8*b/d^3*arccosh(c*x)*ln(1+c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/6144*(1935360*c^9*integrate(1/96*x^7*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) - 1680*c^8*(2*(5*c^2*x^3 - 3*x)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x -
1)/(c^5*d^3)) - 645120*c^8*integrate(1/96*x^6*log(c*x - 1)/(c^6*d^3*x^8 - 3
```

```
*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 630*(c*(2*(5*c^2*x^2 + 3*c*x
- 6)/(c^8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^
5*d^3) + 5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3))*c^7 + 2800*c^6*(2*(c^2*x^3 + x)/(c^6*d^3
*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^
3*d^3)) + 1290240*c^6*integrate(1/96*x^4*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*
d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 315*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/
(c^6*d^3*x^3 - c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3
) + 3*log(c*x - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^
2 + c^2*d^3))*c^5 + 896*c^4*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x
^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 645120*c^4*i
ntegrate(1/96*x^2*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x) + 128*c^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2
*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) - 32
*(105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1)^2 + 210*(c^7*x^7 - 2*c^5
*x^5 + c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(210*c^6*x^6 - 350*c^4*x^4 +
112*c^2*x^2 - 105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1) + 105*(c^7*x
^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x - 1) + 16)*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3) + 6144*integrate(-1/48*(2
10*c^7*x^6 - 350*c^5*x^4 + 112*c^3*x^2 - 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3
)*log(c*x + 1) + 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(c*x - 1) + 16*c)/(
c^7*d^3*x^10 - 3*c^5*d^3*x^8 + 3*c^3*d^3*x^6 - c*d^3*x^4 + (c^6*d^3*x^9 - 3
*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
 + 1/48*a*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6
*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3
))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^
6 - d^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integral
(b*acosh(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x))/d**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3),x)
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3), x)
```

3.55 $\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx$

Optimal. Leaf size=53

$$\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}$$

```
[Out] 2*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+polylog(2,-a*x-
(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
/a/c
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5903, 4267, 2317, 2438}

$$\frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]
]/(a*c) - PolyLog[2, E^ArcCosh[a*x]]/(a*c)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Subst}\left(\int \frac{\log(1}{x}\right.}{}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 1.45

$$-\frac{\cosh^{-1}(ax)\log\left(1-e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\cosh^{-1}(ax)\log\left(1+e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{PolyLog}\left(2, -e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{PolyLog}\left(2, e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] -((ArcCosh[a*x]*Log[1 - E^ArcCosh[a*x]])/(a*c)) + (ArcCosh[a*x]*Log[1 + E^A
rcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]]/(a*c) - PolyLog[2, E^ArcC
osh[a*x]]/(a*c)
```

Maple [C] Result contains complex when optimal does not.
time = 11.24, size = 169, normalized size = 3.19

method	result
derivativedivides	$\frac{\frac{\text{arctanh}(ax)\text{arccosh}(ax)}{c} - \frac{2i\left(\text{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right) - \text{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right) - \text{dilog}\left(1+\frac{}{\sqrt{-}}\right.\right.}{c\left(a^2 x\right.}}{a}$
default	$\frac{\frac{\text{arctanh}(ax)\text{arccosh}(ax)}{c} - \frac{2i\left(\text{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right) - \text{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right) - \text{dilog}\left(1+\frac{}{\sqrt{-}}\right.\right.}{c\left(a^2 x\right.}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/c*arctanh(a*x)*arccosh(a*x)-2*I/c*(arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))-arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+I*(a
*x+1)/(-a^2*x^2+1)^(1/2))+dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2)))*(-a^2*x^2+
1)^(1/2)*(1/2*a*x+1/2)^(1/2)*(1/2*a*x-1/2)^(1/2)/(a^2*x^2-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))
 - log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/(a*c) + 1
/2*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c) + integr
ate(1/2*(log(a*x + 1) - log(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*
sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}\left(ax\right)}{a^{2}x^{2}-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)/(a**2*x**2 - 1), x)/c
```

<u>Giac</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

<u>Mupad</u> [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}(a\,x)}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2),x)
[Out] int(acosh(a*x)/(c - a^2*c*x^2), x)
```

3.56 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^2} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh^{-1}(ax)}{2c^2\,(1-a^2x^2)}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\text{P}$$

```
[Out] 1/2*x*arccosh(a*x)/c^2/(-a^2*x^2+1)+arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2))/a/c^2+1/2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-
1/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-1/2/a/c^2/(a*x-1)^(1/2
)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5901, 5903, 4267, 2317, 2438, 75}

$$\frac{x\cosh^{-1}(ax)}{2c^2\,(1-a^2x^2)}+\frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{1}{2ac^2\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/2*1/(a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x])/(2*c^2*(1 -
a^2*x^2)) + (ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) + PolyLog[2, -E^
ArcCosh[a*x]]/(2*a*c^2) - PolyLog[2, E^ArcCosh[a*x]]/(2*a*c^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^2}\,dx &= \frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)} + \frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx}{2c} \\
&= -\frac{1}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)} - \frac{\mathrm{Subst}\left(\int x\,\mathrm{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{2ac^2} \\
&= -\frac{1}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\mathrm{Su}}{} \\
&= -\frac{1}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\mathrm{Su}}{} \\
&= -\frac{1}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\mathrm{Li}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.58, size = 120, normalized size = 1.10

$$-\frac{\frac{2\left(\sqrt{\frac{-1+ax}{1+ax}}(1+ax)+\cosh^{-1}(ax)\left(ax+(-1+a^2x^2)\log\left(1-e^{\cosh^{-1}(ax)}\right)+(1-a^2x^2)\log\left(1+e^{\cosh^{-1}(ax)}\right)\right)\right)}{-1+a^2x^2}+2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)-2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{4ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((-2*(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x]*(a*x + (-1 + a^2*
x^2)*Log[1 - E^ArcCosh[a*x]] + (1 - a^2*x^2)*Log[1 + E^ArcCosh[a*x]])))/(-1
 + a^2*x^2) + 2*PolyLog[2, -E^ArcCosh[a*x]] - 2*PolyLog[2, E^ArcCosh[a*x]])
/(4*a*c^2)
```

Maple [A]
time = 3.95, size = 161, normalized size = 1.48

method	result
derivativedivides	$\frac{-\frac{ax\,\mathrm{arccosh}(ax)+\sqrt{ax-1}\,\sqrt{ax+1}}{2\left(a^2x^2-1\right)c^2}+\frac{\mathrm{arccosh}(ax)\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}+\frac{\mathrm{polylog}\left(2,-ax-\sqrt{ax-}\right.}{2c^2}}{a}$
default	$\frac{-\frac{ax\,\mathrm{arccosh}(ax)+\sqrt{ax-1}\,\sqrt{ax+1}}{2\left(a^2x^2-1\right)c^2}+\frac{\mathrm{arccosh}(ax)\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}+\frac{\mathrm{polylog}\left(2,-ax-\sqrt{ax-}\right.}{2c^2}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*(a*x*arccosh(a*x)+(a*x-1)^(1/2)*(a*x+1)^(1/2))/(a^2*x^2-1)/c^2+1/
2/c^2*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/2/c^2*polylog(2,
-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-1/2/c^2*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))-1/2/c^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 + 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 4*a*x + 4*(2*a*x - (a^2*x^2 - 1)*log(
a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
)) - 2*(a^2*x^2 - 1)*log(a*x - 1))/(a^3*c^2*x^2 - a*c^2) + 1/4*(log(a*x - 1
)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c^2) - 1/8*log(a*x + 1)/(a
```

```
*c^2) + integrate(-1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*
log(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4 - 2*a^2
*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arccosh(a*x)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**2,x)
[Out] Integral(acosh(a*x)/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(arccosh(a*x)/(a^2*c*x^2 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2)^2,x)
[Out] int(acosh(a*x)/(c - a^2*c*x^2)^2, x)
```

3.57 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=164

$$\frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}}-\frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh^{-1}(ax)}{4c^3\,(1-a^2x^2)^2}+\frac{3x\cosh^{-1}(ax)}{8c^3\,(1-a^2x^2)}+\frac{3\cosh^{-1}(ax)\,\mathrm{ta}}{4}$$

```
[Out] 1/12/a/c^3/(a*x-1)^(3/2)/(a*x+1)^(3/2)+1/4*x*arccosh(a*x)/c^3/(-a^2*x^2+1)^
2+3/8*x*arccosh(a*x)/c^3/(-a^2*x^2+1)+3/4*arccosh(a*x)*arctanh(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))/a/c^3+3/8*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/
a/c^3-3/8*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3/(a*x-1
)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5901, 5903, 4267, 2317, 2438, 75}

$$\frac{3x\cosh^{-1}(ax)}{8c^3\,(1-a^2x^2)}+\frac{x\cosh^{-1}(ax)}{4c^3\,(1-a^2x^2)^2}+\frac{3\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3}{8ac^3\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{12ac^3(ax-1)^{3/2}(ax+1)^{3/2}}+\frac{3\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - 3/(8*a*c^3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (x*ArcCosh[a*x])/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x
])/(8*c^3*(1 - a^2*x^2)) + (3*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^
3) + (3*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) - (3*PolyLog[2, E^ArcCosh[a*
x]])/(8*a*c^3)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^3}\,dx = \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{(3a)\int \frac{x}{(-1+ax)^{3/2}(1-}}{8c^3}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cos}{8c^3(1}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cos}{8c^3(1}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cos}{8c^3(1}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cos}{8c^3(1}$$

Mathematica [A]
time = 1.53, size = 223, normalized size = 1.36

$$\frac{-\frac{2(-2+ax)\sqrt{1+ax}}{(-1+ax)^{3/2}} + \frac{2\sqrt{-1+ax}\,(2+ax)}{(1+ax)^{3/2}} + \frac{6\cosh^{-1}(ax)}{(-1+ax)^2} - \frac{6\cosh^{-1}(ax)}{(1+ax)^2} + 18\left(-\frac{1}{\sqrt{\frac{-1+ax}{1+ax}}} + \frac{\cosh^{-1}(ax)}{1-ax}\right) + 18\left(\sqrt{\frac{-1+ax}{1+ax}} - \frac{\cosh^{-1}(ax)}{1+ax}\right) + 9\cosh^{-1}(ax)\left(\cosh^{-1}(ax) - 4\log\left(1-e^{\cosh^{-1}(ax)}\right)\right) - 9\cosh^{-1}(ax)\left(\cosh^{-1}(ax) - 4\log\left(1+e^{\cosh^{-1}(ax)}\right)\right) + 36\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right) - 36\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{96ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((-2*(-2 + a*x)*Sqrt[1 + a*x])/(-1 + a*x)^(3/2) + (2*Sqrt[-1 + a*x]*(2 + a*
x))/(1 + a*x)^(3/2) + (6*ArcCosh[a*x])/(-1 + a*x)^2 - (6*ArcCosh[a*x])/(1 +
 a*x)^2 + 18*(-(1/Sqrt[(-1 + a*x)/(1 + a*x)]) + ArcCosh[a*x]/(1 - a*x)) + 1
8*(Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x]/(1 + a*x)) + 9*ArcCosh[a*x]*(A
rcCosh[a*x] - 4*Log[1 - E^ArcCosh[a*x]]) - 9*ArcCosh[a*x]*(ArcCosh[a*x] - 4
*Log[1 + E^ArcCosh[a*x]]) + 36*PolyLog[2, -E^ArcCosh[a*x]] - 36*PolyLog[2,
E^ArcCosh[a*x]])/(96*a*c^3)
```

Maple [A]
time = 4.39, size = 205, normalized size = 1.25

method	result
derivativedivides	$-\frac{9\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+9\,\mathrm{arccosh}(ax)a^3x^3-11\sqrt{ax-1}\,\sqrt{ax+1}-15ax\,\mathrm{arccosh}(ax)}{24\left(a^4x^4-2a^2x^2+1\right)c^3}+\frac{3\,\mathrm{arccosh}(ax)\ln\left(1+\right.}{}$

default	$\frac{-\frac{9\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+9\,\mathrm{arccosh}(ax)a^3x^3-11\sqrt{ax-1}\,\sqrt{ax+1}-15ax\,\mathrm{arccosh}(ax)}{24\left(a^4x^4-2a^2x^2+1\right)c^3}+\frac{3\,\mathrm{arccosh}(ax)\ln\left(1\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/24*(9*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+9*arccosh(a*x)*a^3*x^3-11
*(a*x-1)^(1/2)*(a*x+1)^(1/2)-15*a*x*arccosh(a*x))/(a^4*x^4-2*a^2*x^2+1)/c^3
+3/8/c^3*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/8/c^3*polylog
(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/8/c^3*arccosh(a*x)*ln(1-a*x-(a*x-1)^
(1/2)*(a*x+1)^(1/2))-3/8/c^3*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(10*a^3*x^3 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*l
og(a*x - 1)^2 - 14*a*x + 4*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a
*x + 1)*sqrt(a*x - 1)) - 7*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))/(a^5*c^3
*x^4 - 2*a^3*c^3*x^2 + a*c^3) + 3/16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dil
og(-1/2*a*x + 1/2))/(a*c^3) - 7/64*log(a*x + 1)/(a*c^3) + integrate(-1/16*(
6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a^4*x^4
- 2*a^2*x^2 + 1)*log(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3
 - a*c^3*x + (a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x +
 1)*sqrt(a*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)/(c - a^2*c*x^2)^3, x)
```

3.58 $\int x^4 \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=278

$$\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{16c^4}$$

```
[Out] -1/16*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-1/24*x^3*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/6*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)+1/32*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/96*b*x
^4*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/36*b*c*x^6*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)/b/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5926, 5939, 5893, 30}

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^4}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^4*
Sqrt[d - c^2*d*x^2])/(96*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^6*Sqrt[d
- c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(16*c^4) - (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(24*c^2) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int x^4\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\cosh}{\sqrt{-1+cx}\,\sqrt{}}}{6\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}+$$

$$=\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx}}{}$$

$$=\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^6\sqrt{}}{36\sqrt{-1+}}$$

Mathematica [A]

time = 0.79, size = 198, normalized size = 0.71

$$\frac{48acx\sqrt{d-c^2dx^2}\left(-3-2c^2x^2+8c^4x^4\right)-144a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh(2\cosh^{-1}(cx))-9\cosh(4\cosh^{-1}(cx))-2\cosh(6\cosh^{-1}(cx))+12\cosh^{-1}(cx)\left(-3\sinh(2\cosh^{-1}(cx))+3\sinh(4\cosh^{-1}(cx))+\sinh(6\cosh^{-1}(cx))\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}}{2304c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*x*Sqrt[d - c^2*d*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt[d]*
ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^
2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c
*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)))/(2304*c^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(234) = 468$.
time = 5.36, size = 878, normalized size = 3.16

method	result
default	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6 c^{2} d}-\frac{a x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 c^{4} d}+\frac{a x\sqrt{-c^{2} d\,x^{2}+d}}{16 c^{4}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{16 c^{4}\sqrt{c^{2} d}}+b\left(-\frac{\sqrt{-d\,(}}{32\sqrt{c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1/
16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)^(
1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/c^5*arccosh(c*x)^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-
64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-
1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)+1/512*(-d*(
c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c
^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*(-1+4*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^
3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/
2))*(-1+2*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(-
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)
-2*c*x)*(1+2*arccosh(c*x))/(c*x+1)/c^5/(c*x-1)+1/512*(-d*(c^2*x^2-1))^(1/2)
*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*
x))/(c*x+1)/c^5/(c*x-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^
5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))/(c*x+1)/c^5/(c*x-1))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/48*(8*(-c^2*d*x^2 + d)^(3/2)*x^3/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d)*x/c^4
+ 6*(-c^2*d*x^2 + d)^(3/2)*x/(c^4*d) - 3*sqrt(d)*arcsin(c*x)/c^5)*a + b*int
egrate(sqrt(-c^2*d*x^2 + d)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.59 $\int x^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=201

$$\frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\,(a+$$

```
[Out] -1/8*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)+1/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/16*b*c*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/
16*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A]
time = 0.26, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5926, 5939, 5893, 30}

$$-\frac{x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^4*
Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
```

```
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\cos}{\sqrt{-1+cx}}}{4\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2} +$$

$$= \frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2}}{}$$

Mathematica [A]
time = 0.71, size = 151, normalized size = 0.75

$$-\frac{-16acx(-1+2c^2x^2)\sqrt{d-c^2dx^2}+16a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}\,\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/128*(-16*a*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 16*a*Sqrt[d]*ArcTa
n[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^2*d*x
^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCos
h[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(169) = 338$.
time = 4.12, size = 367, normalized size = 1.83

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{ax\sqrt{-c^2dx^2+d}}{8c^2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\mathrm{arccosh}(cx)}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a/
c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/16*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2+1/256*
(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*
(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x
))/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/8*a*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sq
rt(d)*arcsin(c*x)/c^3) + b*integrate(sqrt(-c^2*d*x^2 + d)*x^2*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-d (cx-1) (cx+1)} (a + b \operatorname{acosh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \operatorname{acosh}(c x)) \sqrt{d - c^2 d x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.60 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=124

$$-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5895, 5893, 30}

$$\frac{1}{2}x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
```

```
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{a+b\cosh^{-1}(\ldots}{\sqrt{-1+cx}\,\sqrt{\ldots}}}{2\sqrt{-1+cx}\,\sqrt{1+c\ldots}}$$

$$= -\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \ldots$$

Mathematica [A]
time = 0.39, size = 144, normalized size = 1.16

$$\frac{1}{8}\left(4ax\sqrt{d-c^2dx^2} - \frac{4a\sqrt{d}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{c} - \frac{b\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{c\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/
(Sqrt[d]*(-1 + c^2*x^2))])/c - (b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + C
osh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(104) = 208$.
time = 4.06, size = 278, normalized size = 2.24

method	result
default	$\frac{ax\sqrt{-c^2d\,x^2+d}}{2} + \frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\text{arccosh}(cx)^2}{4\sqrt{cx-1}\,\sqrt{cx+1}\,c} + \frac{\sqrt{-d\,(c^2x^2-\ldots}}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
```

```
1/2)/c*arccosh(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*
x))/(c*x+1)/(c*x-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*
x))/(c*x+1)/(c*x-1)/c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a + b*integrate(sqrt(-
c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.61 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc\sqrt{d-c^2dx^2}\log(x)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] -(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arccosh(c*x))^2*(-c^2
*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*ln(x)*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5924, 29, 5893}

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^
2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
```

```
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(c}{}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x} + \frac{c\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.29, size = 137, normalized size = 1.16

$$-\frac{a\sqrt{d-c^2dx^2}}{x}+ac\sqrt{d}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+\frac{1}{2}bc\sqrt{d-c^2dx^2}\left(-\frac{2\cosh^{-1}(cx)}{cx}+\frac{\cosh^{-1}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))] + (b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c
*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
)/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(102) = 204.
time = 4.63, size = 286, normalized size = 2.42

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{dx}-ac^2x\sqrt{-c^2dx^2+d}-\frac{ac^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arccosh}(cx}{2\sqrt{cx-1}\,\sqrt{cx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(1
/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c-b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x+1)/(c*x-1)*x*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*
x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] -(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

3.62 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{3dx^3}-\frac{bc^3\sqrt{d-c^2dx^2}\,\log(x)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^3-1/6*b*c*(-c^2*d*x^2+d)^(
1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c^3*ln(x)*(-c^2*d*x^2+d)^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5917, 74, 14}

$$-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{3dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^3\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[d - c^2*d*x^2])/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c
^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*d*x^3) - (b*c^3*Sqrt[d - c^2*d*x^2
]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
```

```
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3x^3} - \frac{\left(bc\sqrt{d-c^2d\right.}}{3\sqrt{-1+}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3x^3} - \frac{\left(bc\sqrt{d-c^2d\right.}}{3\sqrt{-1}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a+b\,\mathrm{c}}{3x^3}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 0.74

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{3x^3}-\frac{1}{3}bc\left(\frac{1}{2x^2}+c^2\log(x)\right)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]
))/(3*x^3) - (b*c*(1/(2*x^2) + c^2*Log[x]))/3))/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. 2(99) = 198.
time = 6.69, size = 1017, normalized size = 8.55

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3dx^3}+\frac{2b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}(cx)c^3}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,x^4\mathrm{arccosh}(cx)c^7}{(3c^4x^4-3c^2x^2+1)\sqrt{cx+1}\,\sqrt{cx-1}}+\frac{b\sqrt{-d\left(c^2x\right.}}{(3c^4x^4-3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(3/2)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)*arccosh(c*x)*c^3-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*
x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+b*(-d*(c^2*x^2-1))^
(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-1/6*b*(-
d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6+1/6*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8+b*(-d*(c^2*x^2-1))^(
1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c
^5-3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^6+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c^4-
1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^5-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x
+1)/(c*x-1)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4
*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+1/2*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+1/6*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-5/3*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+
1)/(c*x-1)*arccosh(c*x)-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 150, normalized size = 1.26

$$\frac{\left(c^4d^2\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)+i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+\frac{\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,d}{x^2}\right)bc}{6\,d}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b\,\mathrm{arcosh}\left(cx\right)}{3\,dx^3}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/6*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d
)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*
d/x^2)*b*c/d - 1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(d*x^3) - 1/3*(-c^
2*d*x^2 + d)^(3/2)*a/(d*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(99) = 198.
time = 0.40, size = 462, normalized size = 3.88

$$\left[\frac{2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,(x^4-1)\sqrt{-d}-d}{c^2x^4-x^2}\right)+\sqrt{-c^2dx^2+d}\left(bcx^3-bcx\right)\sqrt{c^2x^2-1}+2\left(ac^4x^4-2\,ac^2x^2+a\right)\sqrt{-c^2dx^2+d}}{6\left(c^2x^5-x^3\right)},\ -\frac{2\left(bc^5x^5-bc^3x^3\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,(x^2+1)\sqrt{d}}{c^2dx^4-(c^2+1)dx^2+d}\right)-2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\sqrt{-c^2dx^2+d}\left(bcx^3-bcx\right)\sqrt{c^2x^2-1}-2\left(ac^4x^4-2\,ac^2x^2+a\right)\sqrt{-c^2dx^2+d}}{6\left(c^2x^5-x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(2*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) + (b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) +
 2*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3), -1/
6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x
^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(b*c^4*x^4
 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - sqr
t(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(a*c^4*x^4 - 2*a*
c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

3.63 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=199

$$-\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{5dx^5}-\frac{2c^2(d-c^2dx^2)^{3}}{}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^5-2/15*c^2*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccosh(c*x))/d/x^3-1/20*b*c*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+1/30*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-2/15*b*c^5*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {277, 270, 5922, 12, 14}

$$-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{5dx^5}-\frac{2c^2(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{15dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^5\log(x)\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*Sqrt[d - c^2*d*x^2])/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*Sqrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x
^2)^(3/2)*(a + b*ArcCosh[c*x]))/(5*d*x^5) - (2*c^2*(d - c^2*d*x^2)^(3/2)*(a
 + b*ArcCosh[c*x]))/(15*d*x^3) - (2*b*c^5*Sqrt[d - c^2*d*x^2]*Log[x])/(15*S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^6}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(c\right.}{15x^3}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(c\right.}{15x^3}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(c\right.}{15x^3}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c}}{}$$

Mathematica [A]
time = 0.14, size = 128, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\,\left(12(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+8c^2x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(3-2c^2x^2+8c^4x^4\log(x)\right)\right)}{60x^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 8*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) - b*c
```

```
*x*(3 - 2*c^2*x^2 + 8*c^4*x^4*Log[x])))/(60*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(167) = 334$.

time = 7.10, size = 1742, normalized size = 8.75

method	result	size
default	Expression too large to display	1742

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7/(c*x+1)/
(c*x-1)*arccosh(c*x)*c^12-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^
4-15*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10-27/5*b*(-d*(c^2*x^2-1
))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^3/(c*x+1)/(c*x-1)*arccosh(c*
x)*c^2-17/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^
3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8+98/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x
^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+12/5*b*(-d*(c
^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x/(c*x+1)/(c*x-1)*arcc
osh(c*x)*c^4-2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)
*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+2/3*b*(-d*(c^2*x^2-1))^(
1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*ar
ccosh(c*x)*c^9+2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+
9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+2/15*b*(-d*(c^2*x^2-1))
^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7*c^12-3/10*b*(-d*(c^2*x^2-1))
^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3*c^8+3/10*b*(-d*(c^2*x^2-1))^
(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x*c^6+1/2*b*(-d*(c^2*x^2-1))^(1/2
)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-2
/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^9/(c*x+1
)/(c*x-1)*c^14+4/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x
^2+9)*x^7/(c*x+1)/(c*x-1)*c^12+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c
^4*x^4-15*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*c^10-3/5*b*(-d*(c^2*x^2-1))^(1/2)/
(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*c^8+3/10*b*(-d*(c^2
*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*c^6+9/
5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^5/(c*x+1)/
(c*x-1)*arccosh(c*x)-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*
c^2*x^2+9)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+4/15*b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^5-2/1
5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5*c^10-6/5
*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c*x+1)^(1/2)
/(c*x-1)^(1/2)*arccosh(c*x)*c^5-9/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5
*c^4*x^4-15*c^2*x^2+9)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-11/12*b*(-d*(c^2*x
```

```
^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^7+21/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9
)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+a*(-1/5/d/x^5*(-c^2*d*x^2+d)^(3/2)-2/
15*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2))
```

Maxima [A]
time = 0.49, size = 146, normalized size = 0.73

$$-\frac{1}{60}\left(8\,c^4\sqrt{-d}\,\log\left(x\right)-\frac{2\,c^2\sqrt{-d}\,x^2-3\,\sqrt{-d}}{x^4}\right)bc-\frac{1}{15}\,b\left(\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}}{dx^5}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{15}\,a\left(\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}}{dx^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/60*(8*c^4*sqrt(-d)*log(x) - (2*c^2*sqrt(-d)*x^2 - 3*sqrt(-d))/x^4)*b*c -
 1/15*b*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/(d
*x^5))*arccosh(c*x) - 1/15*a*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^
2*d*x^2 + d)^(3/2)/(d*x^5))
```

Fricas [A]
time = 0.43, size = 549, normalized size = 2.76

$$\left[\frac{4\,(2\,bc^6x^6-bc^4x^4-4\,bc^2x^2+3\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+4\,(bc^7x^7-bc^5x^5)\sqrt{-d}\,\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,(x^4-1)\sqrt{-d}-d}{c^2x^4-x^2}\right)+(2\,bc^3x^3-(2\,bc^3-3\,bc)x^5-3\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+4\,(2\,ac^6x^6-ac^4x^4-4\,ac^2x^2+3\,a)\sqrt{-c^2dx^2+d}}{60\,(c^2x^7-x^5)},\ -\frac{8\,(bc^7x^7-bc^5x^5)\sqrt{d}\,\arctan\left(\frac{\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,(x^2+1)\sqrt{d}}{c^2dx^4-(c^2+1)dx^2+d}\right)-4\,(2\,bc^6x^6-bc^4x^4-4\,bc^2x^2+3\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(2\,bc^3x^3-(2\,bc^3-3\,bc)x^5-3\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-4\,(2\,ac^6x^6-ac^4x^4-4\,ac^2x^2+3\,a)\sqrt{-c^2dx^2+d}}{60\,(c^2x^7-x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="fricas"
)
```

```
[Out] [1/60*(4*(2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*log((c^2
*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 -
1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 -
3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(2*a*c^6*x^6 - a*c^4*x^
4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/60*(8*(b*c
^7*x^7 - b*c^5*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(
x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(2*b*c^6*x^6 - b*c^
4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
) - (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2
*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**6, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6, x)
```

3.64 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=279

$$-\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{3/2}\,(a+}{7dx^7}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^7-4/35*c^2*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccosh(c*x))/d/x^5-8/105*c^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(
c*x))/d/x^3-1/42*b*c*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/140*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/105*b*c^5
*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/105*b*c^7*ln(x)*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {277, 270, 5922, 12, 14}

$$-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{7dx^7}-\frac{4c^2(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{35dx^5}-\frac{8c^4(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{105dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{8bc^7\log(x)\sqrt{d-c^2dx^2}}{105\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*Sqrt[d - c^2*d*x^2])/(x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*Sqrt[d - c^2*d*x^2])/(140*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^5*Sqr
t[d - c^2*d*x^2])/(105*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)
^(3/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (4*c^2*(d - c^2*d*x^2)^(3/2)*(a +
b*ArcCosh[c*x]))/(35*d*x^5) - (8*c^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c
*x]))/(105*d*x^3) - (8*b*c^7*Sqrt[d - c^2*d*x^2]*Log[x])/(105*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^8}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx\right.}{35x^5}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx\right.}{35x^5}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx\right.}{35x^5}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2b}{105x^2\sqrt{}}$$

Mathematica [A]
time = 0.17, size = 146, normalized size = 0.52

$$\frac{\sqrt{d-c^2dx^2}\,\left(60(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+16c^2x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(3+2c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-bcx\left(10-3c^2x^2-8c^4x^4+32c^6x^6\log(x)\right)\right)}{420x^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(60*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 16*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(3 + 2*c^2*x^2)*(a + b*Ar
cCosh[c*x]) - b*c*x*(10 - 3*c^2*x^2 - 8*c^4*x^4 + 32*c^6*x^6*Log[x])))/(420
*x^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2536 vs. $2(235) = 470$.
time = 7.58, size = 2537, normalized size = 9.09

method	result	size
default	Expression too large to display	2537

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/7/d/x^7*(-c^2*d*x^2+d)^(3/2)+4/7*c^2*(-1/5/d/x^5*(-c^2*d*x^2+d)^(3/2)
-2/15*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2)))+16/15*b*(-d*(c^2*x^2-1))^(1/2)/(280*
c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9*c^16-88/105*b*(-d*(c^2*
x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7*c^14
-302/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c
^2*x^2+225)*x^5*c^12-10/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6
-21*c^4*x^4-315*c^2*x^2+225)*x^3*c^10+20/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^
8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x*c^8-75/14*b*(-d*(c^2*x^2-1)
)^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^6/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*c-469/60*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^
6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+71/28*b*(
-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/
x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+255/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^
8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*c^3-128/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4
-315*c^2*x^2+225)*x^13/(c*x+1)/(c*x-1)*c^20+16/105*b*(-d*(c^2*x^2-1))^(1/2)
/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^11/(c*x+1)/(c*x-1)*
c^18+40/21*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315
*c^2*x^2+225)*x^9/(c*x+1)/(c*x-1)*c^16+214/105*b*(-d*(c^2*x^2-1))^(1/2)/(28
0*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x+1)/(c*x-1)*c^14-
152/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^
2*x^2+225)*x^5/(c*x+1)/(c*x-1)*c^12+128/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c
^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^11*c^18+8/5*b*(-d*(c^2*x^2
-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^4/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11-351/5*b*(-d*(c^2*x^2-1))^(1/2)/(280*
c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^10+64/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^
4-315*c^2*x^2+225)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-56/3*b*(-d*(c^2*x^
2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x+1
```

```
)/(c*x-1)*arccosh(c*x)*c^14-4/15*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*
c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12+3
057/35*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2
*x^2+225)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-594/35*b*(-d*(c^2*x^2-1))^(1/2
)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x/(c*x+1)/(c*x-1)*ar
ccosh(c*x)*c^6+342/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c
^4*x^4-315*c^2*x^2+225)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-585/7*b*(-d*(c
^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^5/(
c*x+1)/(c*x-1)*arccosh(c*x)*c^2+24*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-10
5*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*c^9-64/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x
^4-315*c^2*x^2+225)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15+8*b*(
-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*
x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-73/20*b*(-d*(c^2*x^2-1))^
(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c^7+16/105*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)*c^7-8/105*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^7-30/7*b*(-d*(c^2*x^2-1))^(1/
2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x+1)/(c*x-1)
*c^10+20/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315
*c^2*x^2+225)*x/(c*x+1)/(c*x-1)*c^8+225/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8
*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*
x)-120/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c
^2*x^2+225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+16/3*b*(-d*(c^2*x^
2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c^13
```

Maxima [A]
time = 0.48, size = 207, normalized size = 0.74

$$-\frac{1}{420}\left(32\,c^{6}\sqrt{-d}\,\log\left(x\right)-\frac{8\,c^{4}\sqrt{-d}\,x^{4}+3\,c^{2}\sqrt{-d}\,x^{2}-10\,\sqrt{-d}}{x^{6}}\right)bc-\frac{1}{105}\left(\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{4}}{dx^{3}}+\frac{12\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}}{dx^{5}}+\frac{15\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}{dx^{7}}\right)b\,\mathrm{arcosh}\left(cx\right)-\frac{1}{105}\left(\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{4}}{dx^{3}}+\frac{12\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}}{dx^{5}}+\frac{15\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}{dx^{7}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="maxima"
)
```

```
[Out] -1/420*(32*c^6*sqrt(-d)*log(x) - (8*c^4*sqrt(-d)*x^4 + 3*c^2*sqrt(-d)*x^2 -
 10*sqrt(-d))/x^6)*b*c - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(
-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*b*ar
ccosh(c*x) - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x^2 +
 d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*a
```

Fricas [A]
time = 0.42, size = 615, normalized size = 2.20

$$\left[\frac{4\left(8\,bc^{8}x^{8}-4\,bc^{6}x^{6}-bc^{4}x^{4}-18\,bc^{2}x^{2}+15\,b\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+16\left(bc^{9}x^{9}-bc^{7}x^{7}\right)\sqrt{-d}\,\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}+\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{4}-1)\sqrt{-d}-d}{c^{2}x^{4}-x^{2}}\right)+\left(8\,bc^{5}x^{5}-\left(8\,bc^{5}+3\,bc^{3}-10\,bc\right)x^{7}+3\,bc^{3}x^{3}-10\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}+4\left(8\,ac^{8}x^{8}-4\,ac^{6}x^{6}-ac^{4}x^{4}-18\,ac^{2}x^{2}+15\,a\right)\sqrt{-c^{2}dx^{2}+d}}{420\left(c^{2}x^{9}-x^{7}\right)},-\frac{32\left(bc^{9}x^{9}-bc^{7}x^{7}\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{2}+1)\sqrt{d}}{c^{2}dx^{4}-(c^{2}+1)dx^{2}+d}\right)-4\left(8\,bc^{8}x^{8}-4\,bc^{6}x^{6}-bc^{4}x^{4}-18\,bc^{2}x^{2}+15\,b\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(8\,bc^{5}x^{5}-\left(8\,bc^{5}+3\,bc^{3}-10\,bc\right)x^{7}+3\,bc^{3}x^{3}-10\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-4\left(8\,ac^{8}x^{8}-4\,ac^{6}x^{6}-ac^{4}x^{4}-18\,ac^{2}x^{2}+15\,a\right)\sqrt{-c^{2}dx^{2}+d}}{420\left(c^{2}x^{9}-x^{7}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/420*(4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 16*(b*c^9*x^9 - b*c^7*x^7)
*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (8*b*c^5*x^5 - (8*b*c
^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1) + 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2
+ 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/420*(32*(b*c^9*x^9 - b*c^
7*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt
(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c
^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b
*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(8*a*c^8*x^8 - 4*a*c^6*x^6
 - a*c^4*x^4 - 18*a*c^2*x^2 + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**8, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8, x)
```

3.65 $\int x^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=272

$$\frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx^5\sqrt{d-c^2dx^2}}{175c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^6/d+2/5*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^6
/d^3+8/105*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/315*b
*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/175*b*x^5*(-c^2
*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*c*x^7*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {272, 45, 5922, 12}

$$-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^3}+\frac{2(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d^2}-\frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d}-\frac{bcx^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx^5\sqrt{d-c^2dx^2}}{175c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*x*Sqrt[d - c^2*d*x^2])/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*x
^3*Sqrt[d - c^2*d*x^2])/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^5*Sqr
t[d - c^2*d*x^2])/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^7*Sqrt[d -
c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a +
 b*ArcCosh[c*x]))/(3*c^6*d) + (2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(5*c^6*d^2) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int x^5\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)}{}$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)}{}$$

$$=\frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx}{175c\sqrt{}}$$

Mathematica [A]
time = 0.24, size = 152, normalized size = 0.56

$$\frac{\sqrt{d-c^2dx^2}\left(b\left(8x+\frac{4c^2x^3}{3}+\frac{3c^4x^5}{5}-\frac{15c^6x^7}{7}\right)+15c^3x^4(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(-1+cx)^{3/2}(1+cx)^{3/2}(2+3c^2x^2)\left(a+b\cosh^{-1}(cx)\right)}{c}\right)}{105c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)
/7) + 15*c^3*x^4*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (4
*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(2 + 3*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))
/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(228) = 456$.
time = 4.22, size = 988, normalized size = 3.63

method	result
default	$a\left(-\frac{x^4(-c^2dx^2+d)^{\frac{3}{2}}}{7c^2d}+\frac{-\frac{4x^2(-c^2dx^2+d)^{\frac{3}{2}}}{35c^2d}-\frac{8(-c^2dx^2+d)^{\frac{3}{2}}}{105dc^4}}{c^2}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,(64c^8x^8-144x^6c^6+64\sqrt{cx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/7*x^4*(-c^2*d*x^2+d)^(3/2)/c^2/d+4/7/c^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/
2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2)))+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)
*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4
-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))/(c
*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/
c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccos
h(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1
)*(1+3*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(-16
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5
*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))/(c*x+1)/c^6/(
c*x-1))
```

<u>Maxima</u> [A]
time = 0.47, size = 205, normalized size = 0.75

$$-\frac{1}{105}\left(\frac{15(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)b\,\mathrm{arcosh}(cx)-\frac{1}{105}\left(\frac{15(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)a-\frac{\left(225c^6\sqrt{-d}\,x^7-63c^4\sqrt{-d}\,x^5-140c^2\sqrt{-d}\,x^3-840\sqrt{-d}\,x\right)b}{11025c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/105*(15*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*b*arccosh(c*x) - 1/105*(15*(
```

```
-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*a - 1/11025*(225*c^6*sqrt(-d)*x^7 - 63*c
^4*sqrt(-d)*x^5 - 140*c^2*sqrt(-d)*x^3 - 840*sqrt(-d)*x)*b/c^5
```

Fricas [A]
time = 0.39, size = 203, normalized size = 0.75

$$\frac{105\,(15\,bc^8x^8-18\,bc^6x^6-bc^4x^4-4\,bc^2x^2+8\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(225\,bc^7x^7-63\,bc^5x^5-140\,bc^3x^3-840\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+105\,(15\,ac^8x^8-18\,ac^6x^6-ac^4x^4-4\,ac^2x^2+8\,a)\sqrt{-c^2dx^2+d}}{11025\,(c^8x^2-c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/11025*(105*(15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 8*b)*
sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (225*b*c^7*x^7 - 63*b*c
^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
+ 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 8*a)*sqrt(-c
^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.66 $\int x^3 \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=195

$$\frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}\right.}{3c^4d}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^4/d+1/5*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/15*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+1/45*b*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-1/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {272, 45, 5922, 12}

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^3*
Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^5*Sqrt[d
- c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a
 + b*ArcCosh[c*x]))/(3*c^4*d) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(5*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{x^2(1-cx)(}{}$$

$$=-\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{x^2(1-cx)(}{}$$

$$=\frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^5\sqrt{}}{25\sqrt{-1+}}$$

Mathematica [A]
time = 0.12, size = 128, normalized size = 0.66

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{1}{15}bcx(30+5c^2x^2-9c^4x^4)+2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+3c^2x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\right)}{15c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*((b*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4))/15 + 2*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + 3*c^2*x^2*(-1 + c*x)^(3/2)*(
1 + c*x)^(3/2)*(a + b*ArcCosh[c*x])))/(15*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(163) = 326$.
time = 4.63, size = 640, normalized size = 3.28

method	result

default	$a\left(-\frac{x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(-c^2d\,x^2+d)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d\,(c^2x^2-1)}\,\left(16x^6c^6-28c^4x^4+16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b*(
1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(
-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/
(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^
2-1)*(-1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-
1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c
^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c
*x+1)/c^4/(c*x-1)+1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c
^4/(c*x-1))
```

Maxima [A]
time = 0.49, size = 144, normalized size = 0.74

$$-\frac{1}{15}\,b\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)-\frac{1}{15}\,a\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9\,c^4\sqrt{-d}\,x^5-5\,c^2\sqrt{-d}\,x^3-30\,\sqrt{-d}\,x\right)b}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/15*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c
^4*d))*arccosh(c*x) - 1/15*a*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^
2*d*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(-d)*x^5 - 5*c^2*sqrt(-d)*x^
3 - 30*sqrt(-d)*x)*b/c^3
```

Fricas [A]
time = 0.34, size = 176, normalized size = 0.90

$$\frac{15\,(3\,bc^6x^6-4\,bc^4x^4-bc^2x^2+2\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(9\,bc^5x^5-5\,bc^3x^3-30\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+15\,(3\,ac^6x^6-4\,ac^4x^4-ac^2x^2+2\,a)\sqrt{-c^2dx^2+d}}{225\,(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - 4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*c^2
*x^2 + 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.67 $\int x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=118

$$\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^2/d+1/3*b*x*(-c^2*d*x^2+d)^(
1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {5914, 41}

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}+\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^3*Sqr
t[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)
*(a + b*ArcCosh[c*x]))/(3*c^2*d)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))\,dx = \frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3c^2} - \frac{\left(b\sqrt{d-c^2dx\right.}{3c\sqrt{-1}}$$

$$= \frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{(1-cx)(1+c}{}$$

Mathematica [A]
time = 0.08, size = 98, normalized size = 0.83

$$\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(3-c^2x^2)+3a(-1+c^2x^2)^2+3b(-1+c^2x^2)^2\cosh^{-1}(cx)\right)}{9c^2\,(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*
a*(-1 + c^2*x^2)^2 + 3*b*(-1 + c^2*x^2)^2*ArcCosh[c*x]))/(9*c^2*(-1 + c^2*x
^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(98) = 196.
time = 1.90, size = 356, normalized size = 3.02

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\sqrt{cx+1}\,\sqrt{cx-1}\,xc+\right.}{72(cx+1)c^2(cx-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/c^2/d*(-c^2*d*x^2+d)^(3/2)+b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4
-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^
2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*
x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(-4
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.69

$$-\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}b\,\mathrm{arcosh}\left(cx\right)}{3\,c^{2}d}-\frac{\left(c^{2}\sqrt{-d}\,dx^{3}-3\,\sqrt{-d}\,dx\right)b}{9\,cd}-\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}a}{3\,c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(c^2*d) - 1/9*(c^2*sqrt(-d)*d*x^
3 - 3*sqrt(-d)*d*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Fricas [A]
time = 0.35, size = 142, normalized size = 1.20

$$\frac{3\left(bc^{4}x^{4}-2\,bc^{2}x^{2}+b\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(bc^{3}x^{3}-3\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}+3\left(ac^{4}x^{4}-2\,ac^{2}x^{2}+a\right)\sqrt{-c^{2}dx^{2}+d}}{9\left(c^{4}x^{2}-c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 - 1)) - (b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
+ 3*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.68 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=213

$$-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))-\frac{2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(\ldots}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] (a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-b*c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*polylog(2,-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5926, 5947, 4265, 2317, 2438, 8}

$$-\frac{2\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}+\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))+\frac{ib\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ib\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*Sqrt[d - c^2
*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
I*b*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx)) - \frac{\sqrt{d-c^2dx^2}\,\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\,\sqrt{1-}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx)) - \frac{\sqrt{d-}}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx)) - \frac{2\sqrt{d-}}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx)) - \frac{2\sqrt{d-}}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx)) - \frac{2\sqrt{d-}}{}$$

Mathematica [A]
time = 0.58, size = 233, normalized size = 1.09

$$a\sqrt{d-c^2dx^2} + a\sqrt{d}\,\log(x) - a\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right) + \frac{b\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)
]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*
x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I
*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A]
time = 3.72, size = 394, normalized size = 1.85

method	result
default	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)a + a\sqrt{-c^2d\,x^2+d} + \frac{b\sqrt{-d\,(c^2x^2-1)}\,\mathrm{arccosh}(cx)x^2c^2}{(cx+1)(cx-1)} - \frac{b\sqrt{}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+a*(-c^2*d*x^2+d)^(1/2
)+b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*
x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c*x
+1)/(c*x-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c
^2*d*x^2 + d))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x, x)
```

3.69 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=235

$$-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{ArcTan}\left(e^{\cosh}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(-c^2*d*x^2+d)^(1/
2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+c^2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I
*b*c^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5924, 30, 5947, 4265, 2317, 2438}

$$\frac{c^2\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibc^2\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[d - c^2*d*x^2])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I
/2)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + ((I/2)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCos
h[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^3}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{-1+cx}\,\sqrt{1+cx}} + \cdots$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2x^2} + \cdots$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2x^2} + \cdots$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2x^2} + \cdots$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2x^2} + \cdots$$

Mathematica [A]
time = 0.72, size = 307, normalized size = 1.31

$$\frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{x^2} - ac^2\sqrt{d}\log(x) + ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}} - \cosh^{-1}(cx) + cx\cosh^{-1}(cx) + ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right) - ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right) + ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\,\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right) - ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\,\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{x^2\sqrt{d-c^2dx^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*Log[
d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c
*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
 c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [A]
time = 5.75, size = 438, normalized size = 1.86

method	result
default	$-\frac{a\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{2dx^2} + \frac{a\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)c^2}{2} - \frac{a\sqrt{-c^2dx^2+d}\,c^2}{2} - \frac{b\sqrt{-d\,(c^2x^2-1)}\,\text{arcc}\cdots}{2(cx+1)(cx-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x
^2+d)^(1/2))/x)*c^2-1/2*a*(-c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/x/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/x^2/(c*x+1)/(c*x-1)*ar
ccosh(c*x)-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-
1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(
sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

3.70 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=315

$$-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4x^4}+\frac{c^2\sqrt{d-c^2dx^2}\,(a-}{8x^2}$$

```
[Out] -1/4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4+1/8*c^2*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)/x^2-1/12*b*c*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(
c*x+1)^(1/2)+1/8*b*c^3*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/4*c^4*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x
^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*I*b*c^4*polylog(2,-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/8*I*b*c^4*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5924, 30, 5933, 5947, 4265, 2317, 2438}

$$\frac{c^4\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibc^4\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ibc^4\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*Sqrt[d - c^2*d*x^2])/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(8*x^2) + (c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^A
rcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/8)*b*c^4*Sqrt[d - c^2*
d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((
I/8)*b*c^4*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
 Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
```

```
erQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx &= \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x^5}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4x^4}+\frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^4}\,dx}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\cdots\\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4x^4}+\cdots\\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\cdots\\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\cdots\\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\cdots\\
&= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.70, size = 290, normalized size = 0.92

$$
\frac{1}{24}\left(\frac{3a(-2+c^2x^2)\sqrt{d-c^2dx^2}}{x^4}-3ac^4\sqrt{d}\log(x)+3ac^4\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{b\sqrt{d-c^2dx^2}\left(-2cx+3c^3x^3-6\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)+3c^2x^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-3ic^4x^4\cosh^{-1}(cx)\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\log\left(1+ie^{-\cosh^{-1}(cx)}\right)\right)-3ic^4x^4\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)\right)}{x^4\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] ((3*a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^4 - 3*a*c^4*Sqrt[d]*Log[x] + 3*
a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]
*(-2*c*x + 3*c^3*x^3 - 6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
+ 3*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - (3*I)*c^4*x
^4*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) - (
3*I)*c^4*x^4*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]
])))/(x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Maple [A]
time = 6.39, size = 541, normalized size = 1.72

method	result
default	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{4 d\,x^{4}}-\frac{a\,c^{2} \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 d\,x^{2}}+\frac{a\,c^{4} \sqrt{d}\,\ln \left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)}{8}-\frac{a\,c^{4} \sqrt{-c^{2} d\,x^{2}+d}}{8}+\frac{b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8*
a*c^4*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2*d*
x^2+d)^(1/2)+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+
1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-3/8*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x+1)/x^4/(c*x-1)*arccosh(c*x)-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1
/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/8*(c^4*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^4 - (-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^2) - 2*(-c^2*d*x
^2 + d)^(3/2)/(d*x^4))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5, x)
```

3.71 $\int x^4(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=360

$$\frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3\ldots}{}$$

```
[Out] 1/8*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3/128*d*x*(a+b*arccosh(c*x)
)*(-c^2*d*x^2+d)^(1/2)/c^4-1/64*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)/c^2+1/16*d*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/256*b*d*x^2*(-c
^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/256*b*d*x^4*(-c^2*d*x^2
+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*b*c*d*x^6*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/64*b*c^3*d*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-3/256*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^5
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5930, 5926, 5939, 5893, 30, 74, 14}

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{64c^2}-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{256bc^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^4}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d*x^2*Sqrt[d - c^2*d*x^2])/(256*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d*x^4*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*x
^6*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^8*Sq
rt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d*x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^4) - (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(64*c^2) + (d*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
16 + (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/8 - (3*d*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
```

```
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^4(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d\right.}{}$$

$$=\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{}$$

$$=-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{dx^3\sqrt{d-}}{}$$

$$=\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^8}{64\sqrt{-1}}$$

$$=\frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc}{32\sqrt{-}}$$

Mathematica [A]
time = 3.20, size = 337, normalized size = 0.94

$$\frac{d\left(-576acx\sqrt{d-c^2dx^2}\left(3+2c^2x^2-24c^4x^4+16c^6x^6\right)-1728a\sqrt{d}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(-1+c^2x^2\right)}\right)+\frac{32b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh\left(2\cosh^{-1}(cx)\right)-9\cosh\left(4\cosh^{-1}(cx)\right)-2\cosh\left(6\cosh^{-1}(cx)\right)+12\cosh^{-1}(cx)\left(-3\sinh\left(2\cosh^{-1}(cx)\right)+3\sinh\left(4\cosh^{-1}(cx)\right)+\sinh\left(6\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}+\frac{b\sqrt{d-c^2dx^2}\left(1440\cosh^{-1}(cx)^2-576\cosh\left(2\cosh^{-1}(cx)\right)+144\cosh\left(4\cosh^{-1}(cx)\right)+64\cosh\left(6\cosh^{-1}(cx)\right)+9\cosh\left(8\cosh^{-1}(cx)\right)-24\cosh^{-1}(cx)\left(-48\sinh\left(2\cosh^{-1}(cx)\right)+24\sinh\left(4\cosh^{-1}(cx)\right)+16\sinh\left(6\cosh^{-1}(cx)\right)+3\sinh\left(8\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)}{73728c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-576*a*c*x*Sqrt[d - c^2*d*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
) - 1728*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] + (32*b*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]
] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*S
inh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^2]*(1440*ArcCosh[c
*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*Arc
Cosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*ArcCosh[c
*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8*ArcCosh
[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(73728*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(304) = 608$.
time = 4.37, size = 783, normalized size = 2.18

method	result
default	$-\frac{a\,x^3(-c^2d\,x^2+d)^{\frac{5}{2}}}{8c^2d}-\frac{ax(-c^2d\,x^2+d)^{\frac{5}{2}}}{16c^4d}+\frac{ax(-c^2d\,x^2+d)^{\frac{3}{2}}}{64c^4}+\frac{3adx\sqrt{-c^2d\,x^2+d}}{128c^4}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+}}\right.}{128c^4\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+1
/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/128
*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3
/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*
d-1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-
88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c*x-32*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(c*x))*d/(c*
x+1)/c^5/(c*x-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/c^5/(c*x-1)+1/1
024*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^
5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1
/2)+4*c*x)*(1+4*arccosh(c*x))*d/(c*x+1)/c^5/(c*x-1)-1/16384*(-d*(c^2*x^2-1)
)^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*x^9+256*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*
c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*c^3*x^3-(c*x-1)^(
1/2)*(c*x+1)^(1/2)+8*c*x)*(1+8*arccosh(c*x))*d/(c*x+1)/c^5/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/128*(16*(-c^2*d*x^2 + d)^(5/2)*x^3/(c^2*d) - 2*(-c^2*d*x^2 + d)^(3/2)*x/
c^4 + 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^4*d) - 3*sqrt(-c^2*d*x^2 + d)*d*x/c^4 -
 3*d^(3/2)*arcsin(c*x)/c^5)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*x^4*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acosh}{(cx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.72 $\int x^2(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=281

$$\frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{dx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(\right.}{16c^2}$$

```
[Out] 1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/16*d*x*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
+1/32*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/96*b*c*d
*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c^3*d*x^6*(-c^
2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*(a+b*arccosh(c*x))^2*(-
c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5930, 5926, 5939, 5893, 30, 74, 14}

$$-\frac{dx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{16c^2}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{32bc^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*
d*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^6
*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/8 + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/6 - (d*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(d\sqrt{d\right.}{}$$

$$=\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}dx^3(1-cx)(1+cx)$$

$$=-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{dx\sqrt{d}}{}$$

$$=\frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx}{36\sqrt{-1}}$$

Mathematica [A]
time = 1.29, size = 270, normalized size = 0.96

$$\frac{d\left(-48acx\sqrt{d-c^2dx^2}\left(3-14c^2x^2+8c^4x^4\right)-144a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(-1+c^2x^2\right)}\right)-\frac{18b\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}+\frac{b\sqrt{d-c^2dx^2}\left(72\cosh^{-1}(cx)^2-18\cosh\left(2\cosh^{-1}(cx)\right)+9\cosh\left(4\cosh^{-1}(cx)\right)+2\cosh\left(6\cosh^{-1}(cx)\right)-12\cosh^{-1}(cx)\left(-3\sinh\left(2\cosh^{-1}(cx)\right)+3\sinh\left(4\cosh^{-1}(cx)\right)+\sinh\left(6\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)}{2304c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-48*a*c*x*Sqrt[d - c^2*d*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (18*b*Sqrt
[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*S
inh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d -
c^2*d*x^2]*(72*ArcCosh[c*x]^2 - 18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[
c*x]] + 2*Cosh[6*ArcCosh[c*x]] - 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))))/(2304*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(237) = 474$.
time = 3.78, size = 883, normalized size = 3.14

method	result

default	$-\frac{ax(-c^2 d x^2+d)^{\frac{5}{2}}}{6c^2 d}+\frac{ax(-c^2 d x^2+d)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2 d x^2+d}}{16c^2}+\frac{a d^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{16c^2\sqrt{c^2 d}}+b\left(-\frac{\sqrt{-d\,(c}}{32\sqrt{c3}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*
a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x
^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c
*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/512*
(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2
)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(-1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))
^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x
+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/512*(-d*(c^2*x^
2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4
*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1
)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] 1/48*a*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d)
 + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + b*integrat
e((-c^2*d*x^2 + d)^(3/2)*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acosh}{(cx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.73 $\int \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=200

$$-\frac{5bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3}{8}dx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\,($$

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+3/8*d*x*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/16*b*c^3*d*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3
/16*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(
1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5897, 5895, 5893, 30, 74, 14}

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{3d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{16bc\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (x*(d - c^2*d*x^2)^(3/2)*(a + b
*ArcCosh[c*x]))/4 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

```
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d-\right.}{}$$

$$= \frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-}$$

$$= -\frac{5bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3}{8}dx\sqrt{d-}$$

Mathematica [A]
time = 0.80, size = 235, normalized size = 1.18

$$-\frac{1}{8}adx(-5+2c^2x^2)\sqrt{d-c^2dx^2}-\frac{3ad^{3/2}\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{8c}-\frac{bd\sqrt{d-c^2dx^2}\,(2\cosh^{-1}(cx)^2+\cosh(2\cosh^{-1}(cx))-2\cosh^{-1}(cx)\sinh(2\cosh^{-1}(cx)))}{8c\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}+\frac{bd\sqrt{d-c^2dx^2}\,(8\cosh^{-1}(cx)^2+\cosh(4\cosh^{-1}(cx))-4\cosh^{-1}(cx)\sinh(4\cosh^{-1}(cx)))}{128c\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(a*d*x*(-5 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (3*a*d^(3/2)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/(8*c) - (b*d*Sqrt[d - c^2
*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*Ar
cCosh[c*x]]))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d*Sqrt[d - c^
2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*A
rcCosh[c*x]]))/(128*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(168) = 336$.
time = 3.29, size = 546, normalized size = 2.73

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2dx^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(-\frac{3\sqrt{-d\,(c^2x^2-1)}\,\text{arccosh}(}{16\sqrt{cx-1}\,\sqrt{cx+1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/16*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d-1/256*(-d*(c^2*x^
2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c
*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4
*arccosh(c*x))*d/(c*x+1)/(c*x-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2
*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1
+2*arccosh(c*x))*d/(c*x+1)/(c*x-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*
(1+2*arccosh(c*x))*d/(c*x+1)/(c*x-1)/c-1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x))*d/(c
*x+1)/(c*x-1)/c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,(a+b\operatorname{acosh}{(cx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.74 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=197

$$\frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{3cd\sqrt{d-}}{4b}$$

```
[Out] -(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x-3/2*c^2*d*x*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)+1/4*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+3/4*c*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+b*c*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5928, 5895, 5893, 30, 74, 14}

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bcd\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*c^3*d*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2
*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - ((d - c^2*d*x^2)^(3/2)*(
a + b*ArcCosh[c*x]))/x + (3*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)
/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*Sqrt[d - c^2*d*x^2]*Log[x])/(S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{\left(bcd\sqrt{d-\right.}{\sqrt{-1}}$$

$$= -\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d}}{}$$

$$= \frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) -$$

Mathematica [A]
time = 0.69, size = 223, normalized size = 1.13

$$\frac{1}{8}\left(-\frac{4ad(2+c^2x^2)\sqrt{d-c^2dx^2}}{x}+12acd^{3/2}\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+4bcd\sqrt{d-c^2dx^2}\left(-\frac{2\cosh^{-1}(cx)}{cx}+\frac{\cosh^{-1}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)+\frac{bcd\sqrt{d-c^2dx^2}\,(2\cosh^{-1}(cx)^2+\cosh(2\cosh^{-1}(cx))-2\cosh^{-1}(cx)\sinh(2\cosh^{-1}(cx)))}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-4*a*d*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x + 12*a*c*d^(3/2)*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 4*b*c*d*Sqrt[d - c^2*d*x^2
]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))) + (b*c*d*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Co
sh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(169) = 338$.
time = 3.97, size = 427, normalized size = 2.17

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{dx}-ac^2x(-c^2dx^2+d)^{\frac{3}{2}}-\frac{3ac^2dx\sqrt{-c^2dx^2+d}}{2}-\frac{3ac^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}+\frac{3b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c^
2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+3/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arc
cosh(c*x)^2*c*d-1/2*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^3+1/4*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2
-b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-1/2*
b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x-1/8*b*(-d*(c^
2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*d/(c*x+1)/(c*x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*
x^2 + d)^(3/2)/x)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,(a+b\operatorname{acosh}(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

3.75 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x}-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{3x^3}-\frac{c^3d\sqrt{d\ \cdots}}{2\cdots}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3+c^2*d*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/2*c^3*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-4/3*b*c^3*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5928, 5924, 29, 5893, 74, 14}

$$\frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x}-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{3x^3}-\frac{c^3d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4bc^3d\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*
d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x - ((d - c^2*d*x^2)^(3/2)*(a +
 b*ArcCosh[c*x]))/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^3*d*Sqrt[d - c^2*d*x^2]*Log
[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bcd\sqrt{d-\right.}{3\sqrt{-}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2}}{3x^3}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}$$

Mathematica [A]
time = 0.52, size = 259, normalized size = 1.28

$$\frac{-2bd^2\sqrt{\frac{-1+cx}{1+cx}}\,(1-5c^2x^2+4c^4x^4)\cosh^{-1}(cx)+3bc^3d^2x^3(-1+cx)\cosh^{-1}(cx)^2-6ac^3d^{3/2}x^3\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+d^2\left(bcx(-1+cx)-2a\sqrt{\frac{-1+cx}{1+cx}}\,(1-5c^2x^2+4c^4x^4)+8bc^3x^3(-1+cx)\log(cx)\right)}{6x^3\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4)*ArcCosh[c*
x] + 3*b*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 6*a*c^3*d^(3/2)*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] + d^2*(b*c*x*(-1 + c*x) - 2*a*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4) + 8*b*c^3*x^3*(-1 + c*x)*Log[c*x]))/(6*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(173) = 346.
time = 5.97, size = 1181, normalized size = 5.82

method	result
default	$-\frac{a\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{3d\,x^3}+\frac{2a\,c^2\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{3dx}+\frac{2a\,c^4 x\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3}+a\,c^4 dx\sqrt{-c^2 d\,x^2+d}+\frac{a\,c^4 d^2\arctan\left(\frac{\sqrt{c^2}}{\sqrt{-c^2 d}}\right.}{\sqrt{c^2 d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a*
c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*d)
^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3*d+8/3*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3*d-32*b*(-d*(c^2*x^2-1
))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^7+32*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^8-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^3*c^6+8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5
/(c*x+1)/(c*x-1)*c^8+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)
*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-52*b*(-d*(c^2*x^2-1))^(1/
2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+2/3*b*(-
d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*c^4-4*b*(-d*(c^2*x^2-1))^
(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-10/3*b
*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6-
4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)*c^3+73/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
```

```
*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+3/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(
24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+2/3*b*(-d*(c^2*x^2-
1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-14/3*b*(-d*(c^2*
x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^3
/(c*x+1)/(c*x-1)*arccosh(c*x)-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a + b*integrate((-c
^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

3.76 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=166

$$-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))}{5dx^5}+\frac{bc^5d\sqrt{d-c^2dx^2}}{5\sqrt{-1+cx}\,\sqrt{}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5-1/20*b*c*d*(-c^2*d*x^2+d
)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5*b*c^5*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5917, 74, 272, 45}

$$-\frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))}{5dx^5}-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^5d\log(x)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*Sqrt[d - c^2*d*x^2])/(5*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*
d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(5*d*x^5) + (b*c^5*d*Sqrt[d - c^2*d*x^2]
*Log[x])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-\right.}{5\sqrt{}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-\right.}{}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-\right.}{}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d(1-c}{}$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.57

$$-\frac{d\sqrt{d-c^2dx^2}\left(4(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+bcx(1-4c^2x^2-4c^4x^4\log(x))\right)}{20x^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -1/20*(d*Sqrt[d - c^2*d*x^2]*(4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*Arc
Cosh[c*x]) + b*c*x*(1 - 4*c^2*x^2 - 4*c^4*x^4*Log[x])))/(x^5*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2170 vs. $2(138) = 276$.
time = 6.29, size = 2171, normalized size = 13.08

method	result	size
default	Expression too large to display	2171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x
^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^
8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-
56/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+28/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^
8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4
+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x
^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-2*b*(-d*(c^2*x^2-1))^(1/2)
*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*arccosh(c*x)*c^11-11*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6
+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+14*b*(-d*(c^
2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c*x+1)
/(c*x-1)*arccosh(c*x)*c^8-1/5*a/d/x^5*(-c^2*d*x^2+d)^(5/2)-3/2*b*(-d*(c^2*x
^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c^5+1/5*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d*c^5-2/5*b*(-d*(c^2*x^2-1))^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^5-8/5*b*(-d*(c^2*x^2-1))^(1/
2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arcc
osh(c*x)*c^2+2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-
5*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-b*(-d*(c^2*x^
2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8
-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x
)*c^5+9/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^
2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/20*b*(-d*(c^2*x^2-1))^(1/2)*
d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*c+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^
2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*
c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
c^11-1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)*x^9/(c*x+1)/(c*x-1)*c^14+13/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8
-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*c^12-3/4*b*(-d*(c^2
*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/
(c*x-1)*c^10+7/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x
```

```
^4-5*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^8-1/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*
c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^6+1/5*b*(-d*
(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^5/(c*x
+1)/(c*x-1)*arccosh(c*x)-9/4*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x
^6+10*c^4*x^4-5*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+3/10*b*(-d*(
c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3*c^8-1
/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)*x*c^6+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)*x^7*c^12-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10
*c^4*x^4-5*c^2*x^2+1)*x^5*c^10
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 189, normalized size = 1.14

$$-\frac{\left(2\,c^{6}d^{3}\sqrt{-\frac{1}{c^{4}d}}\,\log\left(x^{2}-\frac{1}{c^{2}}\right)+2i\,(-1)^{-2\,c^{2}dx^{2}+2\,d}\,c^{4}d^{\frac{5}{2}}\log\left(-2\,c^{2}d+\frac{2\,d}{x^{2}}\right)+\frac{3\sqrt{-c^{4}dx^{4}+2\,c^{2}dx^{2}-d}\;c^{2}d^{2}}{x^{2}}-\frac{\sqrt{-c^{4}dx^{4}+2\,c^{2}dx^{2}-d}\;d^{2}}{x^{4}}\right)bc}{20\,d}-\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}b\operatorname{arcosh}\left(cx\right)}{5\,dx^{5}}-\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}a}{5\,dx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/20*(2*c^6*d^3*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 2*I*(-1)^(-2*c^2*d*x^2
 + 2*d)*c^4*d^(5/2)*log(-2*c^2*d + 2*d/x^2) + 3*sqrt(-c^4*d*x^4 + 2*c^2*d*x
^2 - d)*c^2*d^2/x^2 - sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^2/x^4)*b*c/d - 1
/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(d*x^5) - 1/5*(-c^2*d*x^2 + d)^(5/
2)*a/(d*x^5)
```

Fricas [A]
time = 0.42, size = 572, normalized size = 3.45

$$\left[-\frac{4\left(bc^{6}dx^{6}-3\,bc^{4}dx^{4}+3\,bc^{2}dx^{2}-bd\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-2\left(bc^{7}dx^{7}-bc^{5}dx^{5}\right)\sqrt{-d}\,\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}-\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{4}-1)\sqrt{-d}-d}{c^{2}x^{4}-x^{2}}\right)-\left(4\,bc^{3}dx^{3}-\left(4\,bc^{3}-bc\right)dx^{5}-bcdx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}+4\left(ac^{6}dx^{6}-3\,ac^{4}dx^{4}+3\,ac^{2}dx^{2}-ad\right)\sqrt{-c^{2}dx^{2}+d}}{20\left(c^{2}x^{7}-x^{5}\right)},\ \frac{4\left(bc^{7}dx^{7}-bc^{5}dx^{5}\right)\sqrt{d}\,\arctan\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{2}+1)\sqrt{d}}{c^{2}dx^{4}-(c^{2}+1)dx^{2}+d}\right)-4\left(bc^{6}dx^{6}-3\,bc^{4}dx^{4}+3\,bc^{2}dx^{2}-bd\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+\left(4\,bc^{3}dx^{3}-\left(4\,bc^{3}-bc\right)dx^{5}-bcdx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-4\left(ac^{6}dx^{6}-3\,ac^{4}dx^{4}+3\,ac^{2}dx^{2}-ad\right)\sqrt{-c^{2}dx^{2}+d}}{20\left(c^{2}x^{7}-x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] [-1/20*(4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(-
d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) - (4*b*c^3*d*x^3 - (4*b*c^3 -
b*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(a*c^6*d*x
^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 -
x^5), 1/20*(4*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 +
d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
 4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*d*x^5 - b*
```

```
c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(a*c^6*d*x^6 - 3*a*c^4*d*
x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**6, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6, x)
```

3.77 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=247

$$-\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}\,(a+b\,\mathrm{co}}{7dx^7}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^7-2/35*c^2*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))/d/x^5-1/42*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+2/35*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/70*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+2/35*b*c^7*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {277, 270, 5922, 12, 457, 77}

$$-\frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))}{35dx^5}-\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc^7d\log(x)\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b
*c^3*d*Sqrt[d - c^2*d*x^2])/(35*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*
d*Sqrt[d - c^2*d*x^2])/(70*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*
x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(
a + b*ArcCosh[c*x]))/(35*d*x^5) + (2*b*c^7*d*Sqrt[d - c^2*d*x^2]*Log[x])/(3
5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{x^8}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{x^8}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh}{35x^3}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{70x^2\sqrt{}}$$

Mathematica [A]
time = 0.10, size = 136, normalized size = 0.55

$$-\frac{d\sqrt{d-c^2dx^2}\,\left(30(-1+cx)^{5/2}(1+cx)^{5/2}\,(a+b\cosh^{-1}(cx))+12c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}\,(a+b\cosh^{-1}(cx))+bcx(5-12c^2x^2+3c^4x^4-12c^6x^6\log(x))\right)}{210x^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -1/210*(d*Sqrt[d - c^2*d*x^2]*(30*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*A
rcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c
*x]) + b*c*x*(5 - 12*c^2*x^2 + 3*c^4*x^4 - 12*c^6*x^6*Log[x])))/(x^7*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3144 vs. $2(207) = 414$.
time = 6.93, size = 3145, normalized size = 12.73

method	result	size
default	Expression too large to display	3145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -5/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^
4*x^4-105*c^2*x^2+25)*x/(c*x+1)/(c*x-1)*c^8+25/7*b*(-d*(c^2*x^2-1))^(1/2)*d
```

/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^7/(c*x+1
)/(c*x-1)*arccosh(c*x)-421/42*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c
^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*c^5-161/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^
6+154*c^4*x^4-105*c^2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+10/7*b*(-
d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*
c^2*x^2+25)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+2/35*b*(-d*(c^2*x^
2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+2
5)*x^13/(c*x+1)/(c*x-1)*c^20-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*
c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^18+3*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6
+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-6*b*(-d*
(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^
2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-2*b*(-d*(c^2*x^2
-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25
)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15-4*b*(-d*(c^2*x^2-1))^(1
/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^6/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13+44/5*b*(-d*(c^2*x^2-1))^(1/2)*
d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c
^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^10/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17+55/14*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^1
0*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^4/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c^3-25/42*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8
-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-1
/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x
^4-105*c^2*x^2+25)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15+12*b*(-d*(c^2*x^2-1
))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*
x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14-164/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*
c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/(c*x+1)/(c*
x-1)*arccosh(c*x)*c^12+52/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8
*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x
)*c^10+1966/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x
^6+154*c^4*x^4-105*c^2*x^2+25)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-3272/35*b
(-d(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-1
05*c^2*x^2+25)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+472/7*b*(-d*(c^2*x^2-1))^
(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^3
/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-170/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10
*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^5/(c*x+1)/(c*x-1)
*arccosh(c*x)*c^2+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-7
0*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^13-
11/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4
*x^4-105*c^2*x^2+25)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-9/35*b*(-d*(c^2*x
^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+
25)*x^11/(c*x+1)/(c*x-1)*c^18-1/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10

```
-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x+1)/(c*x-1)*c^16
+142/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154
*c^4*x^4-105*c^2*x^2+25)*x^7/(c*x+1)/(c*x-1)*c^14-72/35*b*(-d*(c^2*x^2-1))^
(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5
/(c*x+1)/(c*x-1)*c^12+25/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8
*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c*x+1)/(c*x-1)*c^10-116/10
5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^
4-105*c^2*x^2+25)*x^5*c^12+20/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-3
5*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3*c^10-5/21*b*(-d*(c^2*x
^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+
25)*x*c^8+26/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6
*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7*c^14-2/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(
35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11*c^18+1/
5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^
4-105*c^2*x^2+25)*x^9*c^16+359/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-
35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*c^7+2/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d*c^7-4/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^7+a*(-1/7/d/x^7*(-c^2*d*x^2+d)^(5/2)-
2/35*c^2/d/x^5*(-c^2*d*x^2+d)^(5/2))
```

Maxima [A]
time = 0.49, size = 163, normalized size = 0.66

$$\frac{1}{210}\left(12\,c^{6}\sqrt{-d}\,d\log\left(x\right)-\frac{3\,c^{4}\sqrt{-d}\,dx^{4}-12\,c^{2}\sqrt{-d}\,dx^{2}+5\,\sqrt{-d}\,d}{x^{6}}\right)bc-\frac{1}{35}\,b\left(\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{5}}+\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{7}}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{35}\,a\left(\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{5}}+\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] 1/210*(12*c^6*sqrt(-d)*d*log(x) - (3*c^4*sqrt(-d)*d*x^4 - 12*c^2*sqrt(-d)*d
*x^2 + 5*sqrt(-d)*d)/x^6)*b*c - 1/35*b*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^5
) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))*arccosh(c*x) - 1/35*a*(2*(-c^2*d*x^2
+ d)^(5/2)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))
```

Fricas [A]
time = 0.43, size = 648, normalized size = 2.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [-1/210*(6*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 -
5*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(b*c^9*d*x^9 -
```

```
 b*c^7*d*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (3*b*c^5
*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*
a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)
, 1/210*(12*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)
*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 6
*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 5*b*d)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - (3*b*c^5
- 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1) - 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a
*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acosh}\,(cx))}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**8,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**8, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8, x)
```

3.78 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^{10}}\,dx$

Optimal. Leaf size=328

$$-\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{420x^4\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2bc^7d\sqrt{d-c^2dx^2}}{315x^2\sqrt{-1+cx}\,\sqrt{1}}$$

```
[Out] -1/9*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^9-4/63*c^2*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))/d/x^7-8/315*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(
c*x))/d/x^5-1/72*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)
+5/189*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/420*b
*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/315*b*c^7*d*(
-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/315*b*c^9*d*ln(x)*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {277, 270, 5922, 12, 1265, 907}

$$-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{315dx^5}-\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bc^9d\log(x)\sqrt{d-c^2dx^2}}{315\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^7d\sqrt{d-c^2dx^2}}{315x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{420x^4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -1/72*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b
*c^3*d*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5
*d*Sqrt[d - c^2*d*x^2])/(420*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^7*d
*Sqrt[d - c^2*d*x^2])/(315*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*
x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(9*d*x^9) - (4*c^2*(d - c^2*d*x^2)^(5/2)*(
a + b*ArcCosh[c*x]))/(63*d*x^7) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCo
sh[c*x]))/(315*d*x^5) + (8*b*c^9*d*Sqrt[d - c^2*d*x^2]*Log[x])/(315*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{x^{10}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{x^{10}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-}}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-}}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-}}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-}}{105x^5}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{420x}$$

Mathematica [A]
time = 0.21, size = 154, normalized size = 0.47

$$-\frac{d\sqrt{d-c^2dx^2}\,\left(840(-1+cx)^{5/2}(1+cx)^{5/2}\,(a+b\cosh^{-1}(cx))+96c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}\,(5+2c^2x^2)\,(a+b\cosh^{-1}(cx))+bcx(105-200c^2x^2+18c^4x^4+48c^6x^6-192c^8x^8\log(x))\right)}{7560x^9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -1/7560*(d*Sqrt[d - c^2*d*x^2]*(840*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b
*ArcCosh[c*x]) + 96*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(5 + 2*c^2*x^2
)*(a + b*ArcCosh[c*x]) + b*c*x*(105 - 200*c^2*x^2 + 18*c^4*x^4 + 48*c^6*x^6
 - 192*c^8*x^8*Log[x])))/(x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4261 vs. $2(276) = 552$.
time = 7.26, size = 4262, normalized size = 12.99

method	result	size
default	Expression too large to display	4262

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/9/d/x^9*(-c^2*d*x^2+d)^(5/2)+4/9*c^2*(-1/7/d/x^7*(-c^2*d*x^2+d)^(5/2)
-2/35*c^2/d/x^5*(-c^2*d*x^2+d)^(5/2)))+64/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840
```

*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2
+1225)*x^12/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^21-7700/9*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2+3151/15*b*
(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*
x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-6
0632/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^
8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5/(c*x+1)/(c*x-1)*arccosh(
c*x)*c^14+59884/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10
+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x+1)/(c*x-
1)*arccosh(c*x)*c^12+2069/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945
*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^7/(c*
x+1)/(c*x-1)*c^16-4639/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^
10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5/(c*x+1
)/(c*x-1)*c^14+455/27*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x+1)/(c*
x-1)*c^12+4*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8
*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^8/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*c^17+25915/126*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^2/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*c^7-1285/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c
^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^4/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*c^5+21175/216*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x
^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/
x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1225/72*b*(-d*(c^2*x^2-1))^(1/2)*d/(840
*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2
+1225)/x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+4189/180*b*(-d*(c^2*x^2-1))^(1/2)*
d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c
^2*x^2+1225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15-16/3*b*(-d*(c^2*x^2-1))^(
1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4
725*c^2*x^2+1225)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^19-35/9*b*(-d*(c^2*x^2
-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4
*x^4-4725*c^2*x^2+1225)*x/(c*x+1)/(c*x-1)*c^10+1225/9*b*(-d*(c^2*x^2-1))^(1
/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-47
25*c^2*x^2+1225)/x^9/(c*x+1)/(c*x-1)*arccosh(c*x)+280/9*b*(-d*(c^2*x^2-1))^
(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-1187/60*b*(
-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x
^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^13-829
/56*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-273
0*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^11+8/315*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c
x-1)^(1/2)(c*x+1)^(1/2))^2)*d*c^9-43264/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(84
0*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^
2+1225)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+113594/63*b*(-d*(c^2*x^2-1))^(1

```
/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-47
25*c^2*x^2+1225)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-174520/63*b*(-d*(c^2*x^
2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^
4*x^4-4725*c^2*x^2+1225)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6-64/3*b*(-d*(c
^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+62
10*c^4*x^4-4725*c^2*x^2+1225)*x^13/(c*x+1)/(c*x-1)*arccosh(c*x)*c^22+104/3*
b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^
6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
0+128/315*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x
^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^17/(c*x+1)/(c*x-1)*c^26-1
6/315*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2
730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^15/(c*x+1)/(c*x-1)*c^24-344/1
89*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730
*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^13/(c*x+1)/(c*x-1)*c^22-922/945*
b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^
6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^11/(c*x+1)/(c*x-1)*c^20+2906/945*b*
(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*
x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^9/(c*x+1)/(c*x-1)*c^18-212/15*b*(-d*(
c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6
210*c^4*x^4-4725*c^2*x^2+1225)*x^9/(c*x+1)/(c*x...
```

Maxima [A]
time = 0.50, size = 225, normalized size = 0.69

$$\frac{1}{7560}\left(192\,c^{8}\sqrt{-d}\,d\log\left(x\right)-\frac{48\,c^{6}\sqrt{-d}\,dx^{6}+18\,c^{4}\sqrt{-d}\,dx^{4}-200\,c^{2}\sqrt{-d}\,dx^{2}+105\,\sqrt{-d}\,d}{x^{8}}\right)bc-\frac{1}{315}\,b\left(\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{4}}{dx^{5}}+\frac{20\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{7}}+\frac{35\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{9}}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{315}\,a\left(\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{4}}{dx^{5}}+\frac{20\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{7}}+\frac{35\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] 1/7560*(192*c^8*sqrt(-d)*d*log(x) - (48*c^6*sqrt(-d)*d*x^6 + 18*c^4*sqrt(-d
)*d*x^4 - 200*c^2*sqrt(-d)*d*x^2 + 105*sqrt(-d)*d)/x^8)*b*c - 1/315*b*(8*(-
c^2*d*x^2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) +
35*(-c^2*d*x^2 + d)^(5/2)/(d*x^9))*arccosh(c*x) - 1/315*a*(8*(-c^2*d*x^2 +
d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*x
^2 + d)^(5/2)/(d*x^9))
```

Fricas [A]
time = 0.44, size = 720, normalized size = 2.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [-1/7560*(24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^
4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
1)) - 96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 -
200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1) + 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6
 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^1
1 - x^9), 1/7560*(192*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*arctan(sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x
^2 + d)) - 24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x
^4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3
+ 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^
4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**10,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10, x)
```

3.79 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx$

Optimal. Leaf size=409

$$-\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{1386x^6\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^7d\sqrt{d-c^2d}}{770x^4\sqrt{-1+cx}\,\sqrt{}}$$

```
[Out] -1/11*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^11-2/33*c^2*(-c^2*d*x^2+d
)^(5/2)*(a+b*arccosh(c*x))/d/x^9-8/231*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos
h(c*x))/d/x^7-16/1155*c^6*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5-1/1
10*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/66*b*c^3*d
*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/1386*b*c^5*d*(-c^2*
d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/770*b*c^7*d*(-c^2*d*x^2+d)
^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/1155*b*c^9*d*(-c^2*d*x^2+d)^(1/2)/
x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+16/1155*b*c^11*d*ln(x)*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {277, 270, 5922, 12, 1813, 1634}

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{1155dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{231dx^7}-\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bc^{11}d\log(x)\sqrt{d-c^2dx^2}}{1155\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4bc^9d\sqrt{d-c^2dx^2}}{1155x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^7d\sqrt{d-c^2dx^2}}{770x^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d\sqrt{d-c^2dx^2}}{1386x^6\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -1/110*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*c^3*d*Sqrt[d - c^2*d*x^2])/(66*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*
d*Sqrt[d - c^2*d*x^2])/(1386*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^7*d*S
qrt[d - c^2*d*x^2])/(770*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^9*d*Sqr
t[d - c^2*d*x^2])/(1155*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2
)^(5/2)*(a + b*ArcCosh[c*x]))/(11*d*x^11) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(a
 + b*ArcCosh[c*x]))/(33*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos
h[c*x]))/(231*d*x^7) - (16*c^6*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/
(1155*d*x^5) + (16*b*c^11*d*Sqrt[d - c^2*d*x^2]*Log[x])/(1155*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{x^{12}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{231x^7}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{}{1386}$$

Mathematica [A]
time = 0.29, size = 170, normalized size = 0.42

$$-\frac{d\sqrt{d-c^2dx^2}\left(630(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+12c^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(35+20c^2x^2+8c^4x^4\right)\left(a+b\cosh^{-1}(cx)\right)+bcx\left(63-105c^2x^2+5c^4x^4+9c^6x^6+24c^8x^8-96c^{10}x^{10}\log(x)\right)\right)}{6930x^{11}\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -1/6930*(d*Sqrt[d - c^2*d*x^2]*(630*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b
*ArcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(35 + 20*c^2*x
^2 + 8*c^4*x^4)*(a + b*ArcCosh[c*x]) + b*c*x*(63 - 105*c^2*x^2 + 5*c^4*x^4
+ 9*c^6*x^6 + 24*c^8*x^8 - 96*c^10*x^10*Log[x])))/(x^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5522 vs. $2(345) = 690$.
time = 9.74, size = 5523, normalized size = 13.50

method	result	size
default	Expression too large to display	5523

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [A]
time = 0.53, size = 287, normalized size = 0.70

$$\frac{1}{6930}\left(96\,c^{10}\sqrt{-d}\,d\log\left(x\right) - \frac{24\,c^{8}\sqrt{-d}\,dx^{8} + 9\,c^{6}\sqrt{-d}\,dx^{6} + 5\,c^{4}\sqrt{-d}\,dx^{4} - 105\,c^{2}\sqrt{-d}\,dx^{2} + 63\,\sqrt{-d}\,d}{x^{10}}\right)bc - \frac{1}{1155}\left(\frac{16\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{6}}{dx^{5}} + \frac{40\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{4}}{dx^{7}} + \frac{70\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{9}} + \frac{105\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{11}}\right)b\,\mathrm{arcosh}\left(cx\right) - \frac{1}{1155}\left(\frac{16\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{6}}{dx^{5}} + \frac{40\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{4}}{dx^{7}} + \frac{70\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}c^{2}}{dx^{9}} + \frac{105\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{dx^{11}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] 1/6930*(96*c^10*sqrt(-d)*d*log(x) - (24*c^8*sqrt(-d)*d*x^8 + 9*c^6*sqrt(-d)
*d*x^6 + 5*c^4*sqrt(-d)*d*x^4 - 105*c^2*sqrt(-d)*d*x^2 + 63*sqrt(-d)*d)/x^1
0)*b*c - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d
)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x
^2 + d)^(5/2)/(d*x^11))*b*arccosh(c*x) - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*
c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(
5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*a
```

Fricas [A]
time = 0.43, size = 792, normalized size = 1.94

$$\left[-\frac{6\left(16bc^{12}dx^{12} - 8bc^{10}dx^{10} - 2bc^{8}dx^{8} - bc^{6}dx^{6} - 145bc^{4}dx^{4} + 245bc^{2}dx^{2} - 105bd\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right) - 48\left(bc^{13}dx^{13} - bc^{11}dx^{11}\right)\sqrt{-d}\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}-\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}x^{2}-1}\left(x^{4}-1\right)\sqrt{-d}-d}{c^{2}x^{4}-x^{2}}\right) + \left(24bc^{9}dx^{9} + 9bc^{7}dx^{7} - \left(24bc^{9}+9bc^{7}+5bc^{5}-105bc^{3}+63bc\right)dx^{11} + 5bc^{5}dx^{5} - 105bc^{3}dx^{3} + 63bcdx\right)\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}x^{2}-1} + 6\left(16ac^{12}dx^{12} - 8ac^{10}dx^{10} - 2ac^{8}dx^{8} - ac^{6}dx^{6} - 145ac^{4}dx^{4} + 245ac^{2}dx^{2} - 105ad\right)\sqrt{-c^{2}dx^{2}+d}}{6930\left(c^{2}x^{13}-x^{11}\right)}, \frac{96\left(bc^{13}dx^{13} - bc^{11}dx^{11}\right)\sqrt{d}\arctan\left(\frac{\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}x^{2}-1}\left(x^{2}+1\right)\sqrt{d}}{c^{2}dx^{4}-\left(c^{2}+1\right)dx^{2}+d}\right) - 6\left(16bc^{12}dx^{12} - 8bc^{10}dx^{10} - 2bc^{8}dx^{8} - bc^{6}dx^{6} - 145bc^{4}dx^{4} + 245bc^{2}dx^{2} - 105bd\right)\sqrt{-c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}-1}\right) - \left(24bc^{9}dx^{9} + 9bc^{7}dx^{7} - \left(24bc^{9}+9bc^{7}+5bc^{5}-105bc^{3}+63bc\right)dx^{11} + 5bc^{5}dx^{5} - 105bc^{3}dx^{3} + 63bcdx\right)\sqrt{-c^{2}dx^{2}+d}\sqrt{c^{2}x^{2}-1} - 6\left(16ac^{12}dx^{12} - 8ac^{10}dx^{10} - 2ac^{8}dx^{8} - ac^{6}dx^{6} - 145ac^{4}dx^{4} + 245ac^{2}dx^{2} - 105ad\right)\sqrt{-c^{2}dx^{2}+d}}{6930\left(c^{2}x^{13}-x^{11}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [-1/6930*(6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x
^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(
c*x + sqrt(c^2*x^2 - 1)) - 48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(-d)*log(
(c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^
4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (
24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 -
 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(
16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^
4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11
), 1/6930*(96*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d
)) - 6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 -
145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) - (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7
 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 +
 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 6*(16*a*c^12*d*x^12 -
 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^
2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**12,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12, x)
```

3.80 $\int x^7(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=399

$$\frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bdx^5\sqrt{d-c^2dx^2}}{1925c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^7\sqrt{d-c^2}}{1617c\sqrt{-1+cx}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^8/d+3/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^8/d^2-1/3*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^8
/d^3+1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c^8/d^4+16/1155*b*d*x*(-
c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3465*b*d*x^3*(-c^2*d*x
^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/1925*b*d*x^5*(-c^2*d*x^2+d)^(
1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1617*b*d*x^7*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/297*b*c*d*x^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+1/121*b*c^3*d*x^11*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {272, 45, 5922, 12, 1824}

$$\frac{(d-c^2dx^2)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^8d^4}-\frac{(d-c^2dx^2)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^8d^3}+\frac{3(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^8d^2}-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^8d}-\frac{4bcdx^9\sqrt{d-c^2dx^2}}{297\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bdx^7\sqrt{d-c^2dx^2}}{1617c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^{11}\sqrt{d-c^2dx^2}}{121\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bdx^5\sqrt{d-c^2dx^2}}{1925c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d*x*Sqrt[d - c^2*d*x^2])/(1155*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8
*b*d*x^3*Sqrt[d - c^2*d*x^2])/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*
b*d*x^5*Sqrt[d - c^2*d*x^2])/(1925*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d
*x^7*Sqrt[d - c^2*d*x^2])/(1617*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d*
x^9*Sqrt[d - c^2*d*x^2])/(297*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^11
*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)
^(5/2)*(a + b*ArcCosh[c*x]))/(5*c^8*d) + (3*(d - c^2*d*x^2)^(7/2)*(a + b*Ar
cCosh[c*x]))/(7*c^8*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(3*
c^8*d^3) + ((d - c^2*d*x^2)^(11/2)*(a + b*ArcCosh[c*x]))/(11*c^8*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^7\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^7(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2}{}$$

$$= \frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{}{19}$$

Mathematica [A]
time = 0.16, size = 182, normalized size = 0.46

$$-\frac{d\sqrt{d-c^2dx^2}\left(-b\left(16x+\frac{8c^2x^3}{3}+\frac{6c^4x^5}{5}+\frac{5c^6x^7}{7}-\frac{140c^8x^9}{9}+\frac{105c^{10}x^{11}}{11}\right)+105c^5x^6(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{2(-1+cx)^{5/2}(1+cx)^{5/2}\left(8+20c^2x^2+35c^4x^4\right)\left(a+b\cosh^{-1}(cx)\right)}{c}\right)}{1155c^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/1155*(d*Sqrt[d - c^2*d*x^2]*(-(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 +
 (5*c^6*x^7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11)) + 105*c^5*x^6*(-1 +
 c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x)^(5/2)*(1 +
 c*x)^(5/2)*(8 + 20*c^2*x^2 + 35*c^4*x^4)*(a + b*ArcCosh[c*x]))/c))/(c^7*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1845 vs. $2(335) = 670$.
time = 4.53, size = 1846, normalized size = 4.63

method	result	size
default	Expression too large to display	1846

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/11*x^6*(-c^2*d*x^2+d)^(5/2)/c^2/d+6/11/c^2*(-1/9*x^4*(-c^2*d*x^2+d)^(
5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*
x^2+d)^(5/2))))+b*(-1/247808*(-d*(c^2*x^2-1))^(1/2)*(1-11*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^5*c^5+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-61*c^2*x^2+220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3+1024*c^12*x^12-3328*c^10*x^10+620*c^4*x^4-2352*x^6*c^6+4096*c^8*x^8)
*(-1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*(c^2*x^2-1))^(1/2)*
(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*x^6*
c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)
+1/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25
*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1
))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/3072*(-d*(c^2*x^2-1))^(
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*
(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcc
osh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/30
72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4
+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1
```

```
)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/
c^8/(c*x-1)+1/100352*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*
(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+
576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5+688*x^6*c^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^
4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/
(c*x+1)/c^8/(c*x-1)-1/247808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^11*c^11+1024*c^12*x^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c
^9-3328*c^10*x^10-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+123
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-2352*x^6*c^6-220*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1
)*(1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1))
```

Maxima [A]
time = 0.49, size = 285, normalized size = 0.71

$$-\frac{1}{1155}\left(\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^8d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{1155}\left(\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^8d}\right)a+\frac{\left(33075\,c^{10}\sqrt{-d}\,dx^{11}-53900\,c^8\sqrt{-d}\,dx^9+2475\,c^6\sqrt{-d}\,dx^7+4158\,c^4\sqrt{-d}\,dx^5+9240\,c^2\sqrt{-d}\,dx^3+55440\,\sqrt{-d}\,dx\right)b}{4002075\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)
*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^
(5/2)/(c^8*d))*b*arccosh(c*x) - 1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2
*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2
/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*a + 1/4002075*(33075*c^10*sqr
t(-d)*d*x^11 - 53900*c^8*sqrt(-d)*d*x^9 + 2475*c^6*sqrt(-d)*d*x^7 + 4158*c^
4*sqrt(-d)*d*x^5 + 9240*c^2*sqrt(-d)*d*x^3 + 55440*sqrt(-d)*d*x)*b/c^7
```

Fricas [A]
time = 0.39, size = 275, normalized size = 0.69

$$-\frac{3465\,(105\,bc^{12}dx^{12}-245\,bc^{10}dx^{10}+145\,bc^8dx^8+bc^6dx^6+2\,bc^4dx^4+8\,bc^2dx^2-16\,bd)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(33075\,bc^{11}dx^{11}-53900\,bc^9dx^9+2475\,bc^7dx^7+4158\,bc^5dx^5+9240\,bc^3dx^3+55440\,bcdx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+3465\,(105\,ac^{12}dx^{12}-245\,ac^{10}dx^{10}+145\,ac^8dx^8+ac^6dx^6+2\,ac^4dx^4+8\,ac^2dx^2-16\,ad)\sqrt{-c^2dx^2+d}}{4002075\,(c^{10}x^2-c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/4002075*(3465*(105*b*c^12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8 +
 b*c^6*d*x^6 + 2*b*c^4*d*x^4 + 8*b*c^2*d*x^2 - 16*b*d)*sqrt(-c^2*d*x^2 + d)
```

```
*log(c*x + sqrt(c^2*x^2 - 1)) - (33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9 +
2475*b*c^7*d*x^7 + 4158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(105*a*c^12*d*x^12 - 245*a*c^10*
d*x^10 + 145*a*c^8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^2 - 16
*a*d)*sqrt(-c^2*d*x^2 + d))/(c^10*x^2 - c^8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.81 $\int x^5(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=321

$$\frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^5\sqrt{d-c^2dx^2}}{525c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{10bcdx^7\sqrt{d-c^2dx^2}}{441\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^6/d+2/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^6
/d^3+8/315*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/945
*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*d*x^5
*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-10/441*b*c*d*x^7*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*c^3*d*x^9*(-c^2*d*x^2+d)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$,
Rules used = {272, 45, 5922, 12, 1167}

$$-\frac{(d-c^2dx^2)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^3}+\frac{2(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^2}-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d}-\frac{10bcdx^7\sqrt{d-c^2dx^2}}{441\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdx^5\sqrt{d-c^2dx^2}}{525c\sqrt{cx-1}\sqrt{cx+1}}+\frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dx^9\sqrt{d-c^2dx^2}}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d*x*Sqrt[d - c^2*d*x^2])/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b
*d*x^3*Sqrt[d - c^2*d*x^2])/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x
^5*Sqrt[d - c^2*d*x^2])/(525*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d*x^
7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^9*Sq
rt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^6*d) + (2*(d - c^2*d*x^2)^(7/2)*(a + b*ArcCos
h[c*x]))/(7*c^6*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCosh[c*x]))/(9*c^6*
d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^5\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^5(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2}{}$$

$$= \frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{}{52}$$

Mathematica [A]
time = 0.12, size = 164, normalized size = 0.51

$$-\frac{d\sqrt{d-c^2dx^2}\left(-b\left(8x+\frac{4c^2x^3}{3}+\frac{3c^4x^5}{5}-\frac{50c^6x^7}{7}+\frac{35c^8x^9}{9}\right)+35c^3x^4(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(-1+cx)^{5/2}(1+cx)^{5/2}\left(2+5c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}\right)}{315c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/315*(d*Sqrt[d - c^2*d*x^2]*(-(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (
50*c^6*x^7)/7 + (35*c^8*x^9)/9)) + 35*c^3*x^4*(-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(2 + 5*c^2*x
^2)*(a + b*ArcCosh[c*x]))/c))/(c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(269) = 538$.
time = 3.27, size = 1376, normalized size = 4.29

method	result	size
default	Expression too large to display	1376

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/9*x^4*(-c^2*d*x^2+d)^(5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/
2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2)))+b*(-1/41472*(-d*(c^2*x^2-1))^(1/
2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*x
^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+
9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^6/(c*x
-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-
25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1
))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcco
sh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152
*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/
c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^6/
(c*x-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-1/41472*(-d*(c^2*
```

```
x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+688*x^6*c^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4
-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/(c*x+
1)/c^6/(c*x-1))
```

Maxima [A]
time = 0.48, size = 223, normalized size = 0.69

$$-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^6d}\right)b\,\mathrm{arcosh}\left(cx\right)-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^6d}\right)a+\frac{\left(1225\,c^8\sqrt{-d}\,dx^9-2250\,c^6\sqrt{-d}\,dx^7+189\,c^4\sqrt{-d}\,dx^5+420\,c^2\sqrt{-d}\,dx^3+2520\,\sqrt{-d}\,dx\right)b}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*b*arccosh(c*x) - 1/315*(35*(
-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*a + 1/99225*(1225*c^8*sqrt(-d)*d*x^9 - 2
250*c^6*sqrt(-d)*d*x^7 + 189*c^4*sqrt(-d)*d*x^5 + 420*c^2*sqrt(-d)*d*x^3 +
2520*sqrt(-d)*d*x)*b/c^5
```

Fricas [A]
time = 0.34, size = 245, normalized size = 0.76

$$-\frac{315\left(35\,bc^{10}dx^{10}-85\,bc^8dx^8+53\,bc^6dx^6+bc^4dx^4+4\,bc^2dx^2-8\,bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(1225\,bc^9dx^9-2250\,bc^7dx^7+189\,bc^5dx^5+420\,bc^3dx^3+2520\,bcdx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+315\left(35\,ac^{10}dx^{10}-85\,ac^8dx^8+53\,ac^6dx^6+ac^4dx^4+4\,ac^2dx^2-8\,ad\right)\sqrt{-c^2dx^2+d}}{99225\left(c^8x^2-c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/99225*(315*(35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*d
*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^3*d
*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 315*(35*a*c^1
0*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*x^2 -
8*a*d)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.82 $\int x^3(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=243

$$\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\,\sqrt{1+cx}}-$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^4/d+1/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/35*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+1/105*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-8/175*b*c*d*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4
9*b*c^3*d*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {272, 45, 5922, 12, 380}

$$\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d^2}-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c*d*x^
5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^7*Sq
rt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^4*d) + ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[
c*x]))/(7*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1}{}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1}{}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1}{}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{8bc}{175\sqrt{}}$$

Mathematica [A]
time = 0.13, size = 136, normalized size = 0.56

$$-\frac{d\sqrt{d-c^2dx^2}\left(105a(-1+c^2x^2)^3(2+5c^2x^2)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(210+35c^2x^2-168c^4x^4+75c^6x^6)+105b(-1+c^2x^2)^3(2+5c^2x^2)\cosh^{-1}(cx)\right)}{3675c^4(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/3675*(d*Sqrt[d - c^2*d*x^2]*(105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) - b*
c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x
^6) + 105*b*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCosh[c*x]))/(c^4*(-1 + c^2*
x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(203) = 406$.
time = 2.82, size = 966, normalized size = 3.98

method	result
default	$a\left(-\frac{x^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{7c^{2}d}-\frac{2\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{35d\,c^{4}}\right)+b\left(-\frac{\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\left(64c^{8}x^{8}-144x^{6}c^{6}+64\sqrt{cx+1}\,\sqrt{cx-1}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b*(
-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c
^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(
1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c
^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*
x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(
c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c
*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^
5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1
)-1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64
*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c
^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

Maxima [A]
time = 0.50, size = 161, normalized size = 0.66

$$-\frac{1}{35}\left(\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{2}}{c^{2}d}+\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{c^{4}d}\right)b\,\mathrm{arcosh}\left(cx\right)-\frac{1}{35}\left(\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{2}}{c^{2}d}+\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{c^{4}d}\right)a+\frac{\left(75\,c^{6}\sqrt{-d}\,dx^{7}-168\,c^{4}\sqrt{-d}\,dx^{5}+35\,c^{2}\sqrt{-d}\,dx^{3}+210\,\sqrt{-d}\,dx\right)b}{3675\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4
*d))*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*
d*x^2 + d)^(5/2)/(c^4*d))*a + 1/3675*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(
-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*b/c^3
```

Fricas [A]
time = 0.35, size = 215, normalized size = 0.88

$$-\frac{105\,(5\,bc^8dx^8-13\,bc^6dx^6+9\,bc^4dx^4+bc^2dx^2-2\,bd)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(75\,bc^7dx^7-168\,bc^5dx^5+35\,bc^3dx^3+210\,bcdx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+105\,(5\,ac^8dx^8-13\,ac^6dx^6+9\,ac^4dx^4+ac^2dx^2-2\,ad)\sqrt{-c^2dx^2+d}}{3675\,(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/3675*(105*(5*b*c^8*d*x^8 - 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2
- 2*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^7*d*x^
7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) + 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*x^6 + 9*a*c^4*d*x^4 + a*c
^2*d*x^2 - 2*a*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.83 $\int x(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=165

$$\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^2/d+1/5*b*d*x*(-c^2*d*x^2+d)
^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/15*b*c*d*x^3*(-c^2*d*x^2+d)^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+1/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5914, 41, 200}

$$-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}+\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*x
^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x]))/(5*c^2*d)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{\left(bd\sqrt{c\right.}{}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{\left(bd\sqrt{c\right.}{}$$

$$=\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^5\sqrt{}}{25\sqrt{-1+}}$$

Mathematica [A]
time = 0.13, size = 107, normalized size = 0.65

$$-\frac{d\sqrt{d-c^2dx^2}\left(15a(-1+c^2x^2)^3+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(-15+10c^2x^2-3c^4x^4)+15b(-1+c^2x^2)^3\cosh^{-1}(cx)\right)}{75c^2\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/75*(d*Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(-15 + 10*c^2*x^2 - 3*c^4*x^4) + 15*b*(-1 + c^2*x^2)^3*ArcCos
h[c*x]))/(c^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(137)=274$.
time = 1.40, size = 620, normalized size = 3.76

method	result
default	$-\frac{a\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{5c^2d}+b\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(16x^6c^6-28c^4x^4+16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c^5+13c^2x^2-20\sqrt{cx+1}\right.}{800(cx+1)c^2(cx-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/c^2/d*(-c^2*d*x^2+d)^(5/2)+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*
c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)
```

```
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arc
cosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c
^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2
)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c
^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)
*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(
c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

Maxima [A]
time = 0.26, size = 102, normalized size = 0.62

$$-\frac{(-c^2dx^2+d)^{\frac{5}{2}}b\operatorname{arcosh}(cx)}{5\,c^2d}-\frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5\,c^2d}+\frac{\left(3\,c^4\sqrt{-d}\,d^2x^5-10\,c^2\sqrt{-d}\,d^2x^3+15\,\sqrt{-d}\,d^2x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(
5/2)*a/(c^2*d) + 1/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 1
5*sqrt(-d)*d^2*x)*b/(c*d)
```

Fricas [A]
time = 0.35, size = 185, normalized size = 1.12

$$-\frac{15\,(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(3\,bc^5dx^5-10\,bc^3dx^3+15\,bcdx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+15\,(ac^6dx^6-3\,ac^4dx^4+3\,ac^2dx^2-ad)\sqrt{-c^2dx^2+d}}{75\,(c^4x^2-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/75*(15*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 15
*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(a*c^6*d*x^6 - 3*a*c^
4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acosh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.84 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=292

$$-\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}\,(a+b\,c$$

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+d*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)-4/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9
*b*c^3*d*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d*(a+b*arcc
osh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)+I*b*d*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d*polylog(2,I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A]
time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5930, 5926, 5947, 4265, 2317, 2438, 8, 41}

$$-\frac{2d\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*
d*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])
)/3 - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])
/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I
)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*Sqrt[d - c^2*d*x
^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
```

```
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(d\sqrt{d-c^2d\right.}}{}$$

$$= -\frac{bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}+d\sqrt{d-c^2dx^2}$$

Mathematica [A]
time = 0.80, size = 336, normalized size = 1.15

$$-\frac{1}{3}ad\left(-4+c^2x^2\right)\sqrt{d-c^2dx^2}-\frac{bd\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh\left(3\cosh^{-1}(cx)\right)\right)}{36\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}+ad^{3/2}\log(x)-ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{bd\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/3*(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b*d*Sqrt[d - c^2*d*x^2]*(9
*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*Ar
cCosh[c*x]]))/(36*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a*d^(3/2)*Log[x]
- a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*Sqrt[d - c^2*d*x^2]
*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCos
h[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*Po
lyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A]
time = 3.48, size = 499, normalized size = 1.71

method	result
default	$\frac{(-c^2 d x^2+d)^{\frac{3}{2}} a}{3} - a d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d x^2+d}}{x}\right) + a\sqrt{-c^2 d x^2+d}\, d + \frac{ib\sqrt{-d\,(c^2x^2-1)}\ \mathrm{dilog}}{\sqrt{cx}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)

[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*(-c^2*d*x^2+d)^(1/2)*d+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*c^2+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")

[Out] -1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")

[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,(a+b\operatorname{acosh}{(cx)})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\operatorname{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x, x)
```

3.85 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=311

$$-\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{(d-c^2dx^2)^{3/2}\,(a+}{2x^2}$$

```
[Out] -1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2-3/2*c^2*d*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*c^2*
d*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*c^2*d*polylog(2,-I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*
I*b*c^2*d*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5928, 5926, 5947, 4265, 2317, 2438, 8, 74, 14}

$$\frac{3c^2d\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d*Sqrt[d - c^2*d*x^2])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*
d*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh
[c*x]))/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[
E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)/2)*b*c^2*d*Sqrt[d
 - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (((3*I)/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
```

```
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{\left(bcd\sqrt{d-\right.}{2\sqrt{-}}$$

$$= -\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d-}}{2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c}$$

Mathematica [A]

time = 1.03, size = 500, normalized size = 1.61

$$\frac{1}{2}\left(-\frac{ad(1+2c^2x^2)\sqrt{d-c^2dx^2}}{x^2} - 3ac^2d^{3/2}\log(x) + 3ac^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) - \frac{2bc^2d\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + \frac{bd^2(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}}-\cosh^{-1}(cx)+cx\cosh^{-1}(cx)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{x^2\sqrt{d-c^2dx^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*d*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2) - 3*a*c^2*d^(3/2)*Log[x]
+ 3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*Sqrt[d
- c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(
-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[
c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) + (b*d^2*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x] + c*
x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 -
I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1
 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)
/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcC
osh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [A]
time = 7.22, size = 542, normalized size = 1.74

method	result
default	$-\frac{a\left(-c^{2} d\,x^{2}+d \right)^{\frac{5}{2}}}{2 d\,x^{2}}-\frac{a\,c^{2} \left(-c^{2} d\,x^{2}+d \right)^{\frac{3}{2}}}{2}+\frac{3 a\,c^{2} \ln \left(\frac{2 d +2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right) d^{\frac{3}{2}}}{2}-\frac{3 a\,c^{2} \sqrt{-c^{2} d\,x^{2}+d}\,d}{2}-\frac{b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(5/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(3/2)+3/2*a*c^2*
ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*d^(3/2)-3/2*a*c^2*(-c^2*d*x^2+d)
^(1/2)*d-b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+b*
(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/2*b*(-d*(c^2*x
^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2
)*d/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/x^2/(c*x
+1)/(c*x-1)*arccosh(c*x)-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+3/2*I
*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+
3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 +
 d)^(5/2)/(d*x^2))*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**3,x)
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

3.86 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=321

$$-\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3c^2d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\,(}{4}$$

```
[Out] -1/4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4+3/8*c^2*d*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)/x^2-1/12*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+5/8*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-3/4*c^4*d*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/8*I*b*c^4*d*polylog(2,-I*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-3/8*I*b*c^4*d*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2
*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5928, 5924, 30, 5947, 4265, 2317, 2438, 74, 14}

$$-\frac{3c^4d\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3c^2d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4}-\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d*Sqrt[d - c^2*d*x^2])/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b
*c^3*d*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - ((d - c^2*d*x^2)^(3/2)*(
a + b*ArcCosh[c*x]))/(4*x^4) - (3*c^4*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((3*I)/8)
*b*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^A
rcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
```

```
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))}{x^5}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{x^5}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{4x^4} - \frac{\left(bcd\sqrt{d-\right.}{4\sqrt{-1}} \ldots$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{8x^2} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2}}{4x^4} \ldots$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d}}{} \ldots$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d}}{} \ldots$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d}}{} \ldots$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d}}{} \ldots$$

Mathematica [A]
time = 0.82, size = 574, normalized size = 1.79

$$\frac{-2bcd^2x+2bc^2d^2x^2+15bc^3d^2x^3-15bc^4d^2x^4-6ad^2\sqrt{\frac{-1+cx}{1+cx}}+21ac^2d^2x^2\sqrt{\frac{-1+cx}{1+cx}}-15ac^4d^2x^4\sqrt{\frac{-1+cx}{1+cx}}-6bd^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+21bc^2d^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)-15bc^4d^2x^4\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+9ibc^4d^2x^4\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-9ibc^5d^2x^5\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-9ibc^4d^2x^4\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+9ibc^5d^2x^5\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+9ac^4d^{3/2}x^4\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}\log(x)-9ac^4d^{3/2}x^4\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)-9ibc^4d^2x^4(-1+cx)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+9ibc^4d^2x^4(-1+cx)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)}{24x^4\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^2*x + 2*b*c^2*d^2*x^2 + 15*b*c^3*d^2*x^3 - 15*b*c^4*d^2*x^4 - 6*a
*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 21*a*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)] - 15*a*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x] + 21*b*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCo
sh[c*x] - 15*b*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (9*I)*
b*c^4*d^2*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^5*d^2*x^5*
ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^4*d^2*x^4*ArcCosh[c*x]*L
og[1 + I/E^ArcCosh[c*x]] + (9*I)*b*c^5*d^2*x^5*ArcCosh[c*x]*Log[1 + I/E^Arc
Cosh[c*x]] + 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
x^2]*Log[x] - 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d
*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (9*I)*b*c^4*d^2*x^4*(-1 + c*x)
*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (9*I)*b*c^4*d^2*x^4*(-1 + c*x)*PolyLog[2
, I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
)
```

Maple [A]
time = 5.77, size = 570, normalized size = 1.78

method	result
default	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{4 d \,x^{4}}+\frac{a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8 d \,x^{2}}+\frac{a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{3 a \,c^{4} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right) d^{\frac{3}{2}}}{8}+\frac{3 a \,c^{4} \sqrt{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8*
a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2)
)/x)*d^(3/2)+3/8*a*c^4*(-c^2*d*x^2+d)^(1/2)*d+5/8*b*d*(-d*(c^2*x^2-1))^(1/2
)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+5/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(
1/2)/x/(c*x-1)^(1/2)*c^3-7/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)
*arccosh(c*x)*c^2-1/12*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)
^(1/2)*c+1/4*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^4/(c*x-1)*arccosh(c*x)+3/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d-3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))*c^4*d+3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dil
og(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d-3/8*I*b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)*c^4*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] -1/8*(3*c^4*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - (-c^2*d*x^2 + d)^(3/2)*c^4 - 3*sqrt(-c^2*d*x^2 + d)*c^4*d - (-c^2*d*x^2
+ d)^(5/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d)^(5/2)/(d*x^4))*a + b*integrate(
(-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5, x)
```

3.87 $\int x^4(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=454

$$\frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{-1+cx}\,\sqrt{1+cx}}-$$

```
[Out] 1/16*d*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/10*x^5*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))-3/256*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2
)/c^4-1/128*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/32*d^2*x^
5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/512*b*d^2*x^2*(-c^2*d*x^2+d)^(1
/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/512*b*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-31/960*b*c*d^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)+21/640*b*c^3*d^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-1/100*b*c^5*d^2*x^10*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-3/512*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^5/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5930, 5926, 5939, 5893, 30, 74, 14, 272, 45}

$$\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^2}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}dx^5\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{512bc^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{256c^4}-\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^{10}\sqrt{d-c^2dx^2}}{100\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(512*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(b*d^2*x^4*Sqrt[d - c^2*d*x^2])/(512*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (31*
b*c*d^2*x^6*Sqrt[d - c^2*d*x^2])/(960*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (21*b
*c^3*d^2*x^8*Sqrt[d - c^2*d*x^2])/(640*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*x^10*Sqrt[d - c^2*d*x^2])/(100*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(256*c^4) - (d^2*x^3*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (d^2*x^5*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/32 + (d*x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]
))/16 + (x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/10 - (3*d^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(512*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^4(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{10}d^2x^5(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-$$

$$=\frac{1}{16}d^2x^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{10}d$$

$$=\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}d^2x^5(1-cx)(1+$$

$$=-\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5}{100\sqrt{}}$$

$$=\frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{21bc}{640\sqrt{}}$$

$$=\frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{31}{960}$$

Mathematica [**A**]
time = 5.04, size = 500, normalized size = 1.10

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2880*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-
15 - 10*c^2*x^2 + 248*c^4*x^4 - 336*c^6*x^6 + 128*c^8*x^8) - 43200*a*d^(5/2
)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sq
rt[d]*(-1 + c^2*x^2))] + 1600*b*d^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2
 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]
] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sin
h[6*ArcCosh[c*x]])) + 100*b*d^2*Sqrt[d - c^2*d*x^2]*(1440*ArcCosh[c*x]^2 -
576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x
]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*ArcCosh[c*x]] + 2
4*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8*ArcCosh[c*x]]))
 + b*d^2*Sqrt[d - c^2*d*x^2]*(-50400*ArcCosh[c*x]^2 + 25200*Cosh[2*ArcCosh[
c*x]] - 3600*Cosh[4*ArcCosh[c*x]] - 2600*Cosh[6*ArcCosh[c*x]] - 675*Cosh[8*
ArcCosh[c*x]] - 72*Cosh[10*ArcCosh[c*x]] + 120*ArcCosh[c*x]*(-420*Sinh[2*Ar
```

```
cCosh[c*x]] + 120*Sinh[4*ArcCosh[c*x]] + 130*Sinh[6*ArcCosh[c*x]] + 45*Sinh
[8*ArcCosh[c*x]] + 6*Sinh[10*ArcCosh[c*x]])))/(3686400*c^5*Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(386) = 772$.
time = 4.54, size = 1743, normalized size = 3.84

method	result	size
default	Expression too large to display	1743

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d+
1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3/2
56*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan((c
^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/512*(-d*(c^2*x^2-1))^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*d^2+1/102400*(-d*(c^2*x^2-1))^(1/2
)*(170*c^3*x^3-1536*c^9*x^9-(c*x-1)^(1/2)*(c*x+1)^(1/2)-10*c*x+512*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^10*c^10+1120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-400*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+50*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-
1280*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8-832*c^5*x^5+512*x^11*c^11+1696*c^7
*x^7)*(-1+10*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)-1/32768*(-d*(c^2*x^2-1))
^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272
*c^5*x^5-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-88*c^3*x^3+160*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*c^4+8*c*x-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)-1/12288
*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c
*x))*d^2/(c*x+1)/c^5/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^
3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d^2/(c*x+1)/c
^5/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d^2/(
c*x+1)/c^5/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x
))*d^2/(c*x+1)/c^5/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^
3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x))*d^2/(c*x+1)/c^5
/(c*x-1)-1/12288*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x
)*(1+6*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)-1/32768*(-d*(c^2*x^2-1))^(1/2)
```

```
*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*x^9+256*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+272
*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*c^3*x^3-(c*x-1)^(1/2)*(c
*x+1)^(1/2)+8*c*x)*(1+8*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1)+1/102400*(-d*
(c^2*x^2-1))^(1/2)*(-512*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^10*c^10+512*x^11*c^1
1+1280*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8-1536*c^9*x^9-1120*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^6*c^6+1696*c^7*x^7+400*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-
832*c^5*x^5-50*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+170*c^3*x^3+(c*x-1)^(1/2
)*(c*x+1)^(1/2)-10*c*x)*(1+10*arccosh(c*x))*d^2/(c*x+1)/c^5/(c*x-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/1280*(128*(-c^2*d*x^2 + d)^(7/2)*x^3/(c^2*d) - 8*(-c^2*d*x^2 + d)^(5/2)*
x/c^4 + 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^4*d) - 10*(-c^2*d*x^2 + d)^(3/2)*d*x
/c^4 - 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^4 - 15*d^(5/2)*arcsin(c*x)/c^5)*a +
b*integrate((-c^2*d*x^2 + d)^(5/2)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2*
b*c^2*d^2*x^6 + b*d^2*x^4)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.88 $\int x^2(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=371

$$\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\,\sqrt{1+cx}}-5$$

```
[Out] 5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/8*x^3*(-c^2*d*x^2+d)^(
5/2)*(a+b*arccosh(c*x))-5/128*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
/c^2+5/64*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/256*b*d^2*x^2*(
-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/768*b*c*d^2*x^4*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/288*b*c^3*d^2*x^6*(-c^2*d*x^2
+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/64*b*c^5*d^2*x^8*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/256*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d
)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5930, 5926, 5939, 5893, 30, 74, 14, 272, 45}

$$-\frac{5d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^2}+\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}dx^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{256bc^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5
9*b*c*d^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17
*b*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (5*d^2*x^3*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d*x^3*(d - c^2*d*x^2)^(3/2)*(a
+ b*ArcCosh[c*x]))/48 + (x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/8
- (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
```

```
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(5\right.}{}$$

$$=\frac{5}{48}d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2$$

$$=\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}d^2x^3(1-cx)(1+$$

$$=-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5}{64\sqrt{}}$$

$$=\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{17bc}{288\sqrt{}}$$

Mathematica [A]
time = 3.14, size = 415, normalized size = 1.12

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (192*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-1
5 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) - 2880*a*d^(5/2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] - 576*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c
*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 64*b*d^2*Sqrt[d - c^2*d*x^2]*
(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*
Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*
ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) + b*d^2*Sqrt[d - c^2*d*x^2]*(-1440*A
rcCosh[c*x]^2 + 576*Cosh[2*ArcCosh[c*x]] - 144*Cosh[4*ArcCosh[c*x]] - 64*Co
sh[6*ArcCosh[c*x]] - 9*Cosh[8*ArcCosh[c*x]] + 24*ArcCosh[c*x]*(-48*Sinh[2*A
rcCosh[c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8
*ArcCosh[c*x]])))/(73728*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(315) = 630$.

time = 3.81, size = 1289, normalized size = 3.47

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{7}{2}}}{8c^2d}+\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{48c^2}+\frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{192c^2}+\frac{5a\,d^2x\sqrt{-c^2d\,x^2+d}}{128c^2}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+}}\right.}{128c^2\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/192
*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/12
8*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-
5/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2
*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c
^6-88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c*x-32*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+8*arccosh(c*x))*d^
2/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+
32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c
*x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/1024*(-d*(c^2*x^
2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c
*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4
*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*
x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(-1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*
(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/
```

```
2)-2*c*x)*(1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/1024*(-d*(c^2*x^2-1)
)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arc
cosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^
(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/163
84*(-d*(c^2*x^2-1))^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9
*x^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88
*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)*(1+8*arccosh(c*x))*d^2/(c*x+1)/
c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] 1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d)
 + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 +
15*d^(5/2)*arcsin(c*x)/c^3)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*x^2*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.89 $\int \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=293

$$-\frac{25bcd^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bc^3d^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2(1-c^2x^2)^3\,\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\,(a+b$$

```
[Out] 5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/6*x*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))+5/16*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-25/9
6*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d
^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*(-c^2*x^
2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*(a+b*arc
cosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5897, 5895, 5893, 30, 74, 14, 267}

$$\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{25bcd^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2(1-c^2x^2)^3\,\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
5*b*c^3*d^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d*x*(d - c^
2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/24 + (x*(d - c^2*d*x^2)^(5/2)*(a + b*A
rcCosh[c*x]))/6 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\left(5d^2\sqrt{\right.}}{}$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{24}d^2x(1-cx)(1+cx)\sqrt{d-c^2dx}$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.$$

$$= -\frac{25bcd^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bd^2(1-c}{36c\sqrt{-}}$$

<u>Mathematica</u> [**A**]
time = 1.62, size = 347, normalized size = 1.18

$$\frac{48acd^2x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}\left(33-26c^2x^2+8c^4x^4\right)-720ad^{5/2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(-1+c^2x^2\right)}\right)-288bd^2\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)+36bd^2\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)+bd^2\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh\left(2\cosh^{-1}(cx)\right)-9\cosh\left(4\cosh^{-1}(cx)\right)-2\cosh\left(6\cosh^{-1}(cx)\right)+12\cosh^{-1}(cx)\left(-3\sinh\left(2\cosh^{-1}(cx)\right)+3\sinh\left(4\cosh^{-1}(cx)\right)+\sinh\left(6\cosh^{-1}(cx)\right)\right)\right)}{2304c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(33
- 26*c^2*x^2 + 8*c^4*x^4) - 720*a*d^(5/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 288*b*d^2*
Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] -
Sinh[2*ArcCosh[c*x]])) + 36*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b*d^2*Sqrt[d -
 c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]]
 + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))
```

<u>Maple</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(249) = 498.
time = 2.53, size = 885, normalized size = 3.02

method	result

default	$\frac{ax\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{6}+\frac{5adx\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2d\,x^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16\sqrt{c^2d}}+b\left(-\frac{5\sqrt{-}}{32}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(
-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+b*(-5/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
c*arccosh(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+3
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c-3/512*(-d*(c^2*x^2-1)
)^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arc
cosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2
*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1
+2*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c
*x)*(1+2*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c-3/512*(-d*(c^2*x^2-1))^(1/2)*(
-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x)
)*d^2/(c*x+1)/(c*x-1)/c+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^
5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a + b*integrate((-c^2*d*x
^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.90 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=284

$$\frac{9bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{4}c^2dx\left(d-c^2dx^2\right.$$

```
[Out] -5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-(-c^2*d*x^2+d)^(5/2)*(
a+b*arccosh(c*x))/x-15/8*c^2*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+
9/16*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*
c^5*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/16*c*d^2*(a
+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d
^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5928, 5897, 5895, 5893, 30, 74, 14, 272, 45}

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bcd^2\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{9bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (9*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
b*c^5*d^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*
c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*c^2*d*x*(d - c^2
*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/4 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCo
sh[c*x]))/x + (15*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{\left(bcd^2\sqrt{d\right.}{\sqrt{}}$$

$$= -\frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-}{}$$

$$= -\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{5}{4}c^2d^2x(1-cx)(1+c$$

$$= \frac{9bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{15}{8}c^2d^2x\sqrt{d-}$$

Mathematica [A]
time = 1.15, size = 305, normalized size = 1.07

$$\frac{1}{128}d^2\left(\frac{16a\sqrt{d-c^2dx^2}\,(-8-9c^2x^2+2c^4x^4)}{x} + 240ac\sqrt{d}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + 64bc\sqrt{d-c^2dx^2}\left(-\frac{2\cosh^{-1}(cx)}{cx} + \frac{\cosh^{-1}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right) + \frac{32bc\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)} - \frac{bc\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d - c^2*d*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4))/x + 240*a*c*S
qrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*
Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x]
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (32*b*c*Sqrt[d - c^2*d*x^2]*(2*
ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*c*Sqrt[d - c^2*d*x^2]*(8*Arc
```

```
Cosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/128
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(244) = 488$.
time = 4.01, size = 550, normalized size = 1.94

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx} - a\,c^2x(-c^2d\,x^2+d)^{\frac{5}{2}} - \frac{5a\,c^2dx(-c^2d\,x^2+d)^{\frac{3}{2}}}{4} - \frac{15a\,c^2d^2x\sqrt{-c^2d\,x^2+d}}{8} - \frac{15a\,c^2d^3\,\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c^
2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/16*b*(-d*(c^2*x^2-1
))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^4-b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c*d^2+9/16*b*(-d*(c^2*x^2-1))^(1/2)
*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d^2+1/4*b*(-
d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-11/8*b*(-d*(c
^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c^2*x^
2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)
*arccosh(c*x)*d^2/(c*x+1)/(c*x-1)/x-33/128*b*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(
c*x+1)^(1/2)/(c*x-1)^(1/2)+15/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)^2*c*d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x
 + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a + b*integrate((
-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{5}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

3.91 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=293

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))+\frac{5c^2d(d-c^2dx$$

```
[Out] 5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x-1/3*(-c^2*d*x^2+d)^(5/2
)*(a+b*arccosh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1
/2)-1/6*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*
c^5*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/4*c^3*d^2*(a
+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b
*c^3*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5928, 5895, 5893, 30, 74, 14, 272, 45}

$$\frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{3x}-\frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{3x^3}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))-\frac{5c^3d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{4b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{7bc^3d^2\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^5*d^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*
d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 + (5*c^2*d*(d - c^2*d*x^2
)^(3/2)*(a + b*ArcCosh[c*x]))/(3*x) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh
[c*x]))/(3*x^3) - (5*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4
*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*Log[x])
/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
```

```
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} + \frac{\left(bcd^2\sqrt{\right.}}{3}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x} - \frac{d^2(1-c}{}$$

$$= \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{5c^2d^2(1-cx)(1+cx)\sqrt{}}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{2}c^4d^2x\sqrt{d}$$

Mathematica [A]
time = 0.93, size = 319, normalized size = 1.09

$$\frac{30bc^3d^3x^3(-1+cx)\cosh^{-1}(cx)^2 - 60ac^3d^{5/2}x^3\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + 3bc^3d^3x^3(-1+cx)\cosh\left(2\cosh^{-1}(cx)\right) - 4d^3\left(bcx(1-cx) + a\sqrt{\frac{-1+cx}{1+cx}}\,(2-16c^2x^2+11c^4x^4+3c^6x^6) - 14bc^3x^3(-1+cx)\log(cx)\right) - 2bd^3(-1+cx)\cosh^{-1}(cx)\left(4\sqrt{\frac{-1+cx}{1+cx}}\,(-1-cx+7c^2x^2+7c^3x^3) + 3c^3x^3\sinh\left(2\cosh^{-1}(cx)\right)\right)}{24x^3\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (30*b*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 60*a*c^3*d^(5/2)*x^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sq
rt[d]*(-1 + c^2*x^2))] + 3*b*c^3*d^3*x^3*(-1 + c*x)*Cosh[2*ArcCosh[c*x]] -
4*d^3*(b*c*x*(1 - c*x) + a*Sqrt[(-1 + c*x)/(1 + c*x)]*(2 - 16*c^2*x^2 + 11*
c^4*x^4 + 3*c^6*x^6) - 14*b*c^3*x^3*(-1 + c*x)*Log[c*x]) - 2*b*d^3*(-1 + c*
x)*ArcCosh[c*x]*(4*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 - c*x + 7*c^2*x^2 + 7*c^3
*x^3) + 3*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. 2(249) = 498.
time = 6.12, size = 1407, normalized size = 4.80

method	result	size

default	Expression too large to display	1407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2+1/8*b
*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)-49/6*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6+7/6*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x*c^4+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2
*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/
(c*x+1)/(c*x-1)*arccosh(c*x)*x+49/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^
4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-28/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+7/6*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4+1/3*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-7/3
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)*c^3-21/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*
c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-1/6*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+14/3*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^3+5/2*b*
(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^3-5/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(
c*x)^2*d^2*c^3-7/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(
1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3-203*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6-147*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)*c^7+35*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^
2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+147*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*
c^8+5/2*a*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)+4/3*a*c^4*x*(-c^2*d*x^2+d)^(5/2)+4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+5/3*a
*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+190/3*b*
(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arcc
osh(c*x)*c^4-23/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/
(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1/3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
+ 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x
^2 + d)^(7/2)/(d*x^3))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{5}{2}}\,(a+b\operatorname{acosh}{(cx)})}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

3.92 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=293

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{c^4d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{c^2d(d-c^2dx^2}{}$$

```
[Out] 1/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3-1/5*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))/x^5-c^4*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/
x-1/20*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/30*b
*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*c^5*d^2*(
a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/1
5*b*c^5*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5928, 5924, 29, 5893, 74, 14, 272, 45}

$$-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{c^2d(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{23bc^5d^2\log(x)\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (1
1*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c
^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (c^2*d*(d - c^2*d*x^2)
^(3/2)*(a + b*ArcCosh[c*x]))/(3*x^3) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos
h[c*x]))/(5*x^5) + (c^5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23*b*c^5*d^2*Sqrt[d - c^2*d*x^2]*Log[x])
/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))}{x^6}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd^2\sqrt{d\right.}}{5\sqrt{}}$$

$$= \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{d^2(1-cx)}{}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-}}{3}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{}}{}$$

Mathematica [A]
time = 2.20, size = 400, normalized size = 1.37

$$\frac{d^2\left(8ad\sqrt{\frac{-1+cx}{1+cx}}(-1+c^2x^2)(3-11c^2x^2+23c^4x^4)+120ac^5\sqrt{d}\,x^5\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+40bc^2dx^2(1-cx)\left(cx-2\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)+2c^3x^3\log(cx)\right)-60bc^4dx^4(1-cx)\left(2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-cx\left(\cosh^{-1}(cx)^2+2\log(cx)\right)\right)-bd(1-cx)\left(20\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)+\cosh\left(5\cosh^{-1}(cx)\right)\log(cx)+\cosh\left(3\cosh^{-1}(cx)\right)(-1+5\log(cx))+cx(3+10\log(cx))-5\cosh^{-1}(cx)\sinh\left(3\cosh^{-1}(cx)\right)-\cosh^{-1}(cx)\sinh\left(5\cosh^{-1}(cx)\right)\right)\right)}{120x^5\sqrt{\frac{-1+cx}{1+cx}}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (d^2*(8*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + c^2*x^2)*(3 - 11*c^2*x^2 + 23*
c^4*x^4) + 120*a*c^5*Sqrt[d]*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 40*b*c^2*
d*x^2*(1 - c*x)*(c*x - 2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c
*x] + 2*c^3*x^3*Log[c*x]) - 60*b*c^4*d*x^4*(1 - c*x)*(2*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - b*d*(
1 - c*x)*(20*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + Cosh[5*Arc
Cosh[c*x]]*Log[c*x] + Cosh[3*ArcCosh[c*x]]*(-1 + 5*Log[c*x]) + c*x*(3 + 10*
Log[c*x]) - 5*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Sinh[5*ArcCo
sh[c*x]])))/(120*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. 2(251) = 502.
time = 8.07, size = 2429, normalized size = 8.29

method	result	size
default	Expression too large to display	2429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -a*c^6*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-a*c^6
*d^2*x*(-c^2*d*x^2+d)^(1/2)+2/15*a*c^2/d/x^3*(-c^2*d*x^2+d)^(7/2)-46/15*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^5+1/2*
b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*d^2*c^5
+23/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^5-175/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035
*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
c^5-8/15*a*c^4/d/x*(-c^2*d*x^2+d)^(7/2)-2/3*a*c^6*d*x*(-c^2*d*x^2+d)^(3/2)+
9/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c
^2*x^2+9)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)-69/20*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*c^
6-5819/30*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^
4-75*c^2*x^2+9)*x^9/(c*x+1)/(c*x-1)*c^14+69/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^5-1329/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765
*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+759/
2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2
*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11+1889/12*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^2/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*c^7+141/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c
^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-9/20*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^
2+9)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+18791/60*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x+1)/(c*x-1)*c
^12-943/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^
4-75*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*c^10+207/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*c^
8-1/5*a/d/x^5*(-c^2*d*x^2+d)^(7/2)-8/15*a*c^6*x*(-c^2*d*x^2+d)^(5/2)-115*b*
(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2
+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+777/5*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x/(c*x+1)/
(c*x-1)*arccosh(c*x)*c^4+5318/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-
765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-
1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*
c^2*x^2+9)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+3519*b*(-d*(c^2*x^2-1))^(1
/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x+1)/(c*
x-1)*arccosh(c*x)*c^12-117/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765
*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-959
5/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c
^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10-9602/15*b*(-d*(c^2*x^2-1))^
```

```
(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x+1)/(c*
x-1)*arccosh(c*x)*c^6+1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c
^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*
x)*c^13-1173*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4
*x^4-75*c^2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+1495/3
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*
x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-69/20*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x*c^6+5
819/30*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-7
5*c^2*x^2+9)*x^7*c^12-7153/60*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-76
5*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5*c^10+759/20*b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3*c^8
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/15*(10*(-c^2*d*x^2 + d)^(3/2)*c^6*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^6*d^2*
x + 15*c^5*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^4/x - 2*(-c^2*d
*x^2 + d)^(7/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^5))*a + b*integ
rate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**6, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6, x)
```

3.93 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=219

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2dx^2)^{7/2}\,(a+b\cosh^{-1}(cx))}{7dx^7}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^7-1/42*b*c*d^2*(-c^2*d*x^2
+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/28*b*c^3*d^2*(-c^2*d*x^2+d)^(1/
2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/14*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c^7*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5917, 74, 272, 45}

$$-\frac{(d-c^2dx^2)^{7/2}\,(a+b\cosh^{-1}(cx))}{7dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^7d^2\log(x)\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3
*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(28*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*
b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(14*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d
- c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*d*x^7) - (b*c^7*d^2*Sqrt[d - c^
2*d*x^2]*Log[x])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{\right.}{7}$$
$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{\right.}{}$$
$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{\right.}{}$$
$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3b}{14x^2}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 0.48

$$\frac{d^2\sqrt{d-c^2dx^2}\left(12(-1+cx)^{7/2}(1+cx)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-bcx(2-9c^2x^2+18c^4x^4+12c^6x^6\log(x))\right)}{84x^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCos
h[c*x]) - b*c*x*(2 - 9*c^2*x^2 + 18*c^4*x^4 + 12*c^6*x^6*Log[x])))/(84*x^7*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3774 vs. $2(183) = 366$.
time = 7.07, size = 3775, normalized size = 17.24

method	result	size
default	Expression too large to display	3775

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6
*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12+3*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17-b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21
*c^4*x^4-7*c^2*x^2+1)*x^12/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^19-5*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^
6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15
+5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6
*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c
^13-66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35
*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+330/
7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*
x^6+21*c^4*x^4-7*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-165/7*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+73/42*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7
*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*c^16+55/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c
^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^3/(c*
x+1)/(c*x-1)*arccosh(c*x)*c^4-11/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^1
2-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^5/(c*x+1)/(c
*x-1)*arccosh(c*x)*c^2+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^
10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^13/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^20-3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^
8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*ar
ccosh(c*x)*c^11+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c
^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*a
rccosh(c*x)*c^9-7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35
*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11/(c*x+1)/(c*x-1)*arccosh(c*
x)*c^18+1/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x
^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)+3/14
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x
^6+21*c^4*x^4-7*c^2*x^2+1)*x^13/(c*x+1)/(c*x-1)*c^20-27/28*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c
```

```
^2*x^2+1)*x^11/(c*x+1)/(c*x-1)*c^18+23*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12
*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^16-47*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21
*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1
)*arccosh(c*x)*c^14-83/84*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10
*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7*c^14+17/28*b*(-d*(c
^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4
*x^4-7*c^2*x^2+1)*x^5*c^12-67/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-
21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7/(c*x+1)/(c*x
-1)*c^14+11/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^
8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^12-17/84*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+2
1*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^10+1/42*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2
+1)*x/(c*x+1)/(c*x-1)*c^8+21/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21
*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c^15-1/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x
^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^6/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*c-119/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c
^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^13+47/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-
35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-109
/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^
6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-1/7*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*
c^4*x^4-7*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-3/2*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*
c^4*x^4-7*c^2*x^2+1)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^17-41/28*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x
^4-7*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+23/84*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^
2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/7*a/d/x^7*(-c^2*d*x^2+d)^(7/
2)-5/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 224, normalized size = 1.02

$$\frac{\left(6\,c^8 d^4 \sqrt{-\frac{1}{c^4 d}}\,\log\left(x^2-\frac{1}{c^2}\right)+6i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^6 d^{\frac{7}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+\frac{11\,\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,c^4d^3}{x^2}-\frac{7\,\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,c^2d^3}{x^4}+\frac{2\,\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,d^3}{x^6}\right)bc}{84\,d}-\frac{(-c^2dx^2+d)^{\frac{7}{2}}b\,\mathrm{arcosh}\,(cx)}{7\,dx^7}-\frac{(-c^2dx^2+d)^{\frac{7}{2}}a}{7\,dx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] 1/84*(6*c^8*d^4*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 6*I*(-1)^(-2*c^2*d*x^2
+ 2*d)*c^6*d^(7/2)*log(-2*c^2*d + 2*d/x^2) + 11*sqrt(-c^4*d*x^4 + 2*c^2*d*x
```

```
^2 - d)*c^4*d^3/x^2 - 7*sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*c^2*d^3/x^4 + 2*
sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^3/x^6)*b*c/d - 1/7*(-c^2*d*x^2 + d)^(7
/2)*b*arccosh(c*x)/(d*x^7) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(d*x^7)
```

Fricas [A]
time = 0.43, size = 703, normalized size = 3.21

$$\left[\frac{12\,(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+6\,(bc^9d^2x^9-bc^7d^2x^7)\sqrt{-d}\,\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,(x^4-1)\sqrt{-d}-d}{c^2x^4-x^2}\right)-(18\,bc^5d^2x^5-(18\,bc^5-9\,bc^3+2\,bc)d^2x^7-9\,bc^3d^2x^3+2\,bcd^2x)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+12\,(ac^8d^2x^8-4\,ac^6d^2x^6+6\,ac^4d^2x^4-4\,ac^2d^2x^2+ad^2)\sqrt{-c^2dx^2+d}}{84\,(c^2x^9-x^7)},\ -\frac{12\,(bc^9d^2x^9-bc^7d^2x^7)\sqrt{d}\,\arctan\left(\frac{\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,(x^2+1)\sqrt{d}}{c^2dx^4-(c^2+1)dx^2+d}\right)-12\,(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+(18\,bc^5d^2x^5-(18\,bc^5-9\,bc^3+2\,bc)d^2x^7-9\,bc^3d^2x^3+2\,bcd^2x)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-12\,(ac^8d^2x^8-4\,ac^6d^2x^6+6\,ac^4d^2x^4-4\,ac^2d^2x^2+ad^2)\sqrt{-c^2dx^2+d}}{84\,(c^2x^9-x^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/84*(12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^9*d
^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(
-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2))
- (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^
3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^8*d^2*x^8
 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d
*x^2 + d))/(c^2*x^9 - x^7), -1/84*(12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(
d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x
^4 - (c^2 + 1)*d*x^2 + d)) - 12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*
d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*
x^2 - 1)) + (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*
c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 12*(a*c
^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*s
qrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**8,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8, x)
```

3.94 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx$

Optimal. Leaf size=314

$$-\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2(1-c^2x^2)^4\sqrt{d-}}{72x^8\sqrt{-1+cx}\,\sqrt{1+}}$$

```
[Out] -1/9*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^9-2/63*c^2*(-c^2*d*x^2+d)^
(7/2)*(a+b*arccosh(c*x))/d/x^7-1/189*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+1/42*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-1/21*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/72*b*c*d^2*(-c^2*x^2+1)^4*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-2/63*b*c^9*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {277, 270, 5922, 12, 457, 79, 45}

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{2c^2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^9d^2\log(x)\sqrt{d-c^2dx^2}}{63\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -1/189*(b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(42*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^7*d^2*Sqrt[d - c^2*d*x^2])/(21*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*
d^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(9*d*x^9) - (2*c^2*(d -
c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(63*d*x^7) - (2*b*c^9*d^2*Sqrt[d - c
^2*d*x^2]*Log[x])/(63*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))}{x^{10}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))}{x^{10}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\,(a+b\,\mathrm{co}}{63x^3}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\,(a+b\,\mathrm{co}}{63x^3}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\,(a+b\,\mathrm{co}}{63x^3}$$

$$= -\frac{bcd^2(1-c^2x^2)^4\,\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(c}{21x^5}$$

$$= -\frac{bcd^2(1-c^2x^2)^4\,\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(c}{21x^5}$$

$$= -\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{21x^2}$$

Mathematica [A]
time = 0.11, size = 147, normalized size = 0.47

$$\frac{d^2\sqrt{d-c^2dx^2}\,\left(168(-1+cx)^{7/2}(1+cx)^{7/2}\,(a+b\cosh^{-1}(cx))+48c^2x^2(-1+cx)^{7/2}(1+cx)^{7/2}\,(a+b\cosh^{-1}(cx))-bcx(21-76c^2x^2+90c^4x^4-12c^6x^6+48c^8x^8\log(x))\right)}{1512x^9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(168*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCo
sh[c*x]) + 48*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x])
 - b*c*x*(21 - 76*c^2*x^2 + 90*c^4*x^4 - 12*c^6*x^6 + 48*c^8*x^8*Log[x]))/
(1512*x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5006 vs. 2(266) = 532.
time = 7.21, size = 5007, normalized size = 15.95

method	result	size
default	Expression too large to display	5007

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [A]
time = 0.50, size = 187, normalized size = 0.60

$$-\frac{1}{1512}\left(48\,c^{8}\sqrt{-d}\,d^{2}\log\left(x\right)-\frac{12\,c^{6}\sqrt{-d}\,d^{2}x^{6}-90\,c^{4}\sqrt{-d}\,d^{2}x^{4}+76\,c^{2}\sqrt{-d}\,d^{2}x^{2}-21\,\sqrt{-d}\,d^{2}}{x^{8}}\right)bc-\frac{1}{63}\,b\left(\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{7}{2}}c^{2}}{dx^{7}}+\frac{7\left(-c^{2}dx^{2}+d\right)^{\frac{7}{2}}}{dx^{9}}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{63}\,a\left(\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{7}{2}}c^{2}}{dx^{7}}+\frac{7\left(-c^{2}dx^{2}+d\right)^{\frac{7}{2}}}{dx^{9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] -1/1512*(48*c^8*sqrt(-d)*d^2*log(x) - (12*c^6*sqrt(-d)*d^2*x^6 - 90*c^4*sqr
t(-d)*d^2*x^4 + 76*c^2*sqrt(-d)*d^2*x^2 - 21*sqrt(-d)*d^2)/x^8)*b*c - 1/63*
b*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))
*arccosh(c*x) - 1/63*a*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^
2 + d)^(7/2)/(d*x^9))
```

Fricas [A]
time = 0.45, size = 795, normalized size = 2.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [1/1512*(24*(2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^
4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 - 1)) + 24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*log((c^2*d*x
^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*s
qrt(-d) - d)/(c^2*x^4 - x^2)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*
b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 24*(2*a*c^10*d^2*x^10 - a*c^
8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^
2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/1512*(48*(b*c^11*d^2*x^11 - b
*c^9*d^2*x^9)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 +
1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 24*(2*b*c^10*d^2*x^10 - b*c
^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d
^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (12*b*c^7*d^2*x^7 -
 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*
b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 24*(
2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 2
6*a*c^2*d^2*x^2 + 7*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**10,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10, x)
```

3.95 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^{12}}\,dx$

Optimal. Leaf size=385

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{113bc^5d^2\sqrt{d-c^2dx^2}}{4158x^6\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^7d^2\sqrt{d-c}}{924x^4\sqrt{-1+cx}}$$

```
[Out] -1/11*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^11-4/99*c^2*(-c^2*d*x^2+d
)^(7/2)*(a+b*arccosh(c*x))/d/x^9-8/693*c^4*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos
h(c*x))/d/x^7-1/110*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+23/792*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-113/4158*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/924*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/693*b
*c^9*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/693*b*c^11*
d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {277, 270, 5922, 12, 1265, 907}

$$-\frac{(d-c^2dx^2)^{7/2}(a+b\cosh^{-1}(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\cosh^{-1}(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\cosh^{-1}(cx))}{693dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{8bc^{11}d^2\log(x)\sqrt{d-c^2dx^2}}{693\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc^9d^2\sqrt{d-c^2dx^2}}{693x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^7d^2\sqrt{d-c^2dx^2}}{924x^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{113bc^5d^2\sqrt{d-c^2dx^2}}{4158x^6\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -1/110*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(23*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(792*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (113*b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(4158*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) + (b*c^7*d^2*Sqrt[d - c^2*d*x^2])/(924*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (2*b*c^9*d^2*Sqrt[d - c^2*d*x^2])/(693*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(11*d*x^11) - (4*c^2*(d - c^
2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(99*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(7
/2)*(a + b*ArcCosh[c*x]))/(693*d*x^7) - (8*b*c^11*d^2*Sqrt[d - c^2*d*x^2]*L
og[x])/(693*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))}{x^{12}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\right.}{231x^5}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{}{41}$$

Mathematica [A]
time = 0.14, size = 165, normalized size = 0.43

$$\frac{d^2\sqrt{d-c^2dx^2}\left(7560(-1+cx)^{7/2}(1+cx)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+480c^2x^2(-1+cx)^{7/2}(1+cx)^{7/2}\left(7+2c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-bcx\left(756-2415c^2x^2+2260c^4x^4-90c^6x^6-240c^8x^8+960c^{10}x^{10}\log(x)\right)\right)}{83160x^{11}\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(7560*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcC
osh[c*x]) + 480*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(7 + 2*c^2*x^2)*(a
 + b*ArcCosh[c*x]) - b*c*x*(756 - 2415*c^2*x^2 + 2260*c^4*x^4 - 90*c^6*x^6
- 240*c^8*x^8 + 960*c^10*x^10*Log[x])))/(83160*x^11*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6381 vs. $2(325) = 650$.
time = 8.10, size = 6382, normalized size = 16.58

method	result	size
default	Expression too large to display	6382

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [A]
time = 0.50, size = 251, normalized size = 0.65

$$-\frac{1}{83160}\left(960\,c^{10}\sqrt{-d}\,d^2\log\left(x\right)-\frac{240\,c^8\sqrt{-d}\,d^2x^8+90\,c^6\sqrt{-d}\,d^2x^6-2260\,c^4\sqrt{-d}\,d^2x^4+2415\,c^2\sqrt{-d}\,d^2x^2-756\sqrt{-d}\,d^2}{x^{10}}\right)bc-\frac{1}{693}\,b\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7}+\frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9}+\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{693}\,a\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7}+\frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9}+\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] -1/83160*(960*c^10*sqrt(-d)*d^2*log(x) - (240*c^8*sqrt(-d)*d^2*x^8 + 90*c^6
*sqrt(-d)*d^2*x^6 - 2260*c^4*sqrt(-d)*d^2*x^4 + 2415*c^2*sqrt(-d)*d^2*x^2 -
 756*sqrt(-d)*d^2)/x^10)*b*c - 1/693*b*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7
) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^
11))*arccosh(c*x) - 1/693*a*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^
2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))
```

Fricas [A]
time = 0.42, size = 879, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [1/83160*(120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*
b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 480*(b*c^13*d^2*x^13 - b*c^11*d^
2*x^11)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (240*b*c^9*d^2
*x^9 + 90*b*c^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 -
 756*b*c)*d^2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^
10*d^2*x^10 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a
*c^2*d^2*x^2 + 63*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11), -1/83160*
(960*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*b*c^6*d^2*
x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (240*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (2
40*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2*x^11 - 2260*b*
c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
```

```
- 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*x^2 + 63*a*d^2)*sqr
t(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**12,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12, x)
```

3.96 $\int x^7(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=458

$$\frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bd^2x^5\sqrt{d-c^2dx^2}}{5005c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bd^2x^7\sqrt{d-c^2}}{21021c\sqrt{-1+cx}\,\sqrt{}}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^8/d+1/3*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^8/d^2-3/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c
^8/d^3+1/13*(-c^2*d*x^2+d)^(13/2)*(a+b*arccosh(c*x))/c^8/d^4+16/3003*b*d^2*
x*(-c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/9009*b*d^2*x^3*(-c
^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/5005*b*d^2*x^5*(-c^2*d*
x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/21021*b*d^2*x^7*(-c^2*d*x^2+
d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-53/3861*b*c*d^2*x^9*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+27/1573*b*c^3*d^2*x^11*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/169*b*c^5*d^2*x^13*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {272, 45, 5922, 12, 1824}

$$\frac{(d-c^2dx^2)^{13/2}\left(a+b\cosh^{-1}(cx)\right)}{13c^8d^4}-\frac{3(d-c^2dx^2)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^8d^3}+\frac{(d-c^2dx^2)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^8d^2}-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^8d}-\frac{53bcd^2x^9\sqrt{d-c^2dx^2}}{3861\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bd^2x^7\sqrt{d-c^2dx^2}}{21021c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^{13}\sqrt{d-c^2dx^2}}{169\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{27bc^3d^2x^{11}\sqrt{d-c^2dx^2}}{1573\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bd^2x^5\sqrt{d-c^2dx^2}}{5005c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^2*x*Sqrt[d - c^2*d*x^2])/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(8*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(9009*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (2*b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(5005*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*d^2*x^7*Sqrt[d - c^2*d*x^2])/(21021*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (53*b*c*d^2*x^9*Sqrt[d - c^2*d*x^2])/(3861*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (27*b*c^3*d^2*x^11*Sqrt[d - c^2*d*x^2])/(1573*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (b*c^5*d^2*x^13*Sqrt[d - c^2*d*x^2])/(169*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^8*d) + ((d - c^2*d*x^
2)^(9/2)*(a + b*ArcCosh[c*x]))/(3*c^8*d^2) - (3*(d - c^2*d*x^2)^(11/2)*(a +
 b*ArcCosh[c*x]))/(11*c^8*d^3) + ((d - c^2*d*x^2)^(13/2)*(a + b*ArcCosh[c*x
]))/(13*c^8*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^7\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^7(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8}-\frac{8d}{}$$

$$=-\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8}-\frac{8d}{}$$

$$=-\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8}-\frac{8d}{}$$

$$=\frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

Mathematica [A]
time = 0.18, size = 193, normalized size = 0.42

$$\frac{d^2\sqrt{d-c^2dx^2}\left(b\left(16x+\frac{8c^2x^3}{3}+\frac{6c^4x^5}{5}+\frac{5c^6x^7}{7}-\frac{371c^8x^9}{9}+\frac{567c^{10}x^{11}}{11}-\frac{231c^{12}x^{13}}{13}\right)+231c^5x^6(-1+cx)^{7/2}(1+cx)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{2(-1+cx)^{7/2}(1+cx)^{7/2}(8+28c^2x^2+63c^4x^4)\left(a+b\cosh^{-1}(cx)\right)}{c}\right)}{3003c^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*
x^7)/7 - (371*c^8*x^9)/9 + (567*c^10*x^11)/11 - (231*c^12*x^13)/13) + 231*c
^5*x^6*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x
)^(7/2)*(1 + c*x)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4)*(a + b*ArcCosh[c*x]))
/c))/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2373 vs. $2(386) = 772$.
time = 3.95, size = 2374, normalized size = 5.18

method	result	size
default	Expression too large to display	2374

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/13*x^6*(-c^2*d*x^2+d)^(7/2)/c^2/d+6/13/c^2*(-1/11*x^4*(-c^2*d*x^2+d)^
(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*
d*x^2+d)^(7/2))))+b*(1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-1+4096*c^14*x^14+13
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-9984*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+2
912*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-13312*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^11*c^11+16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+85*c^2*x^2-364*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3+4096*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^13*c^13-15360
*c^12*x^12+22784*c^10*x^10-1204*c^4*x^4+6496*x^6*c^6-16896*c^8*x^8)*(-1+13*
arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*(c^2*x^2-1))^(1/2)*(1-11
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1
232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
11*c^11-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-61*c^2*x^2+220*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+1024*c^12*x^12-3328*c^10*x^10+620*c^4*x^4-2352*x^6
*c^6+4096*c^8*x^8)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-
d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^9*c^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^
4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*
d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6
*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c
```

```
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1
)+3/40960*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)
+5/24576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d^
2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1)+5/24576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*
x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+3/40960*(-d*(c^2*x^2-1))^
(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x
^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^
(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+1
04*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x
))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*x^6*c^6+120*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*
(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+1024*c^12*x
^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*c^10*x^10-2816*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5
*c^5-2352*x^6*c^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2/(c*x+1)
/c^8/(c*x-1)+1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-4096*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^13*c^13+4096*c^14*x^14+13312*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-
15360*c^12*x^12-16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+22784*c^10*x^10+9
984*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-16896*c^8*x^8-2912*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+6496*x^6*c^6+364*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-12
04*c^4*x^4-13*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+85*c^2*x^2-1)*(1+13*arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1))
```

Maxima [A]
time = 0.50, size = 313, normalized size = 0.68

$$-\frac{1}{3003}\left(\frac{231\,(-c^2dx^2+d)^{\frac{7}{2}}x^6}{c^2d}+\frac{126\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^4d}+\frac{56\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^8d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{3003}\left(\frac{231\,(-c^2dx^2+d)^{\frac{7}{2}}x^6}{c^2d}+\frac{126\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^4d}+\frac{56\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^6d}+\frac{16\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^8d}\right)a-\frac{\left(800415\,c^{12}\sqrt{-d}\,d^2x^{13}-2321865\,c^{10}\sqrt{-d}\,d^2x^{11}+1856855\,c^8\sqrt{-d}\,d^2x^9-32175\,c^6\sqrt{-d}\,d^2x^7-54054\,c^4\sqrt{-d}\,d^2x^5-120120\,c^2\sqrt{-d}\,d^2x^3-720720\,\sqrt{-d}\,d^2x\right)b}{135270135\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/(c^2*d) + 126*(-c^2*d*x^2 + d)^(7/2
)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)
^(7/2)/(c^8*d))*b*arccosh(c*x) - 1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/(c^
2*d) + 126*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/2)*x
^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(7/2)/(c^8*d))*a - 1/135270135*(800415*c^1
2*sqrt(-d)*d^2*x^13 - 2321865*c^10*sqrt(-d)*d^2*x^11 + 1856855*c^8*sqrt(-d)
*d^2*x^9 - 32175*c^6*sqrt(-d)*d^2*x^7 - 54054*c^4*sqrt(-d)*d^2*x^5 - 120120
*c^2*sqrt(-d)*d^2*x^3 - 720720*sqrt(-d)*d^2*x)*b/c^7
```

Fricas [**A**]
time = 0.35, size = 353, normalized size = 0.77

$$\frac{45045\,(231\,bc^{14}d^2x^{14}-798\,bc^{12}d^2x^{12}+938\,bc^{10}d^2x^{10}-376\,bc^8d^2x^8-bc^6d^2x^6-2\,bc^4d^2x^4-8\,bc^2d^2x^2+16\,bd^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(800415\,bc^{13}d^2x^{13}-2321865\,bc^{11}d^2x^{11}+1856855\,bc^9d^2x^9-32175\,bc^7d^2x^7-54054\,bc^5d^2x^5-120120\,bc^3d^2x^3-720720\,bcd^2x)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+45045\,(231\,ac^{14}d^2x^{14}-798\,ac^{12}d^2x^{12}+938\,ac^{10}d^2x^{10}-376\,ac^8d^2x^8-ac^6d^2x^6-2\,ac^4d^2x^4-8\,ac^2d^2x^2+16\,ad^2)\sqrt{-c^2dx^2+d}}{135270135\,(c^{10}x^2-c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/135270135*(45045*(231*b*c^14*d^2*x^14 - 798*b*c^12*d^2*x^12 + 938*b*c^10*
d^2*x^10 - 376*b*c^8*d^2*x^8 - b*c^6*d^2*x^6 - 2*b*c^4*d^2*x^4 - 8*b*c^2*d^
2*x^2 + 16*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (8004
15*b*c^13*d^2*x^13 - 2321865*b*c^11*d^2*x^11 + 1856855*b*c^9*d^2*x^9 - 3217
5*b*c^7*d^2*x^7 - 54054*b*c^5*d^2*x^5 - 120120*b*c^3*d^2*x^3 - 720720*b*c*d
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 45045*(231*a*c^14*d^2*x^14 -
 798*a*c^12*d^2*x^12 + 938*a*c^10*d^2*x^10 - 376*a*c^8*d^2*x^8 - a*c^6*d^2*
x^6 - 2*a*c^4*d^2*x^4 - 8*a*c^2*d^2*x^2 + 16*a*d^2)*sqrt(-c^2*d*x^2 + d))/(
c^10*x^2 - c^8)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7315 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 (a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.97 $\int x^5(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=378

$$\frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2x^5\sqrt{d-c^2dx^2}}{1155c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{113bcd^2x^7\sqrt{d-c^2dx^2}}{4851\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^6/d+2/9*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c
^6/d^3+8/693*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4
/2079*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1155
*b*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-113/4851*b*c*
d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/891*b*c^3*d^2*x
^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/121*b*c^5*d^2*x^11*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {272, 45, 5922, 12, 1167}

$$-\frac{(d-c^2dx^2)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^6d^3}+\frac{2(d-c^2dx^2)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^2}-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d}-\frac{113bcd^2x^7\sqrt{d-c^2dx^2}}{4851\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2x^5\sqrt{d-c^2dx^2}}{1155c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^{11}\sqrt{d-c^2dx^2}}{121\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{23bc^3d^2x^9\sqrt{d-c^2dx^2}}{891\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*x*Sqrt[d - c^2*d*x^2])/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4
*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(2079*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(1155*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (113
*b*c*d^2*x^7*Sqrt[d - c^2*d*x^2])/(4851*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23
*b*c^3*d^2*x^9*Sqrt[d - c^2*d*x^2])/(891*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^11*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d
 - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^6*d) + (2*(d - c^2*d*x^2)^(9
/2)*(a + b*ArcCosh[c*x]))/(9*c^6*d^2) - ((d - c^2*d*x^2)^(11/2)*(a + b*ArcC
osh[c*x]))/(11*c^6*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^5\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^5(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6}-\frac{4d^2}{}$$

$$=-\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6}-\frac{4d^2}{}$$

$$=-\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6}-\frac{4d^2}{}$$

$$=\frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{}{1}$$

Mathematica [A]
time = 0.13, size = 175, normalized size = 0.46

$$\frac{d^2\sqrt{d-c^2dx^2}\left(b\left(8x+\frac{4c^2x^3}{3}+\frac{3c^4x^5}{5}-\frac{113c^6x^7}{7}+\frac{161c^8x^9}{9}-\frac{63c^{10}x^{11}}{11}\right)+63c^3x^4(-1+cx)^{7/2}(1+cx)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(-1+cx)^{7/2}(1+cx)^{7/2}\left(2+7c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}\right)}{693c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6
*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11) + 63*c^3*x^4*(-1 + c*x)^(7/2
)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2
)*(2 + 7*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(318) = 636$.
time = 3.38, size = 1840, normalized size = 4.87

method	result	size
default	Expression too large to display	1840

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/11*x^4*(-c^2*d*x^2+d)^(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(
7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2)))+b*(1/247808*(-d*(c^2*x^2-1))^(
1/2)*(1-11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^7*c^7-1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+1024*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-61*c^2*x^2+220*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+1024*c^12*x^12-3328*c^10*x^10+620*c^4*x^
4-2352*x^6*c^6+4096*c^8*x^8)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1
/165888*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x
^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/
c^6/(c*x-1)+1/10240*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c
^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh
(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1024*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1
024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+
arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/10240*(-d*(c^2*
```

```
x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
13*c^2*x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*
x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arc
cosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1/165888*(-d*(c^2*x^2-1))^(1/2)*(-256*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*x^6*c^
6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/247
808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+102
4*c^12*x^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*c^10*x^10-2816*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5-2352*x^6*c^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x
^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2
/(c*x+1)/c^6/(c*x-1))
```

Maxima [A]
time = 0.48, size = 249, normalized size = 0.66

$$-\frac{1}{693}\left(\frac{63\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^2d}+\frac{28\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^6d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{693}\left(\frac{63\,(-c^2dx^2+d)^{\frac{7}{2}}x^4}{c^2d}+\frac{28\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^6d}\right)a-\frac{\left(19845\,c^{10}\sqrt{-d}\,d^2x^{11}-61985\,c^8\sqrt{-d}\,d^2x^9+55935\,c^6\sqrt{-d}\,d^2x^7-2079\,c^4\sqrt{-d}\,d^2x^5-4620\,c^2\sqrt{-d}\,d^2x^3-27720\,\sqrt{-d}\,d^2x\right)b}{2401245\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*b*arccosh(c*x) - 1/693*(63*(
-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*a - 1/2401245*(19845*c^10*sqrt(-d)*d^2*x
^11 - 61985*c^8*sqrt(-d)*d^2*x^9 + 55935*c^6*sqrt(-d)*d^2*x^7 - 2079*c^4*sq
rt(-d)*d^2*x^5 - 4620*c^2*sqrt(-d)*d^2*x^3 - 27720*sqrt(-d)*d^2*x)*b/c^5
```

Fricas [A]
time = 0.37, size = 317, normalized size = 0.84

$$\frac{3465\,(63\,bc^{12}d^2x^{12}-224\,bc^{10}d^2x^{10}+274\,bc^8d^2x^8-116\,bc^6d^2x^6-bc^4d^2x^4-4\,bc^2d^2x^2+8\,bd^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(19845\,bc^{11}d^2x^{11}-61985\,bc^9d^2x^9+55935\,bc^7d^2x^7-2079\,bc^5d^2x^5-4620\,bc^3d^2x^3-27720\,bcd^2x)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+3465\,(63\,ac^{12}d^2x^{12}-224\,ac^{10}d^2x^{10}+274\,ac^8d^2x^8-116\,ac^6d^2x^6-ac^4d^2x^4-4\,ac^2d^2x^2+8\,ad^2)\sqrt{-c^2dx^2+d}}{2401245\,(c^8x^2-c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/2401245*(3465*(63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d^2*x
^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*sqrt(-c
^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (19845*b*c^11*d^2*x^11 - 61985
```

```
*b*c^9*d^2*x^9 + 55935*b*c^7*d^2*x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*d^2*
x^3 - 27720*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(63*a*
c^12*d^2*x^12 - 224*a*c^10*d^2*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6
 - a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + 8*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^8*x^
2 - c^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.98 $\int x^3(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=298

$$\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{19bc^3d^2x^7\sqrt{d-c^2dx^2}}{441\sqrt{-1+cx}\,\sqrt{1+cx}}-$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^4/d+1/9*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/189*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+19/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8
1*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {272, 45, 5922, 12, 380}

$$\frac{(d-c^2dx^2)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^4d^2}-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d}-\frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^9\sqrt{d-c^2dx^2}}{81\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{19bc^3d^2x^7\sqrt{d-c^2dx^2}}{441\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d^2*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^
2*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^
2)^(7/2)*(a + b*ArcCosh[c*x]))/(7*c^4*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*Ar
cCosh[c*x]))/(9*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2}{}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2}{}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2}{}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2}{21\sqrt{-}}$$

Mathematica [A]
time = 0.14, size = 145, normalized size = 0.49

$$\frac{d^2\sqrt{d-c^2dx^2}\left(63a(-1+c^2x^2)^4(2+7c^2x^2)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(126+21c^2x^2-189c^4x^4+171c^6x^6-49c^8x^8)+63b(-1+c^2x^2)^4(2+7c^2x^2)\cosh^{-1}(cx)\right)}{3969c^4\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(63*a*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) + b*c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(126 + 21*c^2*x^2 - 189*c^4*x^4 + 171*c^6*x^6 - 4
9*c^8*x^8) + 63*b*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2)*ArcCosh[c*x]))/(3969*c^4
*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(250) = 500$.
time = 3.74, size = 1102, normalized size = 3.70

method	result
default	$a\left(-\frac{x^2(-c^2dx^2+d)^{\frac{7}{2}}}{9c^2d}-\frac{2(-c^2dx^2+d)^{\frac{7}{2}}}{63dc^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(256c^{10}x^{10}-704c^8x^8+256\sqrt{cx+1}\sqrt{cx-\right.}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b*(
1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^
8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c
^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(
c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(
c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arc
cosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*
c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-
1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh
(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*x^6*c^6+120
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

Maxima [A]

time = 0.48, size = 185, normalized size = 0.62

$$-\frac{1}{63}\left(\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{63}\left(\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)a-\frac{\left(49\,c^8\sqrt{-d}\,d^2x^9-171\,c^6\sqrt{-d}\,d^2x^7+189\,c^4\sqrt{-d}\,d^2x^5-21\,c^2\sqrt{-d}\,d^2x^3-126\,\sqrt{-d}\,d^2x\right)b}{3969\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4
*d))*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*
d*x^2 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqr
t(-d)*d^2*x^7 + 189*c^4*sqrt(-d)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sq
rt(-d)*d^2*x)*b/c^3
```

Fricas [A]
time = 0.44, size = 281, normalized size = 0.94

$$\frac{63\,(7\,bc^{10}d^2x^{10}-26\,bc^8d^2x^8+34\,bc^6d^2x^6-16\,bc^4d^2x^4-bc^2d^2x^2+2\,bd^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(49\,bc^9d^2x^9-171\,bc^7d^2x^7+189\,bc^5d^2x^5-21\,bc^3d^2x^3-126\,bcd^2x)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+63\,(7\,ac^{10}d^2x^{10}-26\,ac^8d^2x^8+34\,ac^6d^2x^6-16\,ac^4d^2x^4-ac^2d^2x^2+2\,ad^2)\sqrt{-c^2dx^2+d}}{3969\,(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 - 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 - 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 + 2*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) - (49*b*c^9*d^2*x^9 - 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 -
 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) +
 63*(7*a*c^10*d^2*x^10 - 26*a*c^8*d^2*x^8 + 34*a*c^6*d^2*x^6 - 16*a*c^4*d^2
*x^4 - a*c^2*d^2*x^2 + 2*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.99 $\int x(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=218

$$\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(d-c^2d$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^2/d+1/7*b*d^2*x*(-c^2*d*x^2+
d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c*d^2*x^3*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-1/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5914, 41, 200}

$$-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2d}+\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d^2*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2
*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d^2*x
^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*x^7*
Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(
7/2)*(a + b*ArcCosh[c*x]))/(7*c^2*d)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
```

```
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\right.}{}$$

$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\right.}{}$$

$$=\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3d^2x^5\sqrt{}}{35\sqrt{-1+c}}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.54

$$\frac{d^2\sqrt{d-c^2dx^2}\left(35a(-1+c^2x^2)^4+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(35-35c^2x^2+21c^4x^4-5c^6x^6)+35b(-1+c^2x^2)^4\cosh^{-1}(cx)\right)}{245c^2\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(35*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x^6) + 35*b*(-1 + c^2*x^2)^
4*ArcCosh[c*x]))/(245*c^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(182)=364$.
time = 1.41, size = 956, normalized size = 4.39

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\left(64c^8x^8-144x^6c^6+64\sqrt{cx+1}\,\sqrt{cx-1}\,x^7c^7+104c^4x^4-112\sqrt{cx+\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a/c^2/d*(-c^2*d*x^2+d)^(7/2)+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*
x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)
/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^
2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*
(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcc
osh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c
^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))
^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*
x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(
1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+10
4*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x)
)*d^2/(c*x+1)/c^2/(c*x-1))
```

Maxima [A]
time = 0.27, size = 118, normalized size = 0.54

$$-\frac{(-c^2dx^2+d)^{\frac{7}{2}}b\operatorname{arcosh}(cx)}{7c^2d}-\frac{(-c^2dx^2+d)^{\frac{7}{2}}a}{7c^2d}-\frac{\left(5c^6\sqrt{-d}\,d^3x^7-21c^4\sqrt{-d}\,d^3x^5+35c^2\sqrt{-d}\,d^3x^3-35\sqrt{-d}\,d^3x\right)b}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccosh(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^(
7/2)*a/(c^2*d) - 1/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 +
35*c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*b/(c*d)
```

Fricas [A]
time = 0.37, size = 241, normalized size = 1.11

$$\frac{35\left(bc^8d^2x^8-4bc^6d^2x^6+6bc^4d^2x^4-4bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(5bc^7d^2x^7-21bc^5d^2x^5+35bc^3d^2x^3-35bcd^2x\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}+35\left(ac^8d^2x^8-4ac^6d^2x^6+6ac^4d^2x^4-4ac^2d^2x^2+ad^2\right)\sqrt{-c^2dx^2+d}}{245\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (5*b*c^7*d
```

```
^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^2*x)*sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1) + 35*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^
2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.100 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=379

$$-\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}+d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(c\right.$$

```
[Out] 1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b
*arccosh(c*x))+d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/45*b*c^3*d^2*x^3*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d^2*(a+b*arccosh(c*x))*arctan(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
I*b*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5930, 5926, 5947, 4265, 2317, 2438, 8, 41, 200}

$$-\frac{2d^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}+d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))+\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))+\frac{1}{3}d(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))+\frac{ibd^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11
*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2])/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]) + (d*(d - c^2*d*x^2)^(3/2)*(a + b*Ar
cCosh[c*x]))/3 + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/5 - (2*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyL
og[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
```

```
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\left(d^2\sqrt{d-\right.}{}$$

$$= \frac{1}{3}d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}d^2(1-cx$$

$$= -\frac{8bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{}}{25\sqrt{-1+c}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{}}{25\sqrt{-1+c}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{}}{25\sqrt{-1+c}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{}}{25\sqrt{-1+c}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{}}{25\sqrt{-1+c}}$$

Mathematica [A]
time = 2.49, size = 471, normalized size = 1.24

$$\frac{1}{15}ad^2\sqrt{d-c^2dx^2}\left(23-11c^2x^2+3c^4x^4\right) - \frac{bd^2\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh\left(3\cosh^{-1}(cx)\right)\right)}{18\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)} + ad^{5/2}\log(x) - ad^{5/2}\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right) + \frac{bd^2\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)} - \frac{bd^2\sqrt{d-c^2dx^2}\left(25\cosh\left(3\cosh^{-1}(cx)\right)+9\left(-50cx+\cosh\left(5\cosh^{-1}(cx)\right)\right)+15\cosh^{-1}(cx)\left(30\sqrt{\frac{-1+cx}{1+cx}}(1+cx)-5\sinh\left(3\cosh^{-1}(cx)\right)-3\sinh\left(5\cosh^{-1}(cx)\right)\right)\right)}{3600\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b*d^2*Sqrt[
d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh
[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d^2*
Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x
*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCo
sh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^A
rcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)) - (b*d^2*Sqrt[d - c^2*d*x^2]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*
c*x + Cosh[5*ArcCosh[c*x]]) + 15*ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]])))/(3600*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A]
time = 3.52, size = 620, normalized size = 1.64

method	result
default	$\frac{(-c^2 d x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d (-c^2 d x^2+d)^{\frac{3}{2}}}{3}-a d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d x^2+d}}{x}\right)+a\sqrt{-c^2 d x^2+d}\, d^2-23b\sqrt{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*(-c^2*d*x^2+d)^(1/2)*d^2-23/15*b*(-d*(c
^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)+1/5*b*(-d*(c^2*x^2-1))^(1
/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4*c^4-1/25*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^5*c^5+11/45*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*x*c-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2-I*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))*d^2+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ar
ccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2+I*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))*d^2+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*
x^2*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2
 + d)*d^2)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x, x)
```

3.101 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=404

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(\right.$$

```
[Out] -5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/2*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/x^2-5/2*c^2*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
-1/2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7/3*b*c^3*d
^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c^5*d^2*x^3*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*c^2*d^2*(a+b*arccosh(c*x))*
arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-5/2*I*b*c^2*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/2*I*b*c^2*d^2*polylog
(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5928, 5930, 5926, 5947, 4265, 2317, 2438, 8, 41, 74, 276}

$$\frac{5c^2d^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{6}c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*
c^3*d^2*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^
2*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (5*c^2*d*(d - c^2*d*x^2)^(3/2)*
(a + b*ArcCosh[c*x]))/6 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(2*x
^2) + (5*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*d
*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((
5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
```

```
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(bcd^2\sqrt{\right.}{2}$$

$$= -\frac{5}{6}c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}}{6\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{}}{9\sqrt{-1+c}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{}}{9\sqrt{-1+c}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{}}{9\sqrt{-1+c}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{}}{9\sqrt{-1+c}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{}}{9\sqrt{-1+c}}$$

<u>Mathematica</u> [A]
time = 2.83, size = 596, normalized size = 1.48

$$\frac{1}{36}d^2\left(\frac{6a\sqrt{d-c^2dx^2}\left(-3-14c^2x^2+2c^4x^4\right)}{x^2}+\frac{bc^2\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh\left(3\cosh^{-1}(cx)\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}-90ac^2\sqrt{d}\log(x)+90ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)-\frac{72bc^2\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}+\frac{18bd(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}}-\cosh^{-1}(cx)+cx\cosh^{-1}(cx)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{x^2\sqrt{d-c^2dx^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*((6*a*Sqrt[d - c^2*d*x^2]*(-3 - 14*c^2*x^2 + 2*c^4*x^4))/x^2 + (b*c^2*
Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*Ar
cCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
- 90*a*c^2*Sqrt[d]*Log[x] + 90*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d
*x^2]] - (72*b*c^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]
*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x
]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*
PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) + (18*b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1
+ c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
```

```
c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2])))/36
```

Maple [A]
time = 5.26, size = 667, normalized size = 1.65

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{2dx^2} - \frac{ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{2} - \frac{5ac^2d(-c^2dx^2+d)^{\frac{3}{2}}}{6} + \frac{5ac^2d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{2} - \frac{5ac^2\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(7/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(5/2)-5/6*a*c^2*
d*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(
1/2))/x)-5/2*a*c^2*(-c^2*d*x^2+d)^(1/2)*d^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*c^2*d^2-1/9*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
x-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/x^2/(c*x+1)/(c*x-1)*arccosh(c*x)+5/2*I*b*(-d*(c^2*
x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))*c^2*d^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d^2-5/2*I*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*c^2*d^2+11/6*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/
(c*x-1)*arccosh(c*x)+1/3*b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*a
rccosh(c*x)*x^4-8/3*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccos
h(c*x)*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(
-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a + b*integrate
((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

3.102 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{x^5}\,dx$

Optimal. Leaf size=407

$$-\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh\right.$$

```
[Out] 5/8*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))/x^4+15/8*c^4*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(
1/2)-1/12*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+9/8*
b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^2*x*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/4*c^4*d^2*(a+b*arccosh(c*
x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+15/8*I*b*c^4*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/8*I*b*c^4*d^2*p
olylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5928, 5926, 5947, 4265, 2317, 2438, 8, 74, 14, 276}

$$-\frac{15c^4d^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{8x^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{4x^4}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))-\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^2*Sqrt[d - c^2*d*x^2])/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9
*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5
*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*c^4*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (5*c^2*d*(d - c^2*d*x^2)^(3/2)*
(a + b*ArcCosh[c*x]))/(8*x^2) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])
)/(4*x^4) - (15*c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^A
rcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((15*I)/8)*b*c^4*d^2*Sqrt
[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (((15*I)/8)*b*c^4*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x
]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
```

```
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))}{x^5}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{\left(bcd^2\sqrt{\right.}}{4}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2} - \frac{d^2(1-}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^5d^2}{4\sqrt{-1}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x}{\sqrt{-1+}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x}{\sqrt{-1+}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x}{\sqrt{-1+}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x}{\sqrt{-1+}}$$

Mathematica [A]
time = 0.97, size = 660, normalized size = 1.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^3*x + 2*b*c^2*d^3*x^2 + 27*b*c^3*d^3*x^3 - 27*b*c^4*d^3*x^4 - 24*
b*c^5*d^3*x^5 + 24*b*c^6*d^3*x^6 - 6*a*d^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 33*
a*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*a*c^4*d^3*x^4*Sqrt[(-1 + c*x)/
(1 + c*x)] - 24*a*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*ArcCosh[c*x] + 33*b*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)]*ArcCosh[c*x] - 3*b*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] -
 24*b*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (45*I)*b*c^4*d^
3*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^5*d^3*x^5*ArcCosh
[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^4*d^3*x^4*ArcCosh[c*x]*Log[1 +
 I/E^ArcCosh[c*x]] + (45*I)*b*c^5*d^3*x^5*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[
```

```
c*x]] + 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
*Log[x] - 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^
2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*P
olyLog[2, (-I)/E^ArcCosh[c*x]] + (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*PolyLog[2,
 I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [A]
time = 5.77, size = 691, normalized size = 1.70

method	result
default	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{4 d \,x^{4}}+\frac{3 a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{8 d \,x^{2}}+\frac{3 a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8}+\frac{5 a \,c^{4} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{15 a \,c^{4} d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d}}{x}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8*
a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d^(5
/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*(-c^2*d*x^2+d)^(1
/2)*d^2+b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2-b
*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/8*b*(-d*(c^
2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)+9/8*b*d^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-11/8*b*d^2*(-d*(c^2*x^2-1))^(1
/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*d^2*(-d*(c^2*x^2-1))^(1/2)/
(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*d^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/
x^4/(c*x-1)*arccosh(c*x)+15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2+15
/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
*c^4*d^2-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(
1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4*d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] -1/8*(15*c^4*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - 3*(-c^2*d*x^2 + d)^(5/2)*c^4 - 5*(-c^2*d*x^2 + d)^(3/2)*c^4*d - 15*sqrt
```

```
(-c^2*d*x^2 + d)*c^4*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^2) + 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^4))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5, x)
```

3.103 $\int \sqrt{1-x^2} \cosh^{-1}(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1-x}\,x^2}{4\sqrt{-1+x}}+\frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x)-\frac{\sqrt{1-x}\cosh^{-1}(x)^2}{4\sqrt{-1+x}}$$

```
[Out] -1/4*x^2*(1-x)^(1/2)/(-1+x)^(1/2)-1/4*arccosh(x)^2*(1-x)^(1/2)/(-1+x)^(1/2)
+1/2*x*arccosh(x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5895, 5893, 30}

$$-\frac{\sqrt{1-x}\,x^2}{4\sqrt{x-1}}+\frac{1}{2}\sqrt{1-x^2}\,x\cosh^{-1}(x)-\frac{\sqrt{1-x}\cosh^{-1}(x)^2}{4\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -1/4*(Sqrt[1 - x]*x^2)/Sqrt[-1 + x] + (x*Sqrt[1 - x^2]*ArcCosh[x])/2 - (Sqr
t[1 - x]*ArcCosh[x]^2)/(4*Sqrt[-1 + x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rubi steps

$$\int \sqrt{1-x^2}\cosh^{-1}(x)\,dx = \frac{\sqrt{1-x^2}\int \sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$
$$= \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\int x\,dx}{2\sqrt{-1+x}\sqrt{1+x}} - \frac{\sqrt{1-x^2}\int \frac{\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{1+x}}}{2\sqrt{-1+x}\sqrt{1+x}}$$
$$= -\frac{x^2\sqrt{1-x^2}}{4\sqrt{-1+x}\sqrt{1+x}} + \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\cosh^{-1}(x)^2}{4\sqrt{-1+x}\sqrt{1+x}}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.82

$$-\frac{\sqrt{-((-1+x)(1+x))}\left(\cosh\left(2\cosh^{-1}(x)\right)+2\cosh^{-1}(x)\left(\cosh^{-1}(x)-\sinh\left(2\cosh^{-1}(x)\right)\right)\right)}{8\sqrt{\frac{-1+x}{1+x}}(1+x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -1/8*(Sqrt[-((-1 + x)*(1 + x))]*(Cosh[2*ArcCosh[x]] + 2*ArcCosh[x]*(ArcCosh
[x] - Sinh[2*ArcCosh[x]])))/(Sqrt[(-1 + x)/(1 + x)]*(1 + x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(50) = 100$.
time = 3.58, size = 152, normalized size = 2.30

method	result
default	$-\frac{\sqrt{-x^2+1}\,\mathrm{arccosh}(x)^2}{4\sqrt{x-1}\sqrt{1+x}} + \frac{\sqrt{-x^2+1}\left(2x^3-2x+2\sqrt{1+x}\sqrt{x-1}\,x^2-\sqrt{x-1}\sqrt{1+x}\right)(-1+2\,\mathrm{arccosh}}{16(1+x)(x-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-x^2+1)^(1/2)/(x-1)^(1/2)/(1+x)^(1/2)*arccosh(x)^2+1/16*(-x^2+1)^(1/2
)*(2*x^3-2*x+2*(1+x)^(1/2)*(x-1)^(1/2)*x^2-(x-1)^(1/2)*(1+x)^(1/2))*(-1+2*a
rccosh(x))/(1+x)/(x-1)+1/16*(-x^2+1)^(1/2)*(-2*(1+x)^(1/2)*(x-1)^(1/2)*x^2+
2*x^3+(x-1)^(1/2)*(1+x)^(1/2)-2*x)*(1+2*arccosh(x))/(1+x)/(x-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arccosh(x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(x-1)(x+1)}\operatorname{acosh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*acosh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)*arccosh(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acosh}(x)\,\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x)*(1 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x)*(1 - x^2)^(1/2), x)
```

3.104 $\int \frac{x^5(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{8bx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}}-\frac{4bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}}-\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}}-\frac{8\sqrt{d-c^2dx^2}\,(a+b\cosh}{15c^6d}$$

```
[Out] -8/15*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)-4/45*b*x^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-8/15*(a+b*arccosh(c*x))*(-c^2*d*x^2+
d)^(1/2)/c^6/d-4/15*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/5*x
^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5938, 5914, 8, 30}

$$-\frac{x^4\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{5c^2d}-\frac{8\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{15c^6d}-\frac{4x^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{15c^4d}-\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{25c\sqrt{d-c^2dx^2}}-\frac{8bx\sqrt{cx-1}\,\sqrt{cx+1}}{15c^5\sqrt{d-c^2dx^2}}-\frac{4bx^3\sqrt{cx-1}\,\sqrt{cx+1}}{45c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (4*b*x
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(45*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^5*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(25*c*Sqrt[d - c^2*d*x^2]) - (8*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(15*c^6*d) - (4*x^2*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(15*c^4*d) - (x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*
c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
```

```
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{5c^2\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3(a+}{\sqrt{-1+}}}{5c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}} - \frac{4x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{15c^4\sqrt{d-c^2dx^2}} - \frac{x^4(1-}{}$$

$$= -\frac{4bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}} - \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1+cx)\,(a}{15c^6\sqrt{d-}}$$

$$= -\frac{8bx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{4bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}} - \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+}}{25c\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.16, size = 140, normalized size = 0.59

$$\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(120+20c^2x^2+9c^4x^4)-15a(-8+4c^2x^2+c^4x^4+3c^6x^6)-15b(-8+4c^2x^2+c^4x^4+3c^6x^6)\cosh^{-1}(cx)\right)}{225c^6d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2
+ 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 15*b*(-8 + 4*c
```

```
^2*x^2 + c^4*x^4 + 3*c^6*x^6)*ArcCosh[c*x]))/(225*c^6*d*(-1 + c*x)*(1 + c*x
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(200) = 400$.
time = 6.72, size = 670, normalized size = 2.84

method	result
default	$a\left(-\frac{x^4\sqrt{-c^2 d\,x^2+d}}{5c^2 d}+\frac{-\frac{4x^2\sqrt{-c^2 d\,x^2+d}}{15c^2 d}-\frac{8\sqrt{-c^2 d\,x^2+d}}{15d\,c^4}}{c^2}\right)+b\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\left(16x^6c^6-\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+
d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*
(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(
-1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^
4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x
^2-1)*(1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1)
)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2
*x^2-1)*(1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1))
```

Maxima [A]
time = 0.48, size = 195, normalized size = 0.83

$$-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b\,\operatorname{arcosh}(cx)-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)a+\frac{\left(9\,c^4\sqrt{-d}\,x^5+20\,c^2\sqrt{-d}\,x^3+120\,\sqrt{-d}\,x\right)b}{225\,c^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4
*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b*arccosh(c*x) - 1/15*(3*sqrt(-c^2*d*
x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x
^2 + d)/(c^6*d))*a + 1/225*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*
sqrt(-d)*x)*b/(c^5*d)
```

Fricas [A]
time = 0.40, size = 176, normalized size = 0.75

$$-\frac{15\left(3\,bc^{6}x^{6}+bc^{4}x^{4}+4\,bc^{2}x^{2}-8\,b\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(9\,bc^{5}x^{5}+20\,bc^{3}x^{3}+120\,bcx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}+15\left(3\,ac^{6}x^{6}+ac^{4}x^{4}+4\,ac^{2}x^{2}-8\,a\right)\sqrt{-c^{2}dx^{2}+d}}{225\left(c^{8}dx^{2}-c^{6}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/225*(15*(3*b*c^6*x^6 + b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 + 20*b*c^3*x^3 + 120*b*c*x)*
sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 + a*c^4*x^4 + 4*a*
c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}(a+b\operatorname{acosh}(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{5}\,\left(a+b\operatorname{acosh}(c\,x)\right)}{\sqrt{d-c^{2}\,d\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.105 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=212

$$-\frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^3\sqrt{d-c^2dx^2}}-\frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}}-\frac{3x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8c^4d}-x^3\sqrt{d-c^2dx^2}$$

```
[Out] -3/16*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/16*b*x^4
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+3/16*(a+b*arccosh(c*x))
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arccos
h(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/4*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5938, 5892, 30}

$$-\frac{x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4c^2d}+\frac{3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^5\sqrt{d-c^2dx^2}}-\frac{3x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8c^4d}-\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}}{16c\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{16c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c*Sqrt[d - c^2*d*x^2]) - (3*x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c^4*d) - (x^3*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x]))/(4*c^2*d) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCos
h[c*x])^2)/(16*b*c^5*Sqrt[d - c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{4c^2\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2(a+}{\sqrt{-1+}}}{4c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}} - \frac{x^3(1-}{}$$

$$= -\frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^3\sqrt{d-c^2dx^2}} - \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)(}{8c^4\sqrt{d-}}$$

Mathematica [A]
time = 0.59, size = 171, normalized size = 0.81

$$\frac{-\frac{16acx(3+2c^2x^2)\sqrt{d-c^2dx^2}}{d} - \frac{48a\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(-16\cosh(2\cosh^{-1}(cx))-\cosh(4\cosh^{-1}(cx))+4\cosh^{-1}(cx)\left(6\cosh^{-1}(cx)+8\sinh(2\cosh^{-1}(cx))+\sinh(4\cosh^{-1}(cx))\right)\right)}{\sqrt{d-c^2dx^2}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d - (48*a*ArcTan[(c*x*Sqrt
[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(-16*Cosh[2*ArcCosh[c*x]] - Cosh[4*ArcCosh[c*x]] + 4*Arc
Cosh[c*x]*(6*ArcCosh[c*x] + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]]))
)/Sqrt[d - c^2*d*x^2])/(128*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(180) = 360$.
time = 6.20, size = 568, normalized size = 2.68

method	result

default	$-\frac{a\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{-c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(-\frac{3\sqrt{-d\,(c^{2}x^{2}-1)}}{1\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3/
8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/16
*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*arcco
sh(c*x)^2-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x
^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x
^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2
)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/d/c^5/(c^2*x^2-1)-1/256*(-d*(c^2*
x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1
+4*arccosh(c*x))/d/c^5/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*a*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4*
d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\,\mathrm{acosh}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.106 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}}-\frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}}-\frac{2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3c^4d}-\frac{x^2\sqrt{d-c^2dx^2}\,(a}{3c^2}$$

```
[Out] -2/3*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))*(-c^
2*d*x^2+d)^(1/2)/c^4/d-1/3*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/
d
```

Rubi [A]

time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5938, 5914, 8, 30}

$$-\frac{x^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3c^2d}-\frac{2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3c^4d}-\frac{bx^3\sqrt{cx-1}\,\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\,\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/(3*c^4*d) - (x^2*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x(a+\cdots}{\sqrt{-1+\cdots}}}{3c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}} - \frac{x^2(1-c\cdots}{\cdots}$$

$$= -\frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a\cdots\right.}{3c^4\sqrt{d-c\cdots}}$$

Mathematica [**A**]
time = 0.13, size = 113, normalized size = 0.72

$$\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(6+c^2x^2)-3a(-2+c^2x^2+c^4x^4)-3b(-2+c^2x^2+c^4x^4)\cosh^{-1}(cx)\right)}{9c^4d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*
a*(-2 + c^2*x^2 + c^4*x^4) - 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]))/(9
*c^4*d*(-1 + c*x)*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(132) = 264.
time = 4.16, size = 382, normalized size = 2.45

method	result
default	$a\left(-\frac{x^{2}\sqrt{-c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2\sqrt{-c^{2}d\,x^{2}+d}}{3d\,c^{4}}\right)+b\left(-\frac{\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\left(4c^{4}x^{4}-5c^{2}x^{2}+4\sqrt{cx+1}\,\sqrt{cx}\right.}{72c^{4}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b*(-
1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^4/d/
(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^4/d/(c^2*x^2-1)
-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^4/d
/(c^2*x^2-1))
```

Maxima [A]
time = 0.48, size = 131, normalized size = 0.84

$$-\frac{1}{3}\,b\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,x^{2}}{c^{2}d}+\frac{2\,\sqrt{-c^{2}dx^{2}+d}}{c^{4}d}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,x^{2}}{c^{2}d}+\frac{2\,\sqrt{-c^{2}dx^{2}+d}}{c^{4}d}\right)+\frac{\left(c^{2}\sqrt{-d}\,x^{3}+6\,\sqrt{-d}\,x\right)b}{9\,c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d))*
arccosh(c*x) - 1/3*a*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2
+ d)/(c^4*d)) + 1/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-d)*x)*b/(c^3*d)
```

Fricas [A]
time = 0.40, size = 146, normalized size = 0.94

$$-\frac{3\,(bc^{4}x^{4}+bc^{2}x^{2}-2\,b)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-(bc^{3}x^{3}+6\,bcx)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}+3\,(ac^{4}x^{4}+ac^{2}x^{2}-2\,a)\sqrt{-c^{2}dx^{2}+d}}{9\,(c^{6}dx^{2}-c^{4}d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/9*(3*(b*c^4*x^4 + b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^3*x^3 + 6*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
```

```
 + 3*(a*c^4*x^4 + a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{acosh}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.107 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2c^2d}+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}$$

```
[Out] -1/4*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/4*(a+b*arcc
osh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*x*(a
+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5938, 5892, 30}

$$-\frac{x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2c^2d}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -1/4*(b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
```

```
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cos}{\sqrt{-1+cx}}$$
$$= -\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+}}{}$$

Mathematica [A]
time = 0.42, size = 141, normalized size = 1.07

$$\frac{-\frac{4acx\sqrt{d-c^2dx^2}}{d} - \frac{4a\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(-\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{d-c^2dx^2}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-4*a*c*x*Sqrt[d - c^2*d*x^2])/d - (4*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x
]])))/Sqrt[d - c^2*d*x^2])/(8*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(112) = 224.
time = 5.30, size = 300, normalized size = 2.27

method	result
default	$-\frac{ax\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b\Bigg(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccosh}(}{4d\,c^3(c^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/16*(-d*(c^2*x^2-1))^(1/2)*
(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1
)^(1/2))*(-1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*
(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/
2)-2*c*x)*(1+2*arccosh(c*x))/d/c^3/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b*int
egrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{acosh}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.108 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=72

$$-\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{c^2d}$$

```
[Out] -b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5914, 8}

$$-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int 1\,dx}{c\sqrt{d-c^2dx^2}}$$
$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}$$

Mathematica [**A**]
time = 0.10, size = 85, normalized size = 1.18

$$\frac{\sqrt{d-c^2dx^2}\left(a-ac^2x^2+bcx\sqrt{-1+cx}\,\sqrt{1+cx}+(b-bc^2x^2)\cosh^{-1}(cx)\right)}{c^2d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
(b - b*c^2*x^2)*ArcCosh[c*x]))/(c^2*d*(-1 + c*x)*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(64) = 128.
time = 1.80, size = 158, normalized size = 2.19

method	result
default	$-\frac{a\sqrt{-c^2d\,x^2+d}}{c^2d}+b\Bigg(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)(-1+\operatorname{arccosh}(cx))}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d\,(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/c^2/d*(-c^2*d*x^2+d)^(1/2)+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/c^2/d/(c^2*x^2-1))
```

Maxima [**A**]
time = 0.27, size = 63, normalized size = 0.88

$$\frac{b\sqrt{-d}\,x}{cd} - \frac{\sqrt{-c^2dx^2+d}\,b\,\operatorname{arcosh}(cx)}{c^2d} - \frac{\sqrt{-c^2dx^2+d}\,a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*x)/(c^2*d) - sqrt(-c^
2*d*x^2 + d)*a/(c^2*d)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.62

$$\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,bcx-(bc^2x^2-b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-(ac^2x^2-a)\sqrt{-c^2dx^2+d}}{c^4dx^2-c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 +
d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{acosh}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.109 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5892\}$

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.00

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Maple [A]
time = 0.46, size = 89, normalized size = 1.68

method	result	size
default	$\frac{a \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\, x^{2}+d}}\right)}{\sqrt{c^{2} d}}-\frac{b \sqrt{-d \left(c x -1\right) \left(c x +1\right)}\, \sqrt{c x -1}\, \sqrt{c x +1}\, \operatorname{arccosh}\left(c x \right)^{2}}{2 c d \left(c^{2} x^{2}-1\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c*x
-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(c*x)
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
 + a*arcsin(c*x)/(c*sqrt(d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(c x \right)}}{\sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2), x)
```

3.110 $\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{ib\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+I*b*polylog(2,
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5946, 4265, 2317, 2438}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]]
)/Sqrt[d - c^2*d*x^2] - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E
^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\,\sqrt{1+\right.}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\,\sqrt{1+\right.}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{-1+cx}\,\sqrt{1+c}{\sqrt{d-}}$$

Mathematica [A]
time = 0.18, size = 153, normalized size = 1.01

$$\frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{ib\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(\cosh^{-1}(cx)\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\log\left(1+ie^{-\cosh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (I*
b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c
*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyL
og[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Maple [A]
time = 3.80, size = 327, normalized size = 2.17

method	result
default	$-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}+\frac{ib\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\ln\left(1+i\left(cx+\sqrt{cx\right.\right.}}{d(c^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+I*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilo
g(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x)
, x) - a*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}(cx)}{x\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)), x)
```

3.111 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{dx}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)}{\sqrt{d-c^2dx^2}}$$

```
[Out] -b*c*ln(x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*
x))*(-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {5917, 29}

$$-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{dx}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log(x)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(d*x)) - (b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)}{\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 1.00

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x} - bc\log(x)\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x]))/x - b*c*Log[x]))/Sqrt[d - c^2*d*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(63) = 126.
time = 2.76, size = 219, normalized size = 3.08

method	result
default	$-\frac{a\sqrt{-c^2d\,x^2+d}}{dx} - \frac{b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)c}{d(c^2x^2-1)} - \frac{b\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arccosh}(cx}{(c^2x^2-1)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/
(c^2*x^2-1)/d*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d+b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 116, normalized size = 1.63

$$-\frac{\left(c^2d\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right) + i\,(-1)^{-2\,c^2dx^2+2\,d}\,\sqrt{d}\,\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)\right)bc}{2\,d} - \frac{\sqrt{-c^2dx^2+d}\,b\,\operatorname{arcosh}(cx)}{dx} - \frac{\sqrt{-c^2dx^2+d}\,a}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d)
*sqrt(d)*log(-2*c^2*d + 2*d/x^2))*b*c/d - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*
x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a/(d*x)
```

Fricas [A]
time = 0.42, size = 265, normalized size = 3.73

$$\left[-\frac{bc\sqrt{-d}\,x\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}+\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{4}-1)\sqrt{-d}-d}{c^{2}x^{4}-x^{2}}\right)+2\sqrt{-c^{2}dx^{2}+d}\,b\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+2\sqrt{-c^{2}dx^{2}+d}\,a}{2\,dx},\ \frac{bc\sqrt{d}\,x\arctan\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{2}+1)\sqrt{d}}{c^{2}dx^{4}-(c^{2}+1)dx^{2}+d}\right)-\sqrt{-c^{2}dx^{2}+d}\,b\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\sqrt{-c^{2}dx^{2}+d}\,a}{dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/2*(b*c*sqrt(-d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2
+ d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + 2*sqrt(-c
^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(d
*x), (b*c*sqrt(d)*x*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)
*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - sqrt(-c^2*d*x^2 + d)*b*log(c*
x + sqrt(c^2*x^2 - 1)) - sqrt(-c^2*d*x^2 + d)*a)/(d*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}{\left(cx \right)}}{x^{2}\sqrt{-d\left(cx - 1\right)\left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.112 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=238

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{ArcTa}}{\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)+c^2*(a+b*arccosh
(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(-c^2*d*x^2+d)^(1/2)-1/2*I*b*c^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*I*b*c^2*polylog(
2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*
x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5932, 5946, 4265, 2317, 2438, 30}

$$\frac{c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2dx^2}-\frac{ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2x\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*d*x^2) + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((I/2)
*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d
 - c^2*d*x^2] + ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^Ar
cCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+cx}\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{}}{}$$

Mathematica [A]
time = 0.71, size = 309, normalized size = 1.30

$$\frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{dx^2}+\frac{ac^2\log(x)}{\sqrt{d}}-\frac{ac^2\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right)}{\sqrt{d}}+\frac{b(1+cx)\left(cx\sqrt{\frac{-1+cx}{1+cx}}-\cosh^{-1}(cx)+cx\cosh^{-1}(cx)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)-ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\operatorname{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+ic^2x^2\sqrt{\frac{-1+cx}{1+cx}}\operatorname{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{x^2\sqrt{d-c^2dx^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a*c^2*Log[x])/Sqrt[d] - (a*c^2*Log[d
 + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (b*(1 + c*x)*(c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 +
 c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(239) = 478.
time = 5.02, size = 489, normalized size = 2.05

method	result

default	$-\frac{a\sqrt{-c^2d\,x^2+d}}{2d\,x^2}-\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arccosh}(cx)c^2}{2d(c^2x^2-1)}-\frac{b\sqrt{-d\,(c^2x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2
*d*x^2+d)^(1/2))/x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x^2*arccosh(c*x)+1/2*I*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*d
ilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) +
 sqrt(-c^2*d*x^2 + d)/(d*x^2))*a + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{x^3\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{x^3\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

3.113 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}-\frac{2c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx}-\frac{2bc^3\sqrt{-1+c}}{3\sqrt{c}}$$

```
[Out] 1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/d/x^3-2/3*c^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/d
/x
```

Rubi [A]
time = 0.15, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5932, 5917, 29, 30}

$$-\frac{2c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}-\frac{2bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\log(x)}{3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*d*x^3) - (2*c^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/
(3*Sqrt[d - c^2*d*x^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
```

```
0] && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^3}\,dx}{3\sqrt{d-c^2dx^2}} + \frac{(2c}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+}{3x\surd}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+}{3x\surd}$$

Mathematica [A]
time = 0.24, size = 174, normalized size = 1.12

$$-\frac{\sqrt{d-c^2dx^2}\left(bcx+6bc^3x^3+2a\sqrt{-1+cx}\,\sqrt{1+cx}+4ac^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}+2b\sqrt{-1+cx}\,\sqrt{1+cx}\,(1+2c^2x^2)\cosh^{-1}(cx)-4bc^3x^3\log(-1+cx)-4bc^3x^3\log\left(1+\frac{1}{-1+cx}\right)\right)}{6dx^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 + 2*a*Sqrt[-1 + c*x]*Sqrt[1
+ c*x] + 4*a*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(1 + 2*c^2*x^2)*ArcCosh[c*x] - 4*b*c^3*x^3*Log[-1 + c*x] - 4*b*c^
3*x^3*Log[1 + (-1 + c*x)^(-1)]))/(d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(131) = 262.
time = 4.98, size = 855, normalized size = 5.52

method	result
default	$a\left(-\frac{\sqrt{-c^{2}d\,x^{2}+d}}{3d\,x^{3}}-\frac{2c^{2}\sqrt{-c^{2}d\,x^{2}+d}}{3dx}\right)-\frac{4b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)c^{3}}{3d(c^{2}x^{2}-1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*(-c^2*d*x^2+d)^(1/2))-4/3*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*
x)*c^3-2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x-1)*(
c*x+1)*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8+2
*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*c^5-2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2
-1)*x^3*arccosh(c*x)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^
2-1)*x*(c*x-1)*(c*x+1)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*
x^2-1)*x^3*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c
^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^
4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-1/3*b*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*
x^4-2*c^2*x^2-1)/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x
^4-2*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1
/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)+2/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)*c^3
```

Maxima [A]
time = 0.48, size = 134, normalized size = 0.86

$$\frac{1}{6}\left(\frac{4\,c^{2}\sqrt{-d}\,\log\left(x\right)}{d}-\frac{\sqrt{-d}}{dx^{2}}\right)bc-\frac{1}{3}\,b\left(\frac{2\,\sqrt{-c^{2}dx^{2}+d}\,c^{2}}{dx}+\frac{\sqrt{-c^{2}dx^{2}+d}}{dx^{3}}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{2\,\sqrt{-c^{2}dx^{2}+d}\,c^{2}}{dx}+\frac{\sqrt{-c^{2}dx^{2}+d}}{dx^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*b*c - 1/3*b*(2*sqrt(-c^2*d
*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/3*a*(2
*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))
```

Fricas [A]
time = 0.55, size = 479, normalized size = 3.09

$$\left[-\frac{2\,(2\,bc^{4}x^{4}-bc^{2}x^{2}-b)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+2\,(bc^{5}x^{5}-bc^{3}x^{3})\sqrt{-d}\,\log\left(\frac{c^{2}dx^{6}+c^{2}dx^{2}-dx^{4}+\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{4}-1)\sqrt{-d}-d}{c^{2}x^{4}-x^{2}}\right)-\sqrt{-c^{2}dx^{2}+d}\,(bcx^{3}-bcx)\sqrt{c^{2}x^{2}-1}+2\,(2\,ac^{4}x^{4}-ac^{2}x^{2}-a)\sqrt{-c^{2}dx^{2}+d}}{6\,(c^{2}dx^{5}-dx^{3})},\ -\frac{4\,(bc^{5}x^{5}-bc^{3}x^{3})\sqrt{d}\,\arctan\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,(x^{2}+1)\sqrt{d}}{c^{2}dx^{4}-(c^{2}+1)dx^{2}+d}\right)-2\,(2\,bc^{4}x^{4}-bc^{2}x^{2}-b)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+\sqrt{-c^{2}dx^{2}+d}\,(bcx^{3}-bcx)\sqrt{c^{2}x^{2}-1}-2\,(2\,ac^{4}x^{4}-ac^{2}x^{2}-a)\sqrt{-c^{2}dx^{2}+d}}{6\,(c^{2}dx^{5}-dx^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(2*(2*b*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) + 2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x
^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)
/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1
) + 2*(2*a*c^4*x^4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^
3), 1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(2*b
*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))
 + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(2*a*c^4*x^
4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{x^{4} \sqrt{-d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

3.114 $\int \frac{x^5 \left(a+b \cosh^{-1}(cx)\right)}{\left(d-c^2 d x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=233

$$-\frac{5bx\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bx^3\sqrt{d-c^2dx^2}}{9c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{c^6d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx\right.}{c^6d^2}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^6/d^3+(a+b*arccosh(c*x))/c^6
/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^6/d^2-5
/3*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*x^3*(
-c^2*d*x^2+d)^(1/2)/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*arctanh(c*x)*(-c^
2*d*x^2+d)^(1/2)/c^6/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {272, 45, 5922, 12, 1167, 212}

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{c^6d^2}+\frac{a+b\cosh^{-1}(cx)}{c^6d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{c^6d^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5bx\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bx^3\sqrt{d-c^2dx^2}}{9c^3d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-5*b*x*Sqrt[d - c^2*d*x^2])/(3*c^5*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
x^3*Sqrt[d - c^2*d*x^2])/(9*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*
ArcCosh[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x]))/(c^6*d^2) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(3*c
^6*d^3) - (b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(c^6*d^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(}{3}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(}{3}$$

$$= \frac{5bx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{5bx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.07, size = 145, normalized size = 0.62

$$\frac{24a-12ac^2x^2-3ac^4x^4+15bcx\sqrt{-1+cx}\,\sqrt{1+cx}+bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}-3b(-8+4c^2x^2+c^4x^4)\cosh^{-1}(cx)+9b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{9c^6d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (24*a - 12*a*c^2*x^2 - 3*a*c^4*x^4 + 15*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
+ b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)*A
rcCosh[c*x] + 9*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(9*c^6*d*Sqrt[
d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(203) = 406$.
time = 4.77, size = 437, normalized size = 1.88

method	result
default	$a\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)-\frac{b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx+}}{9c^3d^2(c^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(
1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2)))-1/9*b*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c
^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3-5/3*b*(-d*(c^2*x^2-1))^(1/2)/c^5/
d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-8/3*b*(-d*(c^2*x^2-1))^(1/2)/
c^6/d^2/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)-1)+1/3*b*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2
-1)*arccosh(c*x)*x^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arcco
sh(c*x)*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*a*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4*
d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/9*b*(((c^4*sqrt(d)*x^4 + 16*c^2*sq
rt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(-c*x + 1) - 3*(c^5*
sqrt(d)*x^5 + 4*c^3*sqrt(d)*x^3 - 8*c*sqrt(d)*x + (c^4*sqrt(d)*x^4 + 4*c^2*
sqrt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x +
```

```
1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/(sqrt(c*x + 1)*c^7*d^2*x + (c*x + 1)*sqrt
(c*x - 1)*c^6*d^2) + 9*integrate(1/9*(3*c^7*sqrt(d)*x^7 + 9*c^5*sqrt(d)*x^5
 - 36*c^3*sqrt(d)*x^3 + 24*c*sqrt(d)*x + (3*c^6*sqrt(d)*x^6 + 8*c^4*sqrt(d)
*x^4 - 52*c^2*sqrt(d)*x^2 + 32*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*log(c*x -
 1)))/(sqrt(-c*x + 1)*((c^7*d^2*x^2 - c^5*d^2)*e^(3/2*log(c*x + 1) + log(c*
x - 1)) + 2*(c^8*d^2*x^3 - c^6*d^2*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1)) +
 (c^9*d^2*x^4 - c^7*d^2*x^2)*sqrt(c*x + 1))), x))
```

Fricas [A]
time = 0.51, size = 489, normalized size = 2.10

$$\left[\frac{12\,(bc^4x^4+4\,bc^2x^2-8\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-9\,(bc^2x^2-b)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)-4\,(bc^3x^3+15\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+12\,(ac^4x^4+4\,ac^2x^2-8\,a)\sqrt{-c^2dx^2+d}}{36\,(c^8d^2x^2-c^6d^2)},\,-\frac{9\,(bc^2x^2-b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)-6\,(bc^4x^4+4\,bc^2x^2-8\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+2\,(bc^3x^3+15\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-6\,(ac^4x^4+4\,ac^2x^2-8\,a)\sqrt{-c^2dx^2+d}}{18\,(c^8d^2x^2-c^6d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/36*(12*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 - 1)) - 9*(b*c^2*x^2 - b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4
 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*s
qrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(b*c^3*x^3 + 15*b*c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^4*x^4 + 4*a*c^2*x^2 -
8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2), -1/18*(9*(b*c^2*x^2 - b
)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*
d*x^4 - d)) - 6*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 6*(a*c^4*x^4 + 4*a*c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^
8*d^2*x^2 - c^6*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a+b\,\mathrm{acosh}(c\,x))}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.115 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\sqrt{-1+cx}\,\sqrt{1+cx}}{4bc^5d\sqrt{d}}$$

```
[Out] x^3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+1/4*b*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-3/4*(a+b*arccosh(c*x))^2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/b/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A]
time = 0.20, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5934, 5938, 5892, 30, 84, 266}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^5d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-c^2x^2\right)}{2c^5d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^3*(
a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (3*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^2)/(4*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*Log[1 - c^2*x^2])/(2*c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\,(a+b\cosh^{-1}(cx))}{2c^4d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+\right.}{}$$

$$= \frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\,(a+}{2c^4d\sqrt{d-c^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\,(a+}{2c^4d\sqrt{d-c^2dx}}$$

Mathematica [A]
time = 0.95, size = 192, normalized size = 0.85

$$\frac{-4acdx(-3+c^2x^2)+12a\sqrt{d}\,\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+bd\left(8cx\cosh^{-1}(cx)-\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(6\cosh^{-1}(cx)^2-\cosh\left(2\cosh^{-1}(cx)\right)+8\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)+2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)}{8c^5d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*c*d*x*(-3 + c^2*x^2) + 12*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*S
qrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(8*c*x*ArcCosh[c*x] - S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*ArcCosh[c*x]
] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x]*Sinh[2*Arc
Cosh[c*x]])))/(8*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(196) = 392.
time = 5.73, size = 445, normalized size = 1.97

method	result
default	$-\frac{a\,x^3}{2c^2d\sqrt{-c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{-c^2d\,x^2+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^4d\sqrt{c^2d}}+\frac{3b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx}}{4d^2c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3/
2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/4*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
osh(c*x)^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^
3-1/4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x
^2-1)*arccosh(c*x)-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh
(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2
*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d)
 + 3*arcsin(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a+b\,\mathrm{acosh}(c\,x))}{{(d-c^2\,d\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.116 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{bx\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}+\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{c^4d^2}-\frac{b\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] (a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)/c^4/d^2-b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-b*arctanh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^4/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2
)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {272, 45, 5922, 12, 396, 212}

$$\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{c^4d^2}+\frac{a+b\cosh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{c^4d^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bx\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((b*x*Sqrt[d - c^2*d*x^2])/(c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a +
b*ArcCosh[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x]))/(c^4*d^2) - (b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(c^4*d^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+c\right.}{}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\,c\right.}{c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\,c\right.}{c^4d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.65

$$\frac{2a-ac^2x^2+bcx\sqrt{-1+cx}\,\sqrt{1+cx}+b(2-c^2x^2)\cosh^{-1}(cx)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(2 - c^2*x^2)*Arc
Cosh[c*x] + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d - c^
2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(134) = 268.
time = 5.29, size = 314, normalized size = 2.09

method	result
default	$a\left(-\frac{x^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2 d\,x^2+d}}\right)+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}(cx)x^2}{c^2d^2\left(c^2x^2-1\right)}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}}{c^3d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b*(-d*(c^2
*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/2
)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2*b*(-d*(c^2*x^2-1))^(1
/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [A]
time = 0.47, size = 157, normalized size = 1.05

$$-\frac{1}{2}\,bc\left(\frac{2\,\sqrt{-d}\,x}{c^4d^2}+\frac{\sqrt{-d}\,\log\left(cx+1\right)}{c^5d^2}-\frac{\sqrt{-d}\,\log\left(cx-1\right)}{c^5d^2}\right)-b\left(\frac{x^2}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}\,c^4d}\right)\mathrm{arcosh}\left(cx\right)-a\left(\frac{x^2}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}\,c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(-
d)*log(c*x - 1)/(c^5*d^2)) - b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(
-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d)
 - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))
```

Fricas [A]
time = 0.42, size = 429, normalized size = 2.86

$$\left[-\frac{4\,\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx-4\left(bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^2x^2-b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)-4\left(ac^2x^2-2\,a\right)\sqrt{-c^2dx^2+d}}{4\left(c^6d^2x^2-c^4d^2\right)},\ -\frac{2\,\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx+\left(bc^2x^2-b\right)\sqrt{d}\,\arctan\left(\frac{2\,\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)-2\left(bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-2\left(ac^2x^2-2\,a\right)\sqrt{-c^2dx^2+d}}{2\left(c^6d^2x^2-c^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - 4*(b*c^2*x^2 - 2*b)
*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*sqrt(-
d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x
^2 - 1)) - 4*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2
), -1/2*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + (b*c^2*x^2 - b)*s
qrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x
^4 - d)) - 2*(b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2
- 1)) - 2*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.117 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5934, 5892, 74, 266}

$$\frac{x(a+b\cosh^{-1}(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} -$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-1+}}{}$$

Mathematica [A]
time = 0.42, size = 159, normalized size = 1.11

$$\frac{2acdx+2a\sqrt{d}\,\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+bd\left(2cx\cosh^{-1}(cx)-\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(\cosh^{-1}(cx)^2+2\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)\right)\right)}{2c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a*c*d*x + 2*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)])))/(2*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(125) = 250$.
time = 5.05, size = 279, normalized size = 1.95

method	result

default	$\frac{ax}{c^{2}d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{c^{2}d\sqrt{c^{2}d}}+\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}(cx)^{2}}{2d^{2}c^{3}(c^{2}x^{2}-1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)
*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*integrat
e(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}(a+b\,\mathrm{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.118 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+b*arctanh(c*x)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5914, 35, 213}

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.27, size = 113, normalized size = 1.49

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2d^2\left(-1+c^2x^2\right)} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d-c^2dx^2}\left(\log(-1+cx)-\log(1+cx)\right)}{2c^2d^2\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c^2*x^2))) + (b
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*(Log[-1 + c*x] - Log[1 +
c*x]))/(2*c^2*d^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(68) = 136.
time = 1.62, size = 198, normalized size = 2.61

method	result
default	$\frac{a}{c^2d\sqrt{-c^2d\,x^2+d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)}{c^2d^2\left(c^2x^2-1\right)} + \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(cx+\sqrt{\right.}}{c^2d^2\left(c^2x^2-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*a
rccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(((c*sqrt(d)*x + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/sqrt(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sq
rt(-c*x + 1))/(sqrt(c*x + 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) -
 integrate((c^2*x^3 + c*x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(s
qrt(-c*x + 1)*((c^2*d^(3/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x -
1)) + 2*(c^3*d^(3/2)*x^3 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1))
 + (c^4*d^(3/2)*x^4 - c^2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + a/(sqrt(-c^2*
d*x^2 + d)*c^2*d)
```

Fricas [A]
time = 0.41, size = 327, normalized size = 4.30

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+(bc^2x^2-b)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4(c^3x^3+cx)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3c^4x^4+3c^2x^2-1}\right)+4\sqrt{-c^2dx^2+d}\,a}{4(c^4d^2x^2-c^2d^2)}, -\frac{(bc^2x^2-b)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+2\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+2\sqrt{-c^2dx^2+d}\,a}{2(c^4d^2x^2-c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 -
 b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2), -1/
2*((b*c^2*x^2 - b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*
c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*
x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.119 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5899, 266}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.86

$$\frac{2acx+2bcx\cosh^{-1}(cx)-b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a*c*x + 2*b*c*x*ArcCosh[c*x] - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^
2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(74) = 148$.
time = 1.88, size = 180, normalized size = 2.14

method	result
default	$\frac{ax}{d\sqrt{-c^2d\,x^2+d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}(cx)}{d^2c\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}(cx)x}{d^2\left(c^2x^2-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d
^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c
/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.83

$$-\frac{bc\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right)}{2\,d}+\frac{bx\,\mathrm{arcosh}\left(cx\right)}{\sqrt{-c^2dx^2+d}\,d}+\frac{ax}{\sqrt{-c^2dx^2+d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*x*arccosh(c*x)/(sqrt(-c^2*
d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2
*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{(cx)}}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{{\left(d - c^2\,d\,x^2\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2), x)
```

3.120 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tan}{d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*arctan(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1
/2)+b*arctanh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-I*b*p
olylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d/(-c^2*d*x^2+d)^(1/2)+I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5936, 5946, 4265, 2317, 2438, 35, 213}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) +
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (I*
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d -
 c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\,\mathrm{S}}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} +$$

Mathematica [A]
time = 1.71, size = 301, normalized size = 1.31

$$-\frac{\frac{a\sqrt{d-c^2dx^2}}{-1+c^2x^2} - a\sqrt{d}\,\log(x) + a\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right) + \frac{ibd\left(i\cosh^{-1}(cx)+\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)-i\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)+\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{d-c^2dx^2}}}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a*Sqrt[d]*Log[x] + a*Sqrt[d]*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (I*b*d*(I*ArcCosh[c*x] + Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(238) = 476.
time = 2.42, size = 511, normalized size = 2.23

method	result

default	$\frac{a}{d\sqrt{-c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}-\frac{b\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arccosh}(cx)}{d^2(c^2x^2-1)}+\frac{ib\sqrt{-d\,(c^2x^2-1)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/
x)-b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arcco
sh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(sq
rt(-c^2*d*x^2 + d)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2
*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x \left(-d (cx - 1) (cx + 1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\, x)}{x \left(d - c^2\, d\, x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)), x)
```

3.121 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{2c^2x(a+b\cosh^{-1}(cx))}{d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{d-c^2dx^2}\,\log(x)}{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc\sqrt{d-c^2dx^2}\,\log(1-c^2x^2)}{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] (-a-b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccosh(c*x))/d/(
-c^2*d*x^2+d)^(1/2)+b*c*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/2*b*c*ln(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {277, 197, 5922, 12, 457, 78}

$$\frac{2c^2x(a+b\cosh^{-1}(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{d^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\,\log(1-c^2x^2)}{2d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCosh
[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/(d^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(2*
d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{-1+2}{x(1-}}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}(}{2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}(}{2d\sqrt{d-}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)}{d\sqrt{d-c^2dx^2}} -$$

Mathematica [A]
time = 0.06, size = 114, normalized size = 0.72

$$\frac{-2a+4ac^2x^2+2b(-1+2c^2x^2)\cosh^{-1}(cx)-2bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{2dx\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a + 4*a*c^2*x^2 + 2*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x] - 2*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*Log[x] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(2*d*x*Sqrt[d - c^2*d*x^2])
```

Maple [A]
time = 3.43, size = 243, normalized size = 1.54

method	result
default	$a\left(-\frac{1}{dx\sqrt{-c^2 d\,x^2+d}}+\frac{2c^2x}{d\sqrt{-c^2 d\,x^2+d}}\right)-\frac{2b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)c}{d^2(c^2x^2-1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))-2*b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c-2
*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+b*(-d*(c^2*x^2
-1))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-
1)*c
```

Maxima [A]
time = 0.27, size = 144, normalized size = 0.91

$$\frac{1}{2}bc\left(\frac{\sqrt{-d}\,\log(cx+1)}{d^2}+\frac{\sqrt{-d}\,\log(cx-1)}{d^2}+\frac{2\sqrt{-d}\,\log(x)}{d^2}\right)+\left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}\,d}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx}\right)b\,\operatorname{arcosh}(cx)+\left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}\,d}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d)
*log(x)/d^2) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*
d*x))*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x
^2 + d)*d*x))*a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2
*x^4 + d^2*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}\left(cx\right)}{x^{2} \left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{x^2\,{\left(d - c^2\,d\,x^2\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

3.122 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=329

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}}+\frac{3c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{d\sqrt{d-c^2dx^2}}$$

```
[Out] 3/2*c^2*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arccosh(c*x))/d
/x^2/(-c^2*d*x^2+d)^(1/2)+1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x/(-c^2*d*x
^2+d)^(1/2)+3*c^2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+b*c^2*arctanh(c*x)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-3/2*I*b*c^2*polylog(2,-I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+
d)^(1/2)+3/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5932, 5936, 5946, 4265, 2317, 2438, 35, 213, 84}

$$\frac{3c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2dx\sqrt{d-c^2dx^2}}+\frac{bc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x*Sqrt[d - c^2*d*x^2]) + (3*c^2*(a
+ b*ArcCosh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])/(2*d*x^
2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (b*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (((3*I)/2)*b*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c
^2*d*x^2]) + (((3*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^A
rcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
```

```
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+c^2x^2)}\,dx}{2d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{-1+cx\right.}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(3c^2\sqrt{-1\right.}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc^2\sqrt{-1+}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+}}{}$$

Mathematica [A]
time = 2.86, size = 405, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{a\left(-1+3c^2x^2\right)\sqrt{d-c^2dx^2}}{d^2x^2\left(-1+c^2x^2\right)}+\frac{3ac^2\log(x)}{d^{3/2}}-\frac{3ac^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{d^{3/2}}-\frac{bc^2\left(-\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{cx}+\left(-1+\frac{1}{c^2x^2}\right)\cosh^{-1}(cx)-2\cosh^{-1}(cx)\cosh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)+3i\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-3i\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)+3i\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-3i\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\sinh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)}{d\sqrt{d-c^2dx^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-((a*(-1 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^2*x^2*(-1 + c^2*x^2))) + (3*
a*c^2*Log[x])/d^(3/2) - (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(3
/2) - (b*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^
2*x^2))*ArcCosh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] -
(3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2
*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2))/(d*Sqrt[d - c^2*d*x^2]))/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(323) = 646$.
time = 4.92, size = 648, normalized size = 1.97

method	result
default	$-\frac{a}{2d\,x^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3a\,c^{2}}{2d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{3a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{3}{2}}}-\frac{3b\sqrt{-d\,(c^{2}x^{2}-1}}{2d^{2}(c^{2}x^{2}-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*a*c^2/d/(-c^2*d*x^2+d)^(1/2)-3/2*a*c^
2/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-3/2*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c
^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(c^2*x^2-1)/x^2*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2-b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*c^2+3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*c^2-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c
^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+3/2*I*
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(
1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2)
 - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a + b*i
ntegrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2
*x^5 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}\left(cx\right)}{x^{3}\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

3.123 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{bc\sqrt{d-c^2dx^2}}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}+\frac{5bc^3\sqrt{d-c^2dx^2}\,\log(x)}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccosh(c*x
))/d/x/(-c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(
1/2)-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^2/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/3*b
*c^3*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c^3*l
n(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$,
Rules used = {277, 197, 5922, 12, 1265, 907}

$$-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^2x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3\log(x)\sqrt{d-c^2dx^2}}{3d^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3\sqrt{d-c^2dx^2}\,\log\left(1-c^2x^2\right)}{2d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d - c^2*d*x^2])/(d^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a
+ b*ArcCosh[c*x])/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[c*x
]))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3*d*Sqrt[
d - c^2*d*x^2]) + (5*b*c^3*Sqrt[d - c^2*d*x^2]*Log[x])/(3*d^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (b*c^3*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(2*d^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+}{3d\sqrt{}}$$

Mathematica [**A**]
time = 0.08, size = 161, normalized size = 0.64

$$\frac{-2a-8ac^2x^2+16ac^4x^4+bcx\sqrt{-1+cx}\,\sqrt{1+cx}+2b(-1-4c^2x^2+8c^4x^4)\cosh^{-1}(cx)-10bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)-3bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(1-c^2x^2)}{6dx^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a - 8*a*c^2*x^2 + 16*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2
*b*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcCosh[c*x] - 10*b*c^3*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Log[x] - 3*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(6*d*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(217) = 434.
time = 4.91, size = 1053, normalized size = 4.21

method	result
default	$a\left(-\frac{1}{3d\,x^3\sqrt{-c^2d\,x^2+d}}+\frac{4c^2\left(-\frac{1}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2c^2x}{d\sqrt{-c^2d\,x^2+d}}\right)}{3}\right)-\frac{16b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{}}{3d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c
^2/d*x/(-c^2*d*x^2+d)^(1/2)))-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^3+32/3*b*(-d*(c^2*x^2-1))^(1/2)
/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*x-1)*(c*x+1)*c^8-32/3*b*(-d*(c^2*x^2-1)
)^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10-16/3*b*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x-1)*(c*x+1)*c^6+16*b*(-d*(c^2*x^2-1))^(
1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(8*c^4*x^4-7*c^2*x^2-1)*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*c^5-64
/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^
6-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*(c*x-1)*(c*x+1
)*c^4-4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6+8/3*b*
(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*arccosh(c*x)*c^3+8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
*x*arccosh(c*x)*c^4-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1
)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x
^4-7*c^2*x^2-1)*x*c^4+4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1
)/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-
1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^
4*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3
+5/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2
*x^6 + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^4 \left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x^4\,{\left(d - c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

3.124 $\int \frac{x^5(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{bx\sqrt{d-c^2dx^2}}{c^5d^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bx\sqrt{d-c^2dx^2}}{6c^5d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}+\frac{a+b\cosh^{-1}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}-\frac{2(a+b\cosh^{-1}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}-$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^6/d/(-c^2*d*x^2+d)^(3/2)-2*(a+b*arccosh(c*x))/c^6/
d^2/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^6/d^3+b*
x*(-c^2*d*x^2+d)^(1/2)/c^5/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*x*(-c^2*d*
x^2+d)^(1/2)/c^5/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/6*b*arctan
h(c*x)*(-c^2*d*x^2+d)^(1/2)/c^6/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {272, 45, 5922, 12, 1171, 396, 212}

$$-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{c^6d^3}-\frac{2(a+b\cosh^{-1}(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}+\frac{11b\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{6c^6d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx\sqrt{d-c^2dx^2}}{c^5d^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bx\sqrt{d-c^2dx^2}}{6c^5d^3\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*x*Sqr
t[d - c^2*d*x^2])/(6*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)) +
(a + b*ArcCosh[c*x])/(3*c^6*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCosh[c*
x]))/(c^6*d^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x]))/(c^6*d^3) + (11*b*Sqrt[d - c^2*d*x^2]*ArcTanh[c*x])/(6*c^6*d^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5(a+b\cosh^{-1}(cx))}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1 \cdots}{3c^6 \cdots}$$

$$= -\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1 \cdots}{3c^6 \cdots}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh \cdots\right.}{3c^2d^2(1-cx)(1+c \cdots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1} \cdots\right.}{3c^4d^2\sqrt{d-c^2d \cdots}}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1} \cdots\right.}{3c^4d^2\sqrt{d-c^2d \cdots}}$$

Mathematica [A]
time = 0.12, size = 167, normalized size = 0.69

$$\frac{16a-24ac^2x^2+6ac^4x^4+5bcx\sqrt{-1+cx}\,\sqrt{1+cx}-6bc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}+2b\left(8-12c^2x^2+3c^4x^4\right)\cosh^{-1}(cx)-11b\sqrt{-1+cx}\,\sqrt{1+cx}\left(-1+c^2x^2\right)\tanh^{-1}(cx)}{6c^6d^2\left(-1+c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (16*a - 24*a*c^2*x^2 + 6*a*c^4*x^4 + 5*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] -
 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x^4
)*ArcCosh[c*x] - 11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c
*x])/(6*c^6*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(213) = 426.
time = 4.94, size = 471, normalized size = 1.94

method	result
default	$a\left(-\frac{x^4}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)x^2}{c^4d^3\left(c^2x^2-1\right)}+\frac{b\sqrt{-d\left(c^2 \cdots\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/
3/d/c^4/(-c^2*d*x^2+d)^(3/2)))-b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)
*arccosh(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x+b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)
+2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)*x^2+1/6*b*(-
d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-5/
3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)+11/6*b*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))-11/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/9*b*(((9*c^4*sqrt(d)*x^4 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(-
c*x + 1) - 3*(3*c^5*sqrt(d)*x^5 - 12*c^3*sqrt(d)*x^3 + 8*c*sqrt(d)*x + (3*c
^4*sqrt(d)*x^4 - 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/((c^8*d^3*x^2 - c
^6*d^3)*(c*x + 1)*sqrt(c*x - 1) + (c^9*d^3*x^3 - c^7*d^3*x)*sqrt(c*x + 1))
+ 9*integrate(1/9*(9*c^7*sqrt(d)*x^7 - 45*c^5*sqrt(d)*x^5 + 60*c^3*sqrt(d)*
x^3 - 24*c*sqrt(d)*x + (9*c^6*sqrt(d)*x^6 - 54*c^4*sqrt(d)*x^4 + 60*c^2*sqr
t(d)*x^2 - 16*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))/(sqrt(-c*x
+ 1)*((c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*e^(3/2*log(c*x + 1) + log(c*x
 - 1)) + 2*(c^10*d^3*x^5 - 2*c^8*d^3*x^3 + c^6*d^3*x)*e^(log(c*x + 1) + 1/2
*log(c*x - 1)) + (c^11*d^3*x^6 - 2*c^9*d^3*x^4 + c^7*d^3*x^2)*sqrt(c*x + 1)
)), x)) - 1/3*a*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2
 + d)^(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d))
```

Fricas [A]
time = 0.41, size = 529, normalized size = 2.18

$$\left[-\frac{8\,(3\,bc^4x^4-12\,bc^2x^2+8\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+11\,(bc^4x^4-2\,bc^2x^2+b)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)-4\,(6\,bc^3x^3-5\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+8\,(3\,ac^4x^4-12\,ac^2x^2+8\,a)\sqrt{-c^2dx^2+d}}{24\,(c^{10}d^3x^4-2\,c^8d^3x^2+c^6d^3)},\ \frac{11\,(bc^4x^4-2\,bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)-4\,(3\,bc^4x^4-12\,bc^2x^2+8\,b)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+2\,(6\,bc^3x^3-5\,bcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-4\,(3\,ac^4x^4-12\,ac^2x^2+8\,a)\sqrt{-c^2dx^2+d}}{12\,(c^{10}d^3x^4-2\,c^8d^3x^2+c^6d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(8*(3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) + 11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*log(-(c^6*d
```

```
*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(
6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 8*(3*a*c^4*
x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x
^2 + c^6*d^3), 1/12*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt
(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 4*(3*b*c^
4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1
)) + 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(
3*a*c^4*x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c
^8*d^3*x^2 + c^6*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a + b\operatorname{acosh}(c\,x))}{(d - c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

3.125 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d\,(d-c^2dx^2)^{3/2}}+\frac{x^3(a+b\cosh^{-1}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}-\frac{x(a+b\cosh^{-1}(cx))}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx}{2bc^5d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(3/2)-x*(a+b*arccosh(c*x))/c^4/d^2/(-c^2*
d*x^2+d)^(1/2)+1/2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/d
^2/(-c^2*d*x^2+d)^(1/2)+2/3*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
5/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5934, 5892, 74, 266, 272, 45}

$$\frac{x^3(a+b\cosh^{-1}(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^2}{2bc^5d^2\sqrt{d-c^2dx^2}}-\frac{x(a+b\cosh^{-1}(cx))}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6c^5d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log(1-c^2x^2)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (x^3*(a + b*ArcCosh[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (x*(a +
b*ArcCosh[c*x]))/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(ca\right.}{(-1+cx)^{3/2}(1+cx)}}{c^2d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{\right.}{}$$

$$= -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1}}{2bc^5}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}\right.}{3c^2d^2(1-cx)(1+cx)}$$

Mathematica [A]
time = 0.48, size = 225, normalized size = 1.00

$$\frac{\frac{2acx(-3+4c^2x^2)\sqrt{d-c^2dx^2}}{(-1+c^2x^2)^2} - 6a\sqrt{d}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \frac{bd\left(-8cx\cosh^{-1}(cx) - \frac{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)+2cx\cosh^{-1}(cx)}{-1+c^2x^2} + \sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(3\cosh^{-1}(cx)^2+8\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)\right)\right)}{\sqrt{d-c^2dx^2}}}{6c^5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 6*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*d*(-8*c
*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x
])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x]^2
+ 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2])/(6*c^
5*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(194) = 388$.
time = 6.00, size = 1519, normalized size = 6.78

method	result	size
default	Expression too large to display	1519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a/c^4
/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c
*x)^2+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2
*x^2-1)*arccosh(c*x)-32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)*c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^6+
32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)*c^2/d^3*arccosh(c*x)*x^7-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6
*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x-1)*(c*x+1)*x^5+8/3*b*(-d*(c^2*x^2-
1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^7+84*
b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/
c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^4-76*b*(-d*(c^2*x^2-1))^(1
/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+
14/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/c^2/d^3*(c*x-1)*(c*x+1)*x^3+4*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-22
/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
```

```
6)/d^3*x^5-220/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^
4-71*c^2*x^2+16)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^2+181/3
*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)
/c^2/d^3*arccosh(c*x)*x^3-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x-1)*(c*x+1)*x-13/2*b*(-d*(c^2*x^2-1
))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2+20/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^
6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*x^3+64/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*
c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*arccosh(c*x)-16*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*
c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)*x+8/3*b*(-d*(c^2*x^2-1))^(1/2)/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c^4/d^3*x-4/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^
4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a +
 b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5
/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\,\mathrm{acosh}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

3.126 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$\frac{bx\sqrt{d-c^2dx^2}}{6c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{5b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{6c^4d^3\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(3/2)+(-a-b*arccosh(c*x))/c^4/d
^2/(-c^2*d*x^2+d)^(1/2)+1/6*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^3/(c*x-1)^(3/2)/
(c*x+1)^(3/2)+5/6*b*arctanh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^4/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {272, 45, 5922, 12, 393, 212}

$$-\frac{a+b\cosh^{-1}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}+\frac{5b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{6c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bx\sqrt{d-c^2dx^2}}{6c^3d^3\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(b*x*Sqrt[d - c^2*d*x^2])/(c^3*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c
^2*x^2)) + (a + b*ArcCosh[c*x])/(3*c^4*d*(d - c^2*d*x^2)^(3/2)) - (a + b*Ar
cCosh[c*x])/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b*Sqrt[d - c^2*d*x^2]*ArcTan
h[c*x])/(6*c^4*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3(a+b\cosh^{-1}(cx))}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\cosh^{-1}(cx))}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2}{3c^4d^2(1}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\cosh^{-1}(cx))}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2}{3c^4d^2(1}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d^2(1-c^2x^2)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3(a+b}{3cd^2(1-cx)(1}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d^2(1-c^2x^2)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3(a+b}{3cd^2(1-cx)(1}$$

Mathematica [A]
time = 0.09, size = 122, normalized size = 0.77

$$\frac{4a-6ac^2x^2-bcx\sqrt{-1+cx}\,\sqrt{1+cx}+b(4-6c^2x^2)\cosh^{-1}(cx)-5b\sqrt{-1+cx}\,\sqrt{1+cx}\,(-1+c^2x^2)\tanh^{-1}(cx)}{6c^4d^2(-1+c^2x^2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(4 - 6*c^2*x^2)
*ArcCosh[c*x] - 5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c*x
])/(6*c^4*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(138) = 276$.
time = 4.22, size = 314, normalized size = 1.99

method	result
default	$a\left(\frac{x^2}{c^2 d(-c^2 d x^2+d)^{\frac{3}{2}}}-\frac{2}{3d c^4(-c^2 d x^2+d)^{\frac{3}{2}}}\right)+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}(cx)x^2}{d^3\left(c^2x^2-1\right)^2c^2}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx+1}}{6d^3\left(c^2x^2-1\right)^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2/3*b*(-d*(c^2
*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)+5/6*b*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))-5/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3
/c^4/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

Maxima [A]
time = 0.27, size = 175, normalized size = 1.11

$$\frac{1}{12}bc\left(\frac{2\sqrt{-d}\,x}{c^6d^3x^2-c^4d^3}+\frac{5\sqrt{-d}\,\log\left(cx+1\right)}{c^5d^3}-\frac{5\sqrt{-d}\,\log\left(cx-1\right)}{c^5d^3}\right)+\frac{1}{3}b\left(\frac{3x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\mathrm{arcosh}\left(cx\right)+\frac{1}{3}a\left(\frac{3x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/12*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(c
^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 1/3*b*(3*x^2/((-c^2*d*x^2 +
d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a*(3
*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

Fricas [A]
time = 0.51, size = 469, normalized size = 2.97

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx+8\left(3\,bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-5\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)+8\left(3\,ac^2x^2-2\,a\right)\sqrt{-c^2dx^2+d}}{24\left(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3\right)},\ \frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx+5\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\left(3\,bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+4\left(3\,ac^2x^2-2\,a\right)\sqrt{-c^2dx^2+d}}{12\left(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*(3*b*c^2*x^2 - 2*
b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^4*x^4 - 2*b*c
^2*x^2 + b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x
^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3
*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*(3*a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c
^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*b*c*x + 5*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x
^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(3*a*c^2*x^
2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

3.127 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d\,(d-c^2dx^2)^{3/2}}+\frac{x^3(a+b\cosh^{-1}(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(1-c^2x^2)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5917, 74, 272, 45}

$$\frac{x^3(a+b\cosh^{-1}(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6c^3d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\log(1-c^2x^2)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (x^3*(a + b*ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{(-1+c^2x^2)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{x}{(-1+c\cdots}\right.}{6d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{\cdots}{c^2(-\cdots}\right.\right.}{6d^2\sqrt{d-c^2d\cdots}}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+\cdots}}{\cdots}$$

Mathematica [A]
time = 0.14, size = 101, normalized size = 0.76

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(-\frac{2x^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b\left(\frac{1}{1-c^2x^2}+\log\left(1-c^2x^2\right)\right)}{c^3}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((-2*x^3*(a + b*ArcCosh[c*x]))/((-1 + c*x)^(3
/2)*(1 + c*x)^(3/2)) + (b*((1 - c^2*x^2)^(-1) + Log[1 - c^2*x^2]))/c^3))/(6
*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(113) = 226$.

time = 7.24, size = 1251, normalized size = 9.41

method	result
default	$a\left(\frac{x}{2c^2d(-c^2dx^2+d)^{\frac{3}{2}}}-\frac{\frac{x}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2dx^2+d}}}{2c^2}\right)+\frac{2b\sqrt{-d(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1}\,\text{ar}}{3d^3c^3(c^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*x/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/2/c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2
/3/d^2*x/(-c^2*d*x^2+d)^(1/2)))+2/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)/(3*
c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*(c*x+1)^(1/2)*arccosh(c*x
)*(c*x-1)^(1/2)*x^6+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8
*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5+1/6*b*(-
d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x
^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*
c/d^3*(c*x+1)^(1/2)*arccosh(c*x)*(c*x-1)^(1/2)*x^4-b*(-d*(c^2*x^2-1))^(1/2)
/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/6*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(
c*x+1)*(c*x-1)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4
*x^4-5*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-1/3*b*(-d*(c^2*x^2-
1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5-4/3*b*(-
d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*
x+1)^(1/2)*arccosh(c*x)*(c*x-1)^(1/2)*x^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c
^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3-1/2*b*(-d*(c^
2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)/d^3*x^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^
6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x+1)^(1/2)*arccosh(c*x)*(c*x-1)^(1
/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2-1)
```

Maxima [A]

time = 0.28, size = 169, normalized size = 1.27

$$\frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^6d^3x^2-c^4d^3}-\frac{\sqrt{-d}\,\log\left(cx+1\right)}{c^4d^3}-\frac{\sqrt{-d}\,\log\left(cx-1\right)}{c^4d^3}\right)-\frac{1}{3}\,b\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^3)
 - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^
2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a*(x/(sqrt(-c^2*d
*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(a + b\operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

3.128 $\int \frac{x(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd\,(d-c^2dx^2)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*x*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*arctanh(c*x)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5914, 41, 205, 213}

$$\frac{a+b\cosh^{-1}(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{6cd^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x(a+b\cosh^{-1}(cx))}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{(-1+c^2x^2)^2}\,dx}{3cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1\right.}{}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+}}{6}$$

Mathematica [A]
time = 0.17, size = 140, normalized size = 1.10

$$\frac{\sqrt{d-c^2dx^2}\left(2a+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,+2b\cosh^{-1}(cx)\right)}{6c^2d^3\left(-1+c^2x^2\right)^2} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d-c^2dx^2}\,(\log(-1+cx)-\log(1+cx))}{12c^2d^3\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*ArcCos
h[c*x]))/(6*c^2*d^3*(-1 + c^2*x^2)^2) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqr
t[d - c^2*d*x^2]*(Log[-1 + c*x] - Log[1 + c*x]))/(12*c^2*d^3*(-1 + c^2*x^2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(107) = 214$.
time = 2.08, size = 249, normalized size = 1.96

method	result
default	$\frac{a}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}} + \frac{b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx+1}\,\sqrt{cx-1}\,x}{6d^3(c^2x^2-1)^2c} + \frac{b\sqrt{-d\,(c^2x^2-1)}\,\mathrm{arccosh}(cx)}{3d^3(c^2x^2-1)^2c^2} + \frac{b\sqrt{-d\,(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-
1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*
x^2-1)^2/c^2*arccosh(c*x)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-1/6*b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2)
, x) + 1/3*a/((-c^2*d*x^2 + d)^(3/2)*c^2*d)
```

<u>Fricas</u> [A]
time = 0.41, size = 421, normalized size = 3.31

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx + 8\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right) - (bc^4x^4-2bc^2x^2+b)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4(c^3x^3+cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-d}-d}{c^6x^6-3c^4x^4+3c^2x^2-1}\right) + 8\sqrt{-c^2dx^2+d}\,a}{24\,(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3)}, \frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx - (bc^4x^4-2bc^2x^2+b)\sqrt{d}\,\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right) + 4\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right) + 4\sqrt{-c^2dx^2+d}\,a}{12\,(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*sqrt(-c^2*d*x^2 +
 d)*b*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)
*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*
d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2
 - 1)) + 8*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3),
 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^4*x^4 - 2*b*c^
2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d
)*x/(c^4*d*x^4 - d)) + 4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)
) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acosh}(cx))}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\operatorname{acosh}(c\,x))}{{(d-c^2\,d\,x^2)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

3.129 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd\,(d-c^2dx^2)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\,(d-c^2dx^2)^{3/2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)
^(1/2)-1/3*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2+d
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5901, 5899, 266, 74, 267}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6cd^2\,(1-c^2x^2)\,\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]
) + (x*(a + b*ArcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcC
osh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Lo
g[1 - c^2*x^2])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,d}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

Mathematica [A]
time = 0.06, size = 132, normalized size = 0.81

$$\frac{-6acx+4ac^3x^3-b\sqrt{-1+cx}\,\sqrt{1+cx}+2bcx\left(-3+2c^2x^2\right)\cosh^{-1}(cx)-2b\sqrt{-1+cx}\,\sqrt{1+cx}\left(-1+c^2x^2\right)\log\left(1-c^2x^2\right)}{6cd^2\left(-1+c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (-6*a*c*x + 4*a*c^3*x^3 - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*c*x*(-3 + 2*
c^2*x^2)*ArcCosh[c*x] - 2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*Log
[1 - c^2*x^2])/(6*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(138) = 276$.

time = 2.31, size = 1074, normalized size = 6.63

method	result
default	$a\left(\frac{x}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2d\,x^2+d}}\right)-\frac{4b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}(cx)}{3d^3c(c^2x^2-1)}+\frac{2b\sqrt{-}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))-4/3*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x
)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*
(c*x-1)*(c*x+1)*x^5-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^6/d^3*x^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^3/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x^4-2*b*(-d*(c^2
*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5
-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(
c*x-1)*(c*x+1)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^
2*x^2-4)*c^4/d^3*x^5-14/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11
*c^2*x^2-4)*c/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x^2+17/3*b*(-d*(
c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*
x^3+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x-1
)*(c*x+1)*x+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c
^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1
/2)-4*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arcc
osh(c*x)*x-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)
/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c
^4*x^4+11*c^2*x^2-4)/d^3*x+2/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^3/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [A]

time = 0.27, size = 157, normalized size = 0.97

$$\frac{1}{6}\,bc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\,\sqrt{-d}\,\log\left(cx+1\right)}{c^2d^3}+\frac{2\,\sqrt{-d}\,\log\left(cx-1\right)}{c^2d^3}\right)+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)\mathrm{arcosh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*d^
3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d)*
```

```
d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a*(2*x/(sqrt(-c^2*d
*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^
3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{{\left(d - c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2), x)
```

3.130 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+(a+b*arccosh(c*x))/d^2/(-c^2*
d*x^2+d)^(1/2)+1/6*b*c*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2
*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+7/6*b*arctanh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-I*b*polylog(2,-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)
^(1/2)+I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5936, 5946, 4265, 2317, 2438, 35, 213, 41, 205}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{bcx\sqrt{cx-1}\,\sqrt{cx+1}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{7b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{6d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) + (a + b*ArcCosh[c*x
])/(d^2*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCo
sh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (7*b*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) - (I*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d
*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(d
^2*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
```

```
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} \\
&= \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} \\
&= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} \\
&= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} \\
&= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} \\
&= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} \\
&= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}
\end{aligned}
$$

Mathematica [A]
time = 5.12, size = 364, normalized size = 1.15

$$
-\frac{a\left(-4+3c^2x^2\right)\sqrt{d-c^2dx^2}}{3d^3\left(-1+c^2x^2\right)^2}+\frac{a\log(x)}{d^{5/2}}-\frac{a\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right)}{d^{5/2}}+\frac{b\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(14\cosh^{-1}(cx)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)-\operatorname{csch}^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)-\frac{1}{2}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\operatorname{csch}^4\left(\frac{1}{2}\cosh^{-1}(cx)\right)-24i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)+24i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)-28\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)-24i\operatorname{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)+24i\operatorname{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)-\operatorname{sech}^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)-\frac{8\cosh^{-1}(cx)\sinh^4\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3}-14\cosh^{-1}(cx)\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)}{24d^2\sqrt{d-c^2dx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(a*(-4 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^3*(-1 + c^2*x^2)^2) + (a*L
og[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(5/2) + (b*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c*x]*Coth[ArcCosh[c*x]/2] - Cs
ch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*C
sch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] +
(24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - 28*Log[Tanh[ArcCosh[c*x]/2]
] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (24*I)*PolyLog[2, I/E^ArcCosh[
c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^4)/((
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*ArcCosh[c*x]*Tanh[ArcCosh[c*x
]/2]))/(24*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [A]
time = 2.77, size = 619, normalized size = 1.95

method	result
default	$\frac{a}{3d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{a}{d^{2}\sqrt{-c^{2}d\,x^{2}+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{5}{2}}}-\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\mathrm{arccosh}(cx)x^{2}}{d^{3}\left(c^{2}x^{2}-1\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^
2*arccosh(c*x)*x^2*c^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*a
rccosh(c*x)+7/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-7/6*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^
2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*di
log(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2) -
 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + b*integrate
(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^
3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x\,{\left(d - c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)), x)
```

3.131 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{bc\sqrt{d-c^2dx^2}}{6d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}-\frac{a+b\cosh^{-1}(cx)}{dx\,(d-c^2dx^2)^{3/2}}+\frac{4c^2x(a+b\cosh^{-1}(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{8c^2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d-c^2dx^2}}$$

```
[Out] (-a-b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccosh(c*x))/d
/(-c^2*d*x^2+d)^(3/2)+8/3*c^2*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
b*c*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/6*b*c*ln(-
c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {277, 198, 197, 5922, 12, 1265, 907}

$$\frac{8c^2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\cosh^{-1}(cx))}{3d\,(d-c^2dx^2)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{dx\,(d-c^2dx^2)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^3\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc\sqrt{d-c^2dx^2}\,\log(1-c^2x^2)}{6d^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d - c^2*d*x^2])/(d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x
^2)) - (a + b*ArcCosh[c*x])/(d*x*(d - c^2*d*x^2)^(3/2)) + (4*c^2*x*(a + b*A
rcCosh[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x]))/
(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/(d^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (5*b*c*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(6*d^
3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\,\text{co}\right.}{3d^2(1-cx)(1+c}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\,\text{co}\right.}{3d^2(1-cx)(1+c}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\,\text{co}\right.}{3d^2(1-cx)(1+c}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\,\text{co}\right.}{3d^2(1-cx)(1+c}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx}{d^2x(1-cx)(1+cx)\sqrt{d}}$$

Mathematica [A]
time = 0.27, size = 147, normalized size = 0.59

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(\frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{4c^2x(-3+2c^2x^2)(a+b\cosh^{-1}(cx))}{3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{-1+c^2x^2} + 6\log(x) + 5\log\left(1-c^2x^2\right)\right)\right)}{d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*(1
+ c*x)^(3/2)) + (4*c^2*x*(-3 + 2*c^2*x^2)*(a + b*ArcCosh[c*x]))/(3*(-1 + c*
x)^(3/2)*(1 + c*x)^(3/2)) - (b*c*((-1 + c^2*x^2)^(-1) + 6*Log[x] + 5*Log[1
- c^2*x^2]))/6))/(d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1349 vs. $2(218) = 436$.
time = 2.95, size = 1350, normalized size = 5.44

method	result	size
default	Expression too large to display	1350

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*
x/(-c^2*d*x^2+d)^(1/2)))-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+32/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x-1)*(c*x+1)*c^8-32/3*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10-80/3*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*(c*x-1)*(c*x+1
)*c^6+112/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9
)*x^7*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)*x^4*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5-64/3*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6+20*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x-1)*
(c*x+1)*c^4-140/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^5*c^6-136/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*x^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3+56*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-
4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x-1
)*(c*x+1)*c^2+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+24*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*c^4+24*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*c-44*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x
*arccosh(c*x)*c^2-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+9*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)+5/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2-1)*c+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2
*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2)
*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x^2\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x^2\,{\left(d - c^2\,d\,x^2\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

3.132 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=479

$$\frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d\,(d-c^2dx^2)^{3/2}}$$

```
[Out] 5/6*c^2*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arccosh(c*x))/d
/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/
2)+3/4*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*d*x^2+d)^(1/2)-1/4*b*c*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+5/12*b*c
^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+5*c^
2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+13/6*b*c^2*arctanh(c*x)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*I*b*c^2*polylog(2,-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1
/2)+5/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {5932, 5936, 5946, 4265, 2317, 2438, 35, 213, 41, 205, 74, 296, 331}

$$\frac{5c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d\,(d-c^2dx^2)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{2dx^2\,(d-c^2dx^2)^{3/2}}-\frac{5ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{3bc\sqrt{cx-1}\,\sqrt{cx+1}}{4d^2x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{4d^2x\,(1-c^2x^2)\sqrt{d-c^2dx^2}}+\frac{13bc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{6d^2\sqrt{d-c^2dx^2}}+\frac{5bc^3x\sqrt{cx-1}\,\sqrt{cx+1}}{12d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (3*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*Sqrt[d - c^2*d*x^2]) - (b*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(5*b*c^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*d^2*(1 - c^2*x^2)*Sqrt[d - c^2
*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x]))/(6*d*(d - c^2*d*x^2)^(3/2)) - (a +
b*ArcCosh[c*x])/(2*d*x^2*(d - c^2*d*x^2)^(3/2)) + (5*c^2*(a + b*ArcCosh[c*x
]))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (13*b*c
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) -
 (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*
x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
```

```
gerQ[m]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+c^2x^2)^2}\,a}{2d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{a}{2d^2x^2(1-c}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1}}{12d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1}}{12d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1}}{12d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1}}{12d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1}}{12d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

Mathematica [A]
time = 6.20, size = 468, normalized size = 0.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((-2*a*Sqrt[d - c^2*d*x^2]*(3 - 20*c^2*x^2 + 15*c^4*x^4))/(x^2*(-1 + c^2*x^
2)^2) + 30*a*c^2*Sqrt[d]*Log[x] - 30*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d -
 c^2*d*x^2]] + (b*c^2*d*((6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + 6
*(1 - 1/(c^2*x^2))*ArcCosh[c*x] + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 -
Coth[ArcCosh[c*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-
```

```
1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] - 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (
30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]]
+ (30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]
- 26*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c
*x]*Tanh[ArcCosh[c*x]/2]^2))/Sqrt[d - c^2*d*x^2])/(12*d^3)
```

Maple [A]
time = 4.62, size = 801, normalized size = 1.67

method	result
default	$-\frac{a}{2d\,x^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{5a\,c^{2}}{6d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{5a\,c^{2}}{2d^{2}\sqrt{-c^{2}d\,x^{2}+d}}-\frac{5a\,c^{2}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{2d^{\frac{5}{2}}}-\frac{5b\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c^
2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+
d)^(1/2))/x)-5/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x^2*arc
cosh(c*x)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^
2*x^2+1)*arccosh(c*x)*c^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x
^2+1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4
*x^4-2*c^2*x^2+1)/x^2*arccosh(c*x)+13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2
-13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))*c^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-
5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*
arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-5/2*I*b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/6*a*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5
/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d)
+ 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + b*integrate(log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^
3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^3 \left(-d\,(cx-1)\,(cx+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

3.133 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=338

$$-\frac{bc\sqrt{d-c^2dx^2}}{6d^3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3\sqrt{d-c^2dx^2}}{6d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}-\frac{2c^2\left(a+b\cosh^{-1}(cx\right.}{dx\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arccosh(c*x))
/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/
2)+16/3*c^4*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-1/6*b*c*(-c^2*d*x
^2+d)^(1/2)/d^3/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c^3*(-c^2*d*x^2+d)^(1
/2)/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3*b*c^3*ln(x)*(-c^2*d*x^
2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/3*b*c^3*ln(-c^2*x^2+1)*(-c^2*d
*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {277, 198, 197, 5922, 12, 1813, 1634}

$$-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^3x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^3\sqrt{d-c^2dx^2}}{6d^3\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)}+\frac{8bc^3\log(x)\sqrt{d-c^2dx^2}}{3d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{4bc^3\sqrt{d-c^2dx^2}\,\log\left(1-c^2x^2\right)}{3d^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d - c^2*d*x^2])/(d^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^3*Sqrt[d - c^2*d*x^2])/(6*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))
 - (a + b*ArcCosh[c*x])/(3*d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*Arc
Cosh[c*x]))/(d*x*(d - c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3
*d*(d - c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d -
 c^2*d*x^2]) + (8*b*c^3*Sqrt[d - c^2*d*x^2]*Log[x])/(3*d^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) + (4*b*c^3*Sqrt[d - c^2*d*x^2]*Log[1 - c^2*x^2])/(3*d^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5922

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]
)], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] &&
(IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\,c\right.}{d^2x(1-cx)(1+}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\,c\right.}{d^2x(1-cx)(1+}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\,c\right.}{d^2x(1-cx)(1+}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\,c\right.}{d^2x(1-cx)(1+}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2x^2\sqrt{d-c^2dx^2}} + \frac{bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.45, size = 218, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}-2a\left(1+6c^2x^2-24c^4x^4+16c^6x^6\right)-2b\left(1+6c^2x^2-24c^4x^4+16c^6x^6\right)\cosh^{-1}(cx)\right)}{6d^3x^3\left(-1+c^2x^2\right)^2}+\frac{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d-c^2dx^2}\left(\log(-1+cx)+\log(1+cx)+2\left(\log\left(-1+\sqrt{1+cx}\right)+\log\left(1+\sqrt{1+cx}\right)\right)\right)}{3d^3\left(-2+2c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*a*(1 + 6*c^2*x
^2 - 24*c^4*x^4 + 16*c^6*x^6) - 2*b*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^
6)*ArcCosh[c*x]))/(6*d^3*x^3*(-1 + c^2*x^2)^2) + (8*b*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*Sqrt[d - c^2*d*x^2]*(Log[-1 + c*x] + Log[1 + c*x] + 2*(Log[-1 +
 Sqrt[1 + c*x]] + Log[1 + Sqrt[1 + c*x]])))/(3*d^3*(-2 + 2*c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1879 vs. 2(295) = 590.
time = 3.47, size = 1880, normalized size = 5.56

method	result	size
default	Expression too large to display	1880

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(3/2)+2*c^2*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2
*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))))-320/3*b*(-
d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^
7*(c*x+1)*(c*x-1)*c^10-128*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*
x^6+35*c^4*x^4-10*c^2*x^2-1)*x^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)*c
^7+176/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*
c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)*c^5+64*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^6*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)*c^9+12*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*
c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*arccosh(c*x)*c^4-2*b*(-d*(c^2
*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*c^3+6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x
^6+35*c^4*x^4-10*c^2*x^2-1)/x*arccosh(c*x)*c^2-64*b*(-d*(c^2*x^2-1))^(1/2)/
d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*arccosh(c*x)*c^10+1
60*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^
2-1)*x^5*arccosh(c*x)*c^8-344/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36
*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3*arccosh(c*x)*c^6-8/3*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*(c*x+1)*(c*x
-1)*c^4+2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10
*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-1/6*b*(-d*(c^2*x^2-1))^(1/2
)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*c-32/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/
(c^2*x^2-1)*arccosh(c*x)*c^3+16/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-
36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x
)*c^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2
-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*c^3+128/3*b*(-d*(c^2*x^2-1))^
(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^9*(c*x+1)*(c*x-
1)*c^12+80*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-1
0*c^2*x^2-1)*x^5*(c*x+1)*(c*x-1)*c^8-40/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*
c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-128/3*b
*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)
*x^11*c^14+448/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4
*x^4-10*c^2*x^2-1)*x^9*c^12-560/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-
36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*c^10+280/3*b*(-d*(c^2*x^2-1))^(1/2)
/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^5*c^8-32/3*b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3*c^6
-8/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*
x^2-1)*x*c^4+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4
*x^4-10*c^2*x^2-1)/x^3*arccosh(c*x)
```

Maxima [A]

time = 0.28, size = 276, normalized size = 0.82

$$\frac{1}{6}\,bc\left(\frac{8\,c^2\sqrt{-d}\,\log\left(cx+1\right)}{d^3}+\frac{8\,c^2\sqrt{-d}\,\log\left(cx-1\right)}{d^3}+\frac{16\,c^2\sqrt{-d}\,\log\left(x\right)}{d^3}+\frac{\sqrt{-d}}{c^2d^3x^4-d^3x^2}\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{6\,c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)b\,\mathrm{arcosh}\left(cx\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{6\,c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*b*c*(8*c^2*sqrt(-d)*log(c*x + 1)/d^3 + 8*c^2*sqrt(-d)*log(c*x - 1)/d^3
+ 16*c^2*sqrt(-d)*log(x)/d^3 + sqrt(-d)/(c^2*d^3*x^4 - d^3*x^2)) + 1/3*(16*
c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c
^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*b*arcco
sh(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 +
 d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/
2)*d*x^3))*a
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d
^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{x^{4} \left(- d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

3.134 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x\cosh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x\,\cdots}{15c^3\,\cdots}$$

```
[Out] 1/5*x*arccosh(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)/c^2/(-a^2*c*x
^2+c)^(3/2)+8/15*x*arccosh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2)+1/20*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^(1/2)+2/15*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)-4/15*ln(-a^2*x^2+1)*
(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5901, 5899, 266, 74, 267}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}-\frac{4\sqrt{ax-1}\,\sqrt{ax+1}\,\log\left(1-a^2x^2\right)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{4x\cosh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x\cosh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2
]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*
c*x^2]) + (x*ArcCosh[a*x])/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x])
/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcCosh[a*x])/(15*c^3*Sqrt[c - a^2*c
*x^2]) - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(15*a*c^3*Sqrt[c
 - a^2*c*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$
$$= \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,d}{5c^3\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x}{15c^3(1-ax}$$
$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.07, size = 116, normalized size = 0.47

$$\frac{4ax(15-20a^2x^2+8a^4x^4)\cosh^{-1}(ax)+\sqrt{-1+ax}\,\sqrt{1+ax}\left(11-8a^2x^2-16(-1+a^2x^2)^2\log\left(1-a^2x^2\right)\right)}{60ac^3\left(-1+a^2x^2\right)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (4*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x] + Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*(11 - 8*a^2*x^2 - 16*(-1 + a^2*x^2)^2*Log[1 - a^2*x^2]))/(60*a*c^3*(-
1 + a^2*x^2)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 3.65, size = 419, normalized size = 1.70

method	result
default	$-\frac{16\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \sqrt{a x -1}\, \sqrt{a x +1}\, \mathrm{arccosh}\left(a x \right)}{15 c^{4} a \left(a^{2} x^{2}-1\right)}-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(8 a^{5} x^{5}-20 a^{3} x^{3}-8 \sqrt{a x +1}\, \sqrt{a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)
*arccosh(a*x)-1/60*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-20*a^3*x^3-8*(a*x+1)^(
1/2)*(a*x-1)^(1/2)*a^4*x^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-8*
(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-64*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^7*x^7-64*a^
8*x^8+248*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^5*x^5+280*a^6*x^6+160*arccosh(a*x)*
a^4*x^4-340*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3-456*a^4*x^4-380*arccosh(a*x
)*a^2*x^2+165*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+328*a^2*x^2+256*arccosh(a*x)-
88)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4
+8/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*
ln((a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2-1)
```

Maxima [A]
time = 0.28, size = 191, normalized size = 0.78

$$-\frac{1}{60}\,a\left(\frac{16\,\sqrt{-\frac{1}{a^{4}c}}\,\log\left(x^{2}-\frac{1}{a^{2}}\right)}{c^{3}}+\frac{3}{\left(a^{6}c^{3}x^{4}\sqrt{-\frac{1}{c}}-2\,a^{4}c^{3}x^{2}\sqrt{-\frac{1}{c}}+a^{2}c^{3}\sqrt{-\frac{1}{c}}\right)c}-\frac{8}{\left(a^{4}c^{2}x^{2}\sqrt{-\frac{1}{c}}-a^{2}c^{2}\sqrt{-\frac{1}{c}}\right)c^{2}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^{2}cx^{2}+c}\,c^{3}}+\frac{4\,x}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}c^{2}}+\frac{3\,x}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}c}\right)\operatorname{arcosh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*a*(16*sqrt(-1/(a^4*c))*log(x^2 - 1/a^2)/c^3 + 3/((a^6*c^3*x^4*sqrt(-1
/c) - 2*a^4*c^3*x^2*sqrt(-1/c) + a^2*c^3*sqrt(-1/c))*c) - 8/((a^4*c^2*x^2*s
qrt(-1/c) - a^2*c^2*sqrt(-1/c))*c^2)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 + c)*c^3
) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c))*arcc
osh(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 + 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{(-c(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**(7/2),x)

[Out] Integral(acosh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)

Giac [A]
time = 0.46, size = 142, normalized size = 0.58

$$\frac{1}{60}\sqrt{-c}\left(\frac{16\log\left(|a^2x^2-1|\right)}{ac^4}-\frac{24\,a^4x^4-56\,a^2x^2+35}{\left(a^2x^2-1\right)^2ac^4}\right)-\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2-1}\right)}{15\left(a^2cx^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] 1/60*sqrt(-c)*(16*log(abs(a^2*x^2 - 1))/(a*c^4) - (24*a^4*x^4 - 56*a^2*x^2 + 35)/((a^2*x^2 - 1)^2*a*c^4)) - 1/15*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(a^2*c*x^2 - c)^3

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/(c - a^2*c*x^2)^(7/2),x)

[Out] int(acosh(a*x)/(c - a^2*c*x^2)^(7/2), x)

3.135 $\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=145

$$-\frac{3x^2\sqrt{-1+ax}}{16a^3\sqrt{1-ax}}-\frac{x^4\sqrt{-1+ax}}{16a\sqrt{1-ax}}-\frac{3x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{8a^4}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{4a^2}+\frac{3\sqrt{-1+ax}\,\cosh}{16a^5\sqrt{1-a}}$$

```
[Out] -3/16*x^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/16*x^4*(a*x-1)^(1/2)/a/(-a*x+1
)^(1/2)+3/16*arccosh(a*x)^2*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-3/8*x*arccosh(
a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5938, 5892, 30}

$$\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^2}{16a^5\sqrt{1-ax}}-\frac{3x^2\sqrt{ax-1}}{16a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{8a^4}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x])/(16*a^3*Sqrt[1 - a*x]) - (x^4*Sqrt[-1 + a*x])/(16*a
*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(8*a^4) - (x^3*Sqrt[
1 - a^2*x^2]*ArcCosh[a*x])/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(16*
a^5*Sqrt[1 - a*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
```

```
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\cosh}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{16a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}}{8a^4\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.19, size = 93, normalized size = 0.64

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(-16\cosh\left(2\cosh^{-1}(ax)\right)-\cosh\left(4\cosh^{-1}(ax)\right)+4\cosh^{-1}(ax)\left(6\cosh^{-1}(ax)+8\sinh\left(2\cosh^{-1}(ax)\right)+\sinh\left(4\cosh^{-1}(ax)\right)\right)\right)}{128a^5\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-16*Cosh[2*ArcCosh[a*x]] - Cosh[4*Ar
cCosh[a*x]] + 4*ArcCosh[a*x]*(6*ArcCosh[a*x] + 8*Sinh[2*ArcCosh[a*x]] + Sin
h[4*ArcCosh[a*x]])))/(128*a^5*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(119) = 238.
time = 7.06, size = 456, normalized size = 3.14

method	result
default	$-\frac{3\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2}{16a^5(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\left(8a^5x^5-12a^3x^3+8\sqrt{ax+1}\,\sqrt{ax-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^2-1/256*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*a^4*x^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2
)*(a*x+1)^(1/2))*(-1+4*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2
)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x
+1)^(1/2))*(-1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*a
^3*x^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1
/2))*(1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/256*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5
-12*a^3*x^3-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^4*x^4+4*a*x+8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+4*arccosh(a*x))/a^5/(a^2
*x^2-1)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.136 $\int \frac{x^3 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2x\sqrt{-1+ax}}{3a^3\sqrt{1-ax}}-\frac{x^3\sqrt{-1+ax}}{9a\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{3a^2}$$

```
[Out] -2/3*x*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/9*x^3*(a*x-1)^(1/2)/a/(-a*x+1)^(1
/2)-2/3*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arccosh(a*x)*(-a^2*x^2+
1)^(1/2)/a^2
```

Rubi [A]

time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5938, 5914, 8, 30}

$$-\frac{2x\sqrt{ax-1}}{3a^3\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{3a^4}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*x*Sqrt[-1 + a*x])/(3*a^3*Sqrt[1 - a*x]) - (x^3*Sqrt[-1 + a*x])/(9*a*Sqr
t[1 - a*x]) - (2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^
2*x^2]*ArcCosh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\cos}{3a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}}{3a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(a}{3a^4\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.10, size = 74, normalized size = 0.67

$$-\frac{ax\sqrt{-1+ax}\,\sqrt{1+ax}\,(6+a^2x^2)-3(-2+a^2x^2+a^4x^4)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/9*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(6 + a^2*x^2) - 3*(-2 + a^2*x^2 + a^
4*x^4)*ArcCosh[a*x])/(a^4*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(90) = 180.
time = 4.53, size = 311, normalized size = 2.83

method	result

default	$-\frac{\sqrt{-a^2x^2+1}\,\left(4a^4x^4-5a^2x^2+4\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+1\right)(-1+3\,\mathrm{arccosh}(ax))}{72a^4(a^2x^2-1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2+4*(a*x+1)^(1/2)*(a*x-1)^(1/2)
*a^3*x^3-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(-1+3*arccosh(a*x))/a^4/(a^2*
x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(
-1+arccosh(a*x))/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^4/(a^2*x^2-1)-1/72*(-a^2*x^2+1)
^(1/2)*(4*a^4*x^4-5*a^2*x^2-4*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3+3*(a*x+1)
^(1/2)*(a*x-1)^(1/2)*a*x+1)*(1+3*arccosh(a*x))/a^4/(a^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 62, normalized size = 0.56

$$\frac{1}{9}\,a\left(\frac{i\,x^3}{a^2}+\frac{6i\,x}{a^4}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\mathrm{arcosh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(I*x^3/a^2 + 6*I*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a
^2*x^2 + 1)/a^4)*arccosh(a*x)
```

Fricas [A]
time = 0.37, size = 101, normalized size = 0.92

$$-\frac{3\,(a^4x^4+a^2x^2-2)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)-(a^3x^3+6\,ax)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}}{9\,(a^6x^2-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 -
 1)) - (a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a
^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,\mathrm{acosh}\,(ax)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.137 $\int \frac{x^2 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{x^2\sqrt{-1+ax}}{4a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2a^2}+\frac{\sqrt{-1+ax}\cosh^{-1}(ax)^2}{4a^3\sqrt{1-ax}}$$

```
[Out] -1/4*x^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+1/4*arccosh(a*x)^2*(a*x-1)^(1/2)/a^
3/(-a*x+1)^(1/2)-1/2*x*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5938, 5892, 30}

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^2}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/4*(x^2*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (x*Sqrt[1 - a^2*x^2]*ArcCosh[
a*x])/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(4*a^3*Sqrt[1 - a*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
```

```
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$
$$= -\frac{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{4a^3\sqrt{1-a^2}}$$

Mathematica [**A**]
time = 0.12, size = 75, normalized size = 0.85

$$-\frac{\sqrt{-((-1+ax)(1+ax))}\left(-\cosh\left(2\cosh^{-1}(ax)\right)+2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)+\sinh\left(2\cosh^{-1}(ax)\right)\right)\right)}{8a^3\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] -1/8*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-Cosh[2*ArcCosh[a*x]] + 2*ArcCosh[a*x]
*(ArcCosh[a*x] + Sinh[2*ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1
 + a*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(72) = 144.
time = 5.95, size = 223, normalized size = 2.53

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2}{4a^3(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\,\left(2a^3x^3-2ax+2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2}{16a^3(a^2x^2-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^2-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-1+2*arccosh(a*x))/a^3/(a^2*x^2-1
```

```
)-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^
2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+2*arccosh(a*x))/a^3/(a^2*x^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.138 $\int \frac{x \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{x\sqrt{-1+ax}}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{a^2}$$

```
[Out] -x*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{5914, 8\}$

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{a^2}-\frac{x\sqrt{ax-1}}{a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x])) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/
a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int 1\,dx}{a\sqrt{1-a^2x^2}}$$

$$= -\frac{x\sqrt{-1+ax}\,\sqrt{1+ax}}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 1.12

$$\frac{-ax\sqrt{-1+ax}\,\sqrt{1+ax}+(-1+a^2x^2)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (-1 + a^2*x^2)*ArcCosh[a*x])/(a^2*Sq
rt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(43) = 86$.
time = 3.02, size = 123, normalized size = 2.51

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)(-1+\operatorname{arccosh}(ax))}{2a^2(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\left(a^2x^2-\sqrt{ax+1}\,\sqrt{\cdots}\right.}{2a^2(a^2x^2-\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(-1+arc
cosh(a*x))/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a
*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^2/(a^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 28, normalized size = 0.57

$$\frac{i\,x}{a} - \frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] I*x/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/a^2
```

Fricas [A]
time = 0.38, size = 72, normalized size = 1.47

$$\frac{\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax+(-a^2x^2+1)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x + (-a^2*x^2 + 1)^(3/2)*log(a*x +
sqrt(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 40, normalized size = 0.82

$$-\frac{i\,x}{a}-\frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -I*x/a - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x\operatorname{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

3.139 $\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{-1+ax}\,\cosh^{-1}(ax)^2}{2a\sqrt{1-ax}}$$

```
[Out] 1/2*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5892}

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{2a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a*x])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 1.26, size = 51, normalized size = 1.59

method	result	size
default	$-\frac{\sqrt{-\left(a x-1\right) \left(a x+1\right)}\, \sqrt{a x-1}\, \sqrt{a x+1}\, \operatorname{arccosh}\left(a x\right)^{2}}{2 a \left(a^{2} x^{2}-1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(a x\right)}{\sqrt{-\left(a x-1\right) \left(a x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

3.140 $\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{-1+ax}\,\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{i\sqrt{-1+ax}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{i\sqrt{-1+ax}\,\text{PolyLog}}{\sqrt{1-ax}}$$

```
[Out] 2*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+
1)^(1/2)-I*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)+I*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)
/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5946, 4265, 2317, 2438}

$$\frac{2\sqrt{ax-1}\,\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{i\sqrt{ax-1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{i\sqrt{ax-1}\,\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - (I*S
qrt[-1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (I*Sqrt[-1 +
 a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\operatorname{Subst}\left(\int x\operatorname{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}} \\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\operatorname{Sul}}{\sqrt{}} \\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\operatorname{Sul}}{\sqrt{1}} \\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{i\sqrt{-1+ax}\,\sqrt{1+ax}\,\operatorname{Li}_2\left(-\right.}{\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 113, normalized size = 1.10

$$
\frac{i\sqrt{-((-1+ax)(1+ax))}\left(\cosh^{-1}(ax)\left(\log\left(1-ie^{-\cosh^{-1}(ax)}\right)-\log\left(1+ie^{-\cosh^{-1}(ax)}\right)\right)+\operatorname{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-\operatorname{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)\right)}{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[-((-1 + a*x)*(1 + a*x))]*(ArcCosh[a*x]*(Log[1 - I/E^ArcCosh[a*x]] -
 Log[1 + I/E^ArcCosh[a*x]]) + PolyLog[2, (-I)/E^ArcCosh[a*x]] - PolyLog[2,
I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(126) = 252.
time = 3.92, size = 270, normalized size = 2.62

method	result
default	$\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)\ln\left(1+i\left(ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)\right)}{a^2x^2-1} - \frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*l
n(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2)))+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*dilog(1
+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a
*x+1)^(1/2)/(a^2*x^2-1)*dilog(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.141 $\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{x}-\frac{a\sqrt{-1+ax}\log(x)}{\sqrt{1-ax}}$$

```
[Out] -a*ln(x)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-arccosh(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5917, 29}

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{x}-\frac{a\sqrt{ax-1}\log(x)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/x) - (a*Sqrt[-1 + a*x]*Log[x])/Sqrt[1 -
a*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 1.19

$$\frac{(-1+a^2x^2)\cosh^{-1}(ax) - ax\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(x)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-1 + a^2*x^2)*ArcCosh[a*x] - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[x])/(x*
Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(42) = 84$.
time = 4.10, size = 168, normalized size = 3.50

method	result
default	$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{arccosh}(ax)a}{a^2x^2-1} - \frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-1\right)\mathrm{ar}}{(a^2x^2-1)x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*
a-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*
x)/(a^2*x^2-1)/x+(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)
*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 73, normalized size = 1.52

$$-\frac{1}{2}\left(a^2\sqrt{-\frac{1}{a^4}}\log\left(x^2-\frac{1}{a^2}\right) + i\,(-1)^{-2\,a^2x^2+2}\log\left(-2\,a^2+\frac{2}{x^2}\right)\right)a - \frac{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sqrt(-1/a^4)*log(x^2 - 1/a^2) + I*(-1)^(-2*a^2*x^2 + 2)*log(-2*a^
2 + 2/x^2))*a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(42) = 84$.
time = 0.37, size = 89, normalized size = 1.85

$$\frac{ax\arctan\left(\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\,(x^2+1)}{a^2x^4-(a^2+1)x^2+1}\right)-\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*arctan(sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)/(a^2*x^4 - (a^2
+ 1)*x^2 + 1)) - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acosh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 80, normalized size = 1.67

$$\frac{1}{2}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{x|a|}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-i\,a\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*a
bs(a) + a)/(x*abs(a)))*log(a*x + sqrt(a^2*x^2 - 1)) - I*a*log(abs(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{acosh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.142 $\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=167

$$\frac{a\sqrt{-1+ax}}{2x\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2x^2}+\frac{a^2\sqrt{-1+ax}\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{ia^2\sqrt{-1+ax}\,\text{PolyI}}{2\sqrt{1}}$$

```
[Out] 1/2*a*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)+a^2*arccosh(a*x)*arctan(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*I*a^2*polylog(2,-I*(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+1/2*I*a^2*polyl
og(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*
arccosh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5932, 5946, 4265, 2317, 2438, 30}

$$\frac{a^2\sqrt{ax-1}\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{ia^2\sqrt{ax-1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2x^2}+\frac{a\sqrt{ax-1}}{2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x])/(2*x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x])/(
2*x^2) + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 -
a*x] - ((I/2)*a^2*Sqrt[-1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
a*x] + ((I/2)*a^2*Sqrt[-1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\right.}{} \\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{S}}{2\sqrt{}} \\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cos}{\sqrt{1-}} \\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cos}{\sqrt{1-}} \\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cos}{\sqrt{1-}}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 234, normalized size = 1.40

$$
\frac{(1+ax)\left(ax\sqrt{\frac{-1+ax}{1+ax}}-\cosh^{-1}(ax)+ax\cosh^{-1}(ax)-ia^2x^2\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)\log\left(1-ie^{-\cosh^{-1}(ax)}\right)+ia^2x^2\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)\log\left(1+ie^{-\cosh^{-1}(ax)}\right)-ia^2x^2\sqrt{\frac{-1+ax}{1+ax}}\,\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)+ia^2x^2\sqrt{\frac{-1+ax}{1+ax}}\,\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)\right)}{2x^2\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 + a*x)*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x] + a*x*ArcCosh[a*x
] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a
*x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCos
h[a*x]] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, (-I)/E^ArcCosh[a*
x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/E^ArcCosh[a*x]]))/(
2*x^2*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 4.99, size = 349, normalized size = 2.09

method	result
default	$-\frac{\left(\operatorname{arccosh}(ax)a^2x^2+\sqrt{ax+1}\,\sqrt{ax-1}\,ax-\operatorname{arccosh}(ax)\right)\sqrt{-a^2x^2+1}}{2(a^2x^2-1)x^2}+\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arccosh(a*x)*a^2*x^2+(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-arccosh(a*x))*(-
a^2*x^2+1)^(1/2)/(a^2*x^2-1)/x^2+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)
^(1/2)*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^
2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(1-I*(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)+I*(-a^2*x^2+1)^(1/2)*(a
*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/
(2*a^2*x^2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1-I*(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^3 \sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.143 $\int \frac{(fx)^{3/2}(a+b\cosh^{-1}(cx))}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{2(fx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f}+\frac{4bc(fx)^{7/2}\sqrt{-1+cx}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-cx}}$$

```
[Out] 2/5*(f*x)^(5/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/f+4/
35*b*c*(f*x)^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)*(c*x-1)^(1/
2)/f^2/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{5948\}$

$$\frac{4bc\sqrt{cx-1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-cx}}+\frac{2(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) + (4*b*c*(f*x)^(7/2)*Sqrt[-1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/
4}, {9/4, 11/4}, c^2*x^2])/(35*f^2*Sqrt[1 - c*x])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$=\frac{2(fx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f}+\frac{4bc(fx)^{7/2}\sqrt{-1+cx}\,\ldots}{35f\ldots}$$

Mathematica [A]
time = 0.08, size = 100, normalized size = 1.02

$$\frac{2}{35}x(fx)^{3/2}\left(7\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)+\frac{2bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{\sqrt{1-c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c
^2*x^2] + (2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4,
7/4}, {9/4, 11/4}, c^2*x^2])/Sqrt[1 - c^2*x^2]))/35
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}}\,(a+b\,\mathrm{arccosh}(cx))}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*x^
2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^{3/2}}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2), x)
```

3.144 $\int \frac{(fx)^{3/2}(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{2(fx)^{5/2}\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))\;{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f\sqrt{d-c^2dx^2}}+\frac{4bc(fx)^{7/2}\sqrt{-1+cx}\,\sqrt{1+cx}\;{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c\right.}{35f^2\sqrt{d-c^2dx^2}}$$

```
[Out] 4/35*b*c*(f*x)^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/f^2/(-c^2*d*x^2+d)^(1/2)+2/5*(f*x)^(5/2)*(a+b*arccosh(c*
x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f/(-c^2*d*x^2+d)
^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {5948}

$$\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}\,(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)(a+b\cosh^{-1}(cx))}{5f\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^
2])/(35*f^2*Sqrt[d - c^2*d*x^2])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{2(fx)^{5/2}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f\sqrt{d-c^2dx^2}}+\frac{4bc(fx)^{7/2}\sqrt{-}}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 115, normalized size = 0.82

$$\frac{2x(fx)^{3/2}\left(7\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)+2bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)\right)}{35\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, 5/4, 9/4, c^2*x^2] + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeome
tricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))/(35*Sqrt[d - c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}}\,(a+b\,\mathrm{arccosh}(cx))}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*
d*x^2 - d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^{3/2}}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2), x)
```

3.145 $\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=429

$$-\frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,(fx)^{2+m}\,(1-c^2x^2)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^3(9+m)(13+2m)(fx)^{4+m}\,(1-c^2}{f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+c}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-3*c^2*d^3*(f*x)^(3+m)*(a+b*arcco
sh(c*x))/f^3/(3+m)+3*c^4*d^3*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)-c^6*d
^3*(f*x)^(7+m)*(a+b*arccosh(c*x))/f^7/(7+m)-b*c*d^3*(m^4+27*m^3+284*m^2+132
9*m+2271)*(f*x)^(2+m)*(-c^2*x^2+1)/f^2/(7+m)^2/(m^2+8*m+15)^2/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+b*c^3*d^3*(9+m)*(13+2*m)*(f*x)^(4+m)*(-c^2*x^2+1)/f^4/(5+m)^
2/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^3*(f*x)^(6+m)*(-c^2*x^2+1)/f^
6/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161
)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2
)/f^2/(7+m)^2/(m^2+3*m+2)/(m^2+8*m+15)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.61, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5921, 12, 1624, 1823, 1281, 470, 372, 371}

$$-\frac{c^6d^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{3bcd^3(35m^3+455m^2+1813m+2161)\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2(m+5)^2(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^3(m^4+27m^3+284m^2+1329m+2271)\left(1-c^2x^2\right)(fx)^{m+2}}{f^2(m+3)^2(m+5)^2(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^3\left(1-c^2x^2\right)(fx)^{m+6}}{f^6(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d^3(m+9)(2m+13)\left(1-c^2x^2\right)(fx)^{m+4}}{f^4(m+5)^2(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*(f*x)^(2 + m)*(1 - c^2*
x^2))/(f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (
b*c^3*d^3*(9 + m)*(13 + 2*m)*(f*x)^(4 + m)*(1 - c^2*x^2))/(f^4*(5 + m)^2*(7
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^3*(f*x)^(6 + m)*(1 - c^2*x
^2))/(f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*(f*x)^(1 + m)*(a +
 b*ArcCosh[c*x]))/(f*(1 + m)) - (3*c^2*d^3*(f*x)^(3 + m)*(a + b*ArcCosh[c*x
]))/(f^3*(3 + m)) + (3*c^4*d^3*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5
+ m)) - (c^6*d^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)) - (3*b*c
*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}\right.}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}\right.}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}\right.}{f^3(3+m)}$$

$$= -\frac{bc^5d^3(fx)^{6+m}\left(1-c^2x^2\right)}{f^6(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-}\right.}{f(1+m)}$$

$$= \frac{bc^3d^3(9+m)(13+2m)(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^3(fx)^{6}}{f^6(7+m)^2\sqrt{-}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}(1-c}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}(1-c}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}(1-c}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 3.74, size = 424, normalized size = 0.99

$$d^3x(fx)^m\left(\frac{a}{1+m}-\frac{3ac^2x^2}{3+m}+\frac{3ac^4x^4}{5+m}-\frac{ac^6x^6}{7+m}+\frac{b\cosh^{-1}(cx)}{1+m}-\frac{3bc^2x^2\cosh^{-1}(cx)}{3+m}+\frac{3bc^4x^4\cosh^{-1}(cx)}{5+m}-\frac{bc^6x^6\cosh^{-1}(cx)}{7+m}-\frac{bcx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3x^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},2+\frac{m}{2};3+\frac{m}{2};c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3bc^5x^5\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},3+\frac{m}{2};4+\frac{m}{2};c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^7x^7\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},4+\frac{m}{2};5+\frac{m}{2};c^2x^2\right)}{(7+m)(8+m)\sqrt{-1+cx}\,\sqrt{1+cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^3*x*(f*x)^m*(a/(1 + m) - (3*a*c^2*x^2)/(3 + m) + (3*a*c^4*x^4)/(5 + m) -
(a*c^6*x^6)/(7 + m) + (b*ArcCosh[c*x])/(1 + m) - (3*b*c^2*x^2*ArcCosh[c*x])
/(3 + m) + (3*b*c^4*x^4*ArcCosh[c*x])/(5 + m) - (b*c^6*x^6*ArcCosh[c*x])/(7
 + m) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c
^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*x^3*Sqrt
[1 - c^2*x^2]*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/((12 + 7*m
 + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hype
```

```
rgeometric2F1[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c^7*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, 4
+ m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^3 (a+b\,\mathrm{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -a*c^6*d^3*f^m*x^7*x^m/(m + 7) + 3*a*c^4*d^3*f^m*x^5*x^m/(m + 5) - 3*a*c^2*
d^3*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) - ((m^3 + 9*m^2 +
 23*m + 15)*b*c^6*d^3*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*f^m*
x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^3*f^m*x^3 - (m^3 + 15*m^2 + 71*m
 + 105)*b*d^3*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^4 + 16*m
^3 + 86*m^2 + 176*m + 105) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3
*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5*d^3*f^m*x^5 + 3*(m^3 + 13*m^2
 + 47*m + 35)*b*c^3*d^3*f^m*x^3 - (m^3 + 15*m^2 + 71*m + 105)*b*c*d^3*f^m*x
)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*m^3 + 86*m
^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^
4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + integ
rate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*f^m*x^8 - 3*(m^3 + 11*m^2 + 31*m
+ 21)*b*c^6*d^3*f^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^3*f^m*x^4 -
(m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2
+ 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))*(f*x)
^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^{3}\left(\int\left(-a(fx)^{m}\right)dx+\int\left(-b(fx)^{m}\operatorname{acosh}(cx)\right)dx+\int 3ac^{2}x^{2}(fx)^{m}\,dx+\int\left(-3ac^{4}x^{4}(fx)^{m}\right)dx+\int ac^{6}x^{6}(fx)^{m}\,dx+\int 3bc^{2}x^{2}(fx)^{m}\operatorname{acosh}(cx)\,dx+\int\left(-3bc^{4}x^{4}(fx)^{m}\operatorname{acosh}(cx)\right)dx+\int bc^{6}x^{6}(fx)^{m}\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] -d**3*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Int
egral(3*a*c**2*x**2*(f*x)**m, x) + Integral(-3*a*c**4*x**4*(f*x)**m, x) + I
ntegral(a*c**6*x**6*(f*x)**m, x) + Integral(3*b*c**2*x**2*(f*x)**m*acosh(c*
x), x) + Integral(-3*b*c**4*x**4*(f*x)**m*acosh(c*x), x) + Integral(b*c**6*
x**6*(f*x)**m*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m, x)
```

3.146 $\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=307

$$-\frac{bcd^2(38+13m+m^2)\,(fx)^{2+m}\,(1-c^2x^2)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^2(fx)^{4+m}\,(1-c^2x^2)}{f^4(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx\right.}{f(1+m)}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-2*c^2*d^2*(f*x)^(3+m)*(a+b*arcco
sh(c*x))/f^3/(3+m)+c^4*d^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)-b*c*d^2
*(m^2+13*m+38)*(f*x)^(2+m)*(-c^2*x^2+1)/f^2/(3+m)^2/(5+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)+b*c^3*d^2*(f*x)^(4+m)*(-c^2*x^2+1)/f^4/(5+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-b*c*d^2*(15*m^2+100*m+149)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m
],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f^2/(m^2+3*m+2)/(m^2+8*m+15)^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {276, 5921, 12, 534, 1281, 470, 372, 371}

$$\frac{c^4d^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd^2(15m^2+100m+149)\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2(m+5)^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2(m^2+13m+38)\,(1-c^2x^2)\,(fx)^{m+2}}{f^2(m+3)^2(m+5)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d^2(1-c^2x^2)\,(fx)^{m+4}}{f^4(m+5)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*(f*x)^(2 + m)*(1 - c^2*x^2))/(f^2*(3 + m)^2*(5
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c^3*d^2*(f*x)^(4 + m)*(1 - c^2*
x^2))/(f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*(f*x)^(1 + m)*(a
+ b*ArcCosh[c*x]))/(f*(1 + m)) - (2*c^2*d^2*(f*x)^(3 + m)*(a + b*ArcCosh[c*
x]))/(f^3*(3 + m)) + (c^4*d^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5 +
 m)) - (b*c*d^2*(149 + 100*m + 15*m^2)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
+ m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
```

```
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-1}\right.}{f^3(3+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-}\right.}{f^3(3+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a+b\cosh^{-}\right.}{f^3(3+m)}$$

$$= \frac{bc^3d^2(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}\right.}{f(1+m)}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2(fx)}{f^4(5+m)^2\sqrt{}}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2(fx)}{f^4(5+m)^2\sqrt{}}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2(fx)}{f^4(5+m)^2\sqrt{}}$$

Mathematica [A]
time = 0.48, size = 315, normalized size = 1.03

$$d^2x(fx)^m\left(\frac{a}{1+m}-\frac{2ac^2x^2}{3+m}+\frac{ac^4x^4}{5+m}+\frac{b\cosh^{-1}(cx)}{1+m}-\frac{2bc^2x^2\cosh^{-1}(cx)}{3+m}+\frac{bc^4x^4\cosh^{-1}(cx)}{5+m}-\frac{bcx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc^3x^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},2+\frac{m}{2};3+\frac{m}{2};c^2x^2\right)}{(12+7m+m^2)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^5x^5\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},3+\frac{m}{2};4+\frac{m}{2};c^2x^2\right)}{(5+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^2*x*(f*x)^m*(a/(1 + m) - (2*a*c^2*x^2)/(3 + m) + (a*c^4*x^4)/(5 + m) + (b
*ArcCosh[c*x])/(1 + m) - (2*b*c^2*x^2*ArcCosh[c*x])/(3 + m) + (b*c^4*x^4*Ar
cCosh[c*x])/(5 + m) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, 1 + m
/2, 2 + m/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*
b*c^3*x^3*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^
2])/((12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*x^5*Sqrt[1 - c
```

```
^2*x^2]*Hypergeometric2F1[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5 + m)*(6 + m)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^2 (a+b\operatorname{arccosh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] a*c^4*d^2*f^m*x^5*x^m/(m + 5) - 2*a*c^2*d^2*f^m*x^3*x^m/(m + 3) + (f*x)^(m
+ 1)*a*d^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*c^4*d^2*f^m*x^5 - 2*(m^2 + 6*m
+ 5)*b*c^2*d^2*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*m +
3)*b*c^5*d^2*f^m*x^5 - 2*(m^2 + 6*m + 5)*b*c^3*d^2*f^m*x^3 + (m^2 + 8*m + 1
5)*b*c*d^2*f^m*x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m^2 + 2
3*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 1
5)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^6*d^2*
f^m*x^6 - 2*(m^2 + 6*m + 5)*b*c^4*d^2*f^m*x^4 + (m^2 + 8*m + 15)*b*c^2*d^2*
f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a(fx)^m\,dx + \int b(fx)^m\operatorname{acosh}(cx)\,dx + \int\left(-2ac^2x^2(fx)^m\right)dx + \int ac^4x^4(fx)^m\,dx + \int\left(-2bc^2x^2(fx)^m\operatorname{acosh}(cx)\right)dx + \int bc^4x^4(fx)^m\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] d**2*(Integral(a*(f*x)**m, x) + Integral(b*(f*x)**m*acosh(c*x), x) + Integr
al(-2*a*c**2*x**2*(f*x)**m, x) + Integral(a*c**4*x**4*(f*x)**m, x) + Integr
al(-2*b*c**2*x**2*(f*x)**m*acosh(c*x), x) + Integral(b*c**4*x**4*(f*x)**m*a
cosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

3.147 $\int (fx)^m (d - c^2dx^2) \left(a + b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=184

$$\frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2}+\frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}-\frac{c^2d(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}-\frac{bcd(7+3}{f^2(1+}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-c^2*d*(f*x)^(3+m)*(a+b*arccosh(c*x
))/f^3/(3+m)+b*c*d*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(3+m)^2-b*c*
d*(7+3*m)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2
+1)^(1/2)/f^2/(3+m)^2/(m^2+3*m+2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {14, 5921, 12, 471, 127, 372, 371}

$$-\frac{c^2d(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd(3m+7)\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}}{f^2(m+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*c*d*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(f^2*(3 + m)^2) + (d*(f*
x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (c^2*d*(f*x)^(3 + m)*(a + b*
ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*d*(7 + 3*m)*(f*x)^(2 + m)*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2
 + m)*(3 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx = \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m} \left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m} \left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx\right.}{f(1+m)}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx\right.}{f(1+m)}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx\right.}{f(1+m)}$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx\right.}{f(1+m)}$$

Mathematica [A]
time = 0.49, size = 193, normalized size = 1.05

$$dx(fx)^m \left(-\frac{bcx\sqrt{1-c^2x^2}\, {}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)}{(2+3m+m^2)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{-\frac{(-3-m+c^2x^2+c^2mx^2)(a+b\cosh^{-1}(cx))}{1+m} + \frac{bc^3x^3\sqrt{1-c^2x^2}\, {}_2F_1\left(\frac{1}{2},2+\frac{m}{2};3+\frac{m}{2};c^2x^2\right)}{(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}}}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d*x*(f*x)^m*(-((b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, 1 + m/2, 2 +
 m/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (-(((-3 -
 m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcCosh[c*x]))/(1 + m)) + (b*c^3*x^3*Sqrt[
1 - c^2*x^2]*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/((4 + m)*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]))/(3 + m))
```

Maple [F]
time = 9.05, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right) \left(a+b\,\mathrm{arccosh}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -a*c^2*d*f^m*x^3*x^m/(m + 3) - (b*c^2*d*f^m*(m + 1)*x^3 - b*d*f^m*(m + 3)*x
)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m + 1
)*a*d/(f*(m + 1)) - integrate((b*c^3*d*f^m*(m + 1)*x^3 - b*c*d*f^m*(m + 3)*
x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^
2*x^2 - m^2 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + integrate((b*c^4*
d*f^m*(m + 1)*x^4 - b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 -
 m^2 - 4*m - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*(f*x)^m, x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \left(-a(fx)^m\right)\,dx + \int \left(-b(fx)^m \operatorname{acosh}(cx)\right)\,dx + \int ac^2x^2(fx)^m\,dx + \int bc^2x^2(fx)^m \operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] -d*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Integr
al(a*c**2*x**2*(f*x)**m, x) + Integral(b*c**2*x**2*(f*x)**m*acosh(c*x), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m, x)
```

3.148 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Mathematica [A]
time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)}{-c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a(fx)^m}{c^2x^2-1}\,dx + \int \frac{b(fx)^m \operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*(f*x)**m/(c**2*x**2 - 1), x) + Integral(b*(f*x)**m*acosh(c*x)/
(c**2*x**2 - 1), x))/d
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2), x)
```

3.149 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}-\frac{bc(fx)^{2+m}\sqrt{1-c^2x^2}\ {}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{2d^2f^2(2+m)\sqrt{-1+cx}\ \sqrt{1+cx}}+\frac{(1-m)\mathrm{Int}\left(\frac{(fx)^m(a+b\cosh^{-1}(cx))}{d-c^2dx^2},\right.}{2d}$$

```
[Out] 1/2*(f*x)^(1+m)*(a+b*arccosh(c*x))/d^2/f/(-c^2*x^2+1)-1/2*b*c*(f*x)^(2+m)*h
ypergeom([3/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^2/f^2/(2+m)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*(1-m)*Unintegrable((f*x)^m*(a+b*arccosh(c*
x))/(-c^2*d*x^2+d),x)/d
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(2*d^2*f*(1 - c^2*x^2)) - (b*c*(f*x)^(
2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, c^2*x
^2])/(2*d^2*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*Defer[Int]
[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\begin{aligned}
\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2f}+\frac{(1-m)\int \frac{(fx}{}}{} \\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(1-m)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx}{2d}+\frac{\left(bc\sqrt{-}\right.}{2d^2} \\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(1-m)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx}{2d}-\frac{\left(bc\sqrt{1-}\right.}{2d^2f} \\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}-\frac{bc(fx)^{2+m}\sqrt{1-c^2x^2}\ {}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};c^2x\right.}{2d^2f^2(2+m)\sqrt{-1+cx}\ \sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 7.87, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\operatorname{arccosh}(cx)\right)}{\left(-c^2d\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a(fx)^m}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b(fx)^m \operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*(f*x)**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*(f*x)**
m*acosh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,(f\,x)^m}{(d-c^2\,d\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

3.150 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^3} dx$

Optimal. Leaf size=294

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}+\frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}-\frac{bc(3-m)(fx)^{2+m}\sqrt{1-c^2x^2}\ {}_2F_1\left(\frac{3}{2},\frac{2+}{2}\right.}{8d^3f^2(2+m)\sqrt{-1+cx}\ \sqrt{1+}}$$

```
[Out] 1/4*(f*x)^(1+m)*(a+b*arccosh(c*x))/d^3/f/(-c^2*x^2+1)^2+1/8*(3-m)*(f*x)^(1+
m)*(a+b*arccosh(c*x))/d^3/f/(-c^2*x^2+1)-1/8*b*c*(3-m)*(f*x)^(2+m)*hypergeo
m([3/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^3/f^2/(2+m)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*(f*x)^(2+m)*hypergeom([5/2, 1+1/2*m],[2+1/2*m
],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^3/f^2/(2+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8
*(1-m)*(3-m)*Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)/d^2
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(4*d^3*f*(1 - c^2*x^2)^2) + ((3 - m)*(
f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(8*d^3*f*(1 - c^2*x^2)) - (b*c*(3 - m)*(
f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 c^2*x^2])/(8*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2
 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, c^2*x^
2])/(4*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*(3 - m)*Def
er[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3f} + \frac{(3-m)\int \frac{(fx}{}}{}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} + \frac{(bc(}{}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} + \frac{((1}{}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} - \frac{bc(f}{}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2} + \frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)} - \frac{bc(3}{}$$

Mathematica [A]
time = 10.55, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arccosh}(cx)\right)}{\left(-c^2d\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

Maple [A]

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas"
)
```

```
[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a(fx)^m}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{b(fx)^m\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*(f*x)**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Int
egral(b*(f*x)**m*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x)
)/d**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,{(f\,x)}^m}{{(d-c^2\,d\,x^2)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3, x)
```

3.151 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=723

$$-\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{15bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5bcd}{f^2(2+m)(4+}$$

[Out] 5*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)/(6+m)+(f*x)^(1+m)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/f/(6+m)+15*d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(6+m)/(m^2+6*m+8)+15*d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1/2)/f/(6+m)/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b*c^3*d^2*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c^3*d^2*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^2*(f*x)^(6+m)*(-c^2*d*x^2+d)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b*c*d^2*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(6+m)/(m^2+5*m+4)/(c*x-1)^(1/2)/(c*x+1)^(1/2)

Rubi [A]
time = 0.58, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5930, 5926, 5949, 32, 74, 14, 276}

$$-\frac{15bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{15d^2\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)(m+6)\left(m^2+3m+2\right)\sqrt{1-cx}\,\sqrt{cx+1}}+\frac{15d^2\sqrt{d-c^2dx^2}\,(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+6)\left(m^2+6m+8\right)}+\frac{\left(d-c^2dx^2\right)^{5/2}(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+6)}+\frac{5d\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)(m+6)}-\frac{bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{15bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)^2(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2\sqrt{d-c^2dx^2}\,(fx)^{m+6}}{f^6(m+6)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc^3d^2\sqrt{d-c^2dx^2}\,(fx)^{m+4}}{f^4(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d^2\sqrt{d-c^2dx^2}\,(fx)^{m+4}}{f^4(m+4)^2(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]

[Out] -((b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - (15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2])/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2])/(f^4*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^2*d*x^2])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d*(f*x)^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(

```
5/2)*(a + b*ArcCosh[c*x]))/(f*(6 + m)) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) -
(15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2
, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*
(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
```

```
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{d^2(fx)^{1+m}(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{f(6+m)}$$

$$= \frac{5d^2(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)(6+m)}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3d^2(f}{f^4(4+m)(6+}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{15bc}{f^2(2+m)^2(4+}$$

Mathematica [A]
time = 0.97, size = 350, normalized size = 0.48

$$\frac{d^2x(fx)^m\sqrt{d-c^2dx^2}\left(\frac{5bcx\left(-\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{4+m}-bcx\left(\frac{1}{2+m}-\frac{2c^2x^2}{4+m}+\frac{c^4x^4}{6+m}\right)-\frac{5(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{4+m}+(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15\left(-bcx+(2+m)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{(2+m)\left(\frac{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcx\,_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{2+m}\right)}{1+m}\right)}{(2+m)^2(4+m)}\right)}{(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((5*b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4
+ m)))/(4 + m) - b*c*x*((2 + m)^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 +
 m)) - (5*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]))/(4 + m) +
(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (15*(-(b*c*x) + (2
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]) - ((2 + m)*((Sqrt[1
- c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, c^2*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*HypergeometricPFQ[{1, 1
 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + m)))/(1 + m)))/((2 +
 m)^2*(4 + m))))/((6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} (a+b\,\mathrm{arccosh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

3.152 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=455

$$-\frac{3bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d(fx)^{4+m}\sqrt{d-}}{f^4(4+m)^2\sqrt{-1+cx}}$$

```
[Out] (f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)+3*d*(f*x)^(1+m)
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(m^2+6*m+8)+3*d*(f*x)^(1+m)*(a+b
*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2+
d)^(1/2)/f/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d*(f*x)^(2
+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*
d*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+b*c^3*d*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-3*b*c*d*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2
+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(m^2+5*m+4)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5930, 5926, 5949, 32, 74, 14}

$$-\frac{3bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3d\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)\left(m^2+3m+2\right)\sqrt{1-cx}\,\sqrt{cx+1}}+\frac{3d\sqrt{d-c^2dx^2}\,(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f\left(m^2+6m+8\right)}+\frac{\left(d-c^2dx^2\right)^{3/2}(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)}-\frac{bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)^2(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3d\sqrt{d-c^2dx^2}\,(fx)^{m+4}}{f^4(m+4)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*(4 + m)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2
+ m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*(f*x)^(4 + m)*Sqrt[d
- c^2*d*x^2])/(f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1
+ m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + ((f*x)
^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)) + (3*d*(f*
x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(4 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sq
rt[1 + c*x]) - (3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m
)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{d(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)}$$

$$= \frac{3d(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f\left(8+6m+m^2\right)} + \frac{d(fx)^{1+m}(1}{}$$

$$= -\frac{3bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd(f}{f^2(2+m)(4}$$

Mathematica [A]
time = 0.59, size = 274, normalized size = 0.60

$$-\frac{dx(fx)^m\sqrt{d-c^2dx^2}\left(\frac{3bcx}{(2+m)^2}+bcx\left(\frac{1}{2+m}-\frac{c^2x^2}{4+m}\right)-\frac{3\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{2+m}+(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{(1+m)(2+m)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(1+m)(2+m)^2}\right)}{(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((3*b*c*x)/(2 + m)^2 + b*c*x*((2 + m)^(-
1) - (c^2*x^2)/(4 + m)) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
x]))/(2 + m) + (-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (3*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/((1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c*x
*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(
(1 + m)*(2 + m)^2)))/((4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\operatorname{arccosh}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)*(f*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

3.153 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=278

$$-\frac{bc(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\,\left(a+b\,\mathrm{c}\right.}{f\left(2+3m+m\right.}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(2+m)+(f*x)^(1+m)*(a+
b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2
+d)^(1/2)/f/(m^2+3*m+2)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*(-c^2*
d*x^2+d)^(1/2)/f^2/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*hype
rgeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1
/2)/f^2/(1+m)/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5926, 5949, 32}

$$-\frac{bc\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f\left(m^2+3m+2\right)\sqrt{1-cx}\,\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\,(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+2)}-\frac{bc\sqrt{d-c^2dx^2}\,(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*
(2 + m)) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*Sqrt[1 -
c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2
+ m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
```

```
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{\sqrt{d-c^2dx^2}\int (fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\,d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}-\frac{\sqrt{d-c^2dx^2}\int\frac{(f}{\sqrt{-}}}{(2+m)\sqrt{-1}}$$

$$=-\frac{bc(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\,(a-}{f(2+m)}$$

Mathematica [A]
time = 0.21, size = 223, normalized size = 0.80

$$\frac{x(fx)^m\sqrt{d-c^2dx^2}\left((1+m)\left(-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,+a(2+m)\left(-1+c^2x^2\right)+b(2+m)\left(-1+c^2x^2\right)\cosh^{-1}(cx)\right)-(2+m)\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)^2(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (
2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 +
m)^2*(-1 + c*x)*(1 + c*x))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m\,\sqrt{-c^2d\,x^2+d}\,\left(a+b\,\mathrm{arccosh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-d(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

3.154 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{(fx)^{1+m}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{d-c^2dx^2}}+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2(1,1+}{f^2(1+m)(2+m)\sqrt{d}}$$

```
[Out] b*c*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(1+m)/(2+m)/(-c^2*d*x^2+d)^(1/2)+(f*x)^(
1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c
^2*x^2+1)^(1/2)/f/(1+m)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {5948}

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f
*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
 m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*
x^2])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{d-c^2dx^2}} + \frac{bc(fx)^{2+}}{}$$

Mathematica [A]
time = 0.06, size = 147, normalized size = 0.84

$$\frac{x(fx)^m\left((2+m)\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (x*(f*x)^m*((2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(
(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\mathrm{arccosh}(cx))}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,(f\,x)^m}{\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

3.155 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}}-\frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{df(1+m)\sqrt{d-c^2dx^2}}+\frac{bc(fx)^{2+m}\sqrt{}}{}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(1/2)+b*c*(f*x)^(2+m)*hyp
ergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/f^2/(2
+m)/(-c^2*d*x^2+d)^(1/2)-b*c*m*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],
[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/f^2/(1+m)/(2+m)
/(-c^2*d*x^2+d)^(1/2)-m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d/f/(1+m)/(-c^2*d*x^2+d)^(1/
2)
```

Rubi [A]
time = 0.18, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$,
Rules used = {5936, 5948, 74, 371}

$$-\frac{bcm\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{df^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{m\sqrt{1-c^2x^2}\,(fx)^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{df(m+1)\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{df^2(m+2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) - (m*(f*x)^(
1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2])/(d*f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2
 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (b*c*m*(f*x)^(2 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m
/2, 2 + m/2}, c^2*x^2])/(d*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}\\ &= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}}-\frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\frac{(fx)^{1+m}}{-1+c^2x^2}\,dx}{df\sqrt{d-c^2dx^2}}-\\ &= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}}-\frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}{df(1+m)\sqrt{d-c^2dx}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 216, normalized size = 0.72

$$\frac{x(fx)^m\left(-m(2+m)\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)+(1+m)\left((2+m)\left(a+b\cosh^{-1}(cx)\right)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)\right)-bcmx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{d(1+m)(2+m)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcCos
h[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, 1 + m/2,
2 + m/2, c^2*x^2]) - b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(d*(1 + m)*(2 + m)
*Sqrt[d - c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (a + b \operatorname{arccosh}(cx))}{(-c^{2} d\, x^{2} + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (a + b \operatorname{acosh}{(cx)})}{(-d (cx - 1) (cx + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^m}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

3.156 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=450

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}}-\frac{(2-m)m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh\right.}{3d^2f(1+m)\sqrt{d-}}$$

```
[Out] 1/3*(f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(3/2)+1/3*(2-m)*(f*x)
^(1+m)*(a+b*arccosh(c*x))/d^2/f/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*(f*x)^(2
+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^2/f^2/(2+m)/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(f*x)^(2+m)*hypergeom([2, 1+1/2*m
],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/f^2/(2+m)/(-c^2*d*x^2+
d)^(1/2)-1/3*b*c*(2-m)*m*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2
*m, 3/2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/f^2/(1+m)/(2+m)/(-c
^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2
, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^2/f/(1+m)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5936, 5948, 74, 371}

$$-\frac{bc(2-m)m\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3d^2f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(m+1)\sqrt{d-c^2dx^2}}+\frac{(2-m)(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{3df\left(d-c^2dx^2\right)^{3/2}}+\frac{bc(2-m)\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{3d^2f^2(m+2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d*f*(d - c^2*d*x^2)^(3/2)) + ((2 -
m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*Sqrt[d - c^2*d*x^2]) - ((2
- m)*m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2
F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(3*d^2*f*(1 + m)*Sqrt[d - c^2*d*x^2
]) + (b*c*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric
2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt[d - c^2*d*x^
2]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[2,
(2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (
b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(3*d^2*f^2*(1 + m)*(
2 + m)*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

```
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{1+m}}{(-1+c^2x^2)}}{3d^2f\sqrt{d-c^2dx^2}}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.50, size = 319, normalized size = 0.71

$$\frac{x(fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}\left(-\frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bcx\,{}_2F_1\left(2,1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)}{2+m}+\frac{(-2+m)\left(m(2+m)\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)-(1+m)\left((2+m)\left(a+b\cosh^{-1}(cx)\right)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};c^2x^2\right)\right)+bcmx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)\sqrt{-1+cx}\,\sqrt{1+cx}}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/((-1 + c*x)
^(3/2)*(1 + c*x)^(3/2))) + (b*c*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, c^
2*x^2])/(2 + m) + ((-2 + m)*(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x
])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - (1 + m)*((2 + m)
*(a + b*ArcCosh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F
1[1, 1 + m/2, 2 + m/2, c^2*x^2]) + b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hyp
ergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1
+ m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)}{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\,(f\,x)^m}{(d - c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2), x)
```

3.157 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx\right.$

Optimal. Leaf size=817

$$-\frac{bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{15bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2(4+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bc\mathrm{d1}^2\mathrm{d2}}{f^2(2+m)}$$

```
[Out] 5*d1*d2*(f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))
/f/(4+m)/(6+m)+(f*x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccos
h(c*x))/f/(6+m)+15*d1^2*d2^2*(f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/
2)*(-c*d2*x+d2)^(1/2)/f/(6+m)/(m^2+6*m+8)+15*d1^2*d2^2*(f*x)^(1+m)*(a+b*arc
cosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2
)*(-c*d2*x+d2)^(1/2)/f/(6+m)/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2
)-b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)/
(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^
(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-5*b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(6+m
)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b*c^3*d1^2*d2^2*(f*x)^(4+m)*(c*
d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+2*b*c^3*d1^2*d2^2*(f*x)^(4+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^4
/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d1^2*d2^2*(f*x)^(6+m)*(c*d1*
x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b
*c*d1^2*d2^2*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*
m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(6+m)/(m^2+5*m
+4)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.04, antiderivative size = 817, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5931, 5927, 5949, 32, 74, 14, 276}

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2
 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - (15*b*c*d1^2*d2^2*(f*x)^(2 +
 m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*(4 + m)*(6 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*
x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (5*b*c^3*d1^2*d2^2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x
])/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d1^2*d2^
2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^4*(4 + m)*(6 + m)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d1^2*d2^2*(f*x)^(6 + m)*Sqrt[d1 + c*d
```

```
1*x]*Sqrt[d2 - c*d2*x])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*
d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[
c*x]))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d1*d2*(f*x)^(1 + m)*(d1 + c*d1*x)^(
3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)
^(1 + m)*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]))/(f*(
6 + m)) + (15*d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(
a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/
(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (15*b*c*d
1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPF
Q[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 +
m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
```

```
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}\left(a+b\right.}{f(6+m)}$$

$$= \frac{5\mathrm{d1}\mathrm{d2}(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}{f(4+m)(6+m)}$$

$$= -\frac{bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bc\mathrm{d1}^2\mathrm{d2}^2(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 1.68, size = 387, normalized size = 0.47

$$\frac{\mathrm{d1}^2\mathrm{d2}^2x(fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(-\frac{bcx\left(\frac{1}{2+m}-\frac{2c^2x^2}{4+m}+\frac{c^4x^4}{6+m}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}+(-1+c^2x^2)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{5\left(\frac{bcx\left(-\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-(-1+cx)(1+cx)(a+b\cosh^{-1}(cx))+\frac{3\left((1+m)\left(-bcx\sqrt{-1+cx}\,\sqrt{1+cx}+a(2+m)(-1+c^2x^2)+b(2+m)(-1+c^2x^2)\cosh^{-1}(cx)\right)-(2+m)\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)^2(-1+cx)(1+cx)}\right)}{4+m}\right)}{6+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1^2*d2^2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((b*c*x*((2 + m)
^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])) + (-1 + c^2*x^2)^2*(a + b*ArcCosh[c*x]) + (5*((b*c*x*(-(2 + m)^(-1)
+ (c^2*x^2)/(4 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (-1 + c*x)*(1 + c*x)
*(a + b*ArcCosh[c*x]) + (3*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (2 +
m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 + m)^2
*(-1 + c*x)*(1 + c*x))))/(4 + m)))/(6 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^{m} (cd1x + d1)^{\frac{5}{2}} (-cd2x + d2)^{\frac{5}{2}} (a + b\,\mathrm{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral((a*c^4*d1^2*d2^2*x^4 - 2*a*c^2*d1^2*d2^2*x^2 + a*d1^2*d2^2 + (b*c^
4*d1^2*d2^2*x^4 - 2*b*c^2*d1^2*d2^2*x^2 + b*d1^2*d2^2)*arccosh(c*x))*sqrt(c
*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(5/2)*(-c*d2*x+d2)**(5/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,{(f\,x)}^m\,{(d_1+c\,d_1\,x)}^{5/2}\,{(d_2-c\,d_2\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2), x)
```

3.158 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}\left(a+b\cosh^{-1}(c\right.$

Optimal. Leaf size=503

$$-\frac{3bc\text{d1d2}(fx)^{2+m}\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc\text{d1d2}(fx)^{2+m}\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}{f^2(2+m)(4+m)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3\text{d1d2}(fx)^{4+m}}{f^4(4+m)}$$

```
[Out] (f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)
+3*d1*d2*(f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2
)/f/(m^2+6*m+8)+3*d1*d2*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(m^3+7*m
^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d1*d2*(f*x)^(2+m)*(c*d1*x+d1)
^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c
*d1*d2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)/(4+m)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+b*c^3*d1*d2*(f*x)^(4+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x
+d2)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d1*d2*(f*x)^(2+m)*
hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(
1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(m^2+5*m+4)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5931, 5927, 5949, 32, 74, 14}

$$-\frac{3bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)\left(m^2+3m+2\right)\sqrt{1-cx}\sqrt{cx+1}}+\frac{3\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f\left(m^2+6m+8\right)}+\frac{(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+4)}+\frac{bc^3\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+4}}{f^4(m+4)^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}}{f^2(m+2)(m+4)\sqrt{cx-1}\sqrt{cx+1}}-\frac{3bc\text{d1d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}}{f^2(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 +
m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d1*d2*(f*x)^(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*c^3*d1*d2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(
f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (
(f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))
/(f*(4 + m)) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(
a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/
(f*(4 + m)*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (3*b*c*d1*d2*(f*x
)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{1, 1 + m/2
, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
```

```
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+\right.}{f(4+m)}$$

$$=\frac{3\mathrm{d1}\mathrm{d2}(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(a\right.}{f\left(8+6m+m^2\right)}$$

$$=-\frac{3bc\mathrm{d1}\mathrm{d2}(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.74, size = 288, normalized size = 0.57

$$\frac{\mathrm{d1}\mathrm{d2}x(fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(-\frac{3bcx}{(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcx\left(-\frac{1}{2+m}+\frac{c^2x^2}{4+m}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3\left(a+b\cosh^{-1}(cx)\right)}{2+m}-(-1+cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)-\frac{3\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{(1+m)(2+m)(-1+cx)(1+cx)}-\frac{3bcx\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(1+m)(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}\right)}{4+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1*d2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((-3*b*c*x)/((2 + m)^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4 + m))
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*(a + b*ArcCosh[c*x]))/(2 + m) - (-1 +
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]) - (3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*(2 +
m)*(-1 + c*x)*(1 + c*x)) - (3*b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}
, {3/2 + m/2, 2 + m/2}, c^2*x^2])/((1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])))/(4 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m\left(cd1x+d1\right)^{\frac{3}{2}}\left(-cd2x+d2\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d1*d2*x^2 - a*d1*d2 + (b*c^2*d1*d2*x^2 - b*d1*d2)*arccosh(
c*x))*sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(3/2)*(-c*d2*x+d2)**(3/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,(d_1 + c\,d_1\,x)^{3/2}\,(d_2 - c\,d_2\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2),x)
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2), x)
```

3.159 $\int (fx)^m \sqrt{\mathbf{d1} + c\mathbf{d1}x} \sqrt{\mathbf{d2} - c\mathbf{d2}x} \left(a + b\cosh^{-1}(cx\right.$

Optimal. Leaf size=302

$$-\frac{bc(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}+\frac{(fx)^{1+m}\sqrt{\mathrm{d}}}{}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(2+m)
+(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*
x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(m^2+3*m+2)/(-c*x+1)^(1/2)/(c*x
+1)^(1/2)-b*c*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m]
,[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(1+
m)/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$,
Rules used = {5927, 5949, 32}

$$-\frac{bc\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f\left(m^2+3m+2\right)\sqrt{1-cx}\sqrt{cx+1}}+\frac{\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+2)}-\frac{bc\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,(fx)^{m+2}}{f^2(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]
*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*Sqrt[1 - c*x]*Sqrt[1 + c*x]) - (b*c*
(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{1, 1 +
 m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
```

```
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(a+b\cosh^{-1}\right.}{f(2+m)}$$

$$= -\frac{bc(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(fx)}{}$$

Mathematica [**A**]
time = 0.16, size = 229, normalized size = 0.76

$$\frac{x(fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left((1+m)\left(-bcx\sqrt{-1+cx}\,\sqrt{1+cx}+a(2+m)\left(-1+c^2x^2\right)+b(2+m)\left(-1+c^2x^2\right)\cosh^{-1}(cx)\right)-(2+m)\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)\right)}{(1+m)(2+m)^2(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),
x]
```

```
[Out] (x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((1 + m)*(-(b*c*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*A
rcCosh[c*x]) - (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])
)/((1 + m)*(2 + m)^2*(-1 + c*x)*(1 + c*x))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m\sqrt{c\mathit{d1}x+\mathit{d1}}\,\sqrt{-c\mathit{d2}x+\mathit{d2}}\,\left(a+b\,\mathrm{arccosh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d_1 (cx + 1)} (fx)^m \sqrt{-d_2 (cx - 1)} (a + b \operatorname{acosh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(1/2)*(-c*d2*x+d2)**(1/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Integral(sqrt(d1*(c*x + 1))*(f*x)**m*sqrt(-d2*(c*x - 1))*(a + b*acosh(c*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,\sqrt{d_1+c\,d_1\,x}\,\sqrt{d_2-c\,d_2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2), x)
```

3.160 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=188

$$\frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}+\frac{bc(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}\,{}_3F_2\left(1,1+\frac{m}{2}\right.}{f^2(1+m)(2+m)\sqrt{\text{d1}+c\text{d1}x}}$$

```
[Out] b*c*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d
2)^(1/2)+(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2
*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2
)
```

Rubi [A]
time = 0.22, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {5949}

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{\text{cd1}x+\text{d1}}\sqrt{\text{d2}-\text{cd2}x}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)\sqrt{\text{cd1}x+\text{d1}}\sqrt{\text{d2}-\text{cd2}x}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]),x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*
d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}$$
$$= \frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} + \frac{bc(fx}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 4.22, size = 264, normalized size = 1.40

$$\frac{2^{-3-m}(fx)^m\left(\frac{cx}{1+cx}\right)^{1-m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\left(2^{3+m}a(1+m)(-1+cx)F_1\left(-m;-m,\frac{1}{2};1-m;\frac{1}{1+cx},\frac{2}{1+cx}\right)+bm\left(\frac{cx}{1+cx}\right)^m\left(-2^{2+m}(-1+cx)\cosh^{-1}(cx)\,{}_2F_1\left(1,\frac{2+m}{2};\frac{3+m}{2};c^2x^2\right)+c(1+m)\sqrt{\pi}\,x\sqrt{\frac{-1+cx}{1+cx}}\,\Gamma(1+m)\,{}_3\tilde{F}_2\left(1,\frac{2+m}{2},\frac{2+m}{2};\frac{3+m}{2},\frac{4+m}{2};c^2x^2\right)\right)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{c^2\mathrm{d1}m(1+m)x\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*
x]),x]
```

```
[Out] (2^(-3 - m)*(f*x)^m*((c*x)/(1 + c*x))^(1 - m)*Sqrt[d1 + c*d1*x]*(2^(3 + m)*
a*(1 + m)*(-1 + c*x)*AppellF1[-m, -m, 1/2, 1 - m, (1 + c*x)^(-1), 2/(1 + c*
x)] + b*m*((c*x)/(1 + c*x))^m*(-(2^(2 + m)*(-1 + c*x)*ArcCosh[c*x]*Hypergeo
metric2F1[1, (2 + m)/2, (3 + m)/2, c^2*x^2]) + c*(1 + m)*Sqrt[Pi]*x*Sqrt[(-
1 + c*x)/(1 + c*x)]*Gamma[1 + m]*HypergeometricPFQRegularized[{1, (2 + m)/2
, (2 + m)/2}, {(3 + m)/2, (4 + m)/2}, c^2*x^2])*Sinh[2*ArcCosh[c*x]]))/(c^2
*d1*m*(1 + m)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d2 - c*d2*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)}{\sqrt{cd1x+d1}\,\sqrt{-cd2x+d2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^2*d1*d2*x^2 - d1*d2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{\sqrt{d_1 (cx + 1)} \sqrt{-d_2 (cx - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2),
x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x - 1)
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,(f\,x)^m}{\sqrt{d_1 + c\,d_1\,x}\,\sqrt{d_2 - c\,d_2\,x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)),
 x)
```

3.161 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}}\,dx$

Optimal. Leaf size=336

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{\mathrm{d1d2}f(1+m)\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}+\frac{bc(fx)\cdots}{\mathrm{c}\cdots}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))/d1/d2/f/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
+b*c*(f*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d1/d2/f^2/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-b*c*m*(f*x
)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d1/d2/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)
^(1/2)-m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2
*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d1/d2/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2
)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5937, 5949, 74, 371}

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{\mathrm{d1d2}f^2(m+1)(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1d2}f(m+1)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1d2}f\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{\mathrm{d1d2}f^2(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]) - (m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(d1*d2*f*(1 + m)*Sqrt[d1 + c*
d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d1*d2*f^2*(2 + m)*Sqr
t[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c
^2*x^2])/(d1*d2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5937

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m +
1))*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d1*d
2*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1
+ e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
 b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m},
 x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && LtQ[p, -1] &&  !Gt
Q[m, 1] && (IntegerQ[m] || EqQ[n, 1])
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} - \frac{m\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}}{\mathrm{d1}\mathrm{d2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{\mathrm{d1}\mathrm{d2}f^2(2+m)\sqrt{\mathrm{d1}+c\mathrm{d1}}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} - \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cos\right.}{\mathrm{d1}\mathrm{d2}f(1+m)\sqrt{\mathrm{d1}-}}$$

Mathematica [F]

time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\cosh^{-1}(cx)\right)}{(\mathrm{d1} + c\mathrm{d1}x)^{3/2}(\mathrm{d2} - c\mathrm{d2}x)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)), x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\,\mathrm{arccosh}(cx)\right)}{(cd1x + d1)^{\frac{3}{2}} \left(-cd2x + d2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m/
(c^4*d1^2*d2^2*x^4 - 2*c^2*d1^2*d2^2*x^2 + d1^2*d2^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(3/2)/(-c*d2*x+d2)**(3/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m}{(d_1+c\,d_1\,x)^{3/2}\,(d_2-c\,d_2\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)),
 x)
```

3.162 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}}\,dx$

Optimal. Leaf size=504

$$\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}-\frac{(2-m)m(fx)^{1+m}\sqrt{1-c^2x^2}\,(}{3\text{d1}^2\text{d2}^2f(1+m}$$

```
[Out] 1/3*(f*x)^(1+m)*(a+b*arccosh(c*x))/d1/d2/f/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(
3/2)+1/3*(2-m)*(f*x)^(1+m)*(a+b*arccosh(c*x))/d1^2/d2^2/f/(c*d1*x+d1)^(1/2)
/(-c*d2*x+d2)^(1/2)+1/3*b*c*(2-m)*(f*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2
*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2/d2^2/f^2/(2+m)/(c*d1*x+d1)^(1
/2)/(-c*d2*x+d2)^(1/2)+1/3*b*c*(f*x)^(2+m)*hypergeom([2, 1+1/2*m],[2+1/2*m]
,c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2/d2^2/f^2/(2+m)/(c*d1*x+d1)^(1/2)
/(-c*d2*x+d2)^(1/2)-1/3*b*c*(2-m)*m*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/
2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2/d2^2/f^
2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-1/3*(2-m)*m*(f*x)^(1+m)*
(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^
2+1)^(1/2)/d1^2/d2^2/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
```

Rubi [A]

time = 0.84, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5937, 5949, 74, 371}

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+1)(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f(m+1)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(2-m)(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}^2\text{d2}^2f\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{3\text{d1}\text{d2}f(c\text{d1}x+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}}+\frac{bc(2-m)\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{3\text{d1}^2\text{d2}^2f^2(m+2)\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1*d2*f*(d1 + c*d1*x)^(3/2)*(d2 - c
*d2*x)^(3/2)) + ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1^2*d2^2*f
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c
^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(3*d1^2*d2^2*f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c
*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2
 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]*Sq
rt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeo
metric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, c^2*x^2])/(3*d1^2*d2^2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
 c*d2*x])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5937

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m +
1))*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d1*d
2*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1
+ e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
 b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m},
 x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && LtQ[p, -1] &&  !Gt
Q[m, 1] && (IntegerQ[m] || EqQ[n, 1])
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)\int \frac{(fx)^m(a+b\cosh^{-1}(c}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}}}{3\mathrm{d1}\mathrm{d2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-}\right.}{3\mathrm{d1}^2\mathrm{d2}^2 f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-}\right.}{3\mathrm{d1}^2\mathrm{d2}^2 f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-}\right.}{3\mathrm{d1}^2\mathrm{d2}^2 f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

Mathematica [F]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)), x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arccosh}(cx)\right)}{(cd1x+d1)^{\frac{5}{2}}\left(-cd2x+d2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^6*d1^3*d2^3*x^6 - 3*c^4*d1^3*d2^3*x^4 + 3*c^2*d1^3*d2^3*x^2 - d1^3*d2^3
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(5/2)/(-c*d2*x+d2)**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(f\,x\right)}^m}{{\left(d_1+c\,d_1\,x\right)}^{5/2}\,{\left(d_2-c\,d_2\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)),
 x)
```

3.163 $\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=128

$$\frac{(fx)^{1+m}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{f(1+m)}+\frac{a(fx)^{2+m}\sqrt{-1+ax}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};a^2x^2\right)}{f^2(1+m)(2+m)\sqrt{1-ax}}$$

```
[Out] (f*x)^(1+m)*arccosh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/f/
(1+m)+a*(f*x)^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],a^
2*x^2)*(a*x-1)^(1/2)/f^2/(1+m)/(2+m)/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5948}

$$\frac{a\sqrt{ax-1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-ax}}+\frac{\cosh^{-1}(ax)(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^
2*x^2])/(f*(1 + m)) + (a*(f*x)^(2 + m)*Sqrt[-1 + a*x]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/(f^2*(1 + m)*(2 + m)*Sq
rt[1 - a*x])
```

Rule 5948

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))
*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]*HypergeometricPFQ[{1,
 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{(fx)^m\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{(fx)^{1+m}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{f(1+m)}+\frac{a(fx)^{2+m}\sqrt{-1+ax}\,\sqrt{1+ax}\,{}_3F}{f^2(1+m)(2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 124, normalized size = 0.97

$$\frac{x(fx)^m \left(\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)+\frac{ax\sqrt{-1+ax}\,\sqrt{1+ax}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};a^2x^2\right)}{(2+m)\sqrt{1-a^2x^2}}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] (x*(f*x)^m*(ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x
^2] + (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])))/(1 + m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{arccosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*(f*x)^m*arccosh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)\,\left(f\,x\right)^m}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2), x)
```

3.164 $\int (c-a^2cx^2)^3 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=266

$$\frac{4322c^3x}{3675}-\frac{1514a^2c^3x^3}{11025}+\frac{234a^4c^3x^5}{6125}-\frac{2}{343}a^6c^3x^7-\frac{32c^3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{35a}+\frac{16c^3(-1+ax)^{3/2}(1-}{105}$$

```
[Out] 4322/3675*c^3*x-1514/11025*a^2*c^3*x^3+234/6125*a^4*c^3*x^5-2/343*a^6*c^3*x
^7+16/105*c^3*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a-12/175*c^3*(a*x-1)
^(5/2)*(a*x+1)^(5/2)*arccosh(a*x)/a+2/49*c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)*ar
ccosh(a*x)/a+16/35*c^3*x*arccosh(a*x)^2+8/35*c^3*x*(-a^2*x^2+1)*arccosh(a*x
)^2+6/35*c^3*x*(-a^2*x^2+1)^2*arccosh(a*x)^2+1/7*c^3*x*(-a^2*x^2+1)^3*arcco
sh(a*x)^2-32/35*c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.45, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5897, 5879, 5915, 8, 41, 200}

$$-\frac{2}{343}a^6c^3x^7+\frac{234a^4c^3x^5}{6125}-\frac{1514a^2c^3x^3}{11025}+\frac{1}{7}c^3x(1-a^2x^2)^3\cosh^{-1}(ax)^2+\frac{6}{35}c^3x(1-a^2x^2)^2\cosh^{-1}(ax)^2+\frac{8}{35}c^3x(1-a^2x^2)\cosh^{-1}(ax)^2+\frac{16}{35}c^3x\cosh^{-1}(ax)^2+\frac{2c^3(ax-1)^{7/2}(ax+1)^{7/2}\cosh^{-1}(ax)}{49a}-\frac{12c^3(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)}{175a}+\frac{16c^3(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{105a}-\frac{32c^3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{35a}+\frac{4322c^3x}{3675}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```

```
[Out] (4322*c^3*x)/3675 - (1514*a^2*c^3*x^3)/11025 + (234*a^4*c^3*x^5)/6125 - (2*
a^6*c^3*x^7)/343 - (32*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(35*a
) + (16*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(105*a) - (12*c^
3*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])/(175*a) + (2*c^3*(-1 + a*x
)^(7/2)*(1 + a*x)^(7/2)*ArcCosh[a*x])/(49*a) + (16*c^3*x*ArcCosh[a*x]^2)/35
 + (8*c^3*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/35 + (6*c^3*x*(1 - a^2*x^2)^2*Arc
Cosh[a*x]^2)/35 + (c^3*x*(1 - a^2*x^2)^3*ArcCosh[a*x]^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(c-a^2cx^2\right)^3\cosh^{-1}(ax)^2\,dx &= \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{1}{7}(6c)\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2\,dx+\frac{1}{7} \\
&= \frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)}{49a}+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2 \\
&= -\frac{12c^3(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{175a}+\frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{49a} \\
&= \frac{2c^3x}{49}-\frac{2}{49}a^2c^3x^3+\frac{6}{245}a^4c^3x^5-\frac{2}{343}a^6c^3x^7+\frac{16c^3(-1+ax)^{3/2}(1+ax)^{3}}{105a} \\
&= \frac{962c^3x}{3675}-\frac{1514a^2c^3x^3}{11025}+\frac{234a^4c^3x^5}{6125}-\frac{2}{343}a^6c^3x^7-\frac{32c^3\sqrt{-1+ax}\,\sqrt{1+}}{35a} \\
&= \frac{4322c^3x}{3675}-\frac{1514a^2c^3x^3}{11025}+\frac{234a^4c^3x^5}{6125}-\frac{2}{343}a^6c^3x^7-\frac{32c^3\sqrt{-1+ax}\,\sqrt{1}}{35a}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 125, normalized size = 0.47

$$\frac{c^3\left(453810ax-52990a^3x^3+14742a^5x^5-2250a^7x^7+210\sqrt{-1+ax}\,\sqrt{1+ax}\,(-2161+757a^2x^2-351a^4x^4+75a^6x^6)\cosh^{-1}(ax)-11025ax(-35+35a^2x^2-21a^4x^4+5a^6x^6)\cosh^{-1}(ax)^2\right)}{385875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(453810*a*x - 52990*a^3*x^3 + 14742*a^5*x^5 - 2250*a^7*x^7 + 210*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 757*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*Ar
cCosh[a*x] - 11025*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[
a*x]^2))/(385875*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^2c\,x^2+c\right)^3 \operatorname{arccosh}(ax)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x)
```

```
[Out] int((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x)
```

Maxima [A]
time = 0.27, size = 178, normalized size = 0.67

$$-\frac{2}{343}\,a^6c^3x^7+\frac{234}{6125}\,a^4c^3x^5-\frac{1514}{11025}\,a^2c^3x^3+\frac{4322}{3675}\,c^3x+\frac{2}{3675}\left(75\,\sqrt{a^2x^2-1}\,a^4c^3x^6-351\,\sqrt{a^2x^2-1}\,a^2c^3x^4+757\,\sqrt{a^2x^2-1}\,c^3x^2-\frac{2161\,\sqrt{a^2x^2-1}\,c^3}{a^2}\right)a\operatorname{arcosh}(ax)-\frac{1}{35}\left(5\,a^6c^3x^7-21\,a^4c^3x^5+35\,a^2c^3x^3-35\,c^3x\right)\operatorname{arcosh}(ax)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/343*a^6*c^3*x^7 + 234/6125*a^4*c^3*x^5 - 1514/11025*a^2*c^3*x^3 + 4322/3
675*c^3*x + 2/3675*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1
)*a^2*c^3*x^4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/
a^2)*a*arccosh(a*x) - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3
 - 35*c^3*x)*arccosh(a*x)^2
```

Fricas [A]
time = 0.39, size = 175, normalized size = 0.66

$$-\frac{2250\,a^7c^3x^7-14742\,a^5c^3x^5+52990\,a^3c^3x^3-453810\,ac^3x+11025\,(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-210\,(75\,a^6c^3x^6-351\,a^4c^3x^4+757\,a^2c^3x^2-2161\,c^3)\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{385875\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/385875*(2250*a^7*c^3*x^7 - 14742*a^5*c^3*x^5 + 52990*a^3*c^3*x^3 - 45381
0*a*c^3*x + 11025*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 210*(75*a^6*c^3*x^6 - 351*a^4*c^3*x^
4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
1)))/a
```

Sympy [A]
time = 1.08, size = 243, normalized size = 0.91

$$\begin{cases} -\frac{a^6c^3x^7\operatorname{acosh}^2(ax)}{7} - \frac{2a^6c^3x^7}{343} + \frac{2a^5c^3x^6\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{49} + \frac{3a^4c^3x^5\operatorname{acosh}^2(ax)}{5} + \frac{234a^4c^3x^5}{6125} - \frac{234a^3c^3x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{1225} - a^2c^3x^3\operatorname{acosh}^2(ax) - \frac{1514a^2c^3x^3}{11025} + \frac{1514ac^3x^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{3675} + c^3x\operatorname{acosh}^2(ax) + \frac{4322c^3x}{3675} - \frac{4322c^3\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{3675a} & \text{for } a \neq 0 \\ -\frac{\pi^2c^3x}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**2/7 - 2*a**6*c**3*x**7/343 + 2*a**5*
c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)/49 + 3*a**4*c**3*x**5*acosh(a*x)**
2/5 + 234*a**4*c**3*x**5/6125 - 234*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acos
h(a*x)/1225 - a**2*c**3*x**3*acosh(a*x)**2 - 1514*a**2*c**3*x**3/11025 + 15
14*a*c**3*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/3675 + c**3*x*acosh(a*x)**2 +
 4322*c**3*x/3675 - 4322*c**3*sqrt(a**2*x**2 - 1)*acosh(a*x)/(3675*a), Ne(a
, 0)), (-pi**2*c**3*x/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^2\,{\left(c-a^2\,c\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2)^3, x)
```

3.165 $\int (c-a^2cx^2)^2 \cosh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=195

$$\frac{298c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{15a}+\frac{8c^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(}{45a}$$

```
[Out] 298/225*c^2*x-76/675*a^2*c^2*x^3+2/125*a^4*c^2*x^5+8/45*c^2*(a*x-1)^(3/2)*(
a*x+1)^(3/2)*arccosh(a*x)/a-2/25*c^2*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arccosh(a*
x)/a+8/15*c^2*x*arccosh(a*x)^2+4/15*c^2*x*(-a^2*x^2+1)*arccosh(a*x)^2+1/5*c
^2*x*(-a^2*x^2+1)^2*arccosh(a*x)^2-16/15*c^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A]
time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5897, 5879, 5915, 8, 41, 200}

$$\frac{2}{125}a^4c^2x^5-\frac{76}{675}a^2c^2x^3+\frac{1}{5}c^2x(1-a^2x^2)^2\cosh^{-1}(ax)^2+\frac{4}{15}c^2x(1-a^2x^2)\cosh^{-1}(ax)^2+\frac{8}{15}c^2x\cosh^{-1}(ax)^2-\frac{2c^2(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)}{25a}+\frac{8c^2(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{45a}-\frac{16c^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{15a}+\frac{298c^2x}{225}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```

```
[Out] (298*c^2*x)/225 - (76*a^2*c^2*x^3)/675 + (2*a^4*c^2*x^5)/125 - (16*c^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(15*a) + (8*c^2*(-1 + a*x)^(3/2)*(1
+ a*x)^(3/2)*ArcCosh[a*x])/(45*a) - (2*c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)
*ArcCosh[a*x])/(25*a) + (8*c^2*x*ArcCosh[a*x]^2)/15 + (4*c^2*x*(1 - a^2*x^2
)*ArcCosh[a*x]^2)/15 + (c^2*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (c-a^2cx^2)^2\cosh^{-1}(ax)^2\,dx &= \frac{1}{5}c^2x(1-a^2x^2)^2\cosh^{-1}(ax)^2+\frac{1}{5}(4c)\int (c-a^2cx^2)\cosh^{-1}(ax)^2\,dx-\frac{1}{5}\\ &= -\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{25a}+\frac{4}{15}c^2x(1-a^2x^2)\cosh^{-1}(ax)^2\\ &= \frac{8c^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{45a}-\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cos}{25a}\\ &= \frac{58c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{15a}\\ &= \frac{298c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax}{15a}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 101, normalized size = 0.52

$$\frac{c^2\left(4470ax-380a^3x^3+54a^5x^5-30\sqrt{-1+ax}\sqrt{1+ax}\,(149-38a^2x^2+9a^4x^4)\cosh^{-1}(ax)+225ax(15-10a^2x^2+3a^4x^4)\cosh^{-1}(ax)^2\right)}{3375a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```

```
[Out] (c^2*(4470*a*x - 380*a^3*x^3 + 54*a^5*x^5 - 30*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x] + 225*a*x*(15 - 10*a^2*x^2 + 3
*a^4*x^4)*ArcCosh[a*x]^2))/(3375*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c\, x^{2}+c \right)^{2} \operatorname{arccosh}\left(a x \right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x)
```

```
[Out] int((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x)
```

Maxima [A]
time = 0.26, size = 134, normalized size = 0.69

$$\frac{2}{125}\, a^{4} c^{2} x^{5} - \frac{76}{675}\, a^{2} c^{2} x^{3} + \frac{298}{225}\, c^{2} x - \frac{2}{225} \left(9 \sqrt{a^{2} x^{2} - 1}\, a^{2} c^{2} x^{4} - 38 \sqrt{a^{2} x^{2} - 1}\, c^{2} x^{2} + \frac{149 \sqrt{a^{2} x^{2} - 1}\, c^{2}}{a^{2}}\right) a \operatorname{arcosh}\left(a x\right) + \frac{1}{15} \left(3 \, a^{4} c^{2} x^{5} - 10 \, a^{2} c^{2} x^{3} + 15 \, c^{2} x\right) \operatorname{arcosh}\left(a x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2/125*a^4*c^2*x^5 - 76/675*a^2*c^2*x^3 + 298/225*c^2*x - 2/225*(9*sqrt(a^2*
x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149*sqrt(a^2*x^2 - 1)
*c^2/a^2)*a*arccosh(a*x) + 1/15*(3*a^4*c^2*x^5 - 10*a^2*c^2*x^3 + 15*c^2*x)
*arccosh(a*x)^2
```

Fricas [A]
time = 0.34, size = 142, normalized size = 0.73

$$\frac{54 \, a^{5} c^{2} x^{5} - 380 \, a^{3} c^{2} x^{3} + 4470 \, a c^{2} x + 225 \, {\left(3 \, a^{5} c^{2} x^{5} - 10 \, a^{3} c^{2} x^{3} + 15 \, a c^{2} x\right)} \log\left(a x + \sqrt{a^{2} x^{2} - 1}\right)^{2} - 30 \, {\left(9 \, a^{4} c^{2} x^{4} - 38 \, a^{2} c^{2} x^{2} + 149 \, c^{2}\right)} \sqrt{a^{2} x^{2} - 1} \log\left(a x + \sqrt{a^{2} x^{2} - 1}\right)}{3375 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(54*a^5*c^2*x^5 - 380*a^3*c^2*x^3 + 4470*a*c^2*x + 225*(3*a^5*c^2*x^
5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 30*(9*a^4
*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x
^2 - 1)))/a
```

Sympy [A]
time = 0.52, size = 182, normalized size = 0.93

$$\begin{cases} \frac{a^{4} c^{2} x^{5} \operatorname{acosh}^{2}{\left(a x \right)}}{5} + \frac{2 a^{4} c^{2} x^{5}}{125} - \frac{2 a^{3} c^{2} x^{4} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}{\left(a x \right)}}{25} - \frac{2 a^{2} c^{2} x^{3} \operatorname{acosh}^{2}{\left(a x \right)}}{3} - \frac{76 a^{2} c^{2} x^{3}}{675} + \frac{76 a c^{2} x^{2} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}{\left(a x \right)}}{225} + c^{2} x \operatorname{acosh}^{2}{\left(a x \right)} + \frac{298 c^{2} x}{225} - \frac{298 c^{2} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}{\left(a x \right)}}{225 a} & \text{for}\: a \neq 0 \\- \frac{\pi^{2} c^{2} x}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**2/5 + 2*a**4*c**2*x**5/125 - 2*a**3*c
**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)/25 - 2*a**2*c**2*x**3*acosh(a*x)**2
/3 - 76*a**2*c**2*x**3/675 + 76*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/
225 + c**2*x*acosh(a*x)**2 + 298*c**2*x/225 - 298*c**2*sqrt(a**2*x**2 - 1)*
acosh(a*x)/(225*a), Ne(a, 0)), (-pi**2*c**2*x/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}(a\,x)^2\,{\left(c-a^2\,c\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2)^2, x)
```

3.166 $\int (c-a^2cx^2)\cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=112

$$\frac{14cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{9a}+\frac{2}{3}cx\cosh^{-1}(ax$$

```
[Out] 14/9*c*x-2/27*a^2*c*x^3+2/9*c*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a+2/
3*c*x*arccosh(a*x)^2+1/3*c*x*(-a^2*x^2+1)*arccosh(a*x)^2-4/3*c*arccosh(a*x)
*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5897, 5879, 5915, 8, 41}

$$-\frac{2}{27}a^2cx^3+\frac{1}{3}cx(1-a^2x^2)\cosh^{-1}(ax)^2+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{2c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{9a}-\frac{4c\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{3a}+\frac{14cx}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```

```
[Out] (14*c*x)/9 - (2*a^2*c*x^3)/27 - (4*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(3*a) + (2*c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(9*a) + (2
*c*x*ArcCosh[a*x]^2)/3 + (c*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
```

```
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx = \frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{1}{3}(2c)\int \cosh^{-1}(ax)^2\,dx+\frac{1}{3}(2ac)\int x\sqrt{-1}$$
$$= \frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{9a}+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{1}{3}cx\left(1-a^2x^2\right.$$
$$= \frac{2cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1}{}$$
$$= \frac{14cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(}{}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.65

$$\frac{c\left(42ax-2a^3x^3+6\sqrt{-1+ax}\,\sqrt{1+ax}\,(-7+a^2x^2)\cosh^{-1}(ax)-9ax(-3+a^2x^2)\cosh^{-1}(ax)^2\right)}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```

```
[Out] (c*(42*a*x - 2*a^3*x^3 + 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcC
osh[a*x] - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^2))/(27*a)
```

Maple [A]
time = 1.18, size = 90, normalized size = 0.80

$$-\frac{c\left(9\mathrm{arccosh}(ax)^2x^3a^3-6\,\mathrm{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,x^2a^2+2a^3x^3-27\mathrm{arccosh}(ax)^2ax+42\,\mathrm{arccosl}\right.}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^2,x)
```

```
[Out] -1/27/a*c*(9*arccosh(a*x)^2*x^3*a^3-6*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)*x^2*a^2+2*a^3*x^3-27*arccosh(a*x)^2*a*x+42*arccosh(a*x)*(a*x-1)^(1/2)*(
a*x+1)^(1/2)-42*a*x)
```

Maxima [A]
time = 0.25, size = 76, normalized size = 0.68

$$-\frac{2}{27}\,a^2cx^3+\frac{2}{9}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{7\sqrt{a^2x^2-1}\,c}{a^2}\right)a\,\mathrm{arcosh}\left(ax\right)-\frac{1}{3}\left(a^2cx^3-3\,cx\right)\mathrm{arcosh}\left(ax\right)^2+\frac{14}{9}\,cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/27*a^2*c*x^3 + 2/9*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)
*a*arccosh(a*x) - 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^2 + 14/9*c*x
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.85

$$-\frac{2\,a^3cx^3-42\,acx+9\left(a^3cx^3-3\,acx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^2cx^2-7\,c\right)\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*c*x^3 - 42*a*c*x + 9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*
x^2 - 1))^2 - 6*(a^2*c*x^2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2
- 1)))/a
```

Sympy [A]
time = 0.20, size = 105, normalized size = 0.94

$$\begin{cases} -\frac{a^2cx^3\,\mathrm{acosh}^2\left(ax\right)}{3}-\frac{2a^2cx^3}{27}+\frac{2acx^2\sqrt{a^2x^2-1}\,\mathrm{acosh}\left(ax\right)}{9}+cx\,\mathrm{acosh}^2\left(ax\right)+\frac{14cx}{9}-\frac{14c\sqrt{a^2x^2-1}\,\mathrm{acosh}\left(ax\right)}{9a} & \text{for}\ a\neq 0\\ -\frac{\pi^2cx}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**2/3 - 2*a**2*c*x**3/27 + 2*a*c*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)/9 + c*x*acosh(a*x)**2 + 14*c*x/9 - 14*c*sqrt(a*
*2*x**2 - 1)*acosh(a*x)/(9*a), Ne(a, 0)), (-pi**2*c*x/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)^2\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2), x)
```

3.167 $\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=98

$$\frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(\right.}}{ac}$$

```
[Out] 2*arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+2*arccosh(a*x
)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*arccosh(a*x)*polylog(2,
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)
^(1/2))/a/c+2*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5903, 4267, 2611, 2320, 6724}

$$\frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (2*ArcCosh[a*x]*PolyLog[
2, -E^ArcCosh[a*x]])/(a*c) - (2*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a
*c) - (2*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (2*PolyLog[3, E^ArcCosh[a*x]]
)/(a*c)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx &= -\frac{\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac} \\
&= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{2\text{S}}{} \\
&= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}}{ac} \\
&= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}}{ac} \\
&= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}}{ac}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 0.97

$$\frac{-\cosh^{-1}(ax)^2\log\left(1-e^{\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)^2\log\left(1+e^{\cosh^{-1}(ax)}\right)+2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^2*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^2*Log[1 + E^ArcCo
sh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 2*ArcCosh[a*x]*Poly
Log[2, E^ArcCosh[a*x]] - 2*PolyLog[3, -E^ArcCosh[a*x]] + 2*PolyLog[3, E^Arc
Cosh[a*x]])/(a*c)
```

Maple [A]
time = 2.31, size = 187, normalized size = 1.91

method	result
derivativedivides	$-\frac{\frac{\operatorname{arccosh}(ax)^2 \ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}-\frac{2\operatorname{arccosh}(ax)\operatorname{polylog}\left(2,ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}+\frac{2\operatorname{polylog}\left(3\right.}{}}{}$
default	$-\frac{\frac{\operatorname{arccosh}(ax)^2 \ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}-\frac{2\operatorname{arccosh}(ax)\operatorname{polylog}\left(2,ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}+\frac{2\operatorname{polylog}\left(3\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/c*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-2/c*arccosh(
a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+2/c*polylog(3,a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))+1/c*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)
)+2/c*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-2/c*polylog(
3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/
(a*c) - integrate(((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*sqrt
(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*log(a*
x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*sqrt(
a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c),x)
[Out] -Integral(acosh(a*x)**2/(a**2*x**2 - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2),x)
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2), x)
```

3.168 $\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)}+\frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\tanh^{-1}(ax)}{ac^2}+\frac{\cosh^{-1}(ax)\mathrm{Poly}}{}$$

```
[Out] 1/2*x*arccosh(a*x)^2/c^2/(-a^2*x^2+1)+arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))/a/c^2-arctanh(a*x)/a/c^2+arccosh(a*x)*polylog(2,-a*x-(a*
x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))/a/c^2-polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+polylo
g(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-arccosh(a*x)/a/c^2/(a*x-1)^(1/2)
/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5901, 5903, 4267, 2611, 2320, 6724, 5915, 35, 213}

$$\frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)}+\frac{\cosh^{-1}(ax)\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\mathrm{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\mathrm{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)}{ac^2\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\tanh^{-1}(ax)}{ac^2}+\frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(ArcCosh[a*x]/(a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])) + (x*ArcCosh[a*x]^2)/(
2*c^2*(1 - a^2*x^2)) + (ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - A
rcTanh[a*x]/(a*c^2) + (ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(a*c^2) -
(ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - PolyLog[3, -E^ArcCosh[a
*x]]/(a*c^2) + PolyLog[3, E^ArcCosh[a*x]]/(a*c^2)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 4267

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 5901

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p + 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 5903

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^2}\,dx &= \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c^2} + \frac{\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx}{2c} \\
&= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x\right.}{2ac^2} \\
&= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \text{tanh} \\
&= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \text{tanh} \\
&= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \text{tanh} \\
&= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \text{tanh}
\end{aligned}$$

Mathematica [A]
time = 0.58, size = 191, normalized size = 1.17

$$\frac{-4\cosh^{-1}(ax)\coth\left(\frac{1}{2}\cosh^{-1}(ax)\right) - \cosh^{-1}(ax)^2\text{csch}^2\left(\frac{1}{2}\cosh^{-1}(ax)\right) - 4\cosh^{-1}(ax)^2\log\left(1-e^{-\cosh^{-1}(ax)}\right) + 4\cosh^{-1}(ax)^2\log\left(1+e^{-\cosh^{-1}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(ax)\right)\right) - 8\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(ax)}\right) + 8\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{-\cosh^{-1}(ax)}\right) - 8\text{PolyLog}\left(3,-e^{-\cosh^{-1}(ax)}\right) + 8\text{PolyLog}\left(3,e^{-\cosh^{-1}(ax)}\right) - \cosh^{-1}(ax)^2\text{sech}^2\left(\frac{1}{2}\cosh^{-1}(ax)\right) + 4\cosh^{-1}(ax)\tanh\left(\frac{1}{2}\cosh^{-1}(ax)\right)}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-4*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] - ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]
^2 - 4*ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] + 4*ArcCosh[a*x]^2*Log[1 +
 E^(-ArcCosh[a*x])] + 8*Log[Tanh[ArcCosh[a*x]/2]] - 8*ArcCosh[a*x]*PolyLog[
2, -E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] - 8*P
olyLog[3, -E^(-ArcCosh[a*x])] + 8*PolyLog[3, E^(-ArcCosh[a*x])] - ArcCosh[a
*x]^2*Sech[ArcCosh[a*x]/2]^2 + 4*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(8*a*c^
2)
```

Maple [A]
time = 3.15, size = 255, normalized size = 1.56

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccosh}(ax)\left(ax\operatorname{arccosh}(ax)+2\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2(a^2x^2-1)c^2}-\frac{\operatorname{arccosh}(ax)^2\ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}-\frac{\operatorname{arccos}}{}}{}$
default	$\frac{-\frac{\operatorname{arccosh}(ax)\left(ax\operatorname{arccosh}(ax)+2\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2(a^2x^2-1)c^2}-\frac{\operatorname{arccosh}(ax)^2\ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}-\frac{\operatorname{arccos}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2/(a^2*x^2-1)*arccosh(a*x)*(a*x*arccosh(a*x)+2*(a*x-1)^(1/2)*(a*x+1
)^(1/2))/c^2-1/2/c^2*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-1
/c^2*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/c^2*polylog(
3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/2/c^2*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))+1/c^2*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1
)^(1/2))-1/c^2*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-2/c^2*arctanh(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c^2*x^2 - a*c^2) - integrate(-1/2
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4
- 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**2/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(a^2*c*x^2 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^2, x)
```

3.169 $\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=258

$$-\frac{x}{12c^3(1-a^2x^2)}+\frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}}-\frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh^{-1}(ax)^2}{4c^3(1-a^2x^2)^2}+\frac{3x\cosh^{-1}(a}{8c^3(1-a^2}$$

```
[Out] -1/12*x/c^3/(-a^2*x^2+1)+1/6*arccosh(a*x)/a/c^3/(a*x-1)^(3/2)/(a*x+1)^(3/2)
+1/4*x*arccosh(a*x)^2/c^3/(-a^2*x^2+1)^2+3/8*x*arccosh(a*x)^2/c^3/(-a^2*x^2
+1)+3/4*arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/6*a
rctanh(a*x)/a/c^3+3/4*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/
2))/a/c^3-3/4*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3
-3/4*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/4*polylog(3,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/4*arccosh(a*x)/a/c^3/(a*x-1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5901, 5903, 4267, 2611, 2320, 6724, 5915, 35, 213, 41, 205}

$$-\frac{x}{12c^3(1-a^2x^2)}+\frac{3x\cosh^{-1}(ax)^2}{8c^3(1-a^2x^2)}+\frac{x\cosh^{-1}(ax)^2}{4c^3(1-a^2x^2)^2}+\frac{3\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\frac{3\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)}{6ac^3(ax-1)^{3/2}(ax+1)^{3/2}}-\frac{5\tanh^{-1}(ax)}{6ac^3}+\frac{3\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/12*x/(c^3*(1 - a^2*x^2)) + ArcCosh[a*x]/(6*a*c^3*(-1 + a*x)^(3/2)*(1 + a
*x)^(3/2)) - (3*ArcCosh[a*x])/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*A
rcCosh[a*x]^2)/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^2)/(8*c^3*(1 - a
^2*x^2)) + (3*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^3) - (5*ArcTan
h[a*x])/(6*a*c^3) + (3*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(4*a*c^3)
- (3*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(4*a*c^3) - (3*PolyLog[3, -E^
ArcCosh[a*x]])/(4*a*c^3) + (3*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
```

```
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^3}\,dx = \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{2c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)^2}{8c^3\left(1-a^2x^2\right)} - \frac{\int \frac{1}{(-1+a^2x^2)^2}\,dx}{6c^3} + \cdots$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cos\cdots}{4c^3(1\cdots}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cos\cdots}{4c^3(1\cdots}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cos\cdots}{4c^3(1\cdots}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cos\cdots}{4c^3(1\cdots}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cos\cdots}{4c^3(1\cdots}$$

Mathematica [A]
time = 3.04, size = 319, normalized size = 1.24

$$-\frac{80\cosh^{-1}(ax)\coth\left(\frac{1}{2}\cosh^{-1}(ax)\right)+2\left(-2+9\cosh^{-1}(ax)^2\right)\operatorname{csch}^2\left(\frac{1}{2}\cosh^{-1}(ax)\right)-2\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\cosh^{-1}(ax)\operatorname{csch}^4\left(\frac{1}{2}\cosh^{-1}(ax)\right)-3\cosh^{-1}(ax)^2\operatorname{csch}^4\left(\frac{1}{2}\cosh^{-1}(ax)\right)-160\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(ax)\right)\right)+72\left(\cosh^{-1}(ax)^2\log\left(1-e^{-\cosh^{-1}(ax)}\right)-\cosh^{-1}(ax)^2\log\left(1+e^{-\cosh^{-1}(ax)}\right)+2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{-\cosh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{-\cosh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,e^{-\cosh^{-1}(ax)}\right)\right)+2\left(-2+9\cosh^{-1}(ax)^2\right)\operatorname{sech}^2\left(\frac{1}{2}\cosh^{-1}(ax)\right)+3\cosh^{-1}(ax)^2\operatorname{sech}^4\left(\frac{1}{2}\cosh^{-1}(ax)\right)-\frac{32\cosh^{-1}(ax)\sinh^4\left(\frac{1}{2}\cosh^{-1}(ax)\right)}{\left(\frac{-1+ax}{1+ax}\right)^{3/2}(1+ax)^3}-80\cosh^{-1}(ax)\tanh\left(\frac{1}{2}\cosh^{-1}(ax)\right)}{192ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/192*(80*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] + 2*(-2 + 9*ArcCosh[a*x]^2)*Cs
ch[ArcCosh[a*x]/2]^2 - 2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]*
Csch[ArcCosh[a*x]/2]^4 - 3*ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]^4 - 160*Log[
Tanh[ArcCosh[a*x]/2]] + 72*(ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] - Arc
Cosh[a*x]^2*Log[1 + E^(-ArcCosh[a*x])] + 2*ArcCosh[a*x]*PolyLog[2, -E^(-Arc
Cosh[a*x])] - 2*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] + 2*PolyLog[3, -
E^(-ArcCosh[a*x])] - 2*PolyLog[3, E^(-ArcCosh[a*x])]) + 2*(-2 + 9*ArcCosh[a
*x]^2)*Sech[ArcCosh[a*x]/2]^2 + 3*ArcCosh[a*x]^2*Sech[ArcCosh[a*x]/2]^4 - (
32*ArcCosh[a*x]*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 + a*x))^(3/2)*(1 +
a*x)^3) - 80*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(a*c^3)
```

Maple [A]
time = 3.49, size = 320, normalized size = 1.24

method	result
derivativedivides	$-\frac{18\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,x^2a^2+9\operatorname{arccosh}(ax)^2x^3a^3-22\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}-15\operatorname{arccosh}(a}{24\left(a^4x^4-2a^2x^2+1\right)c^3}$
default	$-\frac{18\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,x^2a^2+9\operatorname{arccosh}(ax)^2x^3a^3-22\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}-15\operatorname{arccosh}(a}{24\left(a^4x^4-2a^2x^2+1\right)c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/24*(18*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*x^2*a^2+9*arccosh(a
*x)^2*x^3*a^3-22*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-15*arccosh(a*x)^2
*a*x-2*a^3*x^3+2*a*x)/(a^4*x^4-2*a^2*x^2+1)/c^3-5/3/c^3*arctanh(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))-3/8/c^3*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)
^(1/2))-3/4/c^3*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/4
/c^3*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/8/c^3*arccosh(a*x)^2*ln(1
+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/4/c^3*arccosh(a*x)*polylog(2,-a*x-(a*x-
1)^(1/2)*(a*x+1)^(1/2))-3/4/c^3*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
)^2/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-1/8*(6*a^5*x^5 - 16*
a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x +
 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqr
t(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^6*c
^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^2}{{\left(c-a^2\,c\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^3, x)
```

3.170 $\int x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=371

$$-\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4}+\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}+\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4b^2x\sqrt{d-c^2}}{15c^3\sqrt{-1+}}$$

```
[Out] -856/3375*b^2*(-c^2*d*x^2+d)^(1/2)/c^4+22/3375*b^2*x^2*(-c^2*d*x^2+d)^(1/2)
/c^2+2/125*b^2*x^4*(-c^2*d*x^2+d)^(1/2)-2/15*(a+b*arccosh(c*x))^2*(-c^2*d*x
^2+d)^(1/2)/c^4-1/15*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+1/5*
x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(-c^2*d*x^2+d)^(1/
2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/15*b^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(
1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/45*b*x^3*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c*x^5*(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5926, 5939, 5915, 5879, 75, 5883, 102, 12}

$$-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{15c^2}-\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{15c^4}+\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}-\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4}+\frac{4b^2x\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{15c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-856*b^2*Sqrt[d - c^2*d*x^2])/(3375*c^4) + (22*b^2*x^2*Sqrt[d - c^2*d*x^2]
)/(3375*c^2) + (2*b^2*x^4*Sqrt[d - c^2*d*x^2])/125 + (4*a*b*x*Sqrt[d - c^2*
d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*x*Sqrt[d - c^2*d*x^2
]*ArcCosh[c*x])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*x^3*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*
c*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4) - (x^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
```

```
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\,c}{\sqrt{-1+c}}}{5\sqrt{-1+cx}\sqrt{1}}$$

$$=-\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\right.}{15c^2}$$

$$=\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}+\frac{2bx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45c\sqrt{-1+cx}\sqrt{1+cx}}-$$

$$=-\frac{2b^2x^2\sqrt{d-c^2dx^2}}{135c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}+\frac{4abx\sqrt{d-c}}{15c^3\sqrt{-1+cx}}$$

$$=\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}+\frac{4abx\sqrt{d-c^2}}{15c^3\sqrt{-1+cx}}$$

$$=-\frac{8b^2\sqrt{d-c^2dx^2}}{27c^4}+\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2dx^2}$$

$$=-\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4}+\frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2}+\frac{2}{125}b^2x^4\sqrt{d-c^2}$$

Mathematica [A]

time = 0.27, size = 237, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(225a^2(-1+c^2x^2)^2(2+3c^2x^2)-30abcx\sqrt{-1+cx}\sqrt{1+cx}(-30-5c^2x^2+9c^4x^4)+2b^2(428-439c^2x^2-16c^4x^4+27c^6x^6)+30b\left(15a(-1+c^2x^2)^2(2+3c^2x^2)+bcx\sqrt{-1+cx}\sqrt{1+cx}(30+5c^2x^2-9c^4x^4)\right)\cosh^{-1}(cx)+225b^2(-1+c^2x^2)^2(2+3c^2x^2)\cosh^{-1}(cx)^2\right)}{3375c^4(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) - 30*a*b*c*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(428 -
439*c^2*x^2 - 16*c^4*x^4 + 27*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^2*(2 + 3
*c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(30 + 5*c^2*x^2 - 9*c^4*x^4)
)*ArcCosh[c*x] + 225*b^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2)*ArcCosh[c*x]^2))/
(3375*c^4*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(315) = 630$.
time = 2.65, size = 1284, normalized size = 3.46

method	result	size
default	Expression too large to display	1284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b
^2*(1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/(c*x
+1)/c^4/(c*x-1)+1/864*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccos
h(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+
2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+
1/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arcc
osh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c
*x)^2+10*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/800*(-d*(c^2*x^2-1))
^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2
*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*
```

```
x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))
/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/80
0*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1))
```

Maxima [A]
time = 0.50, size = 326, normalized size = 0.88

$$-\frac{1}{15}\,b^2\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)^2-\frac{2}{15}\,ab\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)-\frac{1}{15}\,a^2\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)+\frac{2}{3375}\,b^2\left(\frac{27\,\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,x^4+11\,\sqrt{c^2x^2-1}\,\sqrt{-d}\,x^2-\frac{428\,\sqrt{c^2x^2-1}\,\sqrt{-d}}{c^2}}{c^2}-\frac{15\left(9\,c^4\sqrt{-d}\,x^5-5\,c^2\sqrt{-d}\,x^3-30\,\sqrt{-d}\,x\right)\operatorname{arcosh}(cx)}{c^3}\right)-\frac{2\left(9\,c^4\sqrt{-d}\,x^5-5\,c^2\sqrt{-d}\,x^3-30\,\sqrt{-d}\,x\right)ab}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/15*b^2*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/
(c^4*d))*arccosh(c*x)^2 - 2/15*a*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) +
2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arccosh(c*x) - 1/15*a^2*(3*(-c^2*d*x^2 +
d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*
sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 11*sqrt(c^2*x^2 - 1)*sqrt(-d)*x^2 - 42
8*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/c^2 - 15*(9*c^4*sqrt(-d)*x^5 - 5*c^2*sqrt
(-d)*x^3 - 30*sqrt(-d)*x)*arccosh(c*x)/c^3) - 2/225*(9*c^4*sqrt(-d)*x^5 - 5
*c^2*sqrt(-d)*x^3 - 30*sqrt(-d)*x)*a*b/c^3
```

Fricas [A]
time = 0.40, size = 349, normalized size = 0.94

$$\frac{225\,(3\,b^2c^6x^6-4\,b^2c^4x^4-b^2c^2x^2+2\,b^2)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\,(9\,abc^5x^5-5\,abc^3x^3-30\,abcx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-30\left((9\,b^2c^5x^5-5\,b^2c^3x^3-30\,b^2cx)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-15\,(3\,abc^6x^6-4\,abc^4x^4-abc^2x^2+2\,ab)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+(27\,(25\,a^2+2\,b^2)c^6x^6-4\,(225\,a^2+8\,b^2)c^4x^4-(225\,a^2+878\,b^2)c^2x^2+450\,a^2+856\,b^2)\sqrt{-c^2dx^2+d}}{3375\,(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^2 + 2*b^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 - 5*a*b*c^3*
x^3 - 30*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^5*x
^5 - 5*b^2*c^3*x^3 - 30*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 1
5*(3*a*b*c^6*x^6 - 4*a*b*c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*sqrt(-c^2*d*x^2 + d
))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 - 4*(225*a^2
 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b^2)*c^2*x^2 + 450*a^2 + 856*b^2)*sqrt(-
c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

3.171 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=319

$$-\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d-c^2dx^2}-\frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}}-$$

```
[Out] -1/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)-1/8*
x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arccosh(c*x))^
2*(-c^2*d*x^2+d)^(1/2)-1/64*b^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+1/8*b*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/24*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(
1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5926, 5939, 5893, 5883, 92, 54, 102, 12}

$$\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{8c^2}-\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{24bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d-c^2dx^2}-\frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/64*(b^2*x*Sqrt[d - c^2*d*x^2])/c^2 + (b^2*x^3*Sqrt[d - c^2*d*x^2])/32 -
(b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) + (b*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*c*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8
*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x])^3)/(24*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\,c}{\sqrt{-1+c}}}{4\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{8c^2}$$

$$= \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc}{}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{16c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a\right.}{8c\sqrt{-1+cx}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh}{16c^3\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh}{64c^3\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A]
time = 1.35, size = 241, normalized size = 0.76

$$-\frac{-96a^2cx(-1+2c^2x^2)\sqrt{d-c^2dx^2}+96a^2\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{12ab\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh(4\cosh^{-1}(cx))-4\cosh^{-1}(cx)\sinh(4\cosh^{-1}(cx))\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}+\frac{b^2\sqrt{d-c^2dx^2}\left(32\cosh^{-1}(cx)^3+12\cosh^{-1}(cx)\cosh(4\cosh^{-1}(cx))-3(1+8\cosh^{-1}(cx)^2)\sinh(4\cosh^{-1}(cx))\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}}{768c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/768*(-96*a^2*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 96*a^2*Sqrt[d]*A
rcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (12*a*b*Sqrt[d
- c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh
[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c
^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1
+ 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)))/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(271) = 542$.
time = 3.04, size = 678, normalized size = 2.13

method	result
default	$-\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}+\frac{a^2x\sqrt{-c^2dx^2+d}}{8c^2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\text{arccosh}}{24\sqrt{cx-1}\,\sqrt{cx+1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/
8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*
(-1/24*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^
3+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)/(c*x+1)/c^3/(c*x-1)+1/512
*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)/(c*x+1)/c^3/(c*x-1))+2*a*b*(-1/
16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2+1/
256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1
/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh
(c*x))/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/8*a^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) +
sqrt(d)*arcsin(c*x)/c^3) + integrate(sqrt(-c^2*d*x^2 + d)*b^2*x^2*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2*d*x^2 + d)*a*b*x^2*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

3.172 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=186

$$-\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2}+\frac{2}{27}b^2x^2\sqrt{d-c^2dx^2}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{9\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/c^2/d-14/27*b^2*(-c^2*d*x^2+
d)^(1/2)/c^2+2/27*b^2*x^2*(-c^2*d*x^2+d)^(1/2)+2/3*b*x*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/9*b*c*x^3*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {5914, 5889, 5894, 12, 471, 75}

$$\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{2}{27}b^2x^2\sqrt{d-c^2dx^2}-\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-14*b^2*Sqrt[d - c^2*d*x^2])/(27*c^2) + (2*b^2*x^2*Sqrt[d - c^2*d*x^2])/27
 + (2*b*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (2*b*c*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(
3*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
```

```
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2} - \frac{\left(2b\sqrt{d-c^2\cdots\right.}{\cdots}$$

$$= \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cos\cdots\right.}{9\sqrt{-1+cx}\,\sqrt{1+c\cdots}}$$

$$= \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cos\cdots\right.}{9\sqrt{-1+cx}\,\sqrt{1+c\cdots}}$$

$$= \frac{2}{27}b^2x^2\sqrt{d-c^2dx^2} + \frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcx\cdots}{\cdots}$$

$$= -\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2} + \frac{2}{27}b^2x^2\sqrt{d-c^2dx^2} + \frac{2bx\sqrt{d-c^2dx^2}\left(a+\cdots\right.}{3c\sqrt{-1+cx}\,\cdots}$$

Mathematica [A]
time = 0.21, size = 181, normalized size = 0.97

$$\frac{\sqrt{d-c^2dx^2}\left(-6abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(-3+c^2x^2)+9a^2(-1+c^2x^2)^2+2b^2(7-8c^2x^2+c^4x^4)+6b\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(3-c^2x^2)+3a(-1+c^2x^2)^2\right)\cosh^{-1}(cx)+9b^2(-1+c^2x^2)^2\cosh^{-1}(cx)^2\right)}{27c^2\left(-1+c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(-6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + c^2*x^2
) + 9*a^2*(-1 + c^2*x^2)^2 + 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCosh[c
*x] + 9*b^2*(-1 + c^2*x^2)^2*ArcCosh[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(158) = 316$.
time = 1.92, size = 726, normalized size = 3.90

method	result
default	$-\frac{a^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\sqrt{cx+1}\,\sqrt{cx-1}\,x\cdots\right.}{216(cx+1)c^2(cx-1)}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/c^2/d*(-c^2*d*x^2+d)^(3/2)+b^2*(1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^
4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8
*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccos
h(c*x)^2-2*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+
2)/(c*x+1)/c^2/(c*x-1)+1/216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(
9*arccosh(c*x)^2+6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/72*(-d*(c^
2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-
3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1
)-1/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-
1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*
(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2
/(c*x-1))
```

Maxima [A]
time = 0.28, size = 204, normalized size = 1.10

$$\frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2-1}\,\sqrt{-d}\,dx^2-\frac{7\,\sqrt{c^2x^2-1}\,\sqrt{-d}\,d}{c^2}}{d}-\frac{3\,\left(c^2\sqrt{-d}\,dx^3-3\,\sqrt{-d}\,dx\right)\operatorname{arcosh}\left(cx\right)}{cd}\right)-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\operatorname{arcosh}\left(cx\right)^2}{3\,c^2d}-\frac{2\,\left(-c^2dx^2+d\right)^{\frac{3}{2}}ab\operatorname{arcosh}\left(cx\right)}{3\,c^2d}-\frac{2\,\left(c^2\sqrt{-d}\,dx^3-3\,\sqrt{-d}\,dx\right)ab}{9\,cd}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}a^2}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 - 7*sqrt(c^2*x^2 - 1)*sqrt(-d)*
d/c^2)/d - 3*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*arccosh(c*x)/(c*d)) - 1/
3*(-c^2*d*x^2 + d)^(3/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 + d)^
(3/2)*a*b*arccosh(c*x)/(c^2*d) - 2/9*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*
a*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Fricas [A]
time = 0.38, size = 280, normalized size = 1.51

$$\frac{9\,\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\,\left(abc^3x^3-3\,abcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-6\,\left(\left(b^2c^3x^3-3\,b^2cx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-3\,\left(abc^4x^4-2\,abc^2x^2+ab\right)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(\left(9\,a^2+2\,b^2\right)c^4x^4-2\,\left(9\,a^2+8\,b^2\right)c^2x^2+9\,a^2+14\,b^2\right)\sqrt{-c^2dx^2+d}}{27\,\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 - 3*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqr
t(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 - 3*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2
```

```
*x^2 - 1) - 3*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(-c^2*d*x^2 + d))*log
(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - 2*(9*a^2 + 8*b^2)*c^
2*x^2 + 9*a^2 + 14*b^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

3.173 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$\frac{1}{4}b^2x\sqrt{d-c^2dx^2}+\frac{b^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{4c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\,(a+b$$

```
[Out] 1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1
/2)+1/4*b^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-1/2*b*c*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/6*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5895, 5893, 5883, 92, 54}

$$-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6bc\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{4}b^2x\sqrt{d-c^2dx^2}+\frac{b^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{4c\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d - c^2*d*x^2])/4 + (b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(4*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\sqrt{d-c^2dx^2}\,\int \sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{2}x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2-\frac{\sqrt{d-c^2dx^2}\,\int \frac{(a+b\cosh^{-1}(}{\sqrt{-1+cx}\,\sqrt{}}}{2\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$=-\frac{bcx^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\,\left(a+b\cosh\right.$$

$$=\frac{1}{4}b^2x\sqrt{d-c^2dx^2}\,-\frac{bcx^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{2}x\sqrt{d}$$

$$=\frac{1}{4}b^2x\sqrt{d-c^2dx^2}\,+\frac{b^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{4c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{2\sqrt{-1+c}}$$

Mathematica [A]

time = 0.71, size = 235, normalized size = 1.15

$$\frac{1}{24}\left(12a^2x\sqrt{d-c^2dx^2}-\frac{12a^2\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{c}-\frac{6ab\sqrt{d-c^2dx^2}\,\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{c\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}+\frac{b^2\sqrt{d-c^2dx^2}\,\left(-4\cosh^{-1}(cx)^3-6\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)+\left(3+6\cosh^{-1}(cx)^2\right)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{c\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d - c^2*d*x^2] - (12*a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/c - (6*a*b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[
c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c^2*d*x^2]*(-4*ArcCosh[
c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*ArcCosh[c*x]^2)*Sinh[
2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(172) = 344$.
time = 1.97, size = 527, normalized size = 2.58

method	result
default	$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\mathrm{arccosh}(cx)^3}{6\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{\sqrt{-d\,(c^2x}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/c*arccosh(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh
(c*x)^2-2*arccosh(c*x)+1)/(c*x+1)/(c*x-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-
2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)/(c*x+1)/(c*x-1)/c)+2*a*b*(-1/4*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2+1/16*(-d
*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-
(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))/(c*x+1)/(c*x-1)/c+1/16*(-d
*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-
1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))/(c*x+1)/(c*x-1)/c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + integrate(sqrt(-
c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2*d
*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

3.174 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=402

$$2b^2\sqrt{d-c^2dx^2}-\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}+\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-$$

```
[Out] 2*b^2*(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*
c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c*x*arccosh(c*x)
*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*(a+b*arccosh(c*x))^2*ar
ctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*(a+b*arcco
sh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*polylog
(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5926, 5947, 4265, 2611, 2320, 6724, 5879, 75}

$$-\frac{2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ib\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2ib^2\sqrt{d-c^2dx^2}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ib^2\sqrt{d-c^2dx^2}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+2b^2\sqrt{d-c^2dx^2}-\frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d - c^2*d*x^2] - (2*a*b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (2*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((2*I)*b*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLo
g[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^Arc
Cosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*Sqrt[d - c^2*d*x^2]
*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\cosh^{-1}(c\cdots}{x\sqrt{-1+cx}\sqrt{\cdots}}}{\sqrt{-1+cx}\sqrt{1+cx}}\cdots$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d}\cdots}{\cdots}$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-\cdots}$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh\cdots}{\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh\cdots}{\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh\cdots}{\sqrt{-1+cx}\sqrt{1+\cdots}}$$

Mathematica [A]
time = 0.84, size = 449, normalized size = 1.12

$$a^2\sqrt{d-c^2dx^2} + a^2\sqrt{d}\log(cx) - a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{2ab\sqrt{d-c^2dx^2}\left(-cx+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)+i\cosh^{-1}(cx)\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-i\cosh^{-1}(cx)\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)} + b^2\sqrt{d-c^2dx^2}\left(2+\frac{2cx\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)}{1-cx}+\cosh^{-1}(cx)^2+\frac{i\left(\cosh^{-1}(cx)^2\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\cosh^{-1}(cx)^2\log\left(1+ie^{-\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-2\cosh^{-1}(cx)\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)+2\text{PolyLog}\left(3,-ie^{-\cosh^{-1}(cx)}\right)-2\text{PolyLog}\left(3,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] +
I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCo
sh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x
]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*Sqrt[d - c^2*d*x^2]*(2 +
(2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[c*x]^2
+ (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E
^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh
[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*
PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^2\,\sqrt{-c^2 d\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] -(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c
^2*d*x^2 + d))*a^2 + integrate(sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/x + 2*sqrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x, x)
```

3.175 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2}{}$$

```
[Out] -(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x+c*(a+b*arccosh(c*x))^2*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/3*c*(a+b*arccosh(c*x))^3*(-c^2*
d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c*(a+b*arccosh(c*x))*ln(1+
1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-b^2*c*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5924, 5882, 3799, 2221, 2317, 2438, 5893}

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}+\frac{2bc\sqrt{d-c^2dx^2}\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b^2c\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x) + (c*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*Arc
Cosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{\left(2bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-}}{x}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{3b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [**A**]
time = 1.08, size = 270, normalized size = 1.15

$$-\frac{a^2\sqrt{d-c^2dx^2}}{x}+a^2c\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+abc\sqrt{d-c^2dx^2}\left(-\frac{2\cosh^{-1}(cx)}{cx}+\frac{\cosh^{-1}(cx)^2+2\log(cx)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)+\frac{1}{3}b^2c\sqrt{d-c^2dx^2}\left(\cosh^{-1}(cx)\left(-\frac{3\cosh^{-1}(cx)}{cx}+\frac{\cosh^{-1}(cx)\left(3+\cosh^{-1}(cx)\right)+6\log\left(1+e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}\right)+\frac{3\sqrt{\frac{-1+cx}{1+cx}}\,\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{1-cx}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] + a*b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x
])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))) + (b^2*c*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*((-3*ArcCosh[c*x])/(c*x) +
 (ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2,
 -E^(-2*ArcCosh[c*x])])/(1 - c*x)))/3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(232) = 464.
time = 3.09, size = 582, normalized size = 2.49

method	result
default	$-\frac{a^2(-c^2 d x^2+d)^{\frac{3}{2}}}{dx} - a^2 c^2 x\sqrt{-c^2 d x^2+d} - \frac{a^2 c^2 d \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{\sqrt{c^2 d}} + \frac{b^2 \sqrt{-d\,(c^2 x^2-1)}\, \text{arccosh}}{3\sqrt{cx-1}\,\sqrt{cx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c-b^2*(-d*(c^2*x^2-1)
)^(1/2)*arccosh(c*x)^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-b^2*(-d*(c^2*x^2-1))^(
1/2)*arccosh(c*x)^2/(c*x+1)/(c*x-1)*x*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/(c*x+1)/(c*x-1)/x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)^2*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)*x*
c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)/x+2*a*b*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] -(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(-c^2
*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*sqrt(-c^2*
d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}{(cx)})^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**2,x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}(c\,x)\right)}^2\,\sqrt{d-c^2\,d\,x^2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2,x)
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

3.176 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=427

$$-\frac{bc\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2x^2}+\frac{c^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{\sqrt{-1+cx}\,\sqrt{1+c}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+c^2*(a+b*arccosh(c*x))^2*
arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b^2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+I*b*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b^2*c^2*polylog(3,-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5924, 5883, 94, 211, 5947, 4265, 2611, 2320, 6724}

$$\frac{c^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))^2}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2x^2}+\frac{b^2c^2\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ib^2c^2\sqrt{d-c^2dx^2}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ib^2c^2\sqrt{d-c^2dx^2}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (c^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b^2*c^2*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*c^2*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (I*b*c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E
^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b^2*c^2*Sqrt[d - c^2*d*
x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b
^2*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
```

```
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-}}{x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{2x^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 977 vs. $2(427) = 854$.
time = 75.14, size = 977, normalized size = 2.29

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(a^2*Sqrt[d - c^2*d*x^2])/x^2 - (a^2*c^2*Sqrt[d]*Log[x])/2 + (a^2*c^2*
Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/2 + (a*b*d*(1 + c*x)*(c*x*Sqr
t[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[
(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2
*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqr
t[(-1 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*
x^2]) + (b^2*((-2*c^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/((-1 + c*x)^(3/2)*S
qrt[1 + c*x]) + (2*c*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*
Sqrt[1 + c*x]) + (Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(x^2*(-1 + c*x)) + (c
*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(x - c*x^2) - (I*c^2*Sqrt[(-1 + c*x)/(
1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]])/(-1
 + c*x) + (I*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
]^2*Log[1 + I/E^ArcCosh[c*x]])/(-1 + c*x) + (2*c^2*Sqrt[d]*Log[x])/(-1 + c*
x) - (2*c^3*Sqrt[d]*x*Log[x])/(-1 + c*x) - (2*c^2*Sqrt[d]*Log[Sqrt[d] + Sqr
t[d - c^2*d*x^2]])/(-1 + c*x) + (2*c^3*Sqrt[d]*x*Log[Sqrt[d] + Sqrt[d - c^2
*d*x^2]])/(-1 + c*x) + ((2*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d
*x^2]*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(1 - c*x) + ((2*I)*c^2*
Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*PolyLog[2, I/E^
ArcCosh[c*x]])/(-1 + c*x) + ((2*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d -
c^2*d*x^2]*PolyLog[3, (-I)/E^ArcCosh[c*x]])/(1 - c*x) + ((2*I)*c^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*PolyLog[3, I/E^ArcCosh[c*x]])/(-1 +
c*x))/2
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2} \sqrt{-c^{2} d\,x^{2}+d}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(
sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 + 2*s
qrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^2\,\sqrt{d - c^2\,d\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3,x)
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

3.177 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=336

$$\frac{b^2c^2\sqrt{d-c^2dx^2}}{3x}-\frac{b^2c^3\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{c^3\sqrt{d-c^2d}}{3\sqrt{-}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/d/x^3+1/3*b^2*c^2*(-c^2*d*x^
2+d)^(1/2)/x-1/3*b^2*c^3*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/3*b*c*(-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*c^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/3*b*c^3*(a+b*arccosh(c*x))*ln(1+1/(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/3*b^2*c^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5917, 5912, 5920, 99, 12, 54, 5882, 3799, 2221, 2317, 2438}

$$-\frac{bc(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}{3dx^3}-\frac{c^3\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^3\sqrt{d-c^2dx^2}\,\log\left(e^{-2\cosh^{-1}(cx)}+1\right)(a+b\cosh^{-1}(cx))}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b^2c^2\sqrt{d-c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b^2c^3\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*ArcCosh[
c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c^3*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*d*x^3) - (2*b*c^3*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b^2*c^3*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
```

```
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bc\sqrt{d-c^2\right.}{3\ldots}$$

$$= -\frac{bc(1-c^2x^2)\,\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\ldots}{}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc(1-c^2x^2)\,\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \ldots$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc(1-c^2x^2)\,\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \ldots$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc(1-c^2x^2)\,\sqrt{d\ldots}}{3x^2\sqrt{\ldots}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc(1-c^2x^2)\,\sqrt{d\ldots}}{3x^2\sqrt{\ldots}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc(1-c^2x^2)\,\sqrt{d\ldots}}{3x^2\sqrt{\ldots}}$$

Mathematica [A]
time = 0.70, size = 304, normalized size = 0.90

$$-\frac{d(1+cx)\left(a^2-a^2cx-a^2c^2x^2-b^2c^2x^2+a^2c^3x^3+b^2c^3x^3-abcx\sqrt{\frac{-1+cx}{1+cx}}-b^2\left(-1+cx+c^2x^2+c^3x^3\left(-1+\sqrt{\frac{-1+cx}{1+cx}}\right)\right)\cosh^{-1}(cx)^2-b\cosh^{-1}(cx)\left(bcx\sqrt{\frac{-1+cx}{1+cx}}-2a(-1+cx)^2(1+cx)+2bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-2abc^3x^3\sqrt{\frac{-1+cx}{1+cx}}\log(cx)+b^2c^3x^3\sqrt{\frac{-1+cx}{1+cx}}\,\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)}{3x^3\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d*(1 + c*x)*(a^2 - a^2*c*x - a^2*c^2*x^2 - b^2*c^2*x^2 + a^2*c^3*x^3
+ b^2*c^3*x^3 - a*b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*(-1 + c*x + c^2*x^
2 + c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 - b*ArcCosh[c
*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*a*(-1 + c*x)^2*(1 + c*x) + 2*b*c^
3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x])]) - 2*a*b*c^3*
x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[c*x] + b^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 +
c*x)]*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/(x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(314) = 628$.
time = 4.48, size = 2633, normalized size = 7.84

method	result	size
default	Expression too large to display	2633

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)
*arccosh(c*x)*c^6+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c
*x+1)/(c*x-1)*arccosh(c*x)^2*c^8-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-
3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^2-3*b^2*(-d*(c^2*x^2-1))^(1
/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^6+1/3*a*b*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-a*b*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*c^5-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/
(c*x-1)*c^6-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^
(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*
x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^
4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+2/3*a*b*(-d*(c^2*x^2-1
))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-10/3*a*b*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*
x)*c^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*
x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(
c*x-1)*c^2-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(
1/2)/(c*x-1)^(1/2)*c^7+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x
^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^
4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+b^2*(-d*(c^2*
x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c
*x)*c^3+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*
c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+1/3*b^2*(-d*(c^
2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+
1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6+1/3*b^2*(-d*
(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)
^2-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*
ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-5/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8+10/3
*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arcco
sh(c*x)^2*c^4-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c-1/3*a^2
/d/x^3*(-c^2*d*x^2+d)^(3/2)-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x
```

```
^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+2*a*b*(-d*(c^2*x^2-1
))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8+2*a*b
*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^5-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1
)*x^3*c^6+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c^4-6*a*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccos
h(c*x)*c^6+20/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*
x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
4*x^4-3*c^2*x^2+1)*x^3*arccosh(c*x)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c
^4*x^4-3*c^2*x^2+1)*x*arccosh(c*x)*c^4+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
^3-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c
^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d
)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*
d/x^2)*a*b*c/d + 1/3*b^2*((c^2*sqrt(d)*x^2 - sqrt(d))*sqrt(c*x + 1)*sqrt(-c
*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 - 3*integrate(2/3*((c*
x + 1)*sqrt(c*x - 1)*c^2*sqrt(d)*x + (c^3*sqrt(d)*x^2 - c*sqrt(d))*sqrt(c*x
 + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c*x^4 + sqrt(
c*x + 1)*sqrt(c*x - 1)*x^3), x)) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*b*arccosh(c
*x)/(d*x^3) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**4,x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4,x)
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

3.178 $\int x^3(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=495

$$-\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4}+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}+\frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875}-\frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2}+\frac{4abd}{35c^3\sqrt{-}}$$

```
[Out] 1/7*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-37384/385875*b^2*d*(-c^2*
d*x^2+d)^(1/2)/c^4+3358/385875*b^2*d*x^2*(-c^2*d*x^2+d)^(1/2)/c^2+484/42875
*b^2*d*x^4*(-c^2*d*x^2+d)^(1/2)-2/343*b^2*c^2*d*x^6*(-c^2*d*x^2+d)^(1/2)-2/
35*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^4-1/35*d*x^2*(a+b*arccosh(
c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^
2+d)^(1/2)+4/35*a*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+4/35*b^2*d*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+2/105*b*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-16/175*b*c*d*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+2/49*b*c^3*d*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.78, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5930, 5926, 5939, 5915, 5879, 75, 5883, 102, 12, 5912, 14, 5921, 471}

$$-\frac{dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{35c^2}-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{3}{35}dx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{35c^4}+\frac{4abdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}-\frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2}+\frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875}-\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4}+\frac{4b^2dx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{35c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-37384*b^2*d*Sqrt[d - c^2*d*x^2])/(385875*c^4) + (3358*b^2*d*x^2*Sqrt[d -
c^2*d*x^2])/(385875*c^2) + (484*b^2*d*x^4*Sqrt[d - c^2*d*x^2])/42875 - (2*b
^2*c^2*d*x^6*Sqrt[d - c^2*d*x^2])/343 + (4*a*b*d*x*Sqrt[d - c^2*d*x^2])/(35
*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh
[c*x])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*x^3*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*b*c*d*x
^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(175*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (2*b*c^3*d*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
35*c^4) - (d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(35*c^2) + (3*
d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/35 + (x^4*(d - c^2*d*x^2)
^(3/2)*(a + b*ArcCosh[c*x])^2)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\ldots\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{7}dx^4(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{\left(3d\sqrt{\ldots\right.}{\ldots}$$

$$=-\frac{2bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}\ldots}{49\sqrt{-1+c\ldots}}$$

$$=-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bc^3dx^7\sqrt{d-c^2d\ldots}}{49\sqrt{-1+\ldots}}$$

$$=\frac{6}{875}b^2dx^4\sqrt{d-c^2dx^2}-\frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2}+\frac{2bdx^3\sqrt{d-\ldots}}{105c\ldots}$$

$$=-\frac{2b^2dx^2\sqrt{d-c^2dx^2}}{315c^2}+\frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875}-\frac{2}{343}b^2c^2dx^6\sqrt{\ldots}$$

$$=\frac{22b^2dx^2\sqrt{d-c^2dx^2}}{7875c^2}+\frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875}-\frac{2}{343}b^2c^2dx^6\sqrt{\ldots}$$

$$=-\frac{8b^2d\sqrt{d-c^2dx^2}}{63c^4}+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}+\frac{484b^2dx^4\sqrt{d-\ldots}}{42875}$$

$$=-\frac{856b^2d\sqrt{d-c^2dx^2}}{7875c^4}+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}+\frac{484b^2dx^4\sqrt{\ldots}}{428\ldots}$$

$$=-\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4}+\frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2}+\frac{484b^2dx^4\ldots}{4\ldots}$$

Mathematica [A]
time = 0.34, size = 262, normalized size = 0.53

$$-\frac{d\sqrt{d-c^2dx^2}\left(11025a^2(-1+c^2x^2)^3(2+5c^2x^2)-210abcx\sqrt{-1+cx}\sqrt{1+cx}(210+35c^2x^2-168c^4x^4+75c^6x^6)+2b^2(-18692+20371c^2x^2+499c^4x^4-3303c^6x^6+1125c^8x^8)-210b\left(-105a(-1+c^2x^2)^3(2+5c^2x^2)+bcx\sqrt{-1+cx}\sqrt{1+cx}(210+35c^2x^2-168c^4x^4+75c^6x^6)\right)\cosh^{-1}(cx)+11025b^2(-1+c^2x^2)^3(2+5c^2x^2)\cosh^{-1}(cx)^2\right)}{385875c^4(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/385875*(d*Sqrt[d - c^2*d*x^2]*(11025*a^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2
) - 210*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^
4 + 75*c^6*x^6) + 2*b^2*(-18692 + 20371*c^2*x^2 + 499*c^4*x^4 - 3303*c^6*x^
```

```
6 + 1125*c^8*x^8) - 210*b*(-105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) + b*c*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6))
*ArcCosh[c*x] + 11025*b^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCosh[c*x]^2))
/(c^4*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1951 vs. $2(423) = 846$.

time = 2.39, size = 1952, normalized size = 3.94

method	result	size
default	Expression too large to display	1952

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b
^2*(-1/43904*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/
16000*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)
/c^4/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(
c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)
+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c
*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2
+6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/16000*(-d*(c^2*x^2-1))^(1/2)*(-1
6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25
*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-1/43904*(-d*(c^2*x
^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arcco
sh(c*x)^2+14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1))+2*a*b*(-1/6272*(-d*(c^2
*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c
^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*ar
ccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6
-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccos
```

```
h(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2
*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^
4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2
)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d
*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-1/6272*(-d*(c^
2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*a
rccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

Maxima [A]
time = 0.49, size = 388, normalized size = 0.78

$$-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arcosh}\left(cx\right)^2-\frac{2}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arcosh}\left(cx\right)-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)a^2-\frac{2}{385875}b^2\left(\frac{1125\sqrt{c^2x^2-1}c^4\sqrt{-d}dx^6-2178\sqrt{c^2x^2-1}c^2\sqrt{-d}dx^4-1679\sqrt{c^2x^2-1}\sqrt{-d}dx^2+\frac{18692\sqrt{c^2x^2-1}\sqrt{-d}d}{c^2}}{c^2}-\frac{105\left(75c^6\sqrt{-d}dx^7-168c^4\sqrt{-d}dx^5+35c^2\sqrt{-d}dx^3+210\sqrt{-d}dx\right)\operatorname{arcosh}\left(cx\right)}{c^3}\right)+\frac{2\left(75c^6\sqrt{-d}dx^7-168c^4\sqrt{-d}dx^5+35c^2\sqrt{-d}dx^3+210\sqrt{-d}dx\right)ab}{3675c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4
*d))*b^2*arccosh(c*x)^2 - 2/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(
5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 - 2/385875*b^2*((1
125*sqrt(c^2*x^2 - 1)*c^4*sqrt(-d)*d*x^6 - 2178*sqrt(c^2*x^2 - 1)*c^2*sqrt(
-d)*d*x^4 - 1679*sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 + 18692*sqrt(c^2*x^2 - 1)
*sqrt(-d)*d/c^2)/c^2 - 105*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)*d*x^5
+ 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*arccosh(c*x)/c^3) + 2/3675*(75*
c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*s
qrt(-d)*d*x)*a*b/c^3
```

Fricas [A]
time = 0.43, size = 432, normalized size = 0.87

$$-\frac{11025\left(5b^2c^8dx^8-13b^2c^6dx^6+9b^2c^4dx^4+b^2c^2dx^2-2b^2d\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)^2-210\left(75abc^7dx^7-168abc^5dx^5+35abc^3dx^3+210abcdx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}-210\left(\left(75b^2c^7dx^7-168b^2c^5dx^5+35b^2c^3dx^3+210b^2cdx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}-105\left(5abc^8dx^8-13abc^6dx^6+9abc^4dx^4+abc^2dx^2-2abd\right)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(1125\left(49a^2+2b^2\right)c^8dx^8-9\left(15925a^2+734b^2\right)c^6dx^6+\left(99225a^2+998b^2\right)c^4dx^4+\left(11025a^2+40742b^2\right)c^2dx^2-2\left(11025a^2+18692b^2\right)d\right)\sqrt{-c^2dx^2+d}}{385875\left(c^6x^2-c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] -1/385875*(11025*(5*b^2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b^
2*c^2*d*x^2 - 2*b^2*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2
- 210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 + 210*a*b*c*
d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((75*b^2*c^7*d*x^7 - 168*
b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 + 210*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 105*(5*a*b*c^8*d*x^8 - 13*a*b*c^6*d*x^6 + 9*a*b*c^4*d*x^4 +
 a*b*c^2*d*x^2 - 2*a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)
) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 + 734*b^2)*c^6*d*x^6 +
(99225*a^2 + 998*b^2)*c^4*d*x^4 + (11025*a^2 + 40742*b^2)*c^2*d*x^2 - 2*(11
025*a^2 + 18692*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acosh}{(cx)})^{2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

3.179 $\int x^2(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=441

$$\frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}+\frac{7b^2d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{1152c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdx^2\sqrt{d}}{16}$$

```
[Out] 1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+7/1152*b^2*d*x*(-c^2*d*x^
2+d)^(1/2)/c^2+43/1728*b^2*d*x^3*(-c^2*d*x^2+d)^(1/2)-1/108*b^2*c^2*d*x^5*(
-c^2*d*x^2+d)^(1/2)-1/16*d*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+
1/8*d*x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arccosh(c*
x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*d*x^2*(a+b*a
rccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/48*b*c*d*
x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1
8*b*c^3*d*x^6*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-1/48*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.72, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5930, 5926, 5939, 5893, 5883, 92, 54, 102, 12, 5912, 14, 5921, 471}

$$\frac{bdx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c\sqrt{cx-1}\sqrt{cx+1}}-\frac{dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16c^2}-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x^3(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{48bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18\sqrt{cx-1}\sqrt{cx+1}}+\frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2}-\frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}+\frac{7b^2d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{1152c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (7*b^2*d*x*Sqrt[d - c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d - c^2*d*x
^2])/1728 - (b^2*c^2*d*x^5*Sqrt[d - c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d - c^2
*d*x^2]*ArcCosh[c*x])/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - (7*b*c*d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (b*c^3*d*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
18*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 +
(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/6 - (d*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^3)/(48*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
```

```
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{\left(d\sqrt{\right.}}{}$$

$$=-\frac{bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{12\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{18\sqrt{-1+c}}$$

$$=-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^6\sqrt{d-c^2dx}}{18\sqrt{-1+}}$$

$$=\frac{1}{64}b^2dx^3\sqrt{d-c^2dx^2}-\frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}+\frac{bdx^2\sqrt{d-}}{16c}$$

$$=-\frac{b^2dx\sqrt{d-c^2dx^2}}{32c^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

$$=-\frac{b^2dx\sqrt{d-c^2dx^2}}{128c^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

$$=\frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

$$=\frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2c^2dx^5\sqrt{d}$$

Mathematica [A]
time = 2.92, size = 485, normalized size = 1.10

$$\frac{-288a^2cdx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}(3-14c^2x^2+8c^4x^4)-864a^2d^{3/2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-216abd\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh(4\cosh^{-1}(cx))-4\cosh^{-1}(cx)\sinh(4\cosh^{-1}(cx))\right)-18b^2d\sqrt{d-c^2dx^2}\left(32\cosh^{-1}(cx)^3+12\cosh^{-1}(cx)\cosh(4\cosh^{-1}(cx))-3(1+8\cosh^{-1}(cx)^2)\sinh(4\cosh^{-1}(cx))\right)-12abd\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh(2\cosh^{-1}(cx))-9\cosh(4\cosh^{-1}(cx))-2\cosh(6\cosh^{-1}(cx))+12\cosh^{-1}(cx)(-3\sinh(2\cosh^{-1}(cx))+3\sinh(4\cosh^{-1}(cx))+\sinh(6\cosh^{-1}(cx)))\right)+b^2d\sqrt{d-c^2dx^2}\left(288\cosh^{-1}(cx)^3+12\cosh^{-1}(cx)(-18\cosh(2\cosh^{-1}(cx))+9\cosh(4\cosh^{-1}(cx))+2\cosh(6\cosh^{-1}(cx)))+108\sinh(2\cosh^{-1}(cx))-27\sinh(4\cosh^{-1}(cx))-4\sinh(6\cosh^{-1}(cx))-72\cosh^{-1}(cx)^2(-3\sinh(2\cosh^{-1}(cx))+3\sinh(4\cosh^{-1}(cx))+\sinh(6\cosh^{-1}(cx)))\right)}{13824c^3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-288*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(3
 - 14*c^2*x^2 + 8*c^4*x^4) - 864*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 216*a*b
*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh
[c*x]*Sinh[4*ArcCosh[c*x]]) - 18*b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]
^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4
*ArcCosh[c*x]]) - 12*a*b*d*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*Ar
cCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCo
sh[c*x]])) + b^2*d*Sqrt[d - c^2*d*x^2]*(288*ArcCosh[c*x]^3 + 12*ArcCosh[c*x
]*(-18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[c*x]] + 2*Cosh[6*ArcCosh[c*x
]]) + 108*Sinh[2*ArcCosh[c*x]] - 27*Sinh[4*ArcCosh[c*x]] - 4*Sinh[6*ArcCosh
[c*x]] - 72*ArcCosh[c*x]^2*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]
] + Sinh[6*ArcCosh[c*x]])))/(13824*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(377) = 754$.
time = 2.89, size = 1721, normalized size = 3.90

method	result	size
default	Expression too large to display	1721

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+1
/16*a^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a^2/c^2*d^2/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/48*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^3*d-1/6912*(-d*(c^2*x^2-1))^(1/2)
*(32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-4
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*arccosh(c*x)+1)*d
/(c*x+1)/c^3/(c*x-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)*d/(c*
x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1
```

```
/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*
arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x
)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)+1/1024*(-d*(c^2
*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(
8*arccosh(c*x)^2+4*arccosh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1)-1/6912*(-d*(c^2*x^
2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c
^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(18*arccosh(c*x)^2+6*arcco
sh(c*x)+1)*d/(c*x+1)/c^3/(c*x-1))+2*a*b*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(
32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1
)+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-
1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1
/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2
*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1
/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)+1/512*(-d
*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c
*x)*(1+4*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*
(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^
3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-
1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] 1/48*a^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*
d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + integrat
e((-c^2*d*x^2 + d)^(3/2)*b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 +
 2*(-c^2*d*x^2 + d)^(3/2)*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x
)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

3.180 $\int x(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=348

$$-\frac{16b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}}{75c^2(1-cx)(1+cx)}-\frac{8b^2d(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{225c^2(1-cx)(1+cx)}-\frac{2b^2d(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{125c^2(1-cx)(1+cx)}+\frac{2bdx\sqrt{d-c}}{5c\sqrt{-}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/c^2/d-16/75*b^2*d*(-c^2*x^2+
1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-8/225*b^2*d*(-c^2*x^2+1)^2*(-c
^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-2/125*b^2*d*(-c^2*x^2+1)^3*(-c^2*d*x
^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)+2/5*b*d*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/25*b*c^3*d*x^5*(a+b*arccosh
(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.296$,
Rules used = {5914, 5889, 200, 5894, 12, 534, 1261, 712}

$$\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2d}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{2b^2d(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{125c^2(1-cx)(cx+1)}-\frac{8b^2d(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{225c^2(1-cx)(cx+1)}-\frac{16b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}}{75c^2(1-cx)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-16*b^2*d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*c^2*(1 - c*x)*(1 + c*x))
- (8*b^2*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*c^2*(1 - c*x)*(1 + c*x
)) - (2*b^2*d*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*c^2*(1 - c*x)*(1 +
c*x)) + (2*b*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (4*b*c*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*x^5*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(5/
2)*(a + b*ArcCosh[c*x])^2)/(5*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2} + \frac{\left(2bd\right.}{}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{4bcdx^3\sqrt{d-c^2dx^2}\,(a}{15\sqrt{-1+cx}\,\surd}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{4bcdx^3\sqrt{d-c^2dx^2}\,(a}{15\sqrt{-1+cx}\,\surd}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{4bcdx^3\sqrt{d-c^2dx^2}\,(a}{15\sqrt{-1+cx}\,\surd}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{4bcdx^3\sqrt{d-c^2dx^2}\,(a}{15\sqrt{-1+cx}\,\surd}$$

$$= \frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{4bcdx^3\sqrt{d-c^2dx^2}\,(a}{15\sqrt{-1+cx}\,\surd}$$

$$= -\frac{16b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}}{75c^2(1-cx)(1+cx)} - \frac{8b^2d(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{225c^2(1-cx)(1+cx)}$$

Mathematica [A]
time = 0.30, size = 208, normalized size = 0.60

$$-\frac{d\sqrt{d-c^2dx^2}\left(225a^2(-1+c^2x^2)^3-30abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(15-10c^2x^2+3c^4x^4)+2b^2(-149+187c^2x^2-47c^4x^4+9c^6x^6)-30b\left(-15a(-1+c^2x^2)^3+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(15-10c^2x^2+3c^4x^4)\right)\cosh^{-1}(cx)+225b^2(-1+c^2x^2)^3\cosh^{-1}(cx)^2\right)}{1125c^2\,(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/1125*(d*Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(-149 + 187*c
^2*x^2 - 47*c^4*x^4 + 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCosh[c*x] + 225
*b^2*(-1 + c^2*x^2)^3*ArcCosh[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(308) = 616.
time = 1.14, size = 1270, normalized size = 3.65

method	result	size
default	Expression too large to display	1270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2/c^2/d*(-c^2*d*x^2+d)^(5/2)+b^2*(-1/4000*(-d*(c^2*x^2-1))^(1/2)*(16
*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*
arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1
)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d
*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c
*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*
x-1)-1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/
(c*x+1)/c^2/(c*x-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^
4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x)
)*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-
1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1
)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*
(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(c^2*x^2-1
))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^
2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

Maxima [A]
time = 0.29, size = 278, normalized size = 0.80

$$-\frac{(-c^2dx^2+d)^{\frac{5}{2}}b^2\operatorname{arcosh}(cx)^2}{5\,c^2d}-\frac{2}{1125}\,b^2\left(\frac{9\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,d^2x^4-38\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2x^2+\frac{149\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2}{c^2}}{d}-\frac{15\left(3\,c^4\sqrt{-d}\,d^2x^5-10\,c^2\sqrt{-d}\,d^2x^3+15\sqrt{-d}\,d^2x\right)\operatorname{arcosh}(cx)}{cd}\right)-\frac{2(-c^2dx^2+d)^{\frac{5}{2}}ab\operatorname{arcosh}(cx)}{5\,c^2d}-\frac{(-c^2dx^2+d)^{\frac{5}{2}}a^2}{5\,c^2d}+\frac{2\left(3\,c^4\sqrt{-d}\,d^2x^5-10\,c^2\sqrt{-d}\,d^2x^3+15\sqrt{-d}\,d^2x\right)ab}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/1125*b^2*((9*sqr
t(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 - 38*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2*x^2
 + 149*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/d - 15*(3*c^4*sqrt(-d)*d^2*x^5 -
 10*c^2*sqrt(-d)*d^2*x^3 + 15*sqrt(-d)*d^2*x)*arccosh(c*x)/(c*d)) - 2/5*(-c
^2*d*x^2 + d)^(5/2)*a*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a
^2/(c^2*d) + 2/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 15*sq
rt(-d)*d^2*x)*a*b/(c*d)
```

Fricas [**A**]
time = 0.39, size = 367, normalized size = 1.05

$$-\frac{225\left(b^{2}c^{6}dx^{6}-3\,b^{2}c^{4}dx^{4}+3\,b^{2}c^{2}dx^{2}-b^{2}d\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)^{2}-30\left(3\,abc^{5}dx^{5}-10\,abc^{3}dx^{3}+15\,abcdx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-30\left(\left(3\,b^{2}c^{5}dx^{5}-10\,b^{2}c^{3}dx^{3}+15\,b^{2}cdx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-15\left(abc^{6}dx^{6}-3\,abc^{4}dx^{4}+3\,abc^{2}dx^{2}-abd\right)\sqrt{-c^{2}dx^{2}+d}\right)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+\left(9\left(25\,a^{2}+2\,b^{2}\right)c^{6}dx^{6}-\left(675\,a^{2}+94\,b^{2}\right)c^{4}dx^{4}+\left(675\,a^{2}+374\,b^{2}\right)c^{2}dx^{2}-\left(225\,a^{2}+298\,b^{2}\right)d\right)\sqrt{-c^{2}dx^{2}+d}}{1125\left(c^{4}x^{2}-c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] -1/1125*(225*(b^2*c^6*d*x^6 - 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*sq
rt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(3*a*b*c^5*d*x^5 - 1
0*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30
*((3*b^2*c^5*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1) - 15*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*x^2 -
 a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (9*(25*a^2 + 2
*b^2)*c^6*d*x^6 - (675*a^2 + 94*b^2)*c^4*d*x^4 + (675*a^2 + 374*b^2)*c^2*d*
x^2 - (225*a^2 + 298*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

3.181 $\int \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=336

$$\frac{15}{64}b^2dx\sqrt{d-c^2dx^2}+\frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}+\frac{9b^2d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcdx^2\sqrt{d-c^2dx^2}}{8\sqrt{-1+c}}$$

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+15/64*b^2*d*x*(-c^2*d*x^2+d
)^(1/2)+1/32*b^2*d*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)+3/8*d*x*(a+b*arc
cosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+9/64*b^2*d*arccosh(c*x)*(-c^2*d*x^2+d)^(1
/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/8*b*c*d*x^2*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*d*(-c^2*x^2+1)^2*(a+b*arccos
h(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*d*(a+b*arcco
sh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5897, 5895, 5893, 5883, 92, 54, 5912, 5914, 38}

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{bd(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3bcdx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{15}{64}b^2dx\sqrt{d-c^2dx^2}+\frac{1}{32}b^2dx(1-cx)(cx+1)\sqrt{d-c^2dx^2}+\frac{9b^2d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d - c^2*d*x^2])/64 + (b^2*d*x*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2])/32 + (9*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - (3*b*c*d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 + (x*(d - c^2*d*x^2)^(3/2)*(a + b
*ArcCosh[c*x])^2)/4 - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(8*b*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(3d\sqrt{d\right.}}{}$$

$$= \frac{bd(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3}{8}dx\sqrt{d-c^2dx}$$

$$= \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3bcdx^2\sqrt{d-c^2dx^2}\,(a+}{8\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3}{}$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{9}{}$$

Mathematica [A]
time = 1.98, size = 374, normalized size = 1.11

$$\frac{-96a^2cdx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(-5+2c^2x^2)\sqrt{d-c^2dx^2}-288a^2d^{3/2}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-192abd\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)-32b^2d\sqrt{d-c^2dx^2}\left(4\cosh^{-1}(cx)^3+6\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)-3\left(1+2\cosh^{-1}(cx)^2\right)\sinh\left(2\cosh^{-1}(cx)\right)\right)+12abd\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)+b^2d\sqrt{d-c^2dx^2}\left(32\cosh^{-1}(cx)^3+12\cosh^{-1}(cx)\cosh\left(4\cosh^{-1}(cx)\right)-3\left(1+8\cosh^{-1}(cx)^2\right)\sinh\left(4\cosh^{-1}(cx)\right)\right)}{768c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-96*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-5 + 2*c^2*x^2)*Sqrt[d
 - c^2*d*x^2] - 288*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan
[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 192*a*b*d*Sqrt[d - c
^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*Arc
Cosh[c*x]])) - 32*b^2*d*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c
*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) +
 12*a*b*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[
c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Si
nh[4*ArcCosh[c*x]]))/(768*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(288) = 576$.
time = 2.59, size = 1061, normalized size = 3.16

method	result
default	$\frac{x(-c^2dx^2+d)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{-c^2dx^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\mathrm{arccc}}{8\sqrt{cx-1}\,\sqrt{cx+}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/8*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^3*d-1/512*(-d*
(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*
c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)*d/(c*x+1)/(c*x-1)/c+1/16*(-d*(c^2*x^2
-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(
1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1)*d/(c*x+1)/(c*x-1)/c
+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*
x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*
d/(c*x+1)/(c*x-1)/c-1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c
*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)*d/(c*x
+1)/(c*x-1)/c)+2*a*b*(-3/16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)/c*arccosh(c*x)^2*d-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d/(c*x+1)/(c*x-1)/c+
1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
```

```
x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d/(c*x+1)/(c*x-1)/
c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3
*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d/(c*x+1)/(c*x-1
)/c-1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*
c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x
+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x))*d/(c*x+1)/(c*x-1)/c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c\,x))^2\,\left(d - c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

3.182 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=573

$$\frac{68}{27}b^2d\sqrt{d-c^2dx^2}-\frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2}-\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2b^2cdx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdx}{}$$

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+68/27*b^2*d*(-c^2*d*x^2+d)^(1
/2)-2/27*b^2*c^2*d*x^2*(-c^2*d*x^2+d)^(1/2)+d*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)-2*a*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2
*b^2*c*d*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/
3*b*c*d*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+2/9*b*c^3*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-2*d*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*d*(a+b*arccosh(c*x
))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*d*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*
I*b^2*d*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5930, 5926, 5947, 4265, 2611, 2320, 6724, 5879, 75, 5889, 5894, 12, 471}

$$-\frac{2d\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bc^3dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib^2d\sqrt{d-c^2dx^2}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2ib^2d\sqrt{d-c^2dx^2}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2}+\frac{68}{27}b^2d\sqrt{d-c^2dx^2}-\frac{2b^2cdx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d*x^2*Sqrt[d - c^2*d*x^2])/2
7 - (2*a*b*c*d*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b
^2*c*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (2*b*c*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (2*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
 + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/3 - (2*d*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + ((2*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I
)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*d*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh
```

```
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*P
olyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(d\sqrt{d-c\right.}{}$$

$$= -\frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3dx^3\sqrt{d-c^2dx^2}\,(a}{9\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

$$= -\frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{d-}}{\sqrt{-1+}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2d}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2d}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2d}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [A]
time = 1.92, size = 650, normalized size = 1.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] -1/3*(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b^2*d*Sqrt[d - c^2*d*x^2
]*(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1 + Cosh[2*ArcCosh[c
*x]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c*x - Cosh[3*ArcCosh[
c*x]]))/(-1 + c*x)))/54 - (a*b*d*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x
)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*x^2]*(-(c*x) +
 Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*A
rcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log
[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I
/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*d*Sqrt[d -
c^2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) +
 ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*
x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c
*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^Ar
cCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] -1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (
-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a^2 + integrate((-c^2*d*x
^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d*x^
2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x, x)
```

3.183 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=453

$$-\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2}-\frac{5b^2cd\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd(1-c$$

```
[Out] -(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x-1/4*b^2*c^2*d*x*(-c^2*d*x^2+d)
^(1/2)-3/2*c^2*d*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*ar
ccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*b*c^3*d*x^2
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d*
(-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+c*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/2*c*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+2*b*c*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b^2*c*d*polylog(2,-1
/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5928, 5895, 5893, 5883, 92, 54, 5912, 5919, 5882, 3799, 2221, 2317, 2438, 38}

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{2b\sqrt{cx-1}\sqrt{cx+1}}+\frac{cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}+\frac{2bcd\sqrt{d-c^2dx^2}\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{b^2cd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2}-\frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -1/4*(b^2*c^2*d*x*Sqrt[d - c^2*d*x^2]) - (5*b^2*c*d*Sqrt[d - c^2*d*x^2]*Arc
Cosh[c*x])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d*x^2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*(1 -
c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (3*c^2*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 + (c*d*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x + (c*d*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^3)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b^2*c*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*Arc
Cosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
```

```
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{\left(2bcd\sqrt{d\right.}}{}$$

$$= \frac{bcd(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx}$$

$$= \frac{1}{2}b^2c^2dx\sqrt{d-c^2dx^2} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3dx^2}{}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3dx}{}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3dx}{}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3dx}{}$$

Mathematica [A]
time = 2.67, size = 433, normalized size = 0.96

$$\frac{-12a^2d\sqrt{\frac{-1+cx}{1+cx}}(1+cx)(2+c^2x^2)\sqrt{d-c^2dx^2}+36a^2cd^{3/2}x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-24abd\sqrt{d-c^2dx^2}\left(2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-cx\left(\cosh^{-1}(cx)^2+2\log(cx)\right)\right)-8b^2d\sqrt{d-c^2dx^2}\left(\cosh^{-1}(cx)\left(3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-cx\left(\cosh^{-1}(cx)\left(3+\cosh^{-1}(cx)\right)+6\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)\right)+3cx\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)+6abcdx\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)+b^2cdx\sqrt{d-c^2dx^2}\left(4\cosh^{-1}(cx)^3+6\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)-3\left(1+2\cosh^{-1}(cx)^2\right)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{24x\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (-12*a^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2)*Sqrt[d - c^2*
d*x^2] + 36*a^2*c*d^(3/2)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 24*a*b*d*Sqrt[d - c^2*d*
x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*
x]^2 + 2*Log[c*x])) - 8*b^2*d*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*(3*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]*(3 + ArcCosh[
c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[2, -E^(-2*ArcCosh[
c*x])]) + 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh
[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + b^2*c*d*x*Sqrt[d - c^2*d*x^2
]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCos
h[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/(24*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(421) = 842$.
time = 2.72, size = 942, normalized size = 2.08

method	result
default	$-\frac{a^{2}\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{dx}-a^{2} c^{2} x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}-\frac{3 a^{2} c^{2} d x\sqrt{-c^{2} d\,x^{2}+d}}{2}-\frac{3 a^{2} c^{2} d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{2\sqrt{c^{2} d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a^2*c^2*d*
x*(-c^2*d*x^2+d)^(1/2)-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)
*x^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*x-1/2*b^2*(-d*(c^
2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3+1/2*b^2*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c*d-1/2*b^2*(-d*(c^
2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x+b^2*(-d*(c^2*x^2-1))
^(1/2)*arccosh(c*x)^2*d/(c*x+1)/(c*x-1)/x-b^2*(-d*(c^2*x^2-1))^(1/2)*c*d/(c
*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c*d
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)+b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d+2
*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^3*
d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^2+3/2*a*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c*d-a*b*(-d*(c^2*x^2-1))^(1/2
)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*c^3
*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-a*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/
(c*x-1)*arccosh(c*x)*x-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*
x-1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d/(c*x+1)/(c*x-1)/x+2*
```

```
a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)*c*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] -1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*
x^2 + d)^(3/2)/x)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

3.184 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=630

$$-2b^2c^2d\sqrt{d-c^2dx^2}+\frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3b^2c^3dx\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}}{x\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2-2*b^2*c^2*d*(-c^2*d*x^2+
d)^(1/2)-3/2*c^2*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+3*a*b*c^3*d*x*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*b^2*c^3*d*x*arccosh(c*x)
*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d*(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^3*d*x*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*c^2*d*(a+b*arccosh(
c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+b^2*c^2*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b*c^2*d*(a+b*arccosh(c*x))*po
lylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+3*I*b*c^2*d*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*I
*b^2*c^2*d*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b^2*c^2*d*polylog(3,I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5928, 5926, 5947, 4265, 2611, 2320, 6724, 5879, 75, 5912, 14, 5921, 471, 94, 211}

$$\frac{3c^2d\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))^2}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2-\frac{bcd\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{x\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2}{2x^2}+\frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^3dx\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b^2c^2d\,\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ib^2c^2d\sqrt{d-c^2dx^2}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3ib^2c^2d\sqrt{d-c^2dx^2}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-2b^2c^2d\sqrt{d-c^2dx^2}+\frac{3b^2c^3dx\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -2*b^2*c^2*d*Sqrt[d - c^2*d*x^2] + (3*a*b*c^3*d*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*b^2*c^3*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh
[c*x])^2)/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*Arc
Tan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b^2*c^2*d*Sqrt[d - c
^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - ((3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-
```

```
I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((3*I)*b*c^2*d*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + ((3*I)*b^2*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*
E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((3*I)*b^2*c^2*d*Sqrt[d -
 c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
```

```
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} - \frac{\left(bcd\sqrt{d-\right.}{}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3dx\sqrt{d-c^2dx^2}\,(a+b}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd\sqrt{d-c^2dx^2}\,(a+}{x\sqrt{-1+cx}}$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2d}}{\sqrt{-1+cx}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2d}}{\sqrt{-1+cx}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2d}}{\sqrt{-1+cx}}$$

Mathematica [A]
time = 140.87, size = 1129, normalized size = 1.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-(a^2*c^2*d) - (a^2*d)/(2*x^2))*Sqrt[-(d*(-1 + c^2*x^2))] - (3*a^2*c^2*d^(
3/2)*Log[x])/2 + (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2)
)]])/2 - 2*a*b*c^2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^Ar
cCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)) + (I*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]])
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^2*(((-I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/
(c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^A
```

```
rcCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/
E^ArcCosh[c*x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^Ar
cCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c
*x]]))/Sqrt[-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d*Sqrt[d - c^2*d*x^2]*((4*c^2
)/(-1 + c*x) - (4*c^3*x)/(-1 + c*x) - (2*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2
)*Sqrt[1 + c*x]) + (2*c*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) -
(4*c^3*x*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (4*c^4*x^2*ArcCos
h[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (2*c^2*ArcCosh[c*x]^2)/(-1 + c*x
) + ArcCosh[c*x]^2/(x^2*(-1 + c*x)) - (2*c^3*x*ArcCosh[c*x]^2)/(-1 + c*x) +
 (c*ArcCosh[c*x]^2)/(x - c*x^2) + (2*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1
 + c*x)*Sqrt[-1 + c^2*x^2]) - (2*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 +
 c*x)*Sqrt[-1 + c^2*x^2]) - ((3*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c
*x]^2*Log[1 - I/E^ArcCosh[c*x]])/(-1 + c*x) + ((3*I)*c^2*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*
Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-
1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/
E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog
[3, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)
]*PolyLog[3, I/E^ArcCosh[c*x]])/(-1 + c*x)))/2
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 +
 d)^(5/2)/(d*x^2))*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))^2/x^3 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

3.185 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=426

$$\frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x}-\frac{b^2c^3d\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd(1-c^2x^2)\,\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2d\sqrt{d-}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3+1/3*b^2*c^2*d*(-c^2*d*x^
2+d)^(1/2)/x+c^2*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*
d*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*d*(
-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-4/3*c^3*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-1/3*c^3*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)
^(1/2)/(c*x+1)^(1/2)-8/3*b*c^3*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/3*b
^2*c^3*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5928, 5924, 5882, 3799, 2221, 2317, 2438, 5893, 5912, 5920, 99, 12, 54}

$$\frac{c^2d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{x}-\frac{bcd(1-c^2x^2)\,\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3}-\frac{c^3d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4c^3d\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{8bc^3d\sqrt{d-c^2dx^2}\,\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x}+\frac{4b^2c^3d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b^2c^3d\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*d*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*d*Sqrt[d - c^2*d*x^2]*ArcC
osh[c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x - (4*c^3*d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((d - c^2*
d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c^3*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*c^3*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(
-2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 5924

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*
x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] - Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 2)*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*
d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bcd\sqrt{d\right.}{}$$

$$= -\frac{bcd(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}}{}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd(1-c^2x}{3}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd(1-c^2x}{3}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd(1-c^2x}{3}$$

Mathematica [**A**]
time = 1.37, size = 583, normalized size = 1.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d^2*x) + a*b*c^2*d^2*x^2 - a^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 5*
a^2*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + b^2*c^2*d^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)] - 4*a^2*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*c^4*d^2*x
^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b*d^2*(-1 + c*x)*(-3*a*c^3*x^3 + b*(-Sqrt[(
-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*c^2*x^2*Sqrt[(-1
+ c*x)/(1 + c*x)] + 4*c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)])))*ArcCosh[c
*x]^2 + b^2*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 3*a^2*c^3*d^(3/2)*x^3*S
qrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d^2*(-1 + c*x)*ArcCosh[c*x]*(b*c*x + 2*a*
```

```
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x - 4*c^2*x^2 - 4*c^3*x^3) + 8*b*c^3*x^3*
Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*b*c^3*d^2*x^3*Log[c*x] + 8*a*b*c^4*d^2*
x^4*Log[c*x] - 4*b^2*c^3*d^2*x^3*(-1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/(3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. $2(394) = 788$.
time = 5.64, size = 2879, normalized size = 6.76

method	result	size
default	Expression too large to display	2879

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3*d+20/3*b^2*(-d*(c^2*x^2-1))^(1/
2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-29/3*b^2*(-d*(c^2*x^2
-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+10/3*b^2*(-d*
(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*b^2
*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*c^2+1/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)
*arccosh(c*x)^2-8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x
^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+3*b^2*(-d*(c
^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*c^5+3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*arccosh(c*x)*c^3-a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)^2*c^3*d+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3*d+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/
(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-20/3*a*b*(-d*(
c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6-8/3*a*
b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^3+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^
2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c
^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*
d/(24*c^4*x^4-9*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)+3*a*b*(-d*(c^2*
x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-8/
3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)*c^3*d+32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9
*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^8+16/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-52*
b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)^2*c^6-20/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1
```

```
)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+73/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(2
4*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^4+4/3*b^2*(-d*(c^
2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^4-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*
x-1)*arccosh(c*x)^2*c^2-32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x
^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7-8*b^2*(-d*(c^2*x^2
-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*c^5-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/
(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+8/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3*d-1/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c^3*d+4/3*b^2*
(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*arccosh(c*x)*c^4-16/3*b
^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*arccosh(c*x)*c^6-4
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3*d+a^2*c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*
d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+a^2*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+2/3*a^2*c
^2/d/x*(-c^2*d*x^2+d)^(5/2)-64*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c
^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+64*a*b*(-d*(c^2*
x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^8+24*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-104*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*
x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+146/3*a*b*(-d*(c^2*x^
2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-2
8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)
*arccosh(c*x)*c^2+12*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*
x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^5+4/3*b^2*(-d*(c^2*x^2-1))
^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*c^6-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(5/2
)+2/3*a^2*c^4*x*(-c^2*d*x^2+d)^(3/2)-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*
c^4*x^4-9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9
*c^2*x^2+1)*x*c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a^2 + integrate((-c
^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2*(-
c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

3.186 $\int x^3(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=880

$$-\frac{37384b^2d^2\sqrt{d-c^2dx^2}}{694575c^4}+\frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2}+\frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175}-\frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087}+\frac{4a}{63c^3}$$

```
[Out] 5/63*d*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/9*x^4*(-c^2*d*x^2+d)
^(5/2)*(a+b*arccosh(c*x))^2-37384/694575*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^4+3
358/694575*b^2*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c^2+484/77175*b^2*d^2*x^4*(-c^2
*d*x^2+d)^(1/2)-10/3087*b^2*c^2*d^2*x^6*(-c^2*d*x^2+d)^(1/2)+16/2835*b^2*d^
2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+8/8505*b^2*d^2*(-c
^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+2/4725*b^2*d^2*(-c^2*
x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)-20/3969*b^2*d^2*(-c^2*x^
2+1)^4*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+2/729*b^2*d^2*(-c^2*x^2+1)
^5*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)-2/63*d^2*(a+b*arccosh(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/c^4-1/63*d^2*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/c^2+1/21*d^2*x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+4/63*a*b*d^
2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/63*b^2*d^2*x*arc
cosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/189*b*d^2*
x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2
/21*b*c*d^2*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+38/441*b*c^3*d^2*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)-2/81*b*c^5*d^2*x^9*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.23, antiderivative size = 880, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {5930, 5926, 5939, 5915, 5879, 75, 5883, 102, 12, 5912, 14, 5921, 471, 276, 534, 1265, 911, 1167}

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-37384*b^2*d^2*Sqrt[d - c^2*d*x^2])/(694575*c^4) + (3358*b^2*d^2*x^2*Sqrt[
d - c^2*d*x^2])/(694575*c^2) + (484*b^2*d^2*x^4*Sqrt[d - c^2*d*x^2])/77175
- (10*b^2*c^2*d^2*x^6*Sqrt[d - c^2*d*x^2])/3087 + (4*a*b*d^2*x*Sqrt[d - c^2
*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (16*b^2*d^2*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2])/(2835*c^4*(1 - c*x)*(1 + c*x)) + (8*b^2*d^2*(1 - c^2*x
^2)^2*Sqrt[d - c^2*d*x^2])/(8505*c^4*(1 - c*x)*(1 + c*x)) + (2*b^2*d^2*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(4725*c^4*(1 - c*x)*(1 + c*x)) - (20*b^2*d
```

```
^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(3969*c^4*(1 - c*x)*(1 + c*x)) + (2
*b^2*d^2*(1 - c^2*x^2)^5*Sqrt[d - c^2*d*x^2])/(729*c^4*(1 - c*x)*(1 + c*x))
 + (4*b^2*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(63*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (2*b*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(189*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^5*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (38*b*c^3*d^2*x^7*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
2*b*c^5*d^2*x^9*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(81*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c
^4) - (d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c^2) + (d^2*
x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/21 + (5*d*x^4*(d - c^2*d*x^
2)^(3/2)*(a + b*ArcCosh[c*x])^2)/63 + (x^4*(d - c^2*d*x^2)^(5/2)*(a + b*Arc
Cosh[c*x])^2)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
```

```
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{1}{9}d^2x^4(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-$$

$$=-\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4bc^3d^2x^7\sqrt{d-c^2}}{63\sqrt{-1}}$$

$$=-\frac{8bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{38bc^3d^2x^7\sqrt{d-c}}{441\sqrt{-1}}$$

$$=-\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{38bc^3d^2x^7\sqrt{d-c}}{441\sqrt{-1}}$$

$$=\frac{2}{525}b^2d^2x^4\sqrt{d-c^2dx^2}-\frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087}+\frac{2bd^2x^3\sqrt{}}{18}$$

$$=-\frac{2b^2d^2x^2\sqrt{d-c^2dx^2}}{567c^2}+\frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175}-\frac{10b^2c^2d^2x}{}$$

$$=\frac{22b^2d^2x^2\sqrt{d-c^2dx^2}}{14175c^2}+\frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175}-\frac{10b^2c^2d^2x}{3}$$

$$=-\frac{40b^2d^2\sqrt{d-c^2dx^2}}{567c^4}+\frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2}+\frac{484b^2d^2x}{7}$$

$$=-\frac{856b^2d^2\sqrt{d-c^2dx^2}}{14175c^4}+\frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2}+\frac{484b^2d^2}{}$$

$$=-\frac{37384b^2d^2\sqrt{d-c^2dx^2}}{694575c^4}+\frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2}+\frac{484b^2}{}$$

Mathematica [A]
time = 0.42, size = 288, normalized size = 0.33

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3969a^2(-1+c^2x^2)^4(2+7c^2x^2)-126abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(-126-21c^2x^2+189c^4x^4-171c^6x^6+49c^8x^8)+2b^2(6140-7039c^2x^2-106c^4x^4+2152c^6x^6-1490c^8x^8+343c^{10}x^{10})+126b\left(63a(-1+c^2x^2)^4(2+7c^2x^2)+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(126+21c^2x^2-189c^4x^4+171c^6x^6-49c^8x^8)\right)\cosh^{-1}(cx)+3969b^2(-1+c^2x^2)^4(2+7c^2x^2)\cosh^{-1}(cx)^2\right)}{250047c^4(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*
c^6*x^6 + 49*c^8*x^8) + 2*b^2*(6140 - 7039*c^2*x^2 - 106*c^4*x^4 + 2152*c^6
*x^6 - 1490*c^8*x^8 + 343*c^10*x^10) + 126*b*(63*a*(-1 + c^2*x^2)^4*(2 + 7*
c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(126 + 21*c^2*x^2 - 189*c^4*x
^4 + 171*c^6*x^6 - 49*c^8*x^8))*ArcCosh[c*x] + 3969*b^2*(-1 + c^2*x^2)^4*(2
 + 7*c^2*x^2)*ArcCosh[c*x]^2))/(250047*c^4*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2223 vs. $2(776) = 1552$.
time = 2.39, size = 2224, normalized size = 2.53

method	result	size
default	Expression too large to display	2224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b
^2*(1/373248*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(81*a
rccosh(c*x)^2-18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/175616*(-d*(c^2*
x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^
7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arcco
sh(c*x)^2-14*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1)
)^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arcco
sh(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1))
^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/175616*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-
56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*d^2/(c*x+1)/c
^4/(c*x-1)+1/373248*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8
-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*x^6*c^6+120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-
1)*(81*arccosh(c*x)^2+18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/
```

```
41472*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^9*c^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-28
0*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccos
h(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-
144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^4
/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*
x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcco
sh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^
2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1)
)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c
*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*x^6*c^6+120*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

Maxima [A]
time = 0.50, size = 471, normalized size = 0.54

$$-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b^2\operatorname{arcosh}\left(cx\right)^2-\frac{2}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)ab\operatorname{arcosh}\left(cx\right)-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a^2+\frac{2}{250047}b^2\left(\frac{343\sqrt{c^2x^2-1}\,c^6\sqrt{-d}\,d^2x^8-1147\sqrt{c^2x^2-1}\,c^4\sqrt{-d}\,d^2x^6+1005\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,d^2x^4+899\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2x^2-\frac{6140\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2}{c^2}}{c^2}-\frac{63\left(49c^8\sqrt{-d}\,d^2x^9-171c^6\sqrt{-d}\,d^2x^7+189c^4\sqrt{-d}\,d^2x^5-21c^2\sqrt{-d}\,d^2x^3-126\sqrt{-d}\,d^2x\right)\operatorname{arcosh}\left(cx\right)}{c^3}\right)-\frac{2\left(49c^8\sqrt{-d}\,d^2x^9-171c^6\sqrt{-d}\,d^2x^7+189c^4\sqrt{-d}\,d^2x^5-21c^2\sqrt{-d}\,d^2x^3-126\sqrt{-d}\,d^2x\right)ab}{3969c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4
*d))*b^2*arccosh(c*x)^2 - 2/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(
7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((3
43*sqrt(c^2*x^2 - 1)*c^6*sqrt(-d)*d^2*x^8 - 1147*sqrt(c^2*x^2 - 1)*c^4*sqrt
(-d)*d^2*x^6 + 1005*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 + 899*sqrt(c^2*x
^2 - 1)*sqrt(-d)*d^2*x^2 - 6140*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/c^2 - 6
3*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c^4*sqrt(-d)*d^
2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*arccosh(c*x)/c^3) - 2
/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c^4*sqrt(-d
)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*a*b/c^3
```

Fricas [A]
time = 0.42, size = 558, normalized size = 0.63

$$\frac{3969\left(7b^2c^{10}d^2x^{10}-26b^2c^8d^2x^8+34b^2c^6d^2x^6-16b^2c^4d^2x^4-b^2c^2d^2x^2+2b^2d^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-126\left(49abc^9d^2x^9-171abc^7d^2x^7+189abc^5d^2x^5-21abc^3d^2x^3-126abcd^2x\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-126\left(\left(49b^2c^9d^2x^9-171b^2c^7d^2x^7+189b^2c^5d^2x^5-21b^2c^3d^2x^3-126b^2cd^2x\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-63\left(7abc^{10}d^2x^{10}-26abc^8d^2x^8+34abc^6d^2x^6-16abc^4d^2x^4-abc^2d^2x^2+2abd^2\right)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(343\left(81a^2+2b^2\right)c^{10}d^2x^{10}-2\left(51597a^2+1490b^2\right)c^8d^2x^8+2\left(67473a^2+2152b^2\right)c^6d^2x^6-4\left(15876a^2+53b^2\right)c^4d^2x^4-\left(3969a^2+14078b^2\right)c^2d^2x^2+2\left(3969a^2+6140b^2\right)d^2\right)\sqrt{-c^2dx^2+d}}{250047\left(c^6x^2-c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 - 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 + 2*b^2*d^2)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1))^2 - 126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*d^2
*x^7 + 189*a*b*c^5*d^2*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 126*((49*b^2*c^9*d^2*x^9 - 171*b^2*c^7*d^2
*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 63*(7*a*b*c^10*d^2*x^10 - 26*a*b*c^8*d^2*x
^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^2 + 2*a*b*d^2)
*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (343*(81*a^2 + 2*b^2)
*c^10*d^2*x^10 - 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152
*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*
b^2)*c^2*d^2*x^2 + 2*(3969*a^2 + 6140*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*
x^2 - c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

3.187 $\int x^2(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=841

$$\frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2}+\frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824}-\frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2}+\frac{73b^2d^2x(1-c^2x^2)\sqrt{d-c^2dx^2}}{12288c^2(1-cx)(1+cx)}+\cdots$$

```
[Out] 5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/8*x^3*(-c^2*d*x^2+d)
^(5/2)*(a+b*arccosh(c*x))^2+35/9216*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+215/
13824*b^2*d^2*x^3*(-c^2*d*x^2+d)^(1/2)-5/864*b^2*c^2*d^2*x^5*(-c^2*d*x^2+d)
^(1/2)+73/12288*b^2*d^2*x*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c
*x+1)+73/18432*b^2*d^2*x^3*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)-43/4608*b^2*c^2*d^2*x^5*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)+1/256*b^2*c^4*d^2*x^7*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)
-5/128*d^2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+35/9216*b^2*d^2*arccosh(c*x)*(-c^2*d
*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/128*b*d^2*x^2*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/384*b*c*d^2*x^4
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/144
*b*c^3*d^2*x^6*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-5/384*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c
^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-73/12288*b^2*d^2*arctanh(c*x/(c^2*x^2-1)^(1/
2))*(c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/c^3/(-c*x+1)/(c*x+1)
```

Rubi [A]
time = 1.10, antiderivative size = 841, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$, Rules used = {5930, 5926, 5939, 5893, 5883, 92, 54, 102, 12, 5912, 14, 5921, 471, 272, 45, 534, 1281, 470, 327, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (35*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/(9216*c^2) + (215*b^2*d^2*x^3*Sqrt[d - c
^2*d*x^2])/13824 - (5*b^2*c^2*d^2*x^5*Sqrt[d - c^2*d*x^2])/864 + (73*b^2*d^
2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(12288*c^2*(1 - c*x)*(1 + c*x)) + (7
3*b^2*d^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(18432*(1 - c*x)*(1 + c*x)
) - (43*b^2*c^2*d^2*x^5*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(4608*(1 - c*x)*
(1 + c*x)) + (b^2*c^4*d^2*x^7*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(256*(1 -
c*x)*(1 + c*x)) + (35*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(9216*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
```

```
osh[c*x]))/(128*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (59*b*c*d^2*x^4*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(384*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17*b
*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(144*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(128*c^2) + (5*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/64 + (5*d*x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/48 + (x^3*
(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/8 - (5*d^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^3)/(384*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (73*b^
2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^
2]])/(12288*c^3*(1 - c*x)*(1 + c*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
```

```
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(\ldots\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \ldots$$

$$= -\frac{bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2x^6\sqrt{d-c^2dx\ldots}}{12\sqrt{-1+\ldots}}$$

$$= -\frac{11bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{96\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-\ldots}}{144\sqrt{-\ldots}}$$

$$= -\frac{59bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{384\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-\ldots}}{144\sqrt{-\ldots}}$$

$$= \frac{5}{512}b^2d^2x^3\sqrt{d-c^2dx^2} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} + \frac{b^2c^4d^2x\ldots}{2\ldots}$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{256c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2\ldots$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{1024c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2\ldots$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x\ldots$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x\ldots$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x\ldots$$

Mathematica [A]
time = 3.95, size = 910, normalized size = 1.08

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/884736*(d^2*(34560*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2
] + 34560*a^2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 2718
72*a^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 271872*a^2*
c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^5*x^5
*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^6*x^6*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^7*x^7*Sqrt[(-1 + c*
x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^8*x^8*Sqrt[(-1 + c*x)/(1 +
 c*x)]*Sqrt[d - c^2*d*x^2] + 11520*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 +
 34560*a^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + 34560*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 +
c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 13824*a*
b*Sqrt[d - c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 3456*a*b*Sqrt[d - c^2*d*x^2]*C
osh[4*ArcCosh[c*x]] - 1536*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 2
16*a*b*Sqrt[d - c^2*d*x^2]*Cosh[8*ArcCosh[c*x]] - 6912*b^2*Sqrt[d - c^2*d*x
^2]*Sinh[2*ArcCosh[c*x]] - 864*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*ArcCosh[c*x]]
 + 256*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 27*b^2*Sqrt[d - c^2*d
*x^2]*Sinh[8*ArcCosh[c*x]] + 24*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*(576*b*C
osh[2*ArcCosh[c*x]] + 144*b*Cosh[4*ArcCosh[c*x]] - 64*b*Cosh[6*ArcCosh[c*x]
] + 9*b*Cosh[8*ArcCosh[c*x]] - 1152*a*Sinh[2*ArcCosh[c*x]] - 576*a*Sinh[4*A
rcCosh[c*x]] + 384*a*Sinh[6*ArcCosh[c*x]] - 72*a*Sinh[8*ArcCosh[c*x]]) - 28
8*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2*(-120*a + 48*b*Sinh[2*ArcCosh[c*x]]
+ 24*b*Sinh[4*ArcCosh[c*x]] - 16*b*Sinh[6*ArcCosh[c*x]] + 3*b*Sinh[8*ArcCos
h[c*x]])))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(741) = 1482$.
time = 2.97, size = 2528, normalized size = 3.01

method	result	size
default	Expression too large to display	2528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+5
/192*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a^2/c^2*d^2*x*(-c^2*d*x^2+d)^(1
/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(
1/2))+b^2*(-5/384*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ar
ccosh(c*x)^3*d^2+1/65536*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+12
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^6*c^6-88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c*x-32*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(32*arccos
h(c*x)^2-8*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)-1/6912*(-d*(c^2*x^2-1))^
(1/2)*(32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*
x^3-48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(
```

1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*arccosh(c*x)
+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3
x^3+8(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1
)*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(
c*x)^2-2*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/
2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^
(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/
2048*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x
^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(
1/2)+4*c*x)*(8*arccosh(c*x)^2+4*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)-1/6
912*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*
x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(18*arccosh
(c*x)^2+6*arccosh(c*x)+1)*d^2/(c*x+1)/c^3/(c*x-1)+1/65536*(-d*(c^2*x^2-1))^
(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*x^9+256*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^
4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*c^3*x^3-(c*x-1)^(1/
2)*(c*x+1)^(1/2)+8*c*x)*(32*arccosh(c*x)^2+8*arccosh(c*x)+1)*d^2/(c*x+1)/c^
3/(c*x-1))+2*a*b*(-5/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
/c^3*arccosh(c*x)^2*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7
x^7+128(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^6*c^6-88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*
c*x-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1
+8*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*
c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c
^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)
+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*(-1+4*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*(c^2*x^
2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^
(1/2)*(c*x+1)^(1/2))*(-1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/256*(-d*
(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1
)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/1
024*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^
5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1
/2)+4*c*x)*(1+4*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)-1/2304*(-d*(c^2*x^2-1
))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+
38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d^2/(c*x+1
)/c^3/(c*x-1)+1/16384*(-d*(c^2*x^2-1))^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^8*c^8+128*c^9*x^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7
-160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-

```
1)^(1/2)*x^2*c^2-88*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)*(1+8*arccosh
(c*x))*d^2/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] 1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d)
 + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 +
15*d^(5/2)*arcsin(c*x)/c^3)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*x^2*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*x^2
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
- 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

3.188 $\int x(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=470

$$-\frac{32b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{245c^2(1-cx)(1+cx)}-\frac{16b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{735c^2(1-cx)(1+cx)}-\frac{12b^2d^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{1225c^2(1-cx)(1+cx)}-\frac{2b^2d^2(1-}{343c}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))^2/c^2/d-32/245*b^2*d^2*(-c^2*x
^2+1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-16/735*b^2*d^2*(-c^2*x^2+1)
^2*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-12/1225*b^2*d^2*(-c^2*x^2+1)^3
*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-2/343*b^2*d^2*(-c^2*x^2+1)^4*(-c
^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)+2/7*b*d^2*x*(a+b*arccosh(c*x))*(-c^2
*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+6/35*b*c^3*d^2*x^5*(
a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/49*b*c
^5*d^2*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A]
time = 0.31, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5914, 5889, 200, 5894, 12, 1624, 1813, 1864}

$$\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{7/2}\left(a+b\cosh^{-1}(cx)\right)^2}{7c^2d}-\frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{2b^2d^2(1-c^2x^2)^4\sqrt{d-c^2dx^2}}{343c^2(1-cx)(cx+1)}-\frac{12b^2d^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{1225c^2(1-cx)(cx+1)}-\frac{16b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{735c^2(1-cx)(cx+1)}-\frac{32b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{245c^2(1-cx)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-32*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(245*c^2*(1 - c*x)*(1 + c*x
)) - (16*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(735*c^2*(1 - c*x)*(1
 + c*x)) - (12*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(1225*c^2*(1 -
c*x)*(1 + c*x)) - (2*b^2*d^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(343*c^2*
(1 - c*x)*(1 + c*x)) + (2*b*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*c^3*d^2*x^5*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(2*b*c^5*d^2*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCosh[c*x])^2)/(7*c^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{7c^2} - \frac{\left(2bd\right.}{}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\,(a}{7\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\,(a}{7\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\,(a}{7\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\,(a}{7\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\,(a}{7\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{32b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{245c^2(1-cx)(1+cx)} - \frac{16b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx}}{735c^2(1-cx)(1+cx)}$$

Mathematica [A]
time = 0.36, size = 234, normalized size = 0.50

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3675a^2(-1+c^2x^2)^4-210abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(-35+35c^2x^2-21c^4x^4+5c^6x^6)+2b^2(2161-2918c^2x^2+1108c^4x^4-426c^6x^6+75c^8x^8)+210b\left(35a(-1+c^2x^2)^4+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(35-35c^2x^2+21c^4x^4-5c^6x^6)\right)\cosh^{-1}(cx)+3675b^2(-1+c^2x^2)^4\cosh^{-1}(cx)^2\right)}{25725c^2\,(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3675*a^2*(-1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(21
61 - 2918*c^2*x^2 + 1108*c^4*x^4 - 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*
(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(35 - 35*c^2*x^2 + 21
*c^4*x^4 - 5*c^6*x^6))*ArcCosh[c*x] + 3675*b^2*(-1 + c^2*x^2)^4*ArcCosh[c*x
]^2))/(25725*c^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. $2(418) = 836$.

time = 1.15, size = 1958, normalized size = 4.17

method	result	size
default	Expression too large to display	1958

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^2/c^2/d*(-c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(-d*(c^2*x^2-1))^(1/2)*(64
*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arcc
osh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c
^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccos
h(c*x)^2-10*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^
(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1)/
c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c
^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh
(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1
/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1)/c
^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)
+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/43904*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*
d^2/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-1
44*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/
(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*
x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*
d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(
```

```
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^
2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*
(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4
*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d^2
/(c*x+1)/c^2/(c*x-1))
```

<u>Maxima</u> [A]
time = 0.28, size = 337, normalized size = 0.72

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b^2\operatorname{arcosh}\left(cx\right)^2}{7\,c^2d}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}ab\operatorname{arcosh}\left(cx\right)}{7\,c^2d}+\frac{2}{25725}b^2\left(\frac{75\sqrt{c^2x^2-1}\,c^4\sqrt{-d}\,d^3x^6-351\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,d^3x^4+757\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^3x^2-\frac{2161\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^3}{c^2}}{d}-\frac{105\left(5\,c^6\sqrt{-d}\,d^3x^7-21\,c^4\sqrt{-d}\,d^3x^5+35\,c^2\sqrt{-d}\,d^3x^3-35\sqrt{-d}\,d^3x\right)\operatorname{arcosh}\left(cx\right)}{cd}\right)-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a^2}{7\,c^2d}-\frac{2\left(5\,c^6\sqrt{-d}\,d^3x^7-21\,c^4\sqrt{-d}\,d^3x^5+35\,c^2\sqrt{-d}\,d^3x^3-35\sqrt{-d}\,d^3x\right)ab}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
d)^(7/2)*a*b*arccosh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 - 1)*c^4*
sqrt(-d)*d^3*x^6 - 351*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^3*x^4 + 757*sqrt(c^
2*x^2 - 1)*sqrt(-d)*d^3*x^2 - 2161*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^3/c^2)/d -
105*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*c^2*sqrt(-d)*d^3
*x^3 - 35*sqrt(-d)*d^3*x)*arccosh(c*x)/(c*d)) - 1/7*(-c^2*d*x^2 + d)^(7/2)*
a^2/(c^2*d) - 2/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*
c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*a*b/(c*d)
```

<u>Fricas</u> [A]
time = 0.41, size = 477, normalized size = 1.01

$$\frac{3675\left(b^2c^8d^2x^8-4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4-4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-210\left(5\,abc^7d^2x^7-21\,abc^5d^2x^5+35\,abc^3d^2x^3-35\,abcd^2x\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-210\left(\left(5\,b^2c^7d^2x^7-21\,b^2c^5d^2x^5+35\,b^2c^3d^2x^3-35\,b^2cd^2x\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-35\left(abc^8d^2x^8-4\,abc^6d^2x^6+6\,abc^4d^2x^4-4\,abc^2d^2x^2+abd^2\right)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(75\left(49\,a^2+2\,b^2\right)c^8d^2x^8-12\left(1225\,a^2+71\,b^2\right)c^6d^2x^6+2\left(11025\,a^2+1108\,b^2\right)c^4d^2x^4-4\left(3675\,a^2+1459\,b^2\right)c^2d^2x^2+\left(3675\,a^2+4322\,b^2\right)d^2\right)\sqrt{-c^2dx^2+d}}{25725\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 - 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
)^2 - 210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 - 35
*a*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((5*b^2*c^7*d^2*
x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3 - 35*b^2*c*d^2*x)*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1) - 35*(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6 + 6*a
*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*x^2 + a*b*d^2)*sqrt(-c^2*d*x^2 + d))*log(c*x
```

```
 + sqrt(c^2*x^2 - 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 - 12*(1225*a^2 + 7
1*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 - 4*(3675*a^2 + 1
459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^
4*x^2 - c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(-d(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\operatorname{acosh}{(cx)})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

3.189 $\int \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=486

$$\frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152}+\frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}+\frac{115b^2}{11}$$

```
[Out] 5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/6*x*(-c^2*d*x^2+d)^(5/
2)*(a+b*arccosh(c*x))^2+245/1152*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+65/1728*b^2
*d^2*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(-c*x+1)^2*(c*
x+1)^2*(-c^2*d*x^2+d)^(1/2)+5/16*d^2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^
(1/2)+115/1152*b^2*d^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-5/16*b*c*d^2*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+5/48*b*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/18*b*d^2*(-c^2*x^2+1)^3*(a+b*a
rccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/48*d^2*(a
+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5897, 5895, 5893, 5883, 92, 54, 5912, 5914, 38}

$$\frac{bd^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bd^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{48bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{5bcd^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{5}{24}dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{108}b^2d^2x(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}+\frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152}+\frac{65b^2d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}}{1728}+\frac{115b^2d^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{1152c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2])/1728 + (b^2*d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c
^2*d*x^2])/108 + (115*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(1152*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*(1 - c^2*x^2)^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(18*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^2)/16 + (5*d*x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/24 + (x*(
d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/6 - (5*d^2*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^3)/(48*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right) \int (-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\right.}{}$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{24}d^2x(1-cx$$

$$= \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} + \frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d}}{48c\sqrt{-1}}$$

$$= \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-cx)^2(1+$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728}$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728}$$

Mathematica [A]
time = 2.37, size = 740, normalized size = 1.52

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 9504*a^
2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^3*x^3
*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^4*x^4*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[(-1 + c*x)/(
1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2] - 1440*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 - 4320*a
^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[d]*(-1 + c^2*x^2))] - 4320*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Arc
Tan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 3240*a*b*Sqrt[d -
 c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 324*a*b*Sqrt[d - c^2*d*x^2]*Cosh[4*ArcCo
sh[c*x]] - 24*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 1620*b^2*Sqrt[
d - c^2*d*x^2]*Sinh[2*ArcCosh[c*x]] - 81*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*Arc
Cosh[c*x]] + 4*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 12*b*Sqrt[d -
```

```
c^2*d*x^2]*ArcCosh[c*x]*(270*b*Cosh[2*ArcCosh[c*x]] - 27*b*Cosh[4*ArcCosh[
c*x]] + 2*b*Cosh[6*ArcCosh[c*x]] - 540*a*Sinh[2*ArcCosh[c*x]] + 108*a*Sinh[
4*ArcCosh[c*x]] - 12*a*Sinh[6*ArcCosh[c*x]]) + 72*b*Sqrt[d - c^2*d*x^2]*Arc
Cosh[c*x]^2*(-60*a + 45*b*Sinh[2*ArcCosh[c*x]] - 9*b*Sinh[4*ArcCosh[c*x]] +
 b*Sinh[6*ArcCosh[c*x]])))/(13824*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1736 vs. $2(422) = 844$.
time = 1.86, size = 1737, normalized size = 3.57

method	result	size
default	Expression too large to display	1737

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*d
^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+b^2*(-5/48*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)/c*arccosh(c*x)^3*d^2+1/6912*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64
*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(18*arccosh(c*x)^2-6*arccosh(c*x)+1)*d^2/(c*x+1)/(c*x
-1)/c-3/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)*d^2/(c*x+1)/(c*x-1)/
c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1
)*d^2/(c*x+1)/(c*x-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(
c*x)^2+2*arccosh(c*x)+1)*d^2/(c*x+1)/(c*x-1)/c-3/1024*(-d*(c^2*x^2-1))^(1/2
)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8*arccosh(c*x
)^2+4*arccosh(c*x)+1)*d^2/(c*x+1)/(c*x-1)/c+1/6912*(-d*(c^2*x^2-1))^(1/2)*(
-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3
+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(18*arccosh(c*x)^2+6*arccosh(c*x)+1)*d^
2/(c*x+1)/(c*x-1)/c)+2*a*b*(-5/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)/c*arccosh(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64
*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(-1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c-3/512*(-d*(
c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c
^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*(-1+4*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2
```

```
*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^
(1/2))*(-1+2*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+15/256*(-d*(c^2*x^2-1))^(1
/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)
^(1/2)-2*c*x)*(1+2*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c-3/512*(-d*(c^2*x^2-1
))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*ar
ccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4
*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(
1/2)*(c*x+1)^(1/2)-6*c*x)*(1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a^2 + integrate((-c^2*d*x
^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2
+ d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

3.190 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=836

$$\frac{68}{27}b^2d^2\sqrt{d-c^2dx^2}-\frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2}-\frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{16b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{75(1-cx)(1+cx)}+\frac{8b^2d^2}{}$$

```
[Out] 1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/5*(-c^2*d*x^2+d)^(5/2)*(a
+b*arccosh(c*x))^2+68/27*b^2*d^2*(-c^2*d*x^2+d)^(1/2)-2/27*b^2*c^2*d^2*x^2*
(-c^2*d*x^2+d)^(1/2)+16/75*b^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+
1)/(c*x+1)+8/225*b^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)+2/125*b^2*d^2*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)+d^2*(
a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c*d^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+22/45*b*c^3*d^2*x^3*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d^2*(a+b*
arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*d^2*(a
+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^
2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*d^2*polylog(3,-I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*
I*b*d^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 836, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5930, 5926, 5947, 4265, 2611, 2320, 6724, 5879, 75, 5889, 5894, 12, 471, 200, 534, 1261, 712}

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d^2*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d^2*x^2*Sqrt[d - c^2*d*x^2
])/27 - (2*a*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (16*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*(1 - c*x)*(1 + c*x)) +
 (8*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*(1 - c*x)*(1 + c*x))
+ (2*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*(1 - c*x)*(1 + c*x))
 - (2*b^2*c*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1
```

```
+ c*x]) - (16*b*c*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (22*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^5*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 + (d*(d - c^2*d*x^2)^(3/2)*(a + b*A
rcCosh[c*x])^2)/3 + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/5 - (2*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((
2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*
PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^
2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
```

```
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
```

```
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
```

```
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(d^2\sqrt{\right.}}{}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4bc^3d^2x^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+c}}$$

$$= -\frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{22bc^3d^2x^3\sqrt{d-c^2}}{45\sqrt{-1+}}$$

$$= -\frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{15\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b^2cd^2x\sqrt{}}{\sqrt{-1}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-}}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-}}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-}}{\sqrt{-1+cx}\,\sqrt{1}}$$

Mathematica [A]
time = 5.21, size = 963, normalized size = 1.15

Warning: Unable to verify antiderivative.

[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]

[Out] (a^2*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b^2*d^2*Sqrt[d - c^2*d*x^2]*(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1 + Cosh[2*ArcCosh[c*x]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c*x - Cosh[3*ArcCosh[c*x]]))/(-1 + c*x)))/27 - (a*b*d^2*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(9*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(5/2)*Log[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*d^2*Sqrt[d - c^2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) - (a*b*d^2*Sqrt[d - c^2*d*x^2]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*c*x + Cosh[5*ArcCosh[c*x]]) + 15*ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]])))/(1800*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b^2*d^2*Sqrt[d - c^2*d*x^2]*(13500*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 30*ArcCosh[c*x]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*c*x + Cosh[5*ArcCosh[c*x]])) - 250*Sinh[3*ArcCosh[c*x]] + 225*ArcCosh[c*x]^2*(30*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]]) - 54*Sinh[5*ArcCosh[c*x]]))/(54000*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)

[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] -1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2
 + d)*d^2)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x,x)
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x, x)
```

3.191 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=607

$$-\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2}-\frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}-\frac{89b^2cd^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{15bc^3d^2}{}$$

```
[Out] -5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))^2/x-31/64*b^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-1/32*b^2*c
^2*d^2*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)-15/8*c^2*d^2*x*(a+b*arccosh(
c*x))^2*(-c^2*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/8*b*c^3*d^2*x^2*(a+b*arccosh(c*x))*(-c^2
*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d^2*(-c^2*x^2+1)*(a+b*arcco
sh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*d^2*(-c^2
*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+c*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+5/8*c*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2*b*c*d^2*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b^2*c*d^2*polylog
(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {5928, 5897, 5895, 5893, 5883, 92, 54, 5912, 5914, 38, 5919, 5882, 3799, 2221, 2317, 2438}

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{5cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8b\sqrt{cx-1}\sqrt{cx+1}}+\frac{cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd^2(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bcd^2\sqrt{d-c^2dx^2}\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{b^2cd^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2}-\frac{1}{32}b^2c^2d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}-\frac{89b^2cd^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{64\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (-31*b^2*c^2*d^2*x*Sqrt[d - c^2*d*x^2])/64 - (b^2*c^2*d^2*x*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*
x])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2)/8 + (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d*x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcC
osh[c*x])^2)/4 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x + (5*c*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(8*b*Sqrt[-1 + c*x]*Sqrt[1 +
```

```
c*x]) + (2*b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*A
rcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c*d^2*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{(2bc$$

$$= \frac{bcd^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{5}{4}c^2d^2x(1-c$$

$$= \frac{1}{8}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{bcd^2(1-c^2x^2)\sqrt{d-c}}{\sqrt{-1+c}}$$

$$= \frac{11}{16}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2d}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2d}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2d}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2d}$$

Mathematica [A]
time = 3.85, size = 554, normalized size = 0.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(96*a^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-8 -
 9*c^2*x^2 + 2*c^4*x^4) + 1440*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 768*a
*b*Sqrt[d - c^2*d*x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
 - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - 256*b^2*Sqrt[d - c^2*d*x^2]*(ArcCos
h[c*x]*(3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[
```

```
c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[
2, -E^(-2*ArcCosh[c*x])]) + 384*a*b*c*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh
[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 64*b^2*c*x
*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]
] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) - 12*a*b*c*x*Sqrt[d - c^
2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*A
rcCosh[c*x]]) - b^2*c*x*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh
[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]])
))/(768*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(559) = 1118$.
time = 2.80, size = 1227, normalized size = 2.02

method	result	size
default	Expression too large to display	1227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d^2+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*c
^6*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-11/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c^4
*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*
x)^2*d^2/(c*x+1)/(c*x-1)/x+5/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)^3*c*d^2-b^2*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*arccosh(c*x)^2+1/32*b^2*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c
*x+1)/(c*x-1)*x^5-35/64*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*
x^3+33/64*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*x-33/64*b^2*(-
d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-33/64*a
*b*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)-a^2/d/x*(-c^2*d
*x^2+d)^(7/2)-1/8*a*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*x^4-15/8*a^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a^2*c^2*d^3/(c^2*d)^
(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-5/4*a^2*c^2*d*x*(-c^2*d*
x^2+d)^(3/2)+9/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*x^2-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*arccosh(c*x)*x^4+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+
1)/(c*x-1)*arccosh(c*x)^2*x+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/
(c*x-1)*arccosh(c*x)^2*x^5-11/8*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/
(c*x-1)*arccosh(c*x)^2*x^3+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(
c*x-1)*arccosh(c*x)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d^2/(c*x+1)
/(c*x-1)/x+9/8*a*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)+15/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh
(c*x)^2*c*d^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1
```

```
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d^2-a^2*c^2*x*(-c^2*d*x^2+d)^(5/2)+
b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] -1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x
 + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a^2 + integrate((
-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*
(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}} (a+b\operatorname{acosh}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

3.192 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=890

$$-\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2}+\frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2}+\frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b^2c^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{3(1-cx)(1+cx)}+$$

```
[Out] -5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-1/2*(-c^2*d*x^2+d)^(5/
2)*(a+b*arccosh(c*x))^2/x^2-170/27*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)+5/27*b^
2*c^4*d^2*x^2*(-c^2*d*x^2+d)^(1/2)+5/3*b^2*c^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2
+d)^(1/2)/(-c*x+1)/(c*x+1)+1/9*b^2*c^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1
/2)/(-c*x+1)/(c*x+1)-5/2*c^2*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+
5*a*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b^2*c^3*
d^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d^2
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*
b*c^3*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-2/9*b*c^5*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+5*c^2*d^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*I*b*c^2*d^2*
(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b^2*c^2*d^2*polylog(3,-I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-5*I*b^2*c^2*d^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b*c^2*d^2*(a+b*arccosh(c*x))*p
olylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-b^2*c^2*d^2*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)
*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)
```

Rubi [A]

time = 0.92, antiderivative size = 890, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 22, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5928, 5930, 5926, 5947, 4265, 2611, 2320, 6724, 5879, 75, 5889, 5894, 12, 471, 5912, 276, 5921, 534, 1265, 911, 1167, 211}

$$-\frac{2bd^2x^3\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))c^5}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{27}b^2d^2x^2\sqrt{d-c^2dx^2}c^4+\frac{5b^2d^2x\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd^2x\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))c^3}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{5abd^2x\sqrt{d-c^2dx^2}c^3}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{6}d(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2c^2-\frac{5}{2}d^2\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))^2c^2+\frac{5d^2\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))^2\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)c^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{b^2d^2\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)c^2}{(1-cx)(cx+1)}-\frac{5ibd^2\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)c^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{5ibd^2\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)c^2}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{5ib^2d^2\sqrt{d-c^2dx^2}\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)c^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{5ib^2d^2\sqrt{d-c^2dx^2}\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)c^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{170}{27}b^2d^2\sqrt{d-c^2dx^2}c^2+\frac{b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}c^2}{9(1-cx)(cx+1)}+\frac{5b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}c^2}{3(1-cx)(cx+1)}-\frac{bd^2\sqrt{d-c^2dx^2}(a+b\cosh^{-1}(cx))c}{x\sqrt{cx-1}\sqrt{cx+1}}-\frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-170*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2])/27 + (5*b^2*c^4*d^2*x^2*Sqrt[d - c^2
*d*x^2])/27 + (5*a*b*c^3*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b^2*c^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(3*(1 - c*x)*(1
 + c*x)) + (b^2*c^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(9*(1 - c*x)*(
1 + c*x)) + (5*b^2*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
```

```
*x]*Sqrt[1 + c*x]) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^3*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (5*c^2*d*(d - c^2*d*x^2)^
(3/2)*(a + b*ArcCosh[c*x])^2)/6 - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x
])^2)/(2*x^2) + (5*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTa
n[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c^2*d^2*Sqrt[-1 +
c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/((1 - c*x)*(1 + c*
x)) - ((5*I)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2,
(-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b*c^2*d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3,
 (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((5*I)*b^2*c^2*d^2*
Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
```

```
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5921

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5926

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2))), x] + (-Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/(Sq
rt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 +
 c*x]*Sqrt[-1 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x
^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])
^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &
& IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rule 5930

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m +
 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0
] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bcd^2\sqrt{\right.}}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bc^3d^2x\sqrt{d-c^2dx^2}\,(a-}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}\,(a+}{3\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{}$$

$$= \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5b^2c^3d^2x\sqrt{d-}}{\sqrt{-1+}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{}}{\sqrt{-1+cx}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{}}{\sqrt{-1+cx}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{}}{\sqrt{-1+cx}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{}}{\sqrt{-1+cx}}$$

Mathematica [A]
time = 78.80, size = 1384, normalized size = 1.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^
4*d^2*x^2)/3) - (a*b*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]
))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (5*a^2*c^2*d^(5/2)*Log[x])/2
 + (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/2 - 4*a*b
*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]]
- Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (I*(
PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^3*(((-I)*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/(c^2*x^2) +
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]]
 - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]]
- Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[
-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d^2*Sqrt[d - c^2*d*x^2]*((244*c^2)/(-1 +
c*x) - (244*c^3*x)/(-1 + c*x) - (4*c^4*x^2)/(-1 + c*x) + (4*c^5*x^3)/(-1 +
c*x) - (54*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (54*c*ArcCo
sh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) - (252*c^3*x*ArcCosh[c*x])/((-1
 + c*x)^(3/2)*Sqrt[1 + c*x]) + (252*c^4*x^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)
*Sqrt[1 + c*x]) + (12*c^5*x^3*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]
) - (12*c^6*x^4*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (126*c^2*A
rcCosh[c*x]^2)/(-1 + c*x) + (27*ArcCosh[c*x]^2)/(x^2*(-1 + c*x)) - (126*c^3
*x*ArcCosh[c*x]^2)/(-1 + c*x) - (18*c^4*x^2*ArcCosh[c*x]^2)/(-1 + c*x) + (1
8*c^5*x^3*ArcCosh[c*x]^2)/(-1 + c*x) + (27*c*ArcCosh[c*x]^2)/(x - c*x^2) +
(54*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - (54
*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - ((13
5*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]
])/(-1 + c*x) + ((135*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[
1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]
*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((270*I)*c^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]])/(-1 + c
*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, (-I)/E^ArcCosh[c*x
]])/(-1 + c*x) + ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, I/E^Arc
Cosh[c*x]])/(-1 + c*x)))/54
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(
-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a^2 + integrate
((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 +
2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

3.193 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=638

$$\frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2}+\frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x}+\frac{23b^2c^3d^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{12\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5bc^5d^2x^2\sqrt{d}}{2\sqrt{}}$$

```
[Out] 5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x-1/3*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/3*b^
2*c^2*d^2*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)/x+5/2*c^4*d^2*x*(a+b*arccos
h(c*x))^2*(-c^2*d*x^2+d)^(1/2)+23/12*b^2*c^3*d^2*arccosh(c*x)*(-c^2*d*x^2+d
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b*c^3*d^2*(-c^2*x^2+1)*(
a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*
d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-7/3*c^3*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)-5/6*c^3*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2
)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)-14/3*b*c^3*d^2*(a+b*arccosh(c*x))*ln(1+1/(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+7/3*b^2*c^3*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5928, 5895, 5893, 5883, 92, 54, 5912, 5919, 5882, 3799, 2221, 2317, 2438, 38, 5920, 99, 12}

$$-\frac{bcd^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5c^2d(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}{3x}-\frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}{3x^3}-\frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2-\frac{5c^3d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^3}{6b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{7c^3d^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{7bc^3d^2(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{14bc^3d^2\sqrt{d-c^2dx^2}\,\log\left(e^{-2\cosh^{-1}(cx)}+1\right)(a+b\cosh^{-1}(cx))}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b^2c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}{3x}+\frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2}+\frac{7b^2c^3d^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{23b^2c^3d^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{12\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d - c^2*d*x^2])/12 + (b^2*c^2*d^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2])/(3*x) + (23*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c
*x])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c^5*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^2*x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^2)/2 - (7*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^2*d*(d - c^2*d*x^2)^(3/2
)*(a + b*ArcCosh[c*x])^2)/(3*x) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x
])^2)/(3*x^3) - (5*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b
```

```
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (14*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (7*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
```

```
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5919

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCosh[c*x])/(2*p)), x] + (Dist[d
, Int[(d + e*x^2)^(p - 1)*((a + b*ArcCosh[c*x])/x), x], x] - Dist[b*c*((-d)
^p/(2*p)), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5920

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCosh[c
*x])/(f*(m + 1))), x] + (-Dist[b*c*((-d)^p/(f*(m + 1))), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[2*e*(p/(f^2*(m + 1)
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 5928

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Cosh[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
 + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(
1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ
[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} + \frac{\left(2bcd^2\right.}{}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5c^2d^2(1-cx}{}$$

$$= \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} - \frac{7bc^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2d}}{3\sqrt{-1+cx}}$$

$$= -\frac{7}{6}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} -$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} +$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} +$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} +$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} +$$

Mathematica [A]
time = 2.17, size = 803, normalized size = 1.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-8*a*b*c*d^3*x + 8*a*b*c^2*d^3*x^2 - 8*a^2*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]
+ 64*a^2*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b^2*c^2*d^3*x^2*Sqrt[(-
1 + c*x)/(1 + c*x)] - 44*a^2*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 8*b^2
*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 12*a^2*c^6*d^3*x^6*Sqrt[(-1 + c*x
)/(1 + c*x)] + 20*b^2*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 60*a^2*c^3*d^
(5/2)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d
```

```
 - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 6*a*b*c^3*d^3*x^3*Cosh[2*ArcCosh
[c*x]] + 6*a*b*c^4*d^3*x^4*Cosh[2*ArcCosh[c*x]] - 112*a*b*c^3*d^3*x^3*Log[c
*x] + 112*a*b*c^4*d^3*x^4*Log[c*x] - 56*b^2*c^3*d^3*x^3*(-1 + c*x)*PolyLog[
2, -E^(-2*ArcCosh[c*x])] + 3*b^2*c^3*d^3*x^3*Sinh[2*ArcCosh[c*x]] - 3*b^2*c
^4*d^3*x^4*Sinh[2*ArcCosh[c*x]] + 2*b*d^3*(-1 + c*x)*ArcCosh[c*x]*(4*b*c*x
+ 8*a*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*a*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*
a*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*a*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*
x)] + 3*b*c^3*x^3*Cosh[2*ArcCosh[c*x]] + 56*b*c^3*x^3*Log[1 + E^(-2*ArcCosh
[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcCosh[c*x]]) - 2*b*d^3*(-1 + c*x)*ArcCosh[c*
x]^2*(-30*a*c^3*x^3 + 4*b*(-Sqrt[(-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] + 7*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 7*c^3*x^3*(-1 + Sqrt[
(-1 + c*x)/(1 + c*x)])) + 3*b*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3430 vs. $2(580) = 1160$.

time = 4.30, size = 3431, normalized size = 5.38

method	result	size
default	Expression too large to display	3431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 5*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c^3-5/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
arccosh(c*x)^2*d^2*c^3-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)
/(c*x-1)^(1/2)*x^2-14/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3+28/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^3-1/3*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)*arccosh(c*x)*c+14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)*arccosh(c*x)^2*d^2*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15
*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*
d^2*c^6/(c*x+1)/(c*x-1)*x^3-14/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4
-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3+1/3*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c
*x)^2-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3+a*b*(-d*(c^2*x^2-1))^
(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-a*b*(-d*(c^2*x^2-1))^(1/2)*d
^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)*x+49/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-56/3*a*b*(-d*(c^2*x^2-1))^
(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+7/3*a*b*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-21*a*b*
(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x
```

-1)^(1/2)*c^5-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/
x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+190/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*
c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^4+7/3*b^2*(-d*(c^2
*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)
*c^4-23/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1
)/(c*x-1)*arccosh(c*x)^2*c^2+147*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4
-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^8+49/3*b^2*(-d*(c^2*x^2
-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^8+5/2*a^2*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2
))+5/3*a^2*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1
/2)+4/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+35*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/
(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^
5-46/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(
c*x-1)*arccosh(c*x)*c^2+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-21*b^2*(-d*(c^2*x^2-1))^(1/2)*d^
2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^
5+294*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/
(c*x-1)*arccosh(c*x)*c^8-406*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*
c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+380/3*a*b*(-d*(c^2*x^2-1))^
(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-203*
b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1
)*arccosh(c*x)^2*c^6-56/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2
*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)
*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6+70*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(6
3*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-29
4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-147*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4
*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7+1/2*b
^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3-21*b^2
(-d(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c^7+5*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x
^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c
^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+5*b^2*(
-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(
1/2)*arccosh(c*x)*c^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*arccosh(c*x)*x^2-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+
1)/(c*x-1)*arccosh(c*x)^2*x+56/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4
-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8-71/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/
(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+16/3*b^2*(-d*(c^2*x^2-1))
^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2
*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*c^2+1/4*a*b*
(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)+7/3*a*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x*c^4-49/3*a*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3*c^6-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^
(7/2)+4/3*a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2

```
*c^4/(c*x+1)/(c*x-1)*x-7/3*b^2*(-d*(c^2*x^2-1))...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
+ 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x
^2 + d)^(7/2)/(d*x^3))*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{5}{2}}\,\left(a+b\operatorname{acosh}{(cx)}\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

3.194 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=421

$$-\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}}-\frac{4144b^2(1-cx)(1+cx)}{3375c^6\sqrt{d-c^2dx^2}}-\frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}}-\frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}}-$$

```
[Out] -4144/3375*b^2*(-c*x+1)*(c*x+1)/c^6/(-c^2*d*x^2+d)^(1/2)-272/3375*b^2*x^2*(
-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-2/125*b^2*x^4*(-c*x+1)*(c*x+1)/c^2
/(-c^2*d*x^2+d)^(1/2)-16/15*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*d*x
^2+d)^(1/2)-16/15*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*
d*x^2+d)^(1/2)-8/45*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
3/(-c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c/(-c^2*d*x^2+d)^(1/2)-8/15*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)
/c^6/d-4/15*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/5*x^4*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.39, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5938, 5914, 5879, 75, 5883, 102, 12}

$$-\frac{2bx^5\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2d}-\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{15c^6d}-\frac{16abx\sqrt{cx-1}\,\sqrt{cx+1}}{15c^5\sqrt{d-c^2dx^2}}-\frac{4x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{15c^4d}-\frac{8bx^3\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{45c^3\sqrt{d-c^2dx^2}}-\frac{2b^2x^4(1-cx)(cx+1)}{125c^2\sqrt{d-c^2dx^2}}-\frac{4144b^2(1-cx)(cx+1)}{3375c^6\sqrt{d-c^2dx^2}}-\frac{16b^2x\sqrt{cx-1}\,\sqrt{cx+1}\cosh^{-1}(cx)}{15c^5\sqrt{d-c^2dx^2}}-\frac{272b^2x^2(1-cx)(cx+1)}{3375c^4\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (41
44*b^2*(1 - c*x)*(1 + c*x))/(3375*c^6*Sqrt[d - c^2*d*x^2]) - (272*b^2*x^2*(
1 - c*x)*(1 + c*x))/(3375*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^4*(1 - c*x)*(
1 + c*x))/(125*c^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (8*b*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(45*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(25*c*Sqrt[d - c^2*d*x^
2]) - (8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^6*d) - (4*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4*d) - (x^4*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^2)/(5*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
```

```
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-}}}{5c^2\sqrt{d-c^2dx}}$$

$$= -\frac{2bx^5\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}} - \frac{4x^2(1-cx)(1+cx)\left(a+b\right.}{15c^4\sqrt{d-c^2dx}}$$

$$= -\frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{8bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{45c^3\sqrt{d-c^2dx^2}} - 2$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{8b^2x^2(1-cx)(1+cx)}{135c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1+}{125c^2\sqrt{d-c^2dx}}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1}{125c^2\sqrt{d-c^2}}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{32b^2(1-cx)(1+cx)}{27c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1-}{3375c^4\sqrt{d-c^2dx}}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{4144b^2(1-cx)(1+cx)}{3375c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(}{3375c^4\sqrt{d-c^2}}$$

Mathematica [A]
time = 0.34, size = 255, normalized size = 0.61

$$\frac{\sqrt{d-c^2dx^2}\left(30abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(120+20c^2x^2+9c^4x^4)-225a^2(-8+4c^2x^2+c^4x^4+3c^6x^6)-2b^2(-2072+1936c^2x^2+109c^4x^4+27c^6x^6)+30b\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(120+20c^2x^2+9c^4x^4)-15a(-8+4c^2x^2+c^4x^4+3c^6x^6)\right)\cosh^{-1}(cx)-225b^2(-8+4c^2x^2+c^4x^4+3c^6x^6)\cosh^{-1}(cx)^2\right)}{3375c^6d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(30*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2
*x^2 + 9*c^4*x^4) - 225*a^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 2*b^2*
(-2072 + 1936*c^2*x^2 + 109*c^4*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2 + 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c
```

```
^4*x^4 + 3*c^6*x^6))*ArcCosh[c*x] - 225*b^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c
^6*x^6)*ArcCosh[c*x]^2))/(3375*c^6*d*(-1 + c*x)*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(365) = 730$.
time = 4.26, size = 1314, normalized size = 3.12

method	result	size
default	Expression too large to display	1314

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^
2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b^2*(-1/4000*(-d*(c^2*x^2-1))^(
1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*
x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/c^6/
d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^
2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^
2+2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-1/4000*
(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+
20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)/c^6/d/(c^2*x^2
-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/c^6/d/(c^2*x^2
-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c
^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x
))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^
4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/c^6/d/
(c^2*x^2-1))
```

Maxima [A]

time = 0.50, size = 407, normalized size = 0.97

$$-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arcosh}(cx)^2-\frac{2}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)ab\operatorname{arcosh}(cx)-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)a^2-\frac{2}{3375}b^2\left(\frac{27\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,x^4+136\sqrt{c^2x^2-1}\,\sqrt{-d}\,x^2+\frac{2072\sqrt{c^2x^2-1}\,\sqrt{-d}}{c^2}}{c^4d}-\frac{15\left(9c^4\sqrt{-d}\,x^5+20c^2\sqrt{-d}\,x^3+120\sqrt{-d}\,x\right)\operatorname{arcosh}(cx)}{c^5d}\right)+\frac{2\left(9c^4\sqrt{-d}\,x^5+20c^2\sqrt{-d}\,x^3+120\sqrt{-d}\,x\right)ab}{225c^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4
*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b^2*arccosh(c*x)^2 - 2/15*(3*sqrt(-c^
2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2
*d*x^2 + d)/(c^6*d))*a*b*arccosh(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c
^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d)
)*a^2 - 2/3375*b^2*((27*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 136*sqrt(c^2*x
^2 - 1)*sqrt(-d)*x^2 + 2072*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^4*d) - 15*(9
*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(-d)*x)*arccosh(c*x)/(c^5
*d)) + 2/225*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(-d)*x)*a*
b/(c^5*d)
```

Fricas [A]
time = 0.46, size = 348, normalized size = 0.83

$$-\frac{225\left(3b^2c^6x^6+b^2c^4x^4+4b^2c^2x^2-8b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(9abc^5x^5+20abc^3x^3+120abcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-30\left(\left(9b^2c^5x^5+20b^2c^3x^3+120b^2cx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-15\left(3abc^6x^6+abc^4x^4+4abc^2x^2-8ab\right)\sqrt{-c^2dx^2+d}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(27\left(25a^2+2b^2\right)c^6x^6+\left(225a^2+218b^2\right)c^4x^4+4\left(225a^2+968b^2\right)c^2x^2-1800a^2-4144b^2\right)\sqrt{-c^2dx^2+d}}{3375\left(c^8dx^2-c^6d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3375*(225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*sqrt(-c^
2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 + 20*a*b*c^
3*x^3 + 120*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^
5*x^5 + 20*b^2*c^3*x^3 + 120*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
) - 15*(3*a*b*c^6*x^6 + a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*sqrt(-c^2*d*x^
2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 + (225*
a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 + 968*b^2)*c^2*x^2 - 1800*a^2 - 4144*b^
2)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\operatorname{acosh}(cx)\right)^2}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

3.195 $\int \frac{x^4 (a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=355

$$-\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}}-\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}}+\frac{15b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{64c^5\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{-1+cx}\,\sqrt{1}}{8c^3\sqrt{}}$$

```
[Out] -15/64*b^2*x*(-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-1/32*b^2*x^3*(-c*x+1
)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)+15/64*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*b*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/8*b*x^4*(a+b*arccosh(c*x))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/8*(a+b*arccosh(c*x))^3*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arccosh(c*x
))^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/4*x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)
^(1/2)/c^2/d
```

Rubi [A]
time = 0.31, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5938, 5892, 5883, 92, 54, 102, 12}

$$-\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{8c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{4c^2d}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^3}{8bc^5\sqrt{d-c^2dx^2}}-\frac{3x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{8c^4d}-\frac{3bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{8c^3\sqrt{d-c^2dx^2}}-\frac{b^2x^3(1-cx)(cx+1)}{32c^2\sqrt{d-c^2dx^2}}+\frac{15b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{64c^5\sqrt{d-c^2dx^2}}-\frac{15b^2x(1-cx)(cx+1)}{64c^4\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 - c*x)*(1 + c*x))/(64*c^4*Sqrt[d - c^2*d*x^2]) - (b^2*x^3*(1
- c*x)*(1 + c*x))/(32*c^2*Sqrt[d - c^2*d*x^2]) + (15*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*ArcCosh[c*x])/(64*c^5*Sqrt[d - c^2*d*x^2]) - (3*b*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[d - c^2*d*x
^2]) - (3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8*c^4*d) - (x^3*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*c^2*d) + (Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^3)/(8*b*c^5*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
```

```
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2 \ldots}{\sqrt{- \ldots}}}{4c^2\sqrt{d-c^2dx \ldots}}$$

$$= -\frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\,\text{co}\ldots\right.}{8c^4\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} - \frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{8c^3\sqrt{d-c^2dx^2}} - \frac{bx \ldots}{\ldots}$$

$$= -\frac{3b^2x(1-cx)(1+cx)}{16c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} - \frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}{8c^3\sqrt{d-c^2 \ldots}}$$

$$= -\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} + \frac{3b^2\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}{16c^5\sqrt{d-c^2dx \ldots}}$$

$$= -\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} + \frac{15b^2\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}{64c^5\sqrt{d-c^2d \ldots}}$$

Mathematica [A]
time = 1.05, size = 295, normalized size = 0.83

$$\frac{32a^2c\sqrt{d}\,x(-1+c^2x^2)(3+2c^2x^2)-96a^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+b^2\sqrt{d}\,\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(32\cosh^{-1}(cx)^3-4\cosh^{-1}(cx)\left(16\cosh\left(2\cosh^{-1}(cx)\right)+\cosh\left(4\cosh^{-1}(cx)\right)\right)+32\sinh\left(2\cosh^{-1}(cx)\right)+\sinh\left(4\cosh^{-1}(cx)\right)+8\cosh^{-1}(cx)^2\left(8\sinh\left(2\cosh^{-1}(cx)\right)+\sinh\left(4\cosh^{-1}(cx)\right)\right)\right)-4ab\sqrt{d}\,\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(16\cosh\left(2\cosh^{-1}(cx)\right)+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\left(6\cosh^{-1}(cx)+8\sinh\left(2\cosh^{-1}(cx)\right)+\sinh\left(4\cosh^{-1}(cx)\right)\right)\right)}{256c^5\sqrt{d}\,\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b^2*Sqrt[
d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(32*ArcCosh[c*x]^3 - 4*ArcCosh[c*x]
*(16*Cosh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]]) + 32*Sinh[2*ArcCosh[c*x]]
 + Sinh[4*ArcCosh[c*x]] + 8*ArcCosh[c*x]^2*(8*Sinh[2*ArcCosh[c*x]] + Sinh[4
*ArcCosh[c*x]])) - 4*a*b*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(16*C
osh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*(6*ArcCosh[c*x]
 + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]]))))/(256*c^5*Sqrt[d]*Sqrt[d
 - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(307) = 614$.
time = 4.56, size = 1092, normalized size = 3.08

method	result
default	$-\frac{a^{2} x^{3} \sqrt{-c^{2} d\,x^{2}+d}}{4 c^{2} d}-\frac{3 a^{2} x \sqrt{-c^{2} d\,x^{2}+d}}{8 c^{4} d}+\frac{3 a^{2} \arctan \left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{8 c^{4} \sqrt{c^{2} d}}+b^{2} \left(-\frac{\sqrt{-d\left(c^{2} x^{2}-1\right)}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/2
)+3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^
2*(-1/8*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1
)*arccosh(c*x)^3-1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arccosh(c*x)^2-4*arccosh(c*x)+1)/d/c^5/(c^2
*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*
x)+1)/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*
x)^2+2*arccosh(c*x)+1)/d/c^5/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8*arccosh(c*x)^2+4*a
rccosh(c*x)+1)/d/c^5/(c^2*x^2-1))+2*a*b*(-3/16*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)^2-1/256*(-d*(c^2*x^2-
1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x
-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+4*a
rccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x
+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2*a
rccosh(c*x))/d/c^5/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2*a
rccosh(c*x))/d/c^5/(c^2*x^2-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12
*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(1+4*arccosh(c*x))/d/c^5/(c^2*x
^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/8*a^2*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^
4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + integrate(b^2*x^4*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

3.196 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=292

$$-\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}}-\frac{40b^2(1-cx)(1+cx)}{27c^4\sqrt{d-c^2dx^2}}-\frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}}-\frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-}}{3c^3\sqrt{d-c^2dx^2}}$$

```
[Out] -40/27*b^2*(-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-2/27*b^2*x^2*(-c*x+1)*
(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)-4/3*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/
(-c^2*d*x^2+d)^(1/2)-4/3*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
/(-c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^
4/d-1/3*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5938, 5914, 5879, 75, 5883, 102, 12}

$$-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bx^3\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}}-\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d}-\frac{4abx\sqrt{cx-1}\,\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}-\frac{2b^2x^2(1-cx)(cx+1)}{27c^2\sqrt{d-c^2dx^2}}-\frac{40b^2(1-cx)(cx+1)}{27c^4\sqrt{d-c^2dx^2}}-\frac{4b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (40*b
^2*(1 - c*x)*(1 + c*x))/(27*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^2*(1 - c*x)
*(1 + c*x))/(27*c^2*Sqrt[d - c^2*d*x^2]) - (4*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c*Sqrt[d - c^2*d*x^2]) - (2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^4*d) - (x^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(3*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x(a}{\sqrt{-1}}}{3c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cos\right.}{3c^4\sqrt{d-c^2dx^2}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+}}{9c\sqrt{d}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+}}{3c^3\sqrt{d-c}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{40b^2(1-cx)(1+cx)}{27c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}}$$

Mathematica [**A**]
time = 0.29, size = 201, normalized size = 0.69

$$\frac{\sqrt{d-c^2dx^2}\left(6abcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(6+c^2x^2)-9a^2(-2+c^2x^2+c^4x^4)-2b^2(-20+19c^2x^2+c^4x^4)+6b\left(bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(6+c^2x^2)-3a(-2+c^2x^2+c^4x^4)\right)\cosh^{-1}(cx)-9b^2(-2+c^2x^2+c^4x^4)\cosh^{-1}(cx)^2\right)}{27c^4d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2)
- 9*a^2*(-2 + c^2*x^2 + c^4*x^4) - 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6*b
*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*a*(-2 + c^2*x^2 + c^
4*x^4))*ArcCosh[c*x] - 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]^2))/(27*
c^4*d*(-1 + c*x)*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(252) = 504$.
time = 3.30, size = 752, normalized size = 2.58

method	result
default	$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b^
2*(-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*a
rccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^4/d/(c^2*x
^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-1/216*(-d*(c^2*x^2-1
))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^4/d/(c
^2*x^2-1))+2*a*b*(-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*a
rccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2
*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x)
)/c^4/d/(c^2*x^2-1)-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*
arccosh(c*x))/c^4/d/(c^2*x^2-1))
```

Maxima [A]
time = 0.48, size = 279, normalized size = 0.96

$$-\frac{1}{3}\,b^{2}\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,x^{2}}{c^{2}d}+\frac{2\,\sqrt{-c^{2}dx^{2}+d}}{c^{4}d}\right)\operatorname{arcosh}\left(cx\right)^{2}-\frac{2}{3}\,ab\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,x^{2}}{c^{2}d}+\frac{2\,\sqrt{-c^{2}dx^{2}+d}}{c^{4}d}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{3}\,a^{2}\left(\frac{\sqrt{-c^{2}dx^{2}+d}\,x^{2}}{c^{2}d}+\frac{2\,\sqrt{-c^{2}dx^{2}+d}}{c^{4}d}\right)-\frac{2}{27}\,b^{2}\left(\frac{\sqrt{c^{2}x^{2}-1}\,\sqrt{-d}\,x^{2}+\frac{20\,\sqrt{c^{2}x^{2}-1}\,\sqrt{-d}}{c^{2}}}{c^{2}d}-\frac{3\,\left(c^{2}\sqrt{-d}\,x^{3}+6\,\sqrt{-d}\,x\right)\operatorname{arcosh}\left(cx\right)}{c^{3}d}\right)+\frac{2\,\left(c^{2}\sqrt{-d}\,x^{3}+6\,\sqrt{-d}\,x\right)ab}{9\,c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*b^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)
)*arccosh(c*x)^2 - 2/3*a*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*
d*x^2 + d)/(c^4*d))*arccosh(c*x) - 1/3*a^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d
) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) - 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)
*x^2 + 20*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^2*d) - 3*(c^2*sqrt(-d)*x^3 + 6
*sqrt(-d)*x)*arccosh(c*x)/(c^3*d)) + 2/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-d)*x)*
a*b/(c^3*d)
```

Fricas [A]
time = 0.43, size = 282, normalized size = 0.97

$$-\frac{9\,\left(b^{2}c^{4}x^{4}+b^{2}c^{2}x^{2}-2\,b^{2}\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)^{2}-6\,\left(abc^{3}x^{3}+6\,abcx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-6\,\left(\left(b^{2}c^{3}x^{3}+6\,b^{2}cx\right)\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}-3\,\left(abc^{4}x^{4}+abc^{2}x^{2}-2\,ab\right)\sqrt{-c^{2}dx^{2}+d}\right)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+\left(\left(9\,a^{2}+2\,b^{2}\right)c^{4}x^{4}+\left(9\,a^{2}+38\,b^{2}\right)c^{2}x^{2}-18\,a^{2}-40\,b^{2}\right)\sqrt{-c^{2}dx^{2}+d}}{27\,\left(c^{6}dx^{2}-c^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/27*(9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 + 6*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 + 6*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1) - 3*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*sqrt(-c^2*d*x^2 + d))*lo
g(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + (9*a^2 + 38*b^2)*c^
2*x^2 - 18*a^2 - 40*b^2)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

3.197 $\int \frac{x^2(a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=226

$$-\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{2c\sqrt{d-c^2dx^2}}-x$$

```
[Out] -1/4*b^2*x*(-c*x+1)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)+1/4*b^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arccosh(
c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/6*(a+b*arccosh(c
*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*x*(a+b*ar
ccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5938, 5892, 5883, 92, 54}

$$-\frac{x\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2c^2d}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{2c\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{b^2x(1-cx)(cx+1)}{4c^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{4c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -1/4*(b^2*x*(1 - c*x)*(1 + c*x))/(c^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c*Sqrt[d - c^2*d*x^2]) -
(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*c^2*d) + (Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\ldots}{\sqrt{-1+\ldots}}}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+b\cosh\ldots\right.}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-\ldots}{}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^3\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\ldots}{2\ldots}$$

Mathematica [A]

time = 0.56, size = 228, normalized size = 1.01

$$\frac{-\frac{12a^2cx\sqrt{d-c^2dx^2}}{d}-\frac{12a^2\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{\sqrt{d}}+\frac{b^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(4\cosh^{-1}(cx)^3-6\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)+\left(3+6\cosh^{-1}(cx)^2\right)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{d-c^2dx^2}}+\frac{6ab\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(-\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)}{\sqrt{d-c^2dx^2}}}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-12*a^2*c*x*Sqrt[d - c^2*d*x^2])/d - (12*a^2*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*(4*ArcCosh[c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*A
rcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (6*a*b*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCo
sh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d - c^2*d*x^2])/(24*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(194) = 388$.
time = 4.04, size = 563, normalized size = 2.49

method	result
default	$-\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arc}}{6d\,c^3(c^2x^2-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*arctan((c^2
*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^3-1/16*(-d*(c^2*x^2-1))^
(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*
(c*x+1)^(1/2))*(2*arccosh(c*x)^2-2*arccosh(c*x)+1)/d/c^3/(c^2*x^2-1)-1/16*(
-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*
x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(2*arccosh(c*x)^2+2*arccosh(c*x)+1)/d/c^3/(
c^2*x^2-1))+2*a*b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c
*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-1+2
*arccosh(c*x))/d/c^3/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(1+2
*arccosh(c*x))/d/c^3/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + int
egrate(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d
) + 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

3.198 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{2b^2(1-cx)(1+cx)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,(a+}{c^2d}$$

```
[Out] -2*b^2*(-c*x+1)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)-2*a*b*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2
/d
```

Rubi [A]
time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5914, 5879, 75}

$$-\frac{2abx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d}-\frac{2b^2(1-cx)(cx+1)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b^2*(1
 - c*x)*(1 + c*x))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcCosh[c*x])/(c*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x])^2)/(c^2*d)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int (a+b}{c\sqrt{d-c^2dx^2}}$$

$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b^2\sqrt{\right.}}{}$$

$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}} - \frac{(1-c}{}$$

$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^2\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\,\sqrt{1+c}}{c\sqrt{d-c^2dx}}$$

Mathematica [A]
time = 0.22, size = 149, normalized size = 0.96

$$\frac{\sqrt{d-c^2dx^2}\left(2abcx\sqrt{-1+cx}\,\sqrt{1+cx}+a^2(1-c^2x^2)-2b^2(-1+c^2x^2)+2b\left(a-ac^2x^2+bcx\sqrt{-1+cx}\,\sqrt{1+cx}\right)\cosh^{-1}(cx)+b^2(1-c^2x^2)\cosh^{-1}(cx)^2\right)}{c^2d(-1+cx)(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + a^2*(1 - c^2
*x^2) - 2*b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])*ArcCosh[c*x] + b^2*(1 - c^2*x^2)*ArcCosh[c*x]^2))/(c^2*d*(-1 +
 c*x)*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.
time = 1.92, size = 314, normalized size = 2.03

method	result

default	$-\frac{a^{2}\sqrt{-c^{2}d\,x^{2}+d}}{c^{2}d}+b^{2}\left(-\frac{\sqrt{-d\,(c^{2}x^{2}-1)}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^{2}x^{2}-1\right)\left(\operatorname{arccosh}(cx)^{2}-2\operatorname{arccosh}(cx)\right.}{2c^{2}d(c^{2}x^{2}-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^2/d/(
c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(-
d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccos
h(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^2/d/(c^2*x^2-1))
```

Maxima [A]
time = 0.28, size = 145, normalized size = 0.94

$$2\,b^{2}\left(\frac{\sqrt{-d}\,x\operatorname{arcosh}\left(cx\right)}{cd}-\frac{\sqrt{c^{2}x^{2}-1}\,\sqrt{-d}}{c^{2}d}\right)+\frac{2\,ab\sqrt{-d}\,x}{cd}-\frac{\sqrt{-c^{2}dx^{2}+d}\,b^{2}\operatorname{arcosh}\left(cx\right)^{2}}{c^{2}d}-\frac{2\,\sqrt{-c^{2}dx^{2}+d}\,ab\operatorname{arcosh}\left(cx\right)}{c^{2}d}-\frac{\sqrt{-c^{2}dx^{2}+d}\,a^{2}}{c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2*b^2*(sqrt(-d)*x*arccosh(c*x)/(c*d) - sqrt(c^2*x^2 - 1)*sqrt(-d)/(c^2*d))
+ 2*a*b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b^2*arccosh(c*x)^2/(c^2*d)
- 2*sqrt(-c^2*d*x^2 + d)*a*b*arccosh(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a^
2/(c^2*d)
```

Fricas [A]
time = 0.38, size = 218, normalized size = 1.41

$$\frac{2\,\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,abcx-(b^{2}c^{2}x^{2}-b^{2})\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)^{2}+2\left(\sqrt{-c^{2}dx^{2}+d}\,\sqrt{c^{2}x^{2}-1}\,b^{2}cx-(abc^{2}x^{2}-ab)\sqrt{-c^{2}dx^{2}+d}\right)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-((a^{2}+2\,b^{2})c^{2}x^{2}-a^{2}-2\,b^{2})\sqrt{-c^{2}dx^{2}+d}}{c^{4}dx^{2}-c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a*b*c*x - (b^2*c^2*x^2 - b^2)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*b^2*c*x - (a*b*c^2*x^2 - a*b)*sqrt(-c^2*d*x^2 + d))*log(c
*x + sqrt(c^2*x^2 - 1)) - ((a^2 + 2*b^2)*c^2*x^2 - a^2 - 2*b^2)*sqrt(-c^2*d
*x^2 + d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2*x/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\,\mathrm{acosh}(c\,x))^2}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

3.199 $\int \frac{(a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {5892}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.00

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(45) = 90$.
time = 0.40, size = 149, normalized size = 2.81

method	result
default	$\frac{a^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d\, x^2 + d}}\right)}{\sqrt{c^2 d}} - \frac{b^2 \sqrt{-d\,(cx-1)\,(cx+1)}\, \sqrt{cx-1}\, \sqrt{cx+1}\, \operatorname{arccosh}(cx)^3}{3cd(c^2x^2-1)} - \frac{ab\sqrt{-d\,(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*b^2*(-d*(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(c*x)^3-a*b*(-d*(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(c*x)^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] a^2*arcsin(c*x)/(c*sqrt(d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(c^2*d*x^2 - d), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{\sqrt{- d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2), x)
```

3.200 $\int \frac{(a+b\cosh^{-1}(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

```
[Out] 2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-2*I*b*(a+b*arccosh(c*x))*polylog(2,-I*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+
d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2
+d)^(1/2)-2*I*b^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5946, 4265, 2611, 2320, 6724}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3
, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(2ib\sqrt{-1+cx}\right.}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}}{}$$

Mathematica [**A**]
time = 0.38, size = 315, normalized size = 1.15

$$\frac{a^2\log(cx)}{\sqrt{d}} - \frac{a^2\log\left(d+\sqrt{d}\,\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{2iab\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(\cosh^{-1}(cx)\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\log\left(1+ie^{-\cosh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{d-c^2dx^2}} + \frac{ib^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(-\cosh^{-1}(cx)^2\left(\log\left(1-ie^{-\cosh^{-1}(cx)}\right)-\log\left(1+ie^{-\cosh^{-1}(cx)}\right)\right)-2\cosh^{-1}(cx)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(cx)}\right)\right)-2\text{PolyLog}\left(3,-ie^{-\cosh^{-1}(cx)}\right)+2\text{PolyLog}\left(3,ie^{-\cosh^{-1}(cx)}\right)\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d]
 - ((2*I)*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I
/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c
*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (I*b^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-(ArcCosh[c*x]^2*(Log[1 - I/E^ArcCosh[c*x]] -
 Log[1 + I/E^ArcCosh[c*x]])) - 2*ArcCosh[c*x]*(PolyLog[2, (-I)/E^ArcCosh[c*
x]] - PolyLog[2, I/E^ArcCosh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] + 2
*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccosh}(cx)\right)^2}{x\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) + inte
grate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x)
 + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)), x)
```

3.201 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=186

$$-\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{dx}-\frac{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+$$

```
[Out] -c*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-2*
b*c*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,-1/(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b
*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]

time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5917, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{dx}-\frac{c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}-\frac{2bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^
2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(d*x) - (2*b*c*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/Sqrt
[d - c^2*d*x^2] + (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(-2*Arc
Cosh[c*x])])/Sqrt[d - c^2*d*x^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}}{x}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a\right.}{\sqrt{d-c^2}}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right.}{x\sqrt{d-c^2dx^2}}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right.}{x\sqrt{d-c^2dx^2}}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right.}{x\sqrt{d-c^2dx^2}}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right.}{x\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.52, size = 237, normalized size = 1.27

$$-\frac{a^2\sqrt{-d\left(-1+c^2x^2\right)}}{dx} - 2abc\left(\frac{\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{cdx} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(\log\left(-1+\sqrt{1+cx}\right)+\log\left(1+\sqrt{1+cx}\right)\right)}{\sqrt{d-c^2dx^2}}\right) + \frac{b^2c\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{cx}-2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)}{\sqrt{-d(-1+cx)(1+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[-(d*(-1 + c^2*x^2))])/(d*x)) - 2*a*b*c*((Sqrt[d - c^2*d*x^2]*Ar
cCosh[c*x])/(c*d*x) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(Log[-1 + Sqrt[1 + c*x]
] + Log[1 + Sqrt[1 + c*x]]))/Sqrt[d - c^2*d*x^2]) + (b^2*c*Sqrt[(-1 + c*x)/
(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(-ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) - 2*Log[1 + E^(-2*ArcCosh[c*x])]) + Poly
Log[2, -E^(-2*ArcCosh[c*x])]))/Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(192) = 384.
time = 2.04, size = 513, normalized size = 2.76

method	result

default	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{dx}-\frac{b^2\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2c}{d(c^2x^2-1)}-\frac{b^2\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arcc}}{(c^2x^2-1)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c-b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh
(c*x)^2*x/(c^2*x^2-1)/d*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^
2*x^2-1)/d+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*
x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*x/(c^2*x^2-1)/d*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/
(c^2*x^2-1)/d+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c
^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d)*sqr
t(d)*log(-2*c^2*d + 2*d/x^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^2), x) - 2*sqrt(-c^2*d*x^2 + d)
*a*b*arccosh(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a^2/(d*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2}{x^2\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.202 $\int \frac{(a+b\cosh^{-1}(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{x\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2dx^2}+\frac{c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b}{\sqrt{d}}$$

```
[Out] b*c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)+c
^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^
(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b*c^2*(a+b*arccos
h(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+I*b*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/
2)+I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*
arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.30, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5932, 5946, 4265, 2611, 2320, 6724, 5883, 94, 211}

$$\frac{c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}-\frac{ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{x\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2dx^2}-\frac{b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{ib^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x
^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/Sqrt[
d - c^2*d*x^2] - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/Sqrt[d - c^2*d*x^2] - (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2
] + (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I
*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - (I*b^2*c^2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
```

```
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-}}{x^2}}{\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2x^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2x^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2x^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2x^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2x^2\sqrt{d-c^2dx^2}}$$

Mathematica [**A**]
time = 75.11, size = 697, normalized size = 1.62

$$-\frac{a^2\sqrt{d-c^2dx^2}}{2dx^2}+\frac{a^2c^2\log(x)}{2\sqrt{d}}-\frac{a^2c^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{2\sqrt{d}}-\frac{b^2\cosh^{-1}(cx)^2\left(\frac{\sqrt{d-c^2dx^2}}{x^2}-c^2\sqrt{d}\log(x)+c^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)\right)}{2d}+\ldots$$

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -1/2*(a^2*Sqrt[d - c^2*d*x^2])/(d*x^2) + (a^2*c^2*Log[x])/(2*Sqrt[d]) - (a^
2*c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/(2*Sqrt[d]) - (b^2*ArcCosh[c*x]
^2*(Sqrt[d - c^2*d*x^2]/x^2 - c^2*Sqrt[d]*Log[x] + c^2*Sqrt[d]*Log[d + Sqrt
[d]*Sqrt[d - c^2*d*x^2]]))/(2*d) + (b^2*c*(-((Sqrt[d - c^2*d*x^2]*ArcCosh[c
*x])/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + c*Sqrt[d]*(-Log[x] + Log[Sqrt[d] +
 Sqrt[d - c^2*d*x^2]]) + (c*Sqrt[d]*ArcCosh[c*x]^2*(-Log[x] + Log[d + Sqrt[
d]*Sqrt[d - c^2*d*x^2]]))/2))/d + (a*b*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 +
c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*
x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
 c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(
1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]) - ((I/2)*b^2
*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCo
sh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLo
g[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2
*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d
- c^2*d*x^2]
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}{x^{3}\sqrt{-c^{2} d\,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) +
 sqrt(-c^2*d*x^2 + d)/(d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^3) + 2*a*b*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^3\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2}{x^3\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

3.203 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=328

$$\frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}}-\frac{2c^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*b^2*c^2*(-c*x+1)*(c*x+1)/x/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+b*arccosh(c*
x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcco
sh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*
arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/3*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x^3-2/3*c^2*(a+b*arccosh(c*x))^2*(
-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A]

time = 0.33, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5932, 5917, 5882, 3799, 2221, 2317, 2438, 5883, 97}

$$-\frac{2c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3dx}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3}-\frac{2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{d-c^2dx^2}}-\frac{4bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{d-c^2dx^2}}+\frac{b^2c^2(1-cx)(cx+1)}{3x\sqrt{d-c^2dx^2}}+\frac{2b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[d - c^2*d*x^2]) - (2*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[d - c^2*d*x^2
]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*d*x^3) - (2*c^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/(3*Sqrt[d - c
^2*d*x^2]) + (2*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(-2*ArcC
osh[c*x])])/(3*Sqrt[d - c^2*d*x^2])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
```

```
0] && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}\cdots}{x^3}\cdots}{3\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\cdots\right.}{3x^3\sqrt{d-c^2dx^2}}\cdots$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1\cdots}{\cdots}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+\cdots}}{\cdots}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+\cdots}}{\cdots}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+\cdots}}{\cdots}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+\cdots}}{\cdots}$$

Mathematica [A]
time = 0.89, size = 370, normalized size = 1.13

$$\frac{-a^2-a^2c^2x^2+b^2c^2x^2+2a^2c^4x^4-b^2c^4x^4+abcx\sqrt{-1+cx}\,\sqrt{1+cx}-b^2(1+cx)\left(1-cx+2c^2x^2+2c^3x^3\left(-1+\sqrt{\frac{-1+cx}{1+cx}}\right)\right)\cosh^{-1}(cx)^2+b(1+cx)\cosh^{-1}(cx)\left(bcx\sqrt{\frac{-1+cx}{1+cx}}+2a(-1+cx-2c^2x^2+2c^3x^3)-4bc^3x^3\sqrt{\frac{-1+cx}{1+cx}}\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-4abc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}\log\left(-1+\sqrt{1+cx}\right)-4abc^3x^3\sqrt{-1+cx}\,\sqrt{1+cx}\log\left(1+\sqrt{1+cx}\right)+2b^2c^3x^3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{3x^3\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-a^2 - a^2*c^2*x^2 + b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 + a*b*c*x*S
qrt[-1 + c*x]*Sqrt[1 + c*x] - b^2*(1 + c*x)*(1 - c*x + 2*c^2*x^2 + 2*c^3*x^
3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 + b*(1 + c*x)*ArcCosh[c
*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*a*(-1 + c*x - 2*c^2*x^2 + 2*c^3*x
^3) - 4*b*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x])])
- 4*a*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[-1 + Sqrt[1 + c*x]] - 4*a*
b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 + Sqrt[1 + c*x]] + 2*b^2*c^3*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(
3*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2196 vs. $2(310) = 620$.
time = 3.76, size = 2197, normalized size = 6.70

method	result	size
default	Expression too large to display	2197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x-1)*(c*x+
1)*c^6-b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*c^5+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-
1)*(c*x-1)^(1/2)*arccosh(c*x)^2*(c*x+1)^(1/2)*c^3-b^2*(-d*(c^2*x^2-1))^(1/2
)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*c^3-4/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arcc
osh(c*x)^2*c^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
/(c^2*x^2-1)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+4/3*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c^3-4/3*a*b*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x-1)*(c*x+1)*c^6-2/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*(c*x-1)*(c*x+1)*c^4+4/3*a*b*(-d*(c^2
*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)
^(1/2)*c^3-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*c-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^
2*x^2-1)*x^3*(c*x-1)*arccosh(c*x)*(c*x+1)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)^2*c^4-2/3*b^2*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+4/3*b^2*(-d
*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*arccosh(c*x)^2*c^2+4*a*b*(-
d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*(c*x-1)^(1/2)*arccosh(c*
```

```
x)*(c*x+1)^(1/2)*c^5+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
^3+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*arccosh(c*x
)*c^8-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*arccosh(c*
x)^2*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*arcco
sh(c*x)*c^6+2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*(c*x
-1)^(1/2)*arccosh(c*x)^2*(c*x+1)^(1/2)*c^5-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d
/(3*c^4*x^4-2*c^2*x^2-1)*x*(c*x-1)*arccosh(c*x)*(c*x+1)*c^4-1/3*b^2*(-d*(c^
2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^2*(c*x-1)^(1/2)*arccosh(c*x)*(c
*x+1)^(1/2)*c-4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*ar
ccosh(c*x)*c^6+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*a
rccosh(c*x)*c^4-a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2
*x^2-1)/x*arccosh(c*x)*c^2+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^
2*x^2-1)*x^5*c^8+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x
^3*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4-1/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*c^2+1/3*b^2*(-d*(c^2*
x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)^2+4/3*a*b*(-d*(c^2
*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8-2/3*a*b*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*c^6-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*
c^4*x^4-2*c^2*x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^
2*x^2-1)/x^3*arccosh(c*x)+a^2*(-1/3/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*
(-c^2*d*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*a*b*c - 2/3*a*b*(2*sqrt(-c
^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/3*
a^2*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3)) + b^2
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x
^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^4\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2}{x^4\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

3.204 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=556

$$\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}}+\frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}}+\frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}}+\frac{16b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}}{3c^5d\sqrt{d-c^2dx^2}}$$

```
[Out] 94/27*b^2*(-c*x+1)*(c*x+1)/c^6/d/(-c^2*d*x^2+d)^(1/2)+2/27*b^2*x^2*(-c*x+1)
*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^4*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*
x^2+d)^(1/2)+16/3*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1
/2)+16/3*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d
)^(1/2)-2*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*
x^2+d)^(1/2)+2/9*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d
/(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d/
(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^6/d/(-c^2*d*x^2+d)^(1/2)+8/3*(a+b*arccosh(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/c^6/d^2+4/3*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1
/2)/c^4/d^2
```

Rubi [A]
time = 0.60, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5934, 5938, 5914, 5879, 75, 5883, 102, 12, 5912, 5903, 4267, 2317, 2438}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^6d^2}+\frac{4b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^6d\sqrt{d-c^2dx^2}}+\frac{16abx\sqrt{cx-1}\,\sqrt{cx+1}}{3c^5d\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c^5d\sqrt{d-c^2dx^2}}+\frac{4x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2}+\frac{2bx^3\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}+\frac{94b^2(1-cx)(cx+1)}{27c^6d\sqrt{d-c^2dx^2}}+\frac{16b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^5d\sqrt{d-c^2dx^2}}+\frac{2b^2x^2(1-cx)(cx+1)}{27c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (94
*b^2*(1 - c*x)*(1 + c*x))/(27*c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*x^2*(1 -
c*x)*(1 + c*x))/(27*c^4*d*Sqrt[d - c^2*d*x^2]) + (16*b^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^5*d*Sqrt[d - c^2*d*x^2]) + (2*
b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3*d*Sqrt[d -
c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^2) + (4*x^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d
*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(
c^6*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
```

```
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4}{}$$

$$= -\frac{2b^2x^2(1-cx)(1+cx)}{9c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^5d\sqrt{d-c^2dx^2}} + \frac{2b}{}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{22b^2(1-cx)(1+cx)}{9c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 2.46, size = 358, normalized size = 0.64

$$\frac{-36a^2(-8+4c^2x^2+c^4x^4)+3ab\left(135\cosh^{-1}(cx)-60\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)-3\cosh^{-1}(cx)\cosh\left(4\cosh^{-1}(cx)\right)-72\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)+62\sinh\left(2\cosh^{-1}(cx)\right)+\sinh\left(4\cosh^{-1}(cx)\right)\right)-b^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(378\sqrt{\frac{-1+cx}{1+cx}}(1+cx)-378cx\cosh^{-1}(cx)+189\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)^2-6\cosh^{-1}(cx)\cosh\left(3\cosh^{-1}(cx)\right)-54\cosh^{-1}(cx)^2\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)+216\cosh^{-1}(cx)\log\left(1-e^{-\cosh^{-1}(cx)}\right)-216\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)+216\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-216\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+2\sinh\left(3\cosh^{-1}(cx)\right)+9\cosh^{-1}(cx)^2\sinh\left(3\cosh^{-1}(cx)\right)+54\cosh^{-1}(cx)^2\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)}{108c^6d\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-36*a^2*(-8 + 4*c^2*x^2 + c^4*x^4) + 3*a*b*(135*ArcCosh[c*x] - 60*ArcCosh[
c*x]*Cosh[2*ArcCosh[c*x]] - 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 72*Sqrt[(
```

```
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 62*Sinh[2*ArcCos
h[c*x]] + Sinh[4*ArcCosh[c*x]]) - b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*
(378*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 378*c*x*ArcCosh[c*x] + 189*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*ArcCosh[c*x]*Cosh[3*Arc
Cosh[c*x]] - 54*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] + 216*ArcCosh[c*x]*Log[
1 - E^(-ArcCosh[c*x])] - 216*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 216*
PolyLog[2, -E^(-ArcCosh[c*x])] - 216*PolyLog[2, E^(-ArcCosh[c*x])] + 2*Sinh
[3*ArcCosh[c*x]] + 9*ArcCosh[c*x]^2*Sinh[3*ArcCosh[c*x]] + 54*ArcCosh[c*x]^
2*Tanh[ArcCosh[c*x]/2]))/(108*c^6*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(529) = 1058$.
time = 3.87, size = 1101, normalized size = 1.98

method	result
default	$a^2\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)-\frac{2b^2\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] a^2*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)
^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2)))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))+2/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*x^4+92/
27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*x^2+2*b^2*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2/9*b^2*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2
*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-10/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/c^5/d^2/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-94
/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d^2/(c^2*x^2-1)+1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2
/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-8/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)^2-16/3*a*b*(-d*(c^2*x^2-1))^(
1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)-2/9*a*b*(-d*(c^2*x^2-1))^(1/2)/c^3/d^
2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3-10/3*a*b*(-d*(c^2*x^2-1))^(1/
2)/c^5/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+2/3*a*b*(-d*(c^2*x^2-1
))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^4+8/3*a*b*(-d*(c^2*x^2-1))^(1/2
)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
```

```
2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x
^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*a^2*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^
4*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 + 4*b^2*c
^2*sqrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))^2/(c^8*d^2*x^2 - c^6*d^2) + integrate(-2/3*((4*b^2
*c^3*x^3 - (3*a*b*c^5 - b^2*c^5)*x^5 - 8*b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) +
 (3*b^2*c^4*x^4 - (3*a*b*c^6 - b^2*c^6)*x^6 - 12*b^2*c^2*x^2 + 8*b^2)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^10*d^(3/
2)*x^5 - 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*d^(3/2)*x^4 - 2*c^7*d^(3/
2)*x^2 + c^5*d^(3/2))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a+b\,\mathrm{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

3.205 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=440

$$\frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{x^3(}{c}$$

```
[Out] 1/4*b^2*x*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arccosh(c*x)
)^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/4*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^3*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/b/c^5/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(c*x))*ln(
1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-
c^2*d*x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arccosh(c*x))^
2*(-c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A]

time = 0.52, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5934, 5938, 5892, 5883, 92, 54, 5912, 5913, 3797, 2221, 2317, 2438}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{2bc^5d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^5d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^5d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2c^4d^2}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}}+\frac{b^2x(1-cx)(cx+1)}{4c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 - c*x)*(1 + c*x))/(4*c^4*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) + (x^3*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^5*d*Sqrt[d - c^2*d*x^2]) + (3
*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*c^4*d^2) - (Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(2*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (
2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[
c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(\ldots}{\ldots}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a\ldots}{c^2d\ldots}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{2c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\ldots}{2c^3\ldots}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\ldots}{2c^3\ldots}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\ldots}{2c^3\ldots}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\ldots}{2c^3\ldots}$$

Mathematica [A]
time = 1.30, size = 343, normalized size = 0.78

$$\frac{-4a^2c\sqrt{d}\,x(-3+c^2x^2)+12a^2\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+2ab\sqrt{d}\left(8cx\cosh^{-1}(cx)-\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(6\cosh^{-1}(cx)^2-\cosh\left(2\cosh^{-1}(cx)\right)+8\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)+2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)+b^2\sqrt{d}\left(8cx\cosh^{-1}(cx)^2+8\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)-\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(4\cosh^{-1}(cx)^3-2\cosh^{-1}(cx)\left(\cosh\left(2\cosh^{-1}(cx)\right)-8\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right)+\sinh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)^2\left(4+\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)\right)}{8c^5d^{3/2}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*ArcC
osh[c*x] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*
ArcCosh[c*x]] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x
]*Sinh[2*ArcCosh[c*x]])) + b^2*Sqrt[d]*(8*c*x*ArcCosh[c*x]^2 + 8*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])] - Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(4*ArcCosh[c*x]^3 - 2*ArcCosh[c*x]*(Cosh[2*ArcCosh[c
*x]] - 8*Log[1 - E^(-2*ArcCosh[c*x])]) + Sinh[2*ArcCosh[c*x]] + 2*ArcCosh[c
*x]^2*(4 + Sinh[2*ArcCosh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(414) = 828$.
time = 4.51, size = 1141, normalized size = 2.59

method	result
default	$-\frac{a^{2} x^{3}}{2 c^{2} d \sqrt{-c^{2} d \,x^{2}+d}}+\frac{3 a^{2} x}{2 c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}-\frac{3 a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{2 c^{4} d \sqrt{c^{2} d}}-\frac{b^{2} \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{c}}{2 d^{2} c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/2
)-3/2*a^2/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-
1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*arccosh(c*x)*x^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2
*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)
*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b^2*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)^2+1/2*b^2*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
osh(c*x)^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5
/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)^2*x^3-3/2*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh(c*x)^2*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*arccosh(c*x)+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2
*x^2-1)*x^3-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*x+3/2*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos
h(c*x)^2+a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3-1/
2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*arccosh(c*x)-3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcco
sh(c*x)*x+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/
c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*
d) + 3*arcsin(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

3.206 $\int \frac{x^3 (a+b \cosh^{-1}(cx))^2}{(d-c^2 d x^2)^{3/2}} \, dx$

Optimal. Leaf size=413

$$\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3 d\sqrt{d-c^2dx^2}}+\frac{2b^2(1-cx)(1+cx)}{c^4 d\sqrt{d-c^2dx^2}}+\frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c^3 d\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{-1+cx}\,\sqrt{1+}}{c^3 d\sqrt{a}}$$

```
[Out] 2*b^2*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^2*(a+b*arccosh(c*x))^2/
c^2/d/(-c^2*d*x^2+d)^(1/2)+4*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*
d*x^2+d)^(1/2)+4*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2
*d*x^2+d)^(1/2)-2*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/
(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*poly
log(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d/(
-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^4/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A]
time = 0.42, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5934, 5914, 5879, 75, 5912, 5938, 5903, 4267, 2317, 2438}

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2}+\frac{4b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{4abx\sqrt{cx-1}\,\sqrt{cx+1}}{c^3d\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{2b^2(1-cx)(cx+1)}{c^4d\sqrt{d-c^2dx^2}}+\frac{4b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*(1 - c*x)*(1 + c*x))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b^2*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcCosh[c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^
2*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x])^2)/(c^4*d^2) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2]) +
(2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(c^4*d*Sqr
t[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCos
h[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
```

```
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(}{}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx}}$$

Mathematica [A]
time = 1.07, size = 302, normalized size = 0.73

$$\frac{-2a^2(-2+c^2x^2)+2ab\left(-\cosh^{-1}(cx)\left(-3+\cosh\left(2\cosh^{-1}(cx)\right)\right)-2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\tanh\left(\tfrac{1}{2}\cosh^{-1}(cx)\right)\right)+\sinh\left(2\cosh^{-1}(cx)\right)\right)+b^2\left(2+3\cosh^{-1}(cx)^2-2\cosh\left(2\cosh^{-1}(cx)\right)-\cosh^{-1}(cx)^2\cosh\left(2\cosh^{-1}(cx)\right)-4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\log\left(1-e^{-\cosh^{-1}(cx)}\right)+4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)-4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\mathrm{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\mathrm{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)\right)}{2c^4d\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-2*a^2*(-2 + c^2*x^2) + 2*a*b*(-(ArcCosh[c*x]*(-3 + Cosh[2*ArcCosh[c*x]]))
 - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sinh[
2*ArcCosh[c*x]]) + b^2*(2 + 3*ArcCosh[c*x]^2 - 2*Cosh[2*ArcCosh[c*x]] - Arc
Cosh[c*x]^2*Cosh[2*ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
rcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
))/(2*c^4*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(410) = 820$.
time = 4.46, size = 835, normalized size = 2.02

method	result
default	$a^{2}\left(-\frac{x^{2}}{c^{2} d \sqrt{-c^{2} d\, x^{2}+d}}+\frac{2}{d\, c^{4} \sqrt{-c^{2} d\, x^{2}+d}}\right)+\frac{b^{2} \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \operatorname{arccosh}\left(c x\right)^{2} x^{2}}{c^{2} d^{2} \left(c^{2} x^{2}-1\right)}-\frac{2 b^{2} \sqrt{-d \left(c^{2} x^{2}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b^2*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x^2-2*b^2*(-d*(c^2*x^
2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x+
2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*x^2-2*b^2*(-d*(c^2*x^2-1))
^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/
d^2/(c^2*x^2-1)-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^2*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*poly
log(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^4/d^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*a*b*(-d*(c
^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-2*a*b*(-d*(c^2*x^2-1)
)^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-4*a*b*(-d*(c^2*x^
2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/
(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -a*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(-d)
*log(c*x - 1)/(c^5*d^2)) - 2*a*b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqr
t(-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a^2*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2
*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) - b^2*((c^2*x^2 - 2)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))^2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4*d^(3/2)) - integ
rate(2*(c^4*x^4 - 3*c^2*x^2 + (c^3*x^3 - 2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c^5*d^(3/2)*x^2 - c^3*d^(3/
2))*(c*x + 1)*sqrt(c*x - 1) + (c^6*d^(3/2)*x^3 - c^4*d^(3/2)*x)*sqrt(c*x +
1))*sqrt(-c*x + 1)), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}(c\,x)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

3.207 $\int \frac{x^2 (a+b\cosh^{-1}(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=257

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}-$$

```
[Out] x*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x))^3
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(
c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3/d/(-c^2*d*x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5934, 5892, 5912, 5913, 3797, 2221, 2317, 2438}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^3*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]
) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} -$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1\right.}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+c}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+c}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+c}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+c}}{}$$

Mathematica [A]
time = 1.28, size = 270, normalized size = 1.05

$$\frac{3a^2cdx+3a^2\sqrt{d}\,\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+3abd\left(2cx\cosh^{-1}(cx)-\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(\cosh^{-1}(cx)^2+2\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)\right)\right)-b^2d\left(\cosh^{-1}(cx)\left(-3cx\cosh^{-1}(cx)+\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(\cosh^{-1}(cx)\left(3+\cosh^{-1}(cx)\right)+6\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right)\right)-3\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{3c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (3*a^2*c*d*x + 3*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d
*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3*a*b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)])) - b^2*d*(ArcCosh[c*x]*(-3*c*x*ArcCosh[c*x] + Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 - E^(-2*A
rcCosh[c*x])])) - 3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*A
rcCosh[c*x])]))/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(255) = 510.
time = 3.91, size = 738, normalized size = 2.87

method	result
default	$\frac{a^{2} x}{c^{2} d \sqrt{-c^{2} d\,x^{2}+d}}-\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{c^{2} d \sqrt{c^{2} d}}+\frac{b^{2} \sqrt{-d\left(c^{2} x^{2}-1\right)}\,\sqrt{c x-1}\,\sqrt{c x+1}\,\operatorname{arccosh}(cx)}{3 d^{2} c^{3}\left(c^{2} x^{2}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/
2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^3-b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)^2/d^2/c^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2
*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)
*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))+a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-2*a*b*(-d*(c^2*x^2-1))^
(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + integrat
e(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) +
 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}(c\,x)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

3.208 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{-1+cx}\,\sqrt{1+c\cdots}}{c^2d\sqrt{\cdots}}$$

[Out] (a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5914, 5889, 5903, 4267, 2317, 2438}

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{co}}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{co}}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{co}}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.63, size = 210, normalized size = 1.07

$$\frac{a^2+2ab\cosh^{-1}(cx)+b^2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\log\left(1-e^{-\cosh^{-1}(cx)}\right)-\log\left(1+e^{-\cosh^{-1}(cx)}\right)\right)\right)-2ab\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)-2b^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)+2b^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a^2 + 2*a*b*ArcCosh[c*x] + b^2*ArcCosh[c*x]*(ArcCosh[c*x] - 2*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[
c*x])])) - 2*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]
/2]] - 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c
*x])] + 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c
*x])])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(215) = 430$.
time = 1.28, size = 542, normalized size = 2.77

method	result
default	$\frac{a^2}{c^2d\sqrt{-c^2d\,x^2+d}} - \frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\text{arccosh}(cx)^2}{c^2d^2\left(c^2x^2-1\right)} - \frac{2b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccos}}{c^2d^2\left(c^2x^2-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-
1)*arccosh(c*x)^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*po
lylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c^2/d^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] a^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

3.209 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\,\mathrm{l}}{cd\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(c*x))*ln(1-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*
x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5899, 5913, 3797, 2221, 2317, 2438}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^2)/(c*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c*d*S
qrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*Arc
Cosh[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)\coth(x)\,dx,\right.}{cd\sqrt{d-c^2dx^2}}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} + \frac{\left(4b\sqrt{-1+\right.}{}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+c}}{}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+c}}{}$$
$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+c}}{}$$

Mathematica [A]
time = 0.32, size = 126, normalized size = 0.64

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left((a+b\cosh^{-1}(cx))\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\log\left(1+e^{\cosh^{-1}(cx)}\right)\right)-2b^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)-2b^2\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)\right)}{c}}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2 + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c
*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2*b*Log[1 + E^ArcC
osh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*PolyLog[2, E^ArcCosh
[c*x]]))/c)/(d*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(204) = 408$.
time = 1.56, size = 578, normalized size = 2.92

method	result

default	$\frac{a^2x}{d\sqrt{-c^2d\,x^2+d}}-\frac{b^2\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}(cx)^2}{d^2c(c^2x^2-1)}-\frac{b^2\sqrt{-d\,(c^2x^2-1)}\,\mathrm{arccosh}(c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x/d/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/d^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-
1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c
*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x+2*a*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b^2*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2), x) + 2*a*b*x*arccosh(c*x)/
(sqrt(-c^2*d*x^2 + d)*d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{\left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2),x)
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2), x)
```

3.210 $\int \frac{(a+b\cosh^{-1}(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=471

$$\frac{(a+b\cosh^{-1}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{-1+cx}\,\sqrt{1+c}}{}$$

```
[Out] (a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))^2*arctan(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d
)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-2
*I*b*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*po
lylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/
(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/
2)-2*I*b^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5936, 5946, 4265, 2611, 2320, 6724, 5889, 5903, 4267, 2317, 2438}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\cosh^{-1}(cx))^2}{d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]
) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCos
h[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[2, -E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (
(2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*
Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I
*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{(2}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, c\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(}\right.}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 2.42, size = 577, normalized size = 1.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a^2*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a^2*Sqrt[d]*Log[c*x] + a^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + ((2*I)*a*b*d*(I*ArcCosh[c*x] +
 Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c
*x]] - I*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2
*d*x^2] + (b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + 2*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x]
)] + I*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]^2*Log[1 +
I/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 2*PolyLog[2
, -E^(-ArcCosh[c*x])] + (2*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]]
```

```
- (2*I)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 2*PolyLog[2, E^(-ArcCos
h[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (2*I)*PolyLog[3, I/E^Arc
Cosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x\,(-c^2 d\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -a^2*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(
sqrt(-c^2*d*x^2 + d)*d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))^2/((-c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{x\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)), x)
```

3.211 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}}+\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}-\frac{4bc\sqrt{-1+c}$$

```
[Out] -(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccosh(c*x))^2
/d/(-c^2*d*x^2+d)^(1/2)+2*c*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(-c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-4*b*
c*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5932, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5916, 5569, 4267}

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{2c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}}-\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d
*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E
^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*
d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+c^2x^2)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(2c^2\sqrt{\right.}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(4bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+}{d\sqrt{d-c^2dx^2}}$$

<u>Mathematica</u> [A]
time = 1.14, size = 315, normalized size = 0.92

$$\frac{a^2(-1+2c^2x^2)+2ab\left(c^2x^2\cosh^{-1}(cx)+\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-cx\left(\log(cx)+\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right)\right)\right)+b^2\left(\cosh^{-1}(cx)\left(c^2x^2\cosh^{-1}(cx)+\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-2cx\left(\cosh^{-1}(cx)+\log\left(1-e^{-2\cosh^{-1}(cx)}\right)+\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)\right)\right)+cx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+cx\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{dx\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 + 2*c^2*x^2) + 2*a*b*(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(L
og[c*x] + Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(ArcCosh[c*x]*
(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 2*c*x*(ArcCosh[c*x] + Log[1 - E^(-2*
ArcCosh[c*x])] + Log[1 + E^(-2*ArcCosh[c*x])]))) + c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])] + c*x*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(d*x*Sqrt[d - c^2*d*x^2
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(359) = 718$.
time = 2.19, size = 825, normalized size = 2.42

method	result
default	$a^2\left(-\frac{1}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2c^2x}{d\sqrt{-c^2d\,x^2+d}}\right)-\frac{2b^2\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2 c}{(c^2x^2-1)d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))-2*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)
^2*c-2*b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2*x/(c^2*x^2-1)/d^2*c^2+b^2*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^2*x^2-1)/d^2+2*b^2*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1+c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c
+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*a
rccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2
*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c-4*a*b*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*c-4*a*b*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2
)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] a*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d)*l
og(x)/d^2) + 2*(2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*
d*x))*a*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d
*x^2 + d)*d*x))*a^2 + b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^2\left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2}{x^2\,\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=650

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}}+\frac{3c^2\sqrt{-1+cx}\,\sqrt{}}{}$$

```
[Out] 3/2*c^2*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^
2/d/x^2/(-c^2*d*x^2+d)^(1/2)+b*c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d/x/(-c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^
2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c
^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*c^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^
2*d*x^2+d)^(1/2)-3*I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+3*I*b
*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*c^2*polylog(2,c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
+3*I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-3*I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5932, 5936, 5946, 4265, 2611, 2320, 6724, 5889, 5903, 4267, 2317, 2438, 5912, 94, 211}

$$\frac{3c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}-\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}}+\frac{4bc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d\sqrt{d-c^2dx^2}}+\frac{2b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{2b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ib^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{3ib^2c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(d*x*Sqrt[d - c^2*d
*x^2]) + (3*c^2*(a + b*ArcCosh[c*x])^2)/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*
ArcCosh[c*x])^2/(2*d*x^2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x
^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(d*Sqrt[d -
```

```
c^2*d*x^2]) - ((3*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b*c^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyL
og[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b^2*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
- ((3*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])
/(d*Sqrt[d - c^2*d*x^2])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+c^2x^2)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

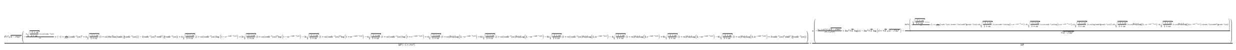
$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\ldots\right.}{2dx^2\sqrt{\ldots}}$$

Mathematica [A]
time = 78.16, size = 979, normalized size = 1.51



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2]*((-2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x])/(c*x) + (-1 + 1/(c^2*x^2))*ArcCosh[c*x]^2 + 4*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 2*ArcCosh[c*x]^2*Cosh[ArcC
osh[c*x]/2]^2 + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 -
 E^(-ArcCosh[c*x])] + (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x]^2*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + (6*I)*Sqrt[(-1 + c*x)/(1 + c
```

```
*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (6*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 4
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + (6*I)
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (6*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, I/E^ArcCosh[c*x]] + 2*Ar
cCosh[c*x]^2*Sinh[ArcCosh[c*x]/2]^2))/(2*d^2*(-1 + c^2*x^2)) + (a*(-((a*(-1
 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(x^2*(-1 + c^2*x^2))) + 3*a*c^2*Sqrt[d]*
Log[x] - 3*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*
(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^2*x^2))*ArcCo
sh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + 2*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3*I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]
*Sinh[ArcCosh[c*x]/2]^2))/Sqrt[d - c^2*d*x^2]))/(2*d^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}{x^{3} \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2)
 - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a^2 + i
ntegrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(3/
2)*x^3) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3
/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^3\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2}{x^3\,{\left(d - c^2\,d\,x^2\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

3.213 $\int \frac{(a+b\cosh^{-1}(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=496

$$\frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{3dx^2\sqrt{d-c^2dx^2}}-\frac{(a+b\cosh^{-1}(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}-\frac{4c^2(a+b\cosh^{-1}(cx$$

```
[Out] 1/3*b^2*c^2*(-c*x+1)*(c*x+1)/d/x/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x)
)^2/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2
+d)^(1/2)+8/3*c^4*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+
b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2/(-c^2*d*x^2+d)^(1/2)+8/3*
c^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
-20/3*b*c^3*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-16/3*b*c^3*(a+b*arccosh
(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c^3*polylo
g(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^
2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5932, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5916, 5569, 4267, 97}

$$-\frac{4c^2(a+b\cosh^{-1}(cx))^2}{3dx\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{3dx^2\sqrt{d-c^2dx^2}}-\frac{(a+b\cosh^{-1}(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\cosh^{-1}(cx))^2}{3d\sqrt{d-c^2dx^2}}+\frac{8c^3\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^2}{3d\sqrt{d-c^2dx^2}}-\frac{16bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{3d\sqrt{d-c^2dx^2}}-\frac{20bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{3d\sqrt{d-c^2dx^2}}+\frac{b^2c^2(1-cx)(cx+1)}{3dx\sqrt{d-c^2dx^2}}-\frac{5b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d*x^2*Sqrt[d - c^2*d*x^2]) - (a
 + b*ArcCosh[c*x])^2/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[
c*x])^2)/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*
d*Sqrt[d - c^2*d*x^2]) + (8*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^2)/(3*d*Sqrt[d - c^2*d*x^2]) - (20*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2
]) - (16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^
(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (5*b^2*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (b
^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt
[d - c^2*d*x^2])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+c^2x^2)}\,dx}{3d\sqrt{d-c^2dx^2}} - \frac{\left(4c^2\sqrt{\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\right.}{3dx\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cos\right.}{3dx^3\sqrt{d}}$$

Mathematica [A]
time = 1.70, size = 529, normalized size = 1.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 - 4*c^2*x^2 + 8*c^4*x^4) + a*b*(6*c^4*x^4*ArcCosh[c*x] + Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*(c*x + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
rcCosh[c*x]) + 2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(5*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(5*Log[c*x] + 3*Log[Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(c^2*x^2 - c^4*x^4 + c*x*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 3*c^4*x^4*ArcCosh[c*x]^2 + (-1 + c*
x)*(1 + c*x)*ArcCosh[c*x]^2 + 5*c^2*x^2*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x]^2
```

```
 - 8*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*c^3*x^
3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c
*x])] - 10*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1
+ E^(-2*ArcCosh[c*x])] + 5*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + 3*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d*x^3*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2866 vs. $2(486) = 972$.
time = 3.64, size = 2867, normalized size = 5.78

method	result	size
default	Expression too large to display	2867

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ar
ccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+64/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^2*(c*x-1)^(1/2)*arccosh(c*x)^2*(c*x
+1)^(1/2)*c^5-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*
(c*x-1)*arccosh(c*x)*(c*x+1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*
x^4-7*c^2*x^2-1)/x^2*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*c+2*b^2*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)*c^3+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7
*c^2*x^2-1)*x^5*(c*x-1)*arccosh(c*x)*(c*x+1)*c^8-32/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x-1)*arccosh(c*x)*(c*x+1)*c^6-64/3
*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10+32*a*b*(-d
*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8-8*a*b*(-d*(c^2*x^2-
1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6-8/3*a*b*(-d*(c^2*x^2-1))^(1/2
)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c
^4*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^
4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8
*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)^2*c^4-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d
^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4+4*b^2*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)^2*c^2-128/3*a*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+16*a*b*(-d*(c^2*x^2
-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4-8/3*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8
*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)*c^2+
128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^2*(c*x-1)^(1
/2)*arccosh(c*x)*(c*x+1)^(1/2)*c^5-7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^
4*x^4-7*c^2*x^2-1)*x*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^
```

```
2*x^2-1)/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x
^3*arccosh(c*x)^2-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-
1)*x^7*c^10+40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5
*c^8-32/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x
^2-1)*arccosh(c*x)*c^3+64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2
*x^2-1)*x^5*(c*x-1)*(c*x+1)*c^8-32/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*
x^4-7*c^2*x^2-1)*x^3*(c*x-1)*(c*x+1)*c^6-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2
/(8*c^4*x^4-7*c^2*x^2-1)*x*(c*x-1)*(c*x+1)*c^4+16/3*a*b*(-d*(c^2*x^2-1))^(1
/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*c^
3-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x^2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*c+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3+10/3*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c
^4*x^4-7*c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*x)^2*(c*x+1)^(1/2)*c^3-8/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x-1)^(1/2)*arccosh(c*
x)*(c*x+1)^(1/2)*c^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+32/3*b^2
*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*x-1)*(c*x+1)*c^8
-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x-1)*(c*
x+1)*c^6-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*c^5-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)^2*c^3+2*b^2*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*c^3+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/d^2/(c^2*x^2-1)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-1/3*
b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*c^3-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7
*arccosh(c*x)*c^10+32*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1
)*x^5*arccosh(c*x)*c^8-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2
*x^2-1)*x^3*arccosh(c*x)^2*c^6+a^2*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2
*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x^4(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{x^4\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

3.214 $\int \frac{x^5 \left(a+b \cosh^{-1}(cx)\right)^2}{\left(d-c^2 d x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=568

$$-\frac{b^2 x^2}{3 c^4 d^2 \sqrt{d-c^2 d x^2}}-\frac{16 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}-\frac{7 b^2 (1-c x) (1+c x)}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{16 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \cosh^{-1}}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}$$

```
[Out] 1/3*x^4*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*x^2/c^4/d^2
/(-c^2*d*x^2+d)^(1/2)-7/3*b^2*(-c*x+1)*(c*x+1)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)
-4/3*x^2*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arccosh(c*x)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arc
cosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x
^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-c^
2*d*x^2+d)^(1/2)-22/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-11/3*b^2*
polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6
/d^2/(-c^2*d*x^2+d)^(1/2)+11/3*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*(a+b*arccos
h(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A]
time = 0.79, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5934, 5914, 5879, 75, 5912, 5938, 5903, 4267, 2317, 2438, 100, 21}

$$\frac{x^4 \left(a+b \cosh^{-1}(cx)\right)^2}{3 c^2 d \left(d-c^2 d x^2\right)^{3/2}}-\frac{8 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)^2}{3 c^6 d^3}-\frac{22 b \sqrt{cx-1} \sqrt{cx+1} \tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right) \left(a+b \cosh^{-1}(cx)\right)}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{16 a b x \sqrt{cx-1} \sqrt{cx+1}}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}+\frac{11 b x \sqrt{cx-1} \sqrt{cx+1} \left(a+b \cosh^{-1}(cx)\right)}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}-\frac{4 x^2 \left(a+b \cosh^{-1}(cx)\right)^2}{3 c^4 d^2 \sqrt{d-c^2 d x^2}}+\frac{b x^3 \sqrt{cx-1} \sqrt{cx+1} \left(a+b \cosh^{-1}(cx)\right)}{3 c^3 d^2 \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}}-\frac{11 b^2 \sqrt{cx-1} \sqrt{cx+1} \text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}+\frac{11 b^2 \sqrt{cx-1} \sqrt{cx+1} \text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{7 b^2 (1-cx) (cx+1)}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{16 b^2 x \sqrt{cx-1} \sqrt{cx+1} \cosh^{-1}(cx)}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}-\frac{b^2 x^2}{3 c^4 d^2 \sqrt{d-c^2 d x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x^2)/(c^4*d^2*Sqrt[d - c^2*d*x^2]) - (16*a*b*x*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*(1 - c*x)*(1 + c*x))/(
3*c^6*d^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Arc
Cosh[c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (11*b*x*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x]))/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b*x^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt
[d - c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3
/2)) - (4*x^2*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (8*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^6*d^2*
Sqrt[d - c^2*d*x^2]) - (11*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^A
```

```
rcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) + (11*b^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
```

```
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(\ldots\right.}{(-1+cx)^{3/2}(1+\ldots}}{3c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{\ldots}{3c^2d\ldots}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{11bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-\ldots}}{\ldots}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{11b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} + \ldots$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{3b^2(1-cx)(1+cx)}{c^6d^2\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \ldots$$

Mathematica [A]
time = 3.62, size = 437, normalized size = 0.77

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/24*(8*a^2*(8 - 12*c^2*x^2 + 3*c^4*x^4) + 2*a*b*(25*ArcCosh[c*x] - 36*Arc
Cosh[c*x]*Cosh[2*ArcCosh[c*x]] + 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 33*S
```

```
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 4*Sinh[2*Ar
cCosh[c*x]] + 11*Log[Tanh[ArcCosh[c*x]/2]]*Sinh[3*ArcCosh[c*x]] - 3*Sinh[4*
ArcCosh[c*x]]) + b^2*(22 + 25*ArcCosh[c*x]^2 - 4*(7 + 9*ArcCosh[c*x]^2)*Cos
h[2*ArcCosh[c*x]] + 3*(2 + ArcCosh[c*x]^2)*Cosh[4*ArcCosh[c*x]] - 66*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 66
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x]
)] + 88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*
x])] - 88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c
*x])] + 8*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + 22*ArcCosh[c*x]*Log[1 - E^(-A
rcCosh[c*x])]*Sinh[3*ArcCosh[c*x]] - 22*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*
x])]*Sinh[3*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(c^6*d*(d
 - c^2*d*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(531) = 1062$.
time = 5.54, size = 1212, normalized size = 2.13

method	result	size
default	Expression too large to display	1212

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-
2/3/d/c^4/(-c^2*d*x^2+d)^(3/2)))+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*
x^2-1)-b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arccosh(c*x)^2*x^2+2*
b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)^2*x^2-11/3*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ar
ccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+11/3*b^2*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))+11/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*a
rccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-11/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*x^2+b^2*(-d
*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^
2-1)^2/c^4*x^2-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh
(c*x)^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arccosh(c*x)*x^2+2
*a*b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)+4*a*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)*x^2+1/3*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-10/3*a*b*
```

```
(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)+11/3*a*b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(1+c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))-11/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*a^2*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^(
3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) - 1/3*(3*b^2*c^4*sqrt(d)*x^
4 - 12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c*x + 1)*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*
d^3) - integrate(2/3*((12*b^2*c^3*x^3 + 3*(a*b*c^5 - b^2*c^5)*x^5 - 8*b^2*c
*x)*(c*x + 1)*sqrt(c*x - 1) + (15*b^2*c^4*x^4 + 3*(a*b*c^6 - b^2*c^6)*x^6 -
 20*b^2*c^2*x^2 + 8*b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(c^12*d^(5/2)*x^7 - 3*c^10*d^(5/2)*x^5 + 3*c^8*d^(5/2)*x
^3 - c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 - 3*c^9*d^(5/2)*x^4 + 3*c^7*d^(5/2)*
x^2 - c^5*d^(5/2))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{acosh}{(cx)})^2}{(-d (cx - 1) (cx + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

3.215 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=482

$$-\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^5/d^2/(-c
^2*d*x^2+d)^(1/2)+1/3*b^2*(-c*x+1)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-x*(a+b*arcc
osh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-4/3*(a+
b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+
1/3*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/d^2/(-c^2*d*x^2+
d)^(1/2)+8/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*b^2*polylog(2
,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(
-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5934, 5892, 5912, 5913, 3797, 2221, 2317, 2438, 91, 12, 79, 54}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^5d^2\sqrt{d-c^2dx^2}}-\frac{4\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{8b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^5d^2\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{4b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}}-\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^5*d^2*Sqrt[d
 - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d^2*
Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c
*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c
*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (x*(a + b*ArcCosh[c*x])^2)/(c^4*d
^2*Sqrt[d - c^2*d*x^2]) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
x])^2)/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x])^3)/(3*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (8*b*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c^5*d^2
*Sqrt[d - c^2*d*x^2]) + (4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2
*ArcCosh[c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}\right.}{(-1+cx)^{3/2}(1+}}{c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3c^2d^2}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x(a+}{c^4d^2}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x(a+}{c^4d^2}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\,c}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(c}{3c^5d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(c}{3c^5d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 1.71, size = 382, normalized size = 0.79

$$\frac{\frac{a^2cx(-3+4c^2x^2)\sqrt{d-c^2dx^2}}{(-1+c^2x^2)^2} - 3a^2\sqrt{d}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \frac{abd\left(-8cx\cosh^{-1}(cx) - \frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+2cx\cosh^{-1}(cx)}{-1+c^2x^2} + \sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(3\cosh^{-1}(cx)^2+8\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right)\right)}{\sqrt{d-c^2dx^2}} + \frac{b^2d\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(-\frac{cx\left(-1+c^2x^2+(-3+4c^2x^2)\cosh^{-1}(cx)^2\right)}{\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3} + \cosh^{-1}(cx)\left(\frac{1}{1-c^2x^2}+\cosh^{-1}(cx)(4+\cosh^{-1}(cx))+8\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right) - 4\,\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{\sqrt{d-c^2dx^2}}}{3c^5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 3*a^2*Sq
rt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (a*b*d*(
-8*c*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh
[c*x])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x
]^2 + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2] +
(b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((c*x*(-1 + c^2*x^2 + (-3 + 4
*c^2*x^2)*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3)) + Ar
```

```
cCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x]*(4 + ArcCosh[c*x]) + 8*Log[1
- E^(-2*ArcCosh[c*x])]) - 4*PolyLog[2, E^(-2*ArcCosh[c*x])]))/Sqrt[d - c^2*
d*x^2])/(3*c^5*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4073 vs. $2(446) = 892$.
time = 4.53, size = 4074, normalized size = 8.45

method	result	size
default	Expression too large to display	4074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 168*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^
2+16)/c/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x^4-440/3*a*b*(-d*(c^2
*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c
*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x^2+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*arc
cosh(c*x)^2*(c*x+1)^(1/2)+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^
6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)
^(1/2)-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(
c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+8/3*b^2*(-d*(c^2*x^2
-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^2-a
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*a
rccosh(c*x)^2+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^3/c^5/(c^2*x^2-1)*arccosh(c*x)+28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8
-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x-1)*(c*x+1)*x^3+8*a*b*(-
d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^
3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-
87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x-1)*(c*x+1)*x-13*a*b*(-d*
(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^
3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*
x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*arccosh(c*x
)*(c*x+1)^(1/2)-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-17*b^2*(-d*(c
^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*x^5-
a^2/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a^2/c^4/d^2/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)+32*b^
2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*
c^2/d^3*arccosh(c*x)^2*x^7+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c
^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7+16/3*b^2*(-d*(c^
2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^
6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x-1)*(c*x+1)*x^5+181/3*b^2*(-d*(c^2
```

```
*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*ar
ccosh(c*x)^2*x^3+40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)*x^3-16*b^2*(-d*(c^2*x^2-1))^(1
/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)^
2*x-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*
x^2+16)/c^4/d^3*arccosh(c*x)*x+64*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87
*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7-16/3*a*b*(-d*(
c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*
x-1)*(c*x+1)*x^5+362/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+11
8*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)*x^3-32*a*b*(-d*(c^2*x^2-1))^(
1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)
*x+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^
2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-64*a*b*(-d*(c^2*x^2-1))^(1/2)
/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*(c*x-1)^(1/2)*arcc
osh(c*x)*(c*x+1)^(1/2)*x^6-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c
^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x-1)*arccosh(c*x)*(c*x+1)*x^5-1/3*
b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*
arccosh(c*x)^3-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^3/c^5/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-8*b^2*(-d*(c^
2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*x^6-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x-1)*(c*x+1)*x^3+21*b^2*(-d*(
c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-8
7*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x-1)*(c*x+1)*x-55/3*b^2*(-d
*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d
^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))-32*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4
*x^4-71*c^2*x^2+16)*c/d^3*(c*x-1)^(1/2)*arccosh(c*x)^2*(c*x+1)^(1/2)*x^6+84
*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)/c/d^3*(c*x-1)^(1/2)*arccosh(c*x)^2*(c*x+1)^(1/2)*x^4+28/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x
-1)*arccosh(c*x)*(c*x+1)*x^3+8*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^
6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(
1/2)*x^4-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^
4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a^2
 + integrate(b^2*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 +
 d)^(5/2) + 2*a*b*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 +
d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{acosh}(c\,x)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

3.216 $\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\cosh^{-1}(c\right.}{3c^4d^2\sqrt{d-c^2d}}$$

```
[Out] 1/3*x^2*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(-c
^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+1/3
*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-
c^2*d*x^2+d)^(1/2)-10/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2
*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
4/d^2/(-c^2*d*x^2+d)^(1/2)+5/3*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5934, 5914, 5889, 5903, 4267, 2317, 2438, 5912, 75}

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{10b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{5b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{5b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^2*
(a + b*ArcCosh[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCosh
[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (10*b*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d
*x^2]) - (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(
3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-}\right.}{(-1+cx)^{3/2}(1}}{3c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{}{3c^2d^2(}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2(a+b}{3c^4d^2}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2(a+b}{3c^4d^2}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2(a+b}{3c^4d^2}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2(a+b}{3c^4d^2}$$

Mathematica [A]
time = 3.00, size = 341, normalized size = 1.01

$$\frac{4a^2(-2+3c^2x^2)-b^2\left(2+2\cosh^{-1}(cx)^2-2(1+3\cosh^{-1}(cx)^2)\cosh\left(2\cosh^{-1}(cx)\right)-15\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)\left(\log\left(1-e^{-\cosh^{-1}(cx)}\right)-\log\left(1+e^{-\cosh^{-1}(cx)}\right)\right)+20\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\mathrm{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-20\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\mathrm{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)-2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)+5\cosh^{-1}(cx)\log\left(1-e^{-\cosh^{-1}(cx)}\right)\sinh\left(3\cosh^{-1}(cx)\right)-5\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)\sinh\left(3\cosh^{-1}(cx)\right)\right)-ab\left(\cosh^{-1}(cx)\left(4-12\cosh\left(2\cosh^{-1}(cx)\right)\right)-2\sinh\left(2\cosh^{-1}(cx)\right)+5\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)\left(-3\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)+\sinh\left(3\cosh^{-1}(cx)\right)\right)\right)}{12c^4d\left(d-c^2dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2*(-2 + 3*c^2*x^2) - b^2*(2 + 2*ArcCosh[c*x]^2 - 2*(1 + 3*ArcCosh[c*x]
^2)*Cosh[2*ArcCosh[c*x]] - 15*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x]*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[c*x])]) + 20*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] - 20*((-1
+ c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] - 2*ArcCo
sh[c*x]*Sinh[2*ArcCosh[c*x]] + 5*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])]*Si
nh[3*ArcCosh[c*x]] - 5*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*ArcCo
sh[c*x]]) - a*b*(ArcCosh[c*x]*(4 - 12*Cosh[2*ArcCosh[c*x]]) - 2*Sinh[2*ArcC
osh[c*x]] + 5*Log[Tanh[ArcCosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x) + Sinh[3*ArcCosh[c*x]]))))/(12*c^4*d*(d - c^2*d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(331) = 662$.
time = 3.50, size = 834, normalized size = 2.48

method	result
default	$a^2\left(\frac{x^2}{c^2 d(-c^2 d x^2+d)^{\frac{3}{2}}}-\frac{2}{3 d c^4(-c^2 d x^2+d)^{\frac{3}{2}}}\right)+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)^2x^2}{d^3(c^2x^2-1)^2c^2}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccos}}{3d^3(c^2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b^2*(-d
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)^2*x^2+1/3*b^2*(-d*(c
^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*x^2-2/3*b^2*(-d
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^3/c^4/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-5/3*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh(
c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c
*x)*x^2+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccos
h(c*x)+5/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(
c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-5/3*a*b*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/6*a*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(
c^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 2/3*a*b*(3*x^2/((-c^2*d*x^2
 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a
^2*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)
) + b^2*integrate(x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2
+ d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

3.217 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+}{3cd^2\left(1-c^2x^2\right)\sqrt{d-}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^3/d^2/(-c^2*d
*x^2+d)^(1/2)+1/3*b^2*(-c*x+1)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x^2*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*x^2+1)/(-c^2*d*x^2
+d)^(1/2)-1/3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^
2*d*x^2+d)^(1/2)+2/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*p
olylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5917, 5912, 5934, 5913, 3797, 2221, 2317, 2438, 91, 12, 79, 54}

$$\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}}-\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^3*d^2*Sqrt[d
- c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3*d^2*
Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c
*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x
])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^2)/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c^3*d^2*Sqrt[d
 - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[
c*x])])/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rule 5934

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(2*e*(p + 1))), x] + (-Dist[f^2*((m - 1)/(2*e*(p + 1)))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], In
t[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(\right.}{(-1+c^2x^2)^2}}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3}{3d^2(1-}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3}{3d^2(1-}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2d}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx}{3c^3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx}{3c^3d^2\sqrt{d-c^2dx^2}}$$

<u>Mathematica</u> **[A]**
time = 1.10, size = 264, normalized size = 0.68

$$\frac{\frac{a^2c^3x^3}{1-c^2x^2} + ab\left(\frac{2c^3x^3\cosh^{-1}(cx)}{1-c^2x^2} + \frac{\sqrt{\frac{-1+cx}{1+cx}}\left(-1+2\left(-1+c^2x^2\right)\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)\right)}{-1+cx}\right) + b^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(-\frac{cx\left(-1+c^2x^2+c^2x^2\cosh^{-1}(cx)^2\right)}{\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3} + \cosh^{-1}(cx)\left(\frac{1}{1-c^2x^2} + \cosh^{-1}(cx) + 2\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right) - \mathrm{PolyLog}\left(2, e^{-2\cosh^{-1}(cx)}\right)\right)}{3c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c^3*x^3)/(1 - c^2*x^2) + a*b*((2*c^3*x^3*ArcCosh[c*x])/(1 - c^2*x^2)
+ (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + 2*(-1 + c^2*x^2)*Log[Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)*(-((c*x*(-1 + c^2*x^2 + c^2*x^2*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^
(3/2)*(1 + c*x)^3)) + ArcCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x] + 2*L
og[1 - E^(-2*ArcCosh[c*x])]) - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c^3*d^2
*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. $2(363) = 726$.
time = 3.83, size = 3466, normalized size = 8.91

method	result	size
default	Expression too large to display	3466

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)/d^3*(c*x+1)*(c*x-1)*x^3+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)^2*x^7+1/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7
-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^
2/d^3*arccosh(c*x)^2*x^5-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^
6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(
c*x+1)^(1/2)+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)/c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)-2/3*a*b*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*ln((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)-1/3*a*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(c*x-1)
*x^5+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x
^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+
4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c
/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)*x^4-b^2*(-d*(c^2*x^2-1))^(1/2
)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*arccosh(c*x)*x^2-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)/d^3*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-
5*c^2*x^2+1)/c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)+1/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(c*x+1)*
(c*x-1)*arccosh(c*x)*x^3-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^3/c^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2/3
*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)
*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(
3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7-2*a*b*
(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3
*arccosh(c*x)*x^5+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)/d^3*(c*x+1)*(c*x-1)*x^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(
```

```
3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)*c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)^2*x^6+b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*arccosh(c*x)*x^4-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*
x)*x^2+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2
*x^2+1)*c^4/d^3*x^7-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*
c^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-
9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3+1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^
3-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x
^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(
c*x-1)*arccosh(c*x)*x^5+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)^2*x^4
-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*arccosh(c*x)^2*x^2-2*a*b*(-d*(c^2*x^2-1
))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*arccosh(c*x)*x^6+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9
*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x^7-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5+1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)^2*x
^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*
x^2-1)*arccosh(c*x)^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5-b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^6+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5
*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+1/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^2/d^3*(c*x+1)*(c*x-1)*
x-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^
3) - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*c^
```

```
2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a^2*(x/(sqrt(
-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrat
e(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^2}{(-d (cx-1)(cx+1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

3.218 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{-1+cx}\,\sqrt{1+c}}{}$$

```
[Out] 1/3*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^2/d^2/(-c^2*d
*x^2+d)^(1/2)+1/3*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/
(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+2/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)
^(1/2)+1/3*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5914, 5889, 5901, 5903, 4267, 2317, 2438, 75}

$$\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*b^2/(c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*
ArcCosh[c*x])^2/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]]
)/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyL
og[2, E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$=\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}+\frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(-1+c^2x^2\right)^2}}{3cd^2\sqrt{d-c^2dx^2}}$$

$$=\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}$$

$$=-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{(a}{3c^2d^2(1-}$$

$$=-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{(a}{3c^2d^2(1-}$$

$$=-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{(a}{3c^2d^2(1-}$$

$$=-\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{(a}{3c^2d^2(1-}$$

Mathematica [A]
time = 1.65, size = 332, normalized size = 1.11

$$\frac{4a^2+b^2\left(-2+4\cosh^{-1}(cx)^2+2\cosh\left(2\cosh^{-1}(cx)\right)-3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)\log\left(1-e^{-\cosh^{-1}(cx)}\right)+3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)+4\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\text{PolyLog}\left(2,-e^{-\cosh^{-1}(cx)}\right)-4\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\text{PolyLog}\left(2,e^{-\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\sinh\left(2\cosh^{-1}(cx)\right)+\cosh^{-1}(cx)\log\left(1-e^{-\cosh^{-1}(cx)}\right)\sinh\left(3\cosh^{-1}(cx)\right)-\cosh^{-1}(cx)\log\left(1+e^{-\cosh^{-1}(cx)}\right)\sinh\left(3\cosh^{-1}(cx)\right)\right)+ab\left(8\cosh^{-1}(cx)+2\sinh\left(2\cosh^{-1}(cx)\right)+\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)\left(-3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+\sinh\left(3\cosh^{-1}(cx)\right)\right)\right)}{12c^2d\left(d-c^2dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2 + b^2*(-2 + 4*ArcCosh[c*x]^2 + 2*Cosh[2*ArcCosh[c*x]] - 3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 3*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] -
 4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] +
 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x
])]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*A
rcCosh[c*x]]) + a*b*(8*ArcCosh[c*x] + 2*Sinh[2*ArcCosh[c*x]] + Log[Tanh[Arc
Cosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sinh[3*ArcCosh[c*x
]])))/(12*c^2*d*(d - c^2*d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(297) = 594$.
time = 1.47, size = 720, normalized size = 2.42

method	result
default	$\frac{a^2}{3c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}} + \frac{b^2\sqrt{-d\,(c^2x^2-1)}\,\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,x}{3d^3(c^2x^2-1)^2c} + \frac{b^2\sqrt{-d\,(c^2x^2-1)}\,x^2}{3d^3(c^2x^2-1)^2} + \frac{b^2\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*
x^2-1)^2/c*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^3/(c^2*x^2-1)^2*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2
-1)^2/c^2*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c
^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*
x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*b^2*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*polylog(2,-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x
+1)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c
^2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+2/3*a*b*(-d
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)+1/3*a*b*(-d*(c^2*x^2
-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)-1)-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
 + a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a + b\operatorname{acosh}(c\,x))^2}{(d - c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(-c^2*d*x^2
+d)^(1/2)+2/3*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*ar
ccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(
1/2)+2/3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2
+d)^(1/2)-4/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*b^2*polylog(2,
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^
2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5901, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5914, 39}

$$\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{2b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b^2*x)/(d^2*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x*(a + b
*ArcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcCosh[c*x])^2
)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])^2)/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*c*d^2*Sqrt[d - c^2
*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x]
)])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3}}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{x(}{3d^2(1-}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh\right.}{3d^2\sqrt{d-}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh\right.}{3d^2\sqrt{d-}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh\right.}{3d^2\sqrt{d-}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh\right.}{3d^2\sqrt{d-}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh\right.}{3d^2\sqrt{d-}}$$

Mathematica [**A**]
time = 0.99, size = 289, normalized size = 0.87

$$\frac{\frac{a^2cx(-3+2c^2x^2)}{-1+c^2x^2}+ab\left(2cx\left(2+\frac{1}{1-c^2x^2}\right)\cosh^{-1}(cx)+\frac{\sqrt{\frac{-1+cx}{1+cx}}\left(-1+(4-4c^2x^2)\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right)}{-1+cx}\right)+b^2\left(-\frac{\cosh^{-1}(cx)\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+cx\cosh^{-1}(cx)\right)}{-1+c^2x^2}+cx(-1+2\cosh^{-1}(cx)^2)-2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1-e^{-2\cosh^{-1}(cx)}\right)\right)+2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\text{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 2*c^2*x^2))/(-1 + c^2*x^2) + a*b*(2*c*x*(2 + (1 - c^2*x^2)^
(-1))*ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + (4 - 4*c^2*x^2)*Log[
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*(-((ArcCosh[c*x]*
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + c*x*ArcCosh[c*x]))/(-1 + c^2*x^2))
+ c*x*(-1 + 2*ArcCosh[c*x]^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x]*(ArcCosh[c*x] + 2*Log[1 - E^(-2*ArcCosh[c*x])]) + 2*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c*d^2*Sqrt[d -
c^2*d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3048 vs. $2(313) = 626$.
time = 2.16, size = 3049, normalized size = 9.21

method	result	size
default	Expression too large to display	3049

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccos
h(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*
x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*x^5-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/
c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+4/3*b^2*(-
d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*
x)*(c*x+1)*(c*x-1)*x^5+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+1
1*c^2*x^2-4)*c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+4/3*b^2
*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arcco
sh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-10/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)
*x^3-13/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^
2/d^3*x^3-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/
d^3*arccosh(c*x)^2*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*
c^2*x^2-4)/d^3*arccosh(c*x)*x-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*
c^4*x^4+11*c^2*x^2-4)*c^6/d^3*x^7+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6
```

```
-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*(c*x+1)*(c*x-1)*x^5-10/3*a*b*(-d*(c^2*x^2
-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(c*x+1)*(c*x-1)*x^3+
a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^2+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4
*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+4/3*a*b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*ln((c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x
^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^2+4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3/d^
3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-4*a*b*(-d*(c^2*x^2-1))^(1/2)
/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5+34/3*a*b*(-d*
(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)
*x^3+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(
c*x+1)*(c*x-1)*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^
2*x^2-4)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*x+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+8/3*b^
2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(
c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^
6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-4
/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)
*arccosh(c*x)^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d^3/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^
2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)*(c*x+1
)*(c*x-1)*x+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/
c/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*(c*x+1)*(c*x-1)*x^
5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3/d^3*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*
c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(c*x+1)*(c*x-1)*x^3-8/3*a*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)-4/3*b^2*(-
d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*arccosh(c*
x)*x^7-2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4
/d^3*arccosh(c*x)^2*x^5+14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x
^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5+17/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(
3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)^2*x^3-16/3*b^2*(-d*
(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)
*x^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d
^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10
*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x+1)*(c*x-1)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(
3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*
c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*x^7+14/3*a*b*(-d*(c^2*x^2-1))^(1/2
)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*x^5-16/3*a*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3-8*a*b*(-d*(c^2*x^2-1
))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)*x+a^2*(1/3*x/
```

```
d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*
d^3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 +
 d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a^2*(2*x/(sqrt(
-c^2*d*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```

3.220 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=597

$$-\frac{b^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}+$$

```
[Out] 1/3*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(-c^2*d*x^2+d)^
(1/2)+(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*x*(a+b*arccosh(
c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+2*(
a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+14/3*b*(a+b*arccosh(c*x))*arctanh(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d
)^(1/2)+7/3*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b*(a+b*arccosh(c*x))*polylog(2,-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d
*x^2+d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-7/3*b^2*pol
ylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c
^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(3,I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2
+d)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5936, 5946, 4265, 2611, 2320, 6724, 5889, 5903, 4267, 2317, 2438, 5901, 75}

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{ArcTan}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}-\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{bcx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}+\frac{14b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{7b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{7b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{2ib^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_3\left(ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{b^2}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*b^2/(d^2*Sqrt[d - c^2*d*x^2]) + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*ArcC
osh[c*x])^2/(3*d*(d - c^2*d*x^2)^(3/2)) + (a + b*ArcCosh[c*x])^2/(d^2*Sqrt[
d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Ar
cTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (14*b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2
*d*x^2]) + (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])
/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) +
```

```
((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^
ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((2*I)*b^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh
[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3}}}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{(a}{3d^2(1-}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{(a+b\cosh}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{(a+b\cosh}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{(a+b\cosh}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{(a+b\cosh}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{(a+b\cosh}{d^2\sqrt{d-}}$$

Mathematica [A]

time = 9.30, size = 806, normalized size = 1.35

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c^
2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2
*x^2))]])/d^(5/2) + (a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c
*x]*Coth[ArcCosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]
*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]]
- 28*Log[Tanh[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (
24*I)*PolyLog[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*
x]*Sinh[ArcCosh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*
ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2]))/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
) + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-4*Coth[ArcCosh[c*x]/2] + 14
*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] - 2*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^
2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Csch[ArcCosh[c*x]/
2]^4)/2 - 56*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (24*I)*ArcCosh[c*x]^
2*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x
]] + 56*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 56*PolyLog[2, -E^(-ArcCos
h[c*x])] - (48*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (48*I)*Arc
Cosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 56*PolyLog[2, E^(-ArcCosh[c*x])] -
 (48*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] + (48*I)*PolyLog[3, I/E^ArcCosh[c*x
]] - 2*ArcCosh[c*x]*Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]^2*Sinh[ArcCosh
[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) + 4*Tanh[ArcCosh[c*x
]/2] - 14*ArcCosh[c*x]^2*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[-(d*(-1 + c*x)
*(1 + c*x))])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}{x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*a^2*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2)
 - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + integrate
(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x) +
2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{x\left(-d(cx-1)(cx+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{x\,(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)), x)
```

3.221 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=476

$$-\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\left(d-c^2dx^2\right)^{3/2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx\right.}{3d\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] -(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccosh(c*x))
^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*c^2*x/d^2/(-c^2*d*x^2+d)^(1/2)+8/3*c^2*x*
(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+8/3*c*(a+b*
arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-4*b*c*
(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*b*c*(a+b*arccosh(c*x))*ln(1-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*
d*x^2+d)^(1/2)-b^2*c*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*c*polylog(2,(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)
^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5932, 5901, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5914, 39, 5936, 5916, 5569, 4267}

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}-\frac{16bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{5b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b^2*c^2*x)/(d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (a +
b*ArcCosh[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2)) + (4*c^2*x*(a + b*ArcCosh[c*x
])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCosh[c*x])^2)/(3*d^2
*Sqrt[d - c^2*d*x^2]) + (8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) -
 (16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d -
 c^2*d*x^2])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(c\cdots}{x(-1+c^2x^2)^2\cdots}}{d^2\sqrt{d-c^2dx^2}}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx\cdots}}\cdots$$

$$= \frac{b^2c^2x}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh\cdots\right.}{3d^2\sqrt{d-c\cdots}}\cdots$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\,c\cdots\right.}{3d^2\sqrt{d\cdots}}\cdots$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\,c\cdots\right.}{3d^2\sqrt{d\cdots}}\cdots$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\,c\cdots\right.}{3d^2\sqrt{d\cdots}}\cdots$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\,c\cdots\right.}{3d^2\sqrt{d\cdots}}\cdots$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\,c\cdots\right.}{3d^2\sqrt{d\cdots}}\cdots$$

<u>Mathematica</u> [A]
time = 2.02, size = 457, normalized size = 0.96

$$\frac{c\left(\frac{a^2(3-12c^2x^2+8c^4x^4)}{cx(-1+c^2x^2)}+ab\left(10cx\cosh^{-1}(cx)-\frac{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+2cx\cosh^{-1}(cx)}{-1+c^2x^2}-2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(-\frac{3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{cx}+3\log(cx)+5\log\left(\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\right)\right)\right)+b^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(\frac{cx\sqrt{\frac{-1+cx}{1+cx}}}{1-cx}+\frac{\cosh^{-1}(cx)}{1-c^2x^2}-8\cosh^{-1}(cx)^2-\frac{cx\cosh^{-1}(cx)^2}{\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3}+\frac{5cx\cosh^{-1}(cx)^2}{\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}+\frac{3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)^2}{cx}-10\cosh^{-1}(cx)\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-6\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)+3\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+5\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(cx)}\right)\right)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (c*((a^2*(3 - 12*c^2*x^2 + 8*c^4*x^4))/(c*x*(-1 + c^2*x^2)) + a*b*(10*c*x*A
rcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x])/(
-1 + c^2*x^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((-3*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) + 3*Log[c*x] + 5*Log[Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)])) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((c*
x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 8*Ar
```

```
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (5*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (3*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 10*ArcCosh[c*x]*
Log[1 - E^(-2*ArcCosh[c*x])] - 6*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])]
+ 3*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 5*PolyLog[2, E^(-2*ArcCosh[c*x])])))
/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3793 vs. $2(470) = 940$.
time = 2.02, size = 3794, normalized size = 7.97

method	result	size
default	Expression too large to display	3794

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^
2*x/(-c^2*d*x^2+d)^(1/2)))-8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c
^4*x^4+26*c^2*x^2-9)*x*(c*x-1)*(c*x+1)*c^2+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d
^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+
48*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c+10/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2-1)*c-88*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-
9)*x*arccosh(c*x)*c^2-3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^
4+26*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-32/3*a*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+64/3*a*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x-1)*(c*x
+1)*c^8-160/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x
^2-9)*x^5*(c*x-1)*(c*x+1)*c^6+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*
c+24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arc
cosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c-3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/
(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*c+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)
*x^7*(c*x-1)*(c*x+1)*c^8-88/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*
c^4*x^4+26*c^2*x^2-9)*x^5*(c*x-1)*(c*x+1)*c^6+80/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x-1)*(c*x+1)*c^4-8/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*c^5-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*
x^4+26*c^2*x^2-9)*x*(c*x-1)*(c*x+1)*c^2+17/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3
/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-16
/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*a
rccosh(c*x)^2*c+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

```
/d^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+b^2*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+40*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x
^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x-1)*(c*x+1)*c^4+10/3*b^2*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*c-160/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*
x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*(c*x-1)*(c*x+1)*c^6-136/3*b^2
*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*arccosh
(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)*(c*x-1)*(c*x+1)*c^2-32/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10+
40*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*c
^8-160/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)
*x^5*c^6+29*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2
-9)*x^3*c^4-5*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x
^2-9)*x*c^2+9*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x
^2-9)/x*arccosh(c*x)^2+56*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*
x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)^2*c^4+48*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/
(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-44*b^2*(-d*(c^2*x^
2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)^2*c^2-8*
b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccos
h(c*x)*c^2-3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*arccosh(c*x)*c^10+224/3*b^2*(-d*(c^2*x^
2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*arccosh(c*x)*c^8-64
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*a
rccosh(c*x)^2*c^6-280/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^
4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^4*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*c^5+40*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)*x^3*arccosh(c*x)*(c*x-1)*(c*x+1)*c^4+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*c^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c
^2*x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-128/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccos
h(c*x)*c^6+112*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*
x^2-9)*x^3*arccosh(c*x)*c^4+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-
25*c^4*x^4+26*c^2*x^2-9)*x^7*arccosh(c*x)*(c*x-1)*(c*x+1)*c^8+2*a*b*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a^2*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/
2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{x^2 \left(-d\,(cx-1)\,(cx+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{x^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

3.222 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=796

$$-\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx\right.}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

```
[Out] 5/6*c^2*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/2*(a+b*arccosh(c*x))^
2/d/x^2/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)+5/2*c^2*(
a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+b*c*(a+b*arccosh(c*x))*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d
)^(1/2)+5*c^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*arctan((c*x-1)
^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+
26/3*b*c^2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+13/3*b^2*c^2*polylog(2,-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+
d)^(1/2)-5*I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+5*I*b*c^2*(
a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-13/3*b^2*c^2*polylog(2,c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)+5*I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5*I*b^2*c^2*polylog(3,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)
```

Rubi [A]

time = 1.01, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 19, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {5932, 5936, 5946, 4265, 2611, 2320, 6724, 5889, 5903, 4267, 2317, 2438, 5901, 75, 5912, 106, 21, 94, 211}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x]))/(d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*b*c^
3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)
*Sqrt[d - c^2*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(6*d*(d - c^2*d*x^2)
```

```
^(3/2)) - (a + b*ArcCosh[c*x])^2/(2*d*x^2*(d - c^2*d*x^2)^(3/2)) + (5*c^2*(
a + b*ArcCosh[c*x])^2)/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d -
c^2*d*x^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*S
qrt[1 + c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (26*b*c^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*
x^2]) + (13*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]
])/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x
^2]) + ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyL
og[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (13*b^2*c^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2])
+ ((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x
]])/(d^2*Sqrt[d - c^2*d*x^2]) - ((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*PolyLog[3, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
```

```
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx}{x^2(-1+c^2x^2)^2}}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)^2}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} -$$

$$= -\frac{b^2c^2}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+c}}{3d^2\,(1}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+}}{3d^2\,(}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+}}{3d^2\,(}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+}}{3d^2\,(}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+}}{3d^2\,(}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+}}{3d^2\,(}$$

<u>Mathematica</u> [**A**]
time = 86.49, size = 1181, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^3*x^2) + (a^2*c^2)/(3*d^3*(-1 + c^2*
x^2)^2) - (2*a^2*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a^2*c^2*Log[x])/(2*d^(5/2)
) - (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (a
*b*c^2*((6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*(-1 + c*x)*(1 +
 c*x)*ArcCosh[c*x])/(c^2*x^2) + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 - Co
```

```
th[ArcCosh[c*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*I)
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]
] - 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (30
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] +
(30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] -
26*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x
]*Tanh[ArcCosh[c*x]/2]^2))/(6*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]) - (b^2*c
^2*Sqrt[d - c^2*d*x^2]*((12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x])/(c*x) + 6*(1 - 1/(c^2*x^2))*ArcCosh[c*x]^2 - 24*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 4*Cosh[ArcCosh[c*x]/2]^2 + 26*
ArcCosh[c*x]^2*Cosh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]
 - ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2]^2 - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (30*I)*Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 5
2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x
])] - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])
] - (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I
)/E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x
]*PolyLog[2, I/E^ArcCosh[c*x]] + 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Po
lyLog[2, E^(-ArcCosh[c*x])] - (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*P
olyLog[3, (-I)/E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)*PolyLog[3, I/E^ArcCosh[c*x]] + 4*Sinh[ArcCosh[c*x]/2]^2 - 26*ArcCosh[c*x]
^2*Sinh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2] - ArcCosh[c
*x]^2*Tanh[ArcCosh[c*x]/2]^2))/(12*d^3*(-1 + c^2*x^2))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^{2}}{x^{3}\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/6*a^2*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^
(5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d
) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + integrate(b^2*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x^3) + 2*a*b*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{x^{3} \left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

3.223 $\int \frac{(a+b\cosh^{-1}(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=562

$$\frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}}-\frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{3d^2x^2(1-c^2x^2)\sqrt{d-c^2dx^2}}-\frac{(a+b\cosh^{-1}(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}-2c^2$$

```
[Out] -1/3*(a+b*arccosh(c*x))^2/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arccosh(c*x
))^2/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+
d)^(3/2)+1/3*b^2*c^2/d^2/x/(-c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(-c^2*d*x
^2+d)^(1/2)+16/3*c^4*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*
c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x^2/(-c^2*x^2+1)/(-c^2
*d*x^2+d)^(1/2)+16/3*c^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^2/(-c^2*d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)-32/3*b*c^3*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*
d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 1.01, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5932, 5901, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5914, 39, 5936, 5916, 5569, 4267, 105, 12}

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))}{3d^2x^2(1-c^2x^2)\sqrt{d-c^2dx^2}}-\frac{2c^2(a+b\cosh^{-1}(cx))^2}{dx(d-c^2dx^2)^{3/2}}-\frac{(a+b\cosh^{-1}(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}+\frac{16c^4x(a+b\cosh^{-1}(cx))^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\cosh^{-1}(cx))^2}{3d(d-c^2dx^2)^{3/2}}+\frac{16c^3\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^2}{3d^2\sqrt{d-c^2dx^2}}-\frac{32bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d-c^2dx^2}}-\frac{32bc^3\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}}-\frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}}-\frac{8b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{8b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*x*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d - c^2
*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*x
^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(3*d*x^3*(d
- c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*ArcCosh[c*x])^2)/(d*x*(d - c^2*d*x^2)^(
3/2)) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (16*
c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (16*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) -
 (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*A
rcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d -
c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcC
```

```
osh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5916

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[-d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5936

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCosh[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1
)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b
*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f
*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || Eq
Q[n, 1])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^2}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}}{x^3(-1+c^2}}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+}}{3d^2x}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} + \frac{8b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 2.50, size = 534, normalized size = 0.95

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))/(x^3*(-1 + c^2*x^2)) + a*b
*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(1/(c^2*x^2) + (1 - c^2*x^2)^(-1)
 + (2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
```

```
)*ArcCosh[c*x])/(c^3*x^3*(-1 + c*x)^3) - 16*Log[c*x] - 16*Log[Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)]) + b^2*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((
c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x))/(c*x) + ArcCosh[c*x]/(c^2*x^2) + ArcCosh[c*x]/(1 - c^2*x^2) - 16*Ar
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (8*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c^3*x^3) + (8*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 16*ArcCosh[c*x]*Log[1 - E
^(-2*ArcCosh[c*x])] - 16*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 8*Poly
Log[2, -E^(-2*ArcCosh[c*x])] + 8*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d^2*S
qrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5246 vs. $2(544) = 1088$.
time = 3.73, size = 5247, normalized size = 9.34

method	result	size
default	Expression too large to display	5247

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*b*c*(8*c^2*sqrt(-d)*log(c*x + 1)/d^3 + 8*c^2*sqrt(-d)*log(c*x - 1)/d^
3 + 16*c^2*sqrt(-d)*log(x)/d^3 + sqrt(-d)/(c^2*d^3*x^4 - d^3*x^2)) + 2/3*(1
6*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6
*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*a
rccosh(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x
^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)
^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2
/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(cx))^2}{x^4(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{x^4\,(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

3.224 $\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=429

$$-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}+\frac{4\sqrt{-1+ax}\,\sqrt{1+}}{15ac^3\left(1-a^2x^2\right.}$$

```
[Out] 1/5*x*arccosh(a*x)^2/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)^2/c^2/(-a^2
*c*x^2+c)^(3/2)-1/3*x/c^3/(-a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(-a*x+1)/(a*x+1)/
(-a^2*c*x^2+c)^(1/2)+8/15*x*arccosh(a*x)^2/c^3/(-a^2*c*x^2+c)^(1/2)+1/10*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^
(1/2)+4/15*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^
2*c*x^2+c)^(1/2)+8/15*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^
2*c*x^2+c)^(1/2)-16/15*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^
2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/15*polylog(2,(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*
c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5901, 5899, 5913, 3797, 2221, 2317, 2438, 5912, 5914, 39, 40}

$$-\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^2}{15c^3\sqrt{c-a^2cx^2}}+\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{15ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}-\frac{16\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\log\left(1-e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{4x\cosh^{-1}(ax)^2}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x\cosh^{-1}(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/3*x/(c^3*Sqrt[c - a^2*c*x^2]) - x/(30*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a
^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(10*a*c^3*(1 - a^2
*x^2)^2*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]
)/(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^2)/(5*c*(c
 - a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x]^2)/(15*c^2*(c - a^2*c*x^2)^(3/2))
+ (8*x*ArcCosh[a*x]^2)/(15*c^3*Sqrt[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]*ArcCosh[a*x]^2)/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (16*Sqrt[-1 + a
*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[1 - E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[
c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, E^(2*ArcCosh[a
*x])])/(15*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
```

```
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,c}{5c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4}{15c^3(1-}$$

$$= -\frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+}}{15ac^3}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-}}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-}}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-}}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-}}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-}}$$

Mathematica [A]
time = 0.94, size = 220, normalized size = 0.51

$$-\frac{ax\left(10+\frac{1}{1-a^2x^2}\right)+2\left(8\sqrt{\frac{-1+ax}{1+ax}}+ax\left(-8+8\sqrt{\frac{-1+ax}{1+ax}}-\frac{3}{(-1+a^2x^2)^2}+\frac{4}{-1+a^2x^2}\right)\right)\cosh^{-1}(ax)^2+\frac{\left(\frac{-1+ax}{1+ax}\right)^{3/2}\cosh^{-1}(ax)\left(-11+8a^2x^2+32(-1+a^2x^2)^2\log\left(1-e^{-2\cosh^{-1}(ax)}\right)\right)}{(-1+ax)^3}-16\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\mathrm{PolyLog}\left(2,e^{-2\cosh^{-1}(ax)}\right)}{30ac^3\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/30*(a*x*(10 + (1 - a^2*x^2)^(-1)) + 2*(8*Sqrt[(-1 + a*x)/(1 + a*x)] + a*
x*(-8 + 8*Sqrt[(-1 + a*x)/(1 + a*x)] - 3/(-1 + a^2*x^2)^2 + 4/(-1 + a^2*x^2
)))*ArcCosh[a*x]^2 + (((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]*(-11 + 8*a^
2*x^2 + 32*(-1 + a^2*x^2)^2*Log[1 - E^(-2*ArcCosh[a*x])]))/(-1 + a*x)^3 - 1
6*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*PolyLog[2, E^(-2*ArcCosh[a*x])])/(a*
c^3*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(395) = 790$.
time = 3.88, size = 794, normalized size = 1.85

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(8 a^{5} x^{5}-20 a^{3} x^{3}-8 \sqrt{a x+1}\, \sqrt{a x-1}\, a^{4} x^{4}+15 a x+16 \sqrt{a x+1}\, \sqrt{a x-1}\, a^{2} x^{2}-8 \sqrt{a x-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-20*a^3*x^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*a^4*x^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(-64*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^7*x^7-64*ar
ccosh(a*x)*a^8*x^8-32*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^7*x^7-32*a^8*x^8+248*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5+280*arccosh(a*x)*a^6*x^6+126
*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^5*x^5+142*a^6*x^6+80*arccosh(a*x)^2*a^4*x^4-
340*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-456*arccosh(a*x)*a^4*x
^4-156*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3-265*a^4*x^4-190*arccosh(a*x)^2*a
^2*x^2+165*a*x*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+328*arccosh(a*x)*a^
2*x^2+62*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+235*a^2*x^2+128*arccosh(a*x)^2-88*
arccosh(a*x)-80)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*
x^2-64)/a/c^4-16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/
a/(a^2*x^2-1)*arccosh(a*x)^2+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2
-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^
2-1)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1
)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))
^(1/2)/c^4/a/(a^2*x^2-1)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^2/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{2,
[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1
,[4,2,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\operatorname{acosh}(a\,x)}^2}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^(7/2), x)
```

3.225 $\int \frac{x^4 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=243

$$-\frac{15x\sqrt{1-ax}\,\sqrt{1+ax}}{64a^4}-\frac{x^3\sqrt{1-ax}\,\sqrt{1+ax}}{32a^2}+\frac{15\sqrt{-1+ax}\,\cosh^{-1}(ax)}{64a^5\sqrt{1-ax}}-\frac{3x^2\sqrt{-1+ax}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-ax}}-$$

```
[Out] 15/64*arccosh(a*x)*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-3/8*x^2*arccosh(a*x)*(a
*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/8*x^4*arccosh(a*x)*(a*x-1)^(1/2)/a/(-a*x+1
)^(1/2)+1/8*arccosh(a*x)^3*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-15/64*x*(-a*x+1
)^(1/2)*(a*x+1)^(1/2)/a^4-1/32*x^3*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-3/8*x*a
rccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arccosh(a*x)^2*(-a^2*x^2+1)^(1
/2)/a^2
```

Rubi [A]
time = 0.22, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5938, 5892, 5883, 92, 54, 102, 12}

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^3}{8a^5\sqrt{1-ax}}+\frac{15\sqrt{ax-1}\,\cosh^{-1}(ax)}{64a^5\sqrt{1-ax}}-\frac{15x\sqrt{1-ax}\,\sqrt{ax+1}}{64a^4}-\frac{3x^2\sqrt{ax-1}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-ax}\,\sqrt{ax+1}}{32a^2}-\frac{x^3\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{4a^2}-\frac{3x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{8a^4}-\frac{x^4\sqrt{ax-1}\,\cosh^{-1}(ax)}{8a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-15*x*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(64*a^4) - (x^3*Sqrt[1 - a*x]*Sqrt[1 +
a*x])/(32*a^2) + (15*Sqrt[-1 + a*x]*ArcCosh[a*x])/(64*a^5*Sqrt[1 - a*x]) -
(3*x^2*Sqrt[-1 + a*x]*ArcCosh[a*x])/(8*a^3*Sqrt[1 - a*x]) - (x^4*Sqrt[-1 +
a*x]*ArcCosh[a*x])/(8*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x
]^2)/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(4*a^2) + (Sqrt[-1 +
a*x]*ArcCosh[a*x]^3)/(8*a^5*Sqrt[1 - a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
```

```
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^2}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+a}}}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-a}{}$$

$$= -\frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\,\sqrt{1+}{8a\sqrt{1-a}}$$

$$= -\frac{3x(1-ax)(1+ax)}{16a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}}$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{16a^5\sqrt{1-a^2x^2}} -$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{15\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{64a^5\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.21, size = 116, normalized size = 0.48

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(32\cosh^{-1}(ax)^3-4\cosh^{-1}(ax)\left(16\cosh\left(2\cosh^{-1}(ax)\right)+\cosh\left(4\cosh^{-1}(ax)\right)\right)+32\sinh\left(2\cosh^{-1}(ax)\right)+\sinh\left(4\cosh^{-1}(ax)\right)+8\cosh^{-1}(ax)^2\left(8\sinh\left(2\cosh^{-1}(ax)\right)+\sinh\left(4\cosh^{-1}(ax)\right)\right)\right)}{256a^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(32*ArcCosh[a*x]^3 - 4*ArcCosh[a*x]*(
16*Cosh[2*ArcCosh[a*x]] + Cosh[4*ArcCosh[a*x]]) + 32*Sinh[2*ArcCosh[a*x]] +
 Sinh[4*ArcCosh[a*x]] + 8*ArcCosh[a*x]^2*(8*Sinh[2*ArcCosh[a*x]] + Sinh[4*A
rcCosh[a*x]])))/(256*a^5*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(199) = 398.
time = 4.58, size = 488, normalized size = 2.01

method	result

default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{arccosh}(ax)^3}{8a^5(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+1}\,\left(8a^5x^5-12a^3x^3+8\sqrt{ax+1}\,\sqrt{ax-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccosh
(a*x)^3-1/512*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*a^4*x^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(8*arccosh(a*x)^2-4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)-1/16*(-
a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-(a*
x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1)/a^5/(a^2*x^2-
1)-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a
^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arccosh(a*x)+1)/a^5
/(a^2*x^2-1)-1/512*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3-8*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*a^4*x^4+4*a*x+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-(a*x-1)^
(1/2)*(a*x+1)^(1/2))*(8*arccosh(a*x)^2+4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{acosh}^2\,(ax)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.226 $\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=177

$$-\frac{40\sqrt{1-ax}\,\sqrt{1+ax}}{27a^4}-\frac{2x^2\sqrt{1-ax}\,\sqrt{1+ax}}{27a^2}-\frac{4x\sqrt{-1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-ax}}-\frac{2x^3\sqrt{-1+ax}\,\cosh^{-1}(ax)}{9a\sqrt{1-ax}}-$$

```
[Out] -4/3*x*arccosh(a*x)*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-2/9*x^3*arccosh(a*x)*(
a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-40/27*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-2/27*x
^2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-2/3*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a
^4-1/3*x^2*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.16, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5938, 5914, 5879, 75, 5883, 102, 12}

$$-\frac{40\sqrt{1-ax}\,\sqrt{ax+1}}{27a^4}-\frac{4x\sqrt{ax-1}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-ax}}-\frac{2x^2\sqrt{1-ax}\,\sqrt{ax+1}}{27a^2}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{3a^4}-\frac{2x^3\sqrt{ax-1}\,\cosh^{-1}(ax)}{9a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(27*a^4) - (2*x^2*Sqrt[1 - a*x]*Sqrt[1 +
a*x])/(27*a^2) - (4*x*Sqrt[-1 + a*x]*ArcCosh[a*x])/(3*a^3*Sqrt[1 - a*x]) -
(2*x^3*Sqrt[-1 + a*x]*ArcCosh[a*x])/(9*a*Sqrt[1 - a*x]) - (2*Sqrt[1 - a^2*x
^2]*ArcCosh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(3*a^2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+}}}{3a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{2x^3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-}{}$$

$$= -\frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\,\sqrt{1}}{9a\sqrt{1-}}$$

$$= -\frac{4(1-ax)(1+ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}}$$

$$= -\frac{40(1-ax)(1+ax)}{27a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax}{3a^3\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.11, size = 123, normalized size = 0.69

$$\left(-\frac{40}{27a^4}-\frac{2x^2}{27a^2}\right)\sqrt{1-a^2x^2} + \frac{2x\sqrt{1-a^2x^2}\,(6+a^2x^2)\cosh^{-1}(ax)}{9a^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\sqrt{1-a^2x^2}\,(2+a^2x^2)\cosh^{-1}(ax)^2}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40/(27*a^4) - (2*x^2)/(27*a^2))*Sqrt[1 - a^2*x^2] + (2*x*Sqrt[1 - a^2*x^2
]*(6 + a^2*x^2)*ArcCosh[a*x])/(9*a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqrt[
1 - a^2*x^2]*(2 + a^2*x^2)*ArcCosh[a*x]^2)/(3*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(145) = 290.
time = 3.26, size = 343, normalized size = 1.94

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(4a^4x^4-5a^2x^2+4\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+1\right)\left(9\mathrm{arccosh}(ax)^2-6\,\mathrm{arc}\right.}{216a^4(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2+4*(a*x+1)^(1/2)*(a*x-1)^(1/2
)*a^3*x^3-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^2-6*arccosh(
a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x+a^2*x^2-1)*(arccosh(a*x)^2-2*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2
*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2
*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*
x^2-4*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x
+1)*(9*arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 105, normalized size = 0.59

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}\left(ax\right)^2+\frac{2\left(-i\sqrt{a^2x^2-1}\,x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)}{27\,a^2}+\frac{2\left(i\,a^2x^3+6i\,x\right)\operatorname{arcosh}\left(ax\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^2
 + 2/27*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)/a^2 + 2/9*(
I*a^2*x^3 + 6*I*x)*arccosh(a*x)/a^3
```

Fricas [A]
time = 0.35, size = 150, normalized size = 0.85

$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)+2\left(a^4x^4+19\,a^2x^2-20\right)\sqrt{-a^2x^2+1}}{27\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^2 - 6*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1)) + 2*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1))/(a
^6*x^2 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\operatorname{acosh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

<u>**Giac**</u> [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.227 $\int \frac{x^2 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=151

$$-\frac{x\sqrt{1-ax}\,\sqrt{1+ax}}{4a^2}+\frac{\sqrt{-1+ax}\,\cosh^{-1}(ax)}{4a^3\sqrt{1-ax}}-\frac{x^2\sqrt{-1+ax}\,\cosh^{-1}(ax)}{2a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{2a^2}+\sqrt{}$$

```
[Out] 1/4*arccosh(a*x)*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/2*x^2*arccosh(a*x)*(a*x
-1)^(1/2)/a/(-a*x+1)^(1/2)+1/6*arccosh(a*x)^3*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1
/2)-1/4*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-1/2*x*arccosh(a*x)^2*(-a^2*x^2+1
)^(1/2)/a^2
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5938, 5892, 5883, 92, 54}

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^3}{6a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{2a^2}-\frac{x\sqrt{1-ax}\,\sqrt{ax+1}}{4a^2}-\frac{x^2\sqrt{ax-1}\,\cosh^{-1}(ax)}{2a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/4*(x*Sqrt[1 - a*x]*Sqrt[1 + a*x])/a^2 + (Sqrt[-1 + a*x]*ArcCosh[a*x])/(4
*a^3*Sqrt[1 - a*x]) - (x^2*Sqrt[-1 + a*x]*ArcCosh[a*x])/(2*a*Sqrt[1 - a*x])
 - (x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a
*x]^3)/(6*a^3*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}}{2a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+}}{}$$

$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\,\mathrm{co}}{2a^2\sqrt{1-a^2x}}$$

$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{4a^3\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{2a\sqrt{1-a^2x^2}}$$

Mathematica [A]

time = 0.13, size = 87, normalized size = 0.58

$$-\frac{\sqrt{-((-1+ax)(1+ax))}\,(4\cosh^{-1}(ax)^3-6\cosh^{-1}(ax)\cosh(2\cosh^{-1}(ax))+(3+6\cosh^{-1}(ax)^2)\sinh(2\cosh^{-1}(ax)))}{24a^3\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/24*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(4*ArcCosh[a*x]^3 - 6*ArcCosh[a*x]*Cos
h[2*ArcCosh[a*x]] + (3 + 6*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]]))/(a^3*Sqrt
[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A]
time = 4.02, size = 239, normalized size = 1.58

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{arccosh}(ax)^3}{6a^3(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+1}\,\left(2a^3x^3-2ax+2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2\right.}{16a^3(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^3-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1
)/a^3/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x-2*(a*x+1)^(1/2)*
(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arcc
osh(a*x)+1)/a^3/(a^2*x^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\operatorname{acosh}(a\,x)}^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.228 $\int \frac{x \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{1-ax}\,\sqrt{1+ax}}{a^2}-\frac{2x\sqrt{-1+ax}\,\cosh^{-1}(ax)}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{a^2}$$

```
[Out] -2*x*arccosh(a*x)*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-2*(-a*x+1)^(1/2)*(a*x+1)^(
1/2)/a^2-arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5914, 5879, 75}

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{a^2}-\frac{2\sqrt{1-ax}\,\sqrt{ax+1}}{a^2}-\frac{2x\sqrt{ax-1}\,\cosh^{-1}(ax)}{a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/a^2 - (2*x*Sqrt[-1 + a*x]*ArcCosh[a*x])/(a
*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)/a^2
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \cosh^{-1}(ax)\,dx}{a\sqrt{1-a^2x^2}}$$

$$= -\frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+\right.}{}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2}\left(-2+\frac{2ax\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}-\cosh^{-1}(ax)^2\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + (2*a*x*ArcCosh[a*x])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - ArcCosh[a*x]^2))/a^2
```

Maple [A]
time = 2.33, size = 139, normalized size = 1.76

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)\left(\operatorname{arccosh}(ax)^2-2\operatorname{arccosh}(ax)+2\right)}{2a^2(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^2-2*arccosh(a*x)+2)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-
(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2*arccosh(a*x)+2)/a^2/(a
^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.28, size = 50, normalized size = 0.63

$$\frac{2i\,x\,\mathrm{arcosh}\,(ax)}{a}-\frac{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)^2}{a^2}-\frac{2i\,\sqrt{a^2x^2-1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*x*arccosh(a*x)/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/a^2 - 2*I*sqrt(a^2
*x^2 - 1)/a^2
```

Fricas [A]
time = 0.35, size = 114, normalized size = 1.44

$$\frac{2\,\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) +
(-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(a^2*x^2 - 1)*sqrt(
-a^2*x^2 + 1))/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,\mathrm{acosh}^2\,(ax)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 76, normalized size = 0.96

$$-\frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^2}-\frac{2i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\sqrt{a^2x^2-1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^2/a^2 - 2*I*(x*log(a*x + s
qrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a)/a
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

3.229 $\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=32

$$\frac{\sqrt{-1+ax}\cosh^{-1}(ax)^3}{3a\sqrt{1-ax}}$$

```
[Out] 1/3*arccosh(a*x)^3*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5892}

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^3}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a*x])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}} \, dx}{\sqrt{1-a^2x^2}} \\ &= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.88, size = 51, normalized size = 1.59

method	result	size
default	$-\frac{\sqrt{-\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a x -1}\, \sqrt{a x +1}\, \mathrm{arccosh}\left(a x \right)^{3}}{3 a \left(a^{2} x^{2}-1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{2}{\left(a x \right)}}{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
[Out] int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

3.230 $\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{-1+ax}\,\cosh^{-1}(ax)^2\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{2i\sqrt{-1+ax}\,\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{-}}{}$$

```
[Out] 2*arccosh(a*x)^2*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*
x+1)^(1/2)-2*I*arccosh(a*x)*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+2*I*arccosh(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+2*I*polylog(3,-I*(a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-2*I*polylog(3,I*(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5946, 4265, 2611, 2320, 6724}

$$\frac{2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{2i\sqrt{ax-1}\,\cosh^{-1}(ax)\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\,\cosh^{-1}(ax)\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{2i\sqrt{ax-1}\,\text{Li}_3\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
2*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
a*x] + ((2*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqr
t[1 - a*x] + ((2*I)*Sqrt[-1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1
- a*x] - ((2*I)*Sqrt[-1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x^2\mathrm{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}\\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(2i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Su}}{}\\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-}}{\sqrt{1-a^2}}\\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-}}{\sqrt{1-a^2}}\\
&= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-}}{\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 151, normalized size = 0.83

$$\frac{i\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(-\cosh^{-1}(ax)^2\left(\log\left(1-ie^{-\cosh^{-1}(ax)}\right)-\log\left(1+ie^{-\cosh^{-1}(ax)}\right)\right)-2\cosh^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)\right)-2\text{PolyLog}\left(3,-ie^{-\cosh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,ie^{-\cosh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-(ArcCosh[a*x]^2*(Log[1 - I/E^ArcC
osh[a*x]] - Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)/E
^ArcCosh[a*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^ArcCos
h[a*x]] + 2*PolyLog[3, I/E^ArcCosh[a*x]]))/Sqrt[1 - a^2*x^2]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.231 $\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=124

$$\frac{a\sqrt{-1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{x}-\frac{2a\sqrt{-1+ax}\,\cosh^{-1}(ax)\log\left(1+e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{a\sqrt{}}{}$$

```
[Out] a*arccosh(a*x)^2*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-2*a*arccosh(a*x)*ln(1+(a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-a*polylog(2,-(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-arccosh(a*x)
^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5917, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{x}-\frac{a\sqrt{ax-1}\,\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{a\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{\sqrt{1-ax}}-\frac{2a\sqrt{ax-1}\,\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}+1\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/Sqrt[1 - a*x] - (Sqrt[1 - a^2*x^2]*ArcCos
h[a*x]^2)/x - (2*a*Sqrt[-1 + a*x]*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])])
/Sqrt[1 - a*x] - (a*Sqrt[-1 + a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])])/Sqrt[1
- a*x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x\tanh(x)\,dx,\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(4a\sqrt{-1+\right.}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}}{}$$

Mathematica [A]
time = 0.34, size = 111, normalized size = 0.90

$$\frac{a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(\cosh^{-1}(ax)\left(-\cosh^{-1}(ax)+\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\cosh^{-1}(ax)}{ax}-2\log\left(1+e^{-2\cosh^{-1}(ax)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)\right)}{\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sqr
t[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 2*Log[1 + E^(-2*Arc
Cosh[a*x])]) + PolyLog[2, -E^(-2*ArcCosh[a*x])]))/Sqrt[-((-1 + a*x)*(1 + a*
x))]
```

Maple [A]
time = 2.59, size = 241, normalized size = 1.94

method	result

default	$-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-1\right)\operatorname{arccosh}(ax)^2}{x(a^2x^2-1)}-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arcc}}{a^2x^2-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^2/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^2*a+2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+(-a^2*x^2+1
)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(2,-(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))^2)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(2*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((sqrt(a*x + 1)*a*x^2 + (a*x + 1)*
sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.232 $\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=296

$$\frac{a\sqrt{-1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{2x^2}+\frac{a^2\sqrt{-1+ax}\,\cosh^{-1}(ax)^2\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{a^2\sqrt{-}}{}$$

```
[Out] a*arccosh(a*x)*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)+a^2*arccosh(a*x)^2*arctan(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-a^2*arctan((a*x-
1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-I*a^2*arccosh(a*x)*pol
ylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+I
*a^2*arccosh(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1
/2)/(-a*x+1)^(1/2)+I*a^2*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a
*x-1)^(1/2)/(-a*x+1)^(1/2)-I*a^2*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5932, 5946, 4265, 2611, 2320, 6724, 5883, 94, 211}

$$-\frac{a^2\sqrt{ax-1}\,\text{ArcTan}\left(\sqrt{ax-1}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}}+\frac{a^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{ia^2\sqrt{ax-1}\,\text{Li}_3\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{2x^2}+\frac{a\sqrt{ax-1}\,\cosh^{-1}(ax)}{x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x])/(x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2]*ArcC
osh[a*x]^2)/(2*x^2) + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a
*x]])/Sqrt[1 - a*x] - (a^2*Sqrt[-1 + a*x]*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*
x]])/Sqrt[1 - a*x] - (I*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, (-I)*E^A
rcCosh[a*x]])/Sqrt[1 - a*x] + (I*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2,
 I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (I*a^2*Sqrt[-1 + a*x]*PolyLog[3, (-I)*E
^ArcCosh[a*x]])/Sqrt[1 - a*x] - (I*a^2*Sqrt[-1 + a*x]*PolyLog[3, I*E^ArcCos
h[a*x]])/Sqrt[1 - a*x]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
```

```
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-\right.}{}\\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\right.}{}\\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}}{}\\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}}{}\\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}}{}\\
&= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.72, size = 233, normalized size = 0.79

$$
\frac{ia^2\sqrt{-((-1+ax)(1+ax))}\left(\frac{2i\cosh^{-1}(ax)}{ax}+\frac{i\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\cosh^{-1}(ax)^2}{a^2x^2}-4i\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(ax)\right)\right)+\cosh^{-1}(ax)^2\log\left(1-ie^{-\cosh^{-1}(ax)}\right)-\cosh^{-1}(ax)^2\log\left(1+ie^{-\cosh^{-1}(ax)}\right)+2\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\cosh^{-1}(ax)}\right)+2\mathrm{PolyLog}\left(3,-ie^{-\cosh^{-1}(ax)}\right)-2\mathrm{PolyLog}\left(3,ie^{-\cosh^{-1}(ax)}\right)\right)}{2\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/2)*a^2*Sqrt[-((-1 + a*x)*(1 + a*x))]*(((2*I)*ArcCosh[a*x])/(a*x) + (I*S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^2)/(a^2*x^2) - (4*I)*ArcTa
n[Tanh[ArcCosh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - ArcCos
h[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^ArcCo
sh[a*x]] - 2*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 2*PolyLog[3, (-I)/
E^ArcCosh[a*x]] - 2*PolyLog[3, I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*
x)]*(1 + a*x))
```

Maple [**F**]
time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccosh}(ax)^2}{x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{acosh}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.233 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=1154

$$-\frac{10b^2c^2d^2(fx)^{3+m}\sqrt{d-c^2dx^2}}{f^3(4+m)^3(6+m)}-\frac{2b^2c^2d^2(52+15m+m^2)(fx)^{3+m}(1-c^2x^2)\sqrt{d-c^2dx^2}}{f^3(4+m)^2(6+m)^3(1-cx)(1+cx)}+\frac{2b^2c^4d^2(fx)^{5+}}{f^5(6+}$$

```
[Out] 5*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/f/(4+m)/(6+m)+(f*
x)^(1+m)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/f/(6+m)-10*b^2*c^2*d^2*(
f*x)^(3+m)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3/(6+m)-2*b^2*c^2*d^2*(m^2+15*m+5
2)*(f*x)^(3+m)*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^2/(6+m)^3/(-c*x+
1)/(c*x+1)+2*b^2*c^4*d^2*(f*x)^(5+m)*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/f^5/
(6+m)^3/(-c*x+1)/(c*x+1)+15*d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))^2*(-c^2*d*x^
2+d)^(1/2)/f/(6+m)/(m^2+6*m+8)-2*b*c*d^2*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/f^2/(2+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-30*b*c*d^2*(f
*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-10*b*c*d^2*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*
d*x^2+d)^(1/2)/f^2/(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+10*b*c^3*d
^2*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(6+m)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+4*b*c^3*d^2*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*
d*x^2+d)^(1/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b*c^5*d^2*(f*x
)^(6+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-30*b^2*c^2*d^2*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2
*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(4+m
)/(6+m)/(-c*x+1)/(c*x+1)-10*b^2*c^2*d^2*(10+3*m)*(f*x)^(3+m)*hypergeom([1/2
, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f
^3/(4+m)^3/(6+m)/(m^2+5*m+6)/(-c*x+1)/(c*x+1)-2*b^2*c^2*d^2*(15*m^2+130*m+2
64)*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1
)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^2/(6+m)^3/(m^2+5*m+6)/(-c*x+1)/(c*x+
1)+15*d^3*Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
/(6+m)/(m^2+6*m+8)
```

Rubi [A]
time = 1.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-10*b^2*c^2*d^2*(f*x)^(3 + m)*Sqrt[d - c^2*d*x^2])/(f^3*(4 + m)^3*(6 + m))
 - (2*b^2*c^2*d^2*(52 + 15*m + m^2)*(f*x)^(3 + m)*(1 - c^2*x^2)*Sqrt[d - c^
```

```
2*d*x^2])/(f^3*(4 + m)^2*(6 + m)^3*(1 - c*x)*(1 + c*x)) + (2*b^2*c^4*d^2*(f
*x)^(5 + m)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(f^5*(6 + m)^3*(1 - c*x)*(1
+ c*x)) - (2*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/(f^2*(2 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (30*b*c*d^2*(f*x)^(2
 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)^2*(4 + m)*(6 +
 m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (10*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*c^3*d
^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^4*(4 + m)*(6
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(f*(6
+ m)*(8 + 6*m + m^2)) + (5*d*(f*x)^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*Arc
Cosh[c*x])^2)/(f*(4 + m)*(6 + m)) + ((f*x)^(1 + m)*(d - c^2*d*x^2)^(5/2)*(a
 + b*ArcCosh[c*x])^2)/(f*(6 + m)) - (30*b^2*c^2*d^2*(f*x)^(3 + m)*Sqrt[1 -
c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c
^2*x^2])/(f^3*(2 + m)^2*(3 + m)*(4 + m)*(6 + m)*(1 - c*x)*(1 + c*x)) - (10*
b^2*c^2*d^2*(10 + 3*m)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*
Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3 + m)
*(4 + m)^3*(6 + m)*(1 - c*x)*(1 + c*x)) - (2*b^2*c^2*d^2*(264 + 130*m + 15*
m^2)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3 + m)*(4 + m)^2*(6 + m)
^3*(1 - c*x)*(1 + c*x)) + (15*d^3*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^
2)/Sqrt[d - c^2*d*x^2], x])/((6 + m)*(8 + 6*m + m^2))
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\,\mathrm{co}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2, x]
```

Maple [**A**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} (a+b\,\text{arccosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

3.234 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=584

$$-\frac{2b^2c^2d(fx)^{3+m}\sqrt{d-c^2dx^2}}{f^3(4+m)^3}-\frac{6bcd(fx)^{2+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd(fx)^{2+m}\sqrt{d-c^2dx^2}\left(a\right.}{f^2(2+m)(4+m)\sqrt{-1+}}$$

```
[Out] (f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/f/(4+m)-2*b^2*c^2*d*(
f*x)^(3+m)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3+3*d*(f*x)^(1+m)*(a+b*arccosh(c*
x))^2*(-c^2*d*x^2+d)^(1/2)/f/(m^2+6*m+8)-6*b*c*d*(f*x)^(2+m)*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b
*c*d*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(4+m)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c^3*d*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-6*b^2*c^2*d*(f*x)^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*
d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(4+m)/(-c*x+1)/(c*x+1)-2*b^2*c^2*d*(10+3*m
)*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^
(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3/(m^2+5*m+6)/(-c*x+1)/(c*x+1)+3*d^2*U
nintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)/(m^2+6*m+8
)
```

Rubi [A]

time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-2*b^2*c^2*d*(f*x)^(3 + m)*Sqrt[d - c^2*d*x^2])/(f^3*(4 + m)^3) - (6*b*c*d
*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)^2*(4
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (2*b*c^3*d*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^
4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1 + m)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^2)/(f*(8 + 6*m + m^2)) + ((f*x)^(1 + m)*(d - c
^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/(f*(4 + m)) - (6*b^2*c^2*d*(f*x)^(3
 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/
2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)^2*(3 + m)*(4 + m)*(1 - c*x)*(1 + c*x))
 - (2*b^2*c^2*d*(10 + 3*m)*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x
^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/(f^3*(2 + m)*(3
```

```
+ m)*(4 + m)^3*(1 - c*x)*(1 + c*x)) + (3*d^2*Defer[Int][((f*x)^m*(a + b*Arc
Cosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x])/(8 + 6*m + m^2)
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

3.235 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=240

$$-\frac{2bc(fx)^{2+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{f(2+m)}-\frac{2b^2c^2(fx)^{3+m}\sqrt{1-}}{f^3(2+}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/f/(2+m)-2*b*c*(f*x)^(
2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-2*b^2*c^2*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*
x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(-c*x+1)/(c*
x+1)+d*Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)/(2
+m)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-2*b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f^2*(2 + m
)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x])^2)/(f*(2 + m)) - (2*b^2*c^2*(f*x)^(3 + m)*Sqrt[1 - c^2*x^2
]*Sqrt[d - c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2]
)/(f^3*(2 + m)^2*(3 + m)*(1 - c*x)*(1 + c*x)) + (d*Defer[Int][((f*x)^m*(a +
 b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x])/(2 + m)
```

Rubi steps

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{\sqrt{d-c^2dx^2}\int (fx)^m \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-c^2 d\, x^2+d}\, \left(a+b \operatorname{arccosh}(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b \operatorname{acosh}{(cx)}\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

3.236 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\operatorname{arccosh}(cx)\right)^2}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^2*d*x^2 - d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\mathrm{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,(f\,x)^m}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

3.237 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^2}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,(f\,x)^m}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

3.238 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(d-c^2dx^2)^{5/2}},x\right)$$

[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(d-c^2dx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(d-c^2dx^2)^{5/2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(d-c^2dx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]

[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^2}{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2\,(f\,x)^m}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(5/2),x)
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(5/2), x)
```

3.239 $\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
[Out] Defer[Int][((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*arccosh(c*x)^2/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{acosh}^2(cx)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(c*x)**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*acosh(c*x)**2/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(c\,x)^2\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
[Out] int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.240 $\int (c-a^2cx^2)^3 \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=505

$$-\frac{976c^3\sqrt{-1+ax}\,\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{7104c^3(1-a^2x^2)}{42875a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{1184c^3(\cdots}{42875a\sqrt{-1\cdots}}$$

```
[Out] 4322/1225*c^3*x*arccosh(a*x)-1514/3675*a^2*c^3*x^3*arccosh(a*x)+702/6125*a^
4*c^3*x^5*arccosh(a*x)-6/343*a^6*c^3*x^7*arccosh(a*x)+8/35*c^3*(a*x-1)^(3/2
)*(a*x+1)^(3/2)*arccosh(a*x)^2/a-18/175*c^3*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arc
cosh(a*x)^2/a+3/49*c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)*arccosh(a*x)^2/a+16/35*c
^3*x*arccosh(a*x)^3+8/35*c^3*x*(-a^2*x^2+1)*arccosh(a*x)^3+6/35*c^3*x*(-a^2
*x^2+1)^2*arccosh(a*x)^3+1/7*c^3*x*(-a^2*x^2+1)^3*arccosh(a*x)^3+7104/42875
*c^3*(-a^2*x^2+1)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1184/42875*c^3*(-a^2*x^2+1)
^2/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2664/214375*c^3*(-a^2*x^2+1)^3/a/(a*x-1)^(
1/2)/(a*x+1)^(1/2)+6/2401*c^3*(-a^2*x^2+1)^4/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-
976/315*c^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+16/315*a*c^3*x^2*(a*x-1)^(1/2)*(a
*x+1)^(1/2)-48/35*c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 1.00, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5897, 5879, 5915, 75, 5889, 5894, 12, 471, 200, 534, 1261, 712, 1624, 1813, 1864}

$$-\frac{6}{343}a^6c^3x^7\cosh^{-1}(ax)+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{6c^3(1-a^2x^2)^4}{2401a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2664c^3(1-a^2x^2)^3}{214375a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1184c^3(1-a^2x^2)^2}{42875a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{7104c^3(1-a^2x^2)}{42875a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{7}c^3x(1-a^2x^2)^3\cosh^{-1}(ax)^3+\frac{6}{35}c^3x(1-a^2x^2)^2\cosh^{-1}(ax)^3+\frac{8}{35}c^3x(1-a^2x^2)\cosh^{-1}(ax)^3+\frac{16}{315}ac^3x^2\sqrt{ax-1}\,\sqrt{ax+1}-\frac{976c^3\sqrt{ax-1}\,\sqrt{ax+1}}{315a}+\frac{16}{35}c^3x\cosh^{-1}(ax)^3+\frac{4322c^3x\cosh^{-1}(ax)}{1225}+\frac{3c^3(ax-1)^{7/2}(ax+1)^{7/2}\cosh^{-1}(ax)^2}{49a}-\frac{18c^3(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)^2}{175a}+\frac{8c^3(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)^2}{35a}-\frac{48c^3\sqrt{ax-1}\,\sqrt{ax+1}\cosh^{-1}(ax)^2}{35a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```

```
[Out] (-976*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(315*a) + (16*a*c^3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x])/315 + (7104*c^3*(1 - a^2*x^2))/(42875*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (1184*c^3*(1 - a^2*x^2)^2)/(42875*a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]) + (2664*c^3*(1 - a^2*x^2)^3)/(214375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 + (6*c^3*(1 - a^2*x^2)^4)/(2401*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (4322*c^
3*x*ArcCosh[a*x])/1225 - (1514*a^2*c^3*x^3*ArcCosh[a*x])/3675 + (702*a^4*c^
3*x^5*ArcCosh[a*x])/6125 - (6*a^6*c^3*x^7*ArcCosh[a*x])/343 - (48*c^3*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(35*a) + (8*c^3*(-1 + a*x)^(3/2)*(1
 + a*x)^(3/2)*ArcCosh[a*x]^2)/(35*a) - (18*c^3*(-1 + a*x)^(5/2)*(1 + a*x)^(
5/2)*ArcCosh[a*x]^2)/(175*a) + (3*c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2)*ArcC
osh[a*x]^2)/(49*a) + (16*c^3*x*ArcCosh[a*x]^3)/35 + (8*c^3*x*(1 - a^2*x^2)*
ArcCosh[a*x]^3)/35 + (6*c^3*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^3)/35 + (c^3*x*(
1 - a^2*x^2)^3*ArcCosh[a*x]^3)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
```

```
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^3+\frac{1}{7}(6c)\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^3\,dx+\frac{1}{7}($$

$$= \frac{3c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3$$

$$= \frac{6}{49}c^3x\cosh^{-1}(ax)-\frac{6}{49}a^2c^3x^3\cosh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\cosh^{-1}(ax)-\frac{6}{343}a$$

$$= \frac{402c^3x\cosh^{-1}(ax)}{1225}-\frac{318a^2c^3x^3\cosh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}$$

$$= \frac{962c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-$$

$$= \frac{4322c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-$$

$$= -\frac{96c^3\sqrt{-1+ax}\,\sqrt{1+ax}}{35a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{4322c^3x\,\mathrm{co}}{122}$$

$$= -\frac{976c^3\sqrt{-1+ax}\,\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{96}{1715a\sqrt{-}}$$

$$= -\frac{976c^3\sqrt{-1+ax}\,\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{710}{42875a\sqrt{}}$$

Mathematica [A]
time = 0.16, size = 179, normalized size = 0.35

$$\frac{c^3\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\,(-22329151+747937a^2x^2-134541a^4x^4+16875a^6x^6)-210ax(-226905+26495a^2x^2-7371a^4x^4+1125a^6x^6)\cosh^{-1}(ax)+11025\sqrt{-1+ax}\,\sqrt{1+ax}\,(-2161+757a^2x^2-351a^4x^4+75a^6x^6)\cosh^{-1}(ax)^2-385875ax(-35+35a^2x^2-21a^4x^4+5a^6x^6)\cosh^{-1}(ax)^3\right)}{13505625a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```

```
[Out] (c^3*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-22329151 + 747937*a^2*x^2 - 134541*a
^4*x^4 + 16875*a^6*x^6) - 210*a*x*(-226905 + 26495*a^2*x^2 - 7371*a^4*x^4 +
 1125*a^6*x^6)*ArcCosh[a*x] + 11025*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 7
57*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*ArcCosh[a*x]^2 - 385875*a*x*(-35 + 3
5*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[a*x]^3))/(13505625*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^2 c\,x^2+c\right)^3 \operatorname{arccosh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x)
```

```
[Out] int((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x)
```

Maxima [A]
time = 0.27, size = 276, normalized size = 0.55

$$\frac{1}{1225}\left(75\sqrt{a^2x^2-1}\,a^4c^3x^6-351\sqrt{a^2x^2-1}\,a^2c^3x^4+757\sqrt{a^2x^2-1}\,c^3x^2-\frac{2161\sqrt{a^2x^2-1}\,c^3}{a^2}\right)a\operatorname{arcosh}(ax)^2-\frac{1}{35}\left(5a^6c^3x^7-21a^4c^3x^5+35a^2c^3x^3-35c^3x\right)\operatorname{arcosh}(ax)^3+\frac{2}{13505625}\left(16875\sqrt{a^2x^2-1}\,a^4c^3x^6-134541\sqrt{a^2x^2-1}\,a^2c^3x^4+747937\sqrt{a^2x^2-1}\,c^3x^2-\frac{22329151\sqrt{a^2x^2-1}\,c^3}{a^2}-\frac{105\left(1125a^6c^3x^7-7371a^4c^3x^5+26495a^2c^3x^3-226905c^3x\right)\operatorname{arcosh}(ax)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/1225*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1)*a^2*c^3*x^
4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/a^2)*a*arcco
sh(a*x)^2 - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 35*c^3*
x)*arccosh(a*x)^3 + 2/13505625*(16875*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 13454
1*sqrt(a^2*x^2 - 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 - 1)*c^3*x^2 - 223291
51*sqrt(a^2*x^2 - 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 - 7371*a^4*c^3*x^5 + 2
6495*a^2*c^3*x^3 - 226905*c^3*x)*arccosh(a*x)/a)*a
```

Fricas [A]
time = 0.35, size = 248, normalized size = 0.49

$$-\frac{385875\left(5a^7c^3x^7-21a^5c^3x^5+35a^3c^3x^3-35ac^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-11025\left(75a^6c^3x^6-351a^4c^3x^4+757a^2c^3x^2-2161c^3\right)\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2+210\left(1125a^7c^3x^7-7371a^5c^3x^5+26495a^3c^3x^3-226905ac^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-2\left(16875a^6c^3x^6-134541a^4c^3x^4+747937a^2c^3x^2-22329151c^3\right)\sqrt{a^2x^2-1}}{13505625\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13505625*(385875*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a
*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 11025*(75*a^6*c^3*x^6 - 351*a^4*c^
3*x^4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^
2 - 1))^2 + 210*(1125*a^7*c^3*x^7 - 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 -
226905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(16875*a^6*c^3*x^6 - 13454
1*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 - 22329151*c^3)*sqrt(a^2*x^2 - 1))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 1.48, size = 367, normalized size = 0.73

$$\begin{cases} - \frac{a^{6} c^{3} x^{7} \operatorname{acosh}^{3}{\left(a x \right)}}{7} - \frac{6 a^{6} c^{3} x^{7} \operatorname{acosh}{\left(a x \right)}}{343} + \frac{3 a^{5} c^{3} x^{6} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}^{2}{\left(a x \right)}}{49} + \frac{6 a^{5} c^{3} x^{6} \sqrt{a^{2} x^{2} - 1}}{2401} + \frac{3 a^{4} c^{3} x^{5} \operatorname{acosh}^{3}{\left(a x \right)}}{5} + \frac{702 a^{4} c^{3} x^{5} \operatorname{acosh}{\left(a x \right)}}{6125} - \frac{351 a^{3} c^{3} x^{4} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}^{2}{\left(a x \right)}}{1225} - \frac{29898 a^{3} c^{3} x^{4} \sqrt{a^{2} x^{2} - 1}}{1500625} - a^{2} c^{3} x^{3} \operatorname{acosh}^{3}{\left(a x \right)} - \frac{1514 a^{2} c^{3} x^{3} \operatorname{acosh}{\left(a x \right)}}{3675} + \frac{757 a c^{3} x^{2} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}^{2}{\left(a x \right)}}{1225} + \frac{1495874 a c^{3} x^{2} \sqrt{a^{2} x^{2} - 1}}{13505625} + c^{3} x \operatorname{acosh}^{3}{\left(a x \right)} + \frac{4322 c^{3} x \operatorname{acosh}{\left(a x \right)}}{1225} - \frac{2161 c^{3} \sqrt{a^{2} x^{2} - 1} \operatorname{acosh}^{2}{\left(a x \right)}}{1225 a} - \frac{44658302 c^{3} \sqrt{a^{2} x^{2} - 1}}{13505625 a} & \text{for}\: a \neq 0 \\- \frac{i \pi^{3} c^{3} x}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**3/7 - 6*a**6*c**3*x**7*acosh(a*x)/34
3 + 3*a**5*c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/49 + 6*a**5*c**3*x**
6*sqrt(a**2*x**2 - 1)/2401 + 3*a**4*c**3*x**5*acosh(a*x)**3/5 + 702*a**4*c*
*3*x**5*acosh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)
**2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)/1500625 - a**2*c**3*x**
3*acosh(a*x)**3 - 1514*a**2*c**3*x**3*acosh(a*x)/3675 + 757*a*c**3*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)**2/1225 + 1495874*a*c**3*x**2*sqrt(a**2*x**2 -
1)/13505625 + c**3*x*acosh(a*x)**3 + 4322*c**3*x*acosh(a*x)/1225 - 2161*c**
3*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
 - 1)/(13505625*a), Ne(a, 0)), (-I*pi**3*c**3*x/8, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(a\,x)^3\,{\left(c-a^2\,c\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^3, x)
```

3.241 $\int (c-a^2cx^2)^2 \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=388

$$-\frac{488c^2\sqrt{-1+ax}\,\sqrt{1+ax}}{135a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{16c^2(1-a^2x^2)}{125a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{8c^2(1-a}{375a\sqrt{-1+ax}}$$

```
[Out] 298/75*c^2*x*arccosh(a*x)-76/225*a^2*c^2*x^3*arccosh(a*x)+6/125*a^4*c^2*x^5
*arccosh(a*x)+4/15*c^2*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)^2/a-3/25*c^
2*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arccosh(a*x)^2/a+8/15*c^2*x*arccosh(a*x)^3+4/
15*c^2*x*(-a^2*x^2+1)*arccosh(a*x)^3+1/5*c^2*x*(-a^2*x^2+1)^2*arccosh(a*x)^
3+16/125*c^2*(-a^2*x^2+1)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+8/375*c^2*(-a^2*x^2
+1)^2/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+6/625*c^2*(-a^2*x^2+1)^3/a/(a*x-1)^(1/2
)/(a*x+1)^(1/2)-488/135*c^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+8/135*a*c^2*x^2*(
a*x-1)^(1/2)*(a*x+1)^(1/2)-8/5*c^2*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a
```

Rubi [A]
time = 0.60, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5897, 5879, 5915, 75, 5889, 5894, 12, 471, 200, 534, 1261, 712}

$$\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6c^2(1-a^2x^2)^3}{625a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{8c^2(1-a^2x^2)^2}{375a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{16c^2(1-a^2x^2)}{125a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{5}c^2x(1-a^2x^2)^2\cosh^{-1}(ax)^3+\frac{4}{15}c^2x(1-a^2x^2)\cosh^{-1}(ax)^3+\frac{8}{135}ac^2x^2\sqrt{ax-1}\,\sqrt{ax+1}-\frac{488c^2\sqrt{ax-1}\,\sqrt{ax+1}}{135a}+\frac{8}{15}c^2x\cosh^{-1}(ax)^3+\frac{298}{75}c^2x\cosh^{-1}(ax)-\frac{3c^2(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)^2}{25a}+\frac{4c^2(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)^2}{15a}-\frac{8c^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```

```
[Out] (-488*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(135*a) + (8*a*c^2*x^2*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/135 + (16*c^2*(1 - a^2*x^2))/(125*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (8*c^2*(1 - a^2*x^2)^2)/(375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (
6*c^2*(1 - a^2*x^2)^3)/(625*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (298*c^2*x*Ar
cCosh[a*x])/75 - (76*a^2*c^2*x^3*ArcCosh[a*x])/225 + (6*a^4*c^2*x^5*ArcCosh
[a*x])/125 - (8*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a) + (4
*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(15*a) - (3*c^2*(-1 +
 a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]^2)/(25*a) + (8*c^2*x*ArcCosh[a*x]^
3)/15 + (4*c^2*x*(1 - a^2*x^2)*ArcCosh[a*x]^3)/15 + (c^2*x*(1 - a^2*x^2)^2*
ArcCosh[a*x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3+\frac{1}{5}(4c)\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx-\frac{1}{5}(3\cdots$$

$$= -\frac{3c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^2}{25a}+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^3\cdots$$

$$= \frac{6}{25}c^2x\cosh^{-1}(ax)-\frac{4}{25}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)+\frac{4c^2(-\cdots}{}$$

$$= \frac{58}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{8c^2\cdots}{}$$

$$= \frac{298}{75}c^2x\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{8c^2\cdots}{}$$

$$= -\frac{16c^2\sqrt{-1+ax}\,\sqrt{1+ax}}{5a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{298}{75}c^2x\cos\cdots$$

$$= -\frac{488c^2\sqrt{-1+ax}\,\sqrt{1+ax}}{135a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{298}{75}c^2x\,c\cdots$$

$$= -\frac{488c^2\sqrt{-1+ax}\,\sqrt{1+ax}}{135a}+\frac{8}{135}ac^2x^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{16c\cdots}{125a\sqrt{-\cdots}}$$

Mathematica [A]
time = 0.12, size = 147, normalized size = 0.38

$$\frac{c^2\left(-2\sqrt{-1+ax}\,\sqrt{1+ax}\,(31841-842a^2x^2+81a^4x^4)+30ax(2235-190a^2x^2+27a^4x^4)\cosh^{-1}(ax)-225\sqrt{-1+ax}\,\sqrt{1+ax}\,(149-38a^2x^2+9a^4x^4)\cosh^{-1}(ax)^2+1125ax(15-10a^2x^2+3a^4x^4)\cosh^{-1}(ax)^3\right)}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(31841 - 842*a^2*x^2 + 81*a^4*x^4) +
30*a*x*(2235 - 190*a^2*x^2 + 27*a^4*x^4)*ArcCosh[a*x] - 225*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2 + 1125*a*x*(15
- 10*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^3))/(16875*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^2c\,x^2+c\right)^2\operatorname{arccosh}(ax)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x)
```

```
[Out] int((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x)
```

Maxima [A]
time = 0.28, size = 210, normalized size = 0.54

$$-\frac{1}{75}\left(9\sqrt{a^2x^2-1}\,a^2c^2x^4-38\sqrt{a^2x^2-1}\,c^2x^2+\frac{149\sqrt{a^2x^2-1}\,c^2}{a^2}\right)a\,\mathrm{arcosh}\left(ax\right)^2+\frac{1}{15}\left(3\,a^4c^2x^5-10\,a^2c^2x^3+15\,c^2x\right)\mathrm{arcosh}\left(ax\right)^3-\frac{2}{16875}\left(81\sqrt{a^2x^2-1}\,a^2c^2x^4-842\sqrt{a^2x^2-1}\,c^2x^2-\frac{15\left(27\,a^4c^2x^5-190\,a^2c^2x^3+2235\,c^2x\right)\mathrm{arcosh}\left(ax\right)}{a}+\frac{31841\sqrt{a^2x^2-1}\,c^2}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 - 1)*c^2/a^2)*a*arccosh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 - 10*a^2
*c^2*x^3 + 15*c^2*x)*arccosh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 - 1)*a^2*c^2
*x^4 - 842*sqrt(a^2*x^2 - 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 - 190*a^2*c^2*x^3
 + 2235*c^2*x)*arccosh(a*x)/a + 31841*sqrt(a^2*x^2 - 1)*c^2/a^2)*a
```

Fricas [A]
time = 0.34, size = 204, normalized size = 0.53

$$\frac{1125\left(3\,a^5c^2x^5-10\,a^3c^2x^3+15\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-225\left(9\,a^4c^2x^4-38\,a^2c^2x^2+149\,c^2\right)\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2+30\left(27\,a^5c^2x^5-190\,a^3c^2x^3+2235\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-2\left(81\,a^4c^2x^4-842\,a^2c^2x^2+31841\,c^2\right)\sqrt{a^2x^2-1}}{16875\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 - 1))^3 - 225*(9*a^4*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 30*(27*a^5*c^2*x^5 - 190*a^3*c^2*x
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(81*a^4*c^2*x^4 - 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 - 1))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 274, normalized size = 0.71

$$\begin{cases}\frac{a^4c^2x^5\,\mathrm{acosh}^3\left(ax\right)}{5}+\frac{6a^4c^2x^5\,\mathrm{acosh}\left(ax\right)}{125}-\frac{3a^3c^2x^4\sqrt{a^2x^2-1}\,\mathrm{acosh}^2\left(ax\right)}{25}-\frac{6a^3c^2x^4\sqrt{a^2x^2-1}}{625}-\frac{2a^2c^2x^3\,\mathrm{acosh}^3\left(ax\right)}{3}-\frac{76a^2c^2x^3\,\mathrm{acosh}\left(ax\right)}{225}+\frac{38ac^2x^2\sqrt{a^2x^2-1}\,\mathrm{acosh}^2\left(ax\right)}{75}+\frac{1684ac^2x^2\sqrt{a^2x^2-1}}{16875}+c^2x\,\mathrm{acosh}^3\left(ax\right)+\frac{298c^2x\,\mathrm{acosh}\left(ax\right)}{75}-\frac{149c^2\sqrt{a^2x^2-1}\,\mathrm{acosh}^2\left(ax\right)}{75a}-\frac{63682c^2\sqrt{a^2x^2-1}}{16875a} & \text{for}\ a\neq 0\\ -\frac{i\pi^3c^2x}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**3,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**3/5 + 6*a**4*c**2*x**5*acosh(a*x)/125
 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/25 - 6*a**3*c**2*x**4
*sqrt(a**2*x**2 - 1)/625 - 2*a**2*c**2*x**3*acosh(a*x)**3/3 - 76*a**2*c**2*
x**3*acosh(a*x)/225 + 38*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/75 +
 1684*a*c**2*x**2*sqrt(a**2*x**2 - 1)/16875 + c**2*x*acosh(a*x)**3 + 298*c*
*2*x*acosh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(75*a) - 63
```

```
682*c**2*sqrt(a**2*x**2 - 1)/(16875*a), Ne(a, 0)), (-I*pi**3*c**2*x/8, True
))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^2, x)
```

3.242 $\int (c-a^2cx^2)\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=175

$$-\frac{122c\sqrt{-1+ax}\,\sqrt{1+ax}}{27a}+\frac{2}{27}acx^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1}}{}$$

```
[Out] 14/3*c*x*arccosh(a*x)-2/9*a^2*c*x^3*arccosh(a*x)+1/3*c*(a*x-1)^(3/2)*(a*x+1
)^(3/2)*arccosh(a*x)^2/a+2/3*c*x*arccosh(a*x)^3+1/3*c*x*(-a^2*x^2+1)*arccos
h(a*x)^3-122/27*c*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+2/27*a*c*x^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)-2*c*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.30, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5897, 5879, 5915, 75, 5889, 5894, 12, 471}

$$-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{2}{27}acx^2\sqrt{ax-1}\,\sqrt{ax+1}-\frac{122c\sqrt{ax-1}\,\sqrt{ax+1}}{27a}+\frac{2}{3}cx\cosh^{-1}(ax)^3+\frac{14}{3}cx\cosh^{-1}(ax)+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)^2}{3a}-\frac{2c\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-122*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (2*a*c*x^2*Sqrt[-1 + a*x]*Sq
rt[1 + a*x])/27 + (14*c*x*ArcCosh[a*x])/3 - (2*a^2*c*x^3*ArcCosh[a*x])/9 -
(2*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + (c*(-1 + a*x)^(3/2)*(
1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(3*a) + (2*c*x*ArcCosh[a*x]^3)/3 + (c*x*(1 -
 a^2*x^2)*ArcCosh[a*x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
```

```
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5894

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx=\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{1}{3}(2c)\int\cosh^{-1}(ax)^3\,dx+(ac)\int x\sqrt{-1+}$$
$$=\frac{c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^2}{3a}+\frac{2}{3}cx\cosh^{-1}(ax)^3+\frac{1}{3}cx\left(1-a^2x^2\right.$$
$$=\frac{2}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax}{a}$$
$$=\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(a}{a}$$
$$=-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}}{a}+\frac{2}{27}acx^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(a$$
$$=-\frac{122c\sqrt{-1+ax}\,\sqrt{1+ax}}{27a}+\frac{2}{27}acx^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-}$$

Mathematica [A]
time = 0.11, size = 109, normalized size = 0.62

$$\frac{c\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\,(-61+a^2x^2)-6ax(-21+a^2x^2)\cosh^{-1}(ax)+9\sqrt{-1+ax}\,\sqrt{1+ax}\,(-7+a^2x^2)\cosh^{-1}(ax)^2-9ax(-3+a^2x^2)\cosh^{-1}(ax)^3\right)}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-61 + a^2*x^2) - 6*a*x*(-21 + a^2*x^2)*
ArcCosh[a*x] + 9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcCosh[a*x]^2
 - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a)
```

Maple [A]
time = 1.18, size = 140, normalized size = 0.80

$$-\frac{c\left(9x^3a^3\mathrm{arccosh}(ax)^3-9x^2a^2\mathrm{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}+6\,\mathrm{arccosh}(ax)\,x^3a^3-27\mathrm{arccosh}(ax)^3\,xa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^3,x)
```

```
[Out] -1/27/a*c*(9*x^3*a^3*arccosh(a*x)^3-9*x^2*a^2*arccosh(a*x)^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)+6*arccosh(a*x)*x^3*a^3-27*arccosh(a*x)^3*x*a-2*(a*x+1)^(1/2)*
(a*x-1)^(1/2)*a^2*x^2+63*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-126*a*x
*arccosh(a*x)+122*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A]
time = 0.26, size = 124, normalized size = 0.71

$$\frac{1}{3}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{7\sqrt{a^2x^2-1}\,c}{a^2}\right)a\,\mathrm{arcosh}\,(ax)^2-\frac{1}{3}\left(a^2cx^3-3\,cx\right)\mathrm{arcosh}\,(ax)^3+\frac{2}{27}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{3\left(a^2cx^3-21\,cx\right)\mathrm{arcosh}\,(ax)}{a}-\frac{61\sqrt{a^2x^2-1}\,c}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)*a*arccosh(a*x)^2
- 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^3 + 2/27*(sqrt(a^2*x^2 - 1)*c*x^2 -
3*(a^2*c*x^3 - 21*c*x)*arccosh(a*x)/a - 61*sqrt(a^2*x^2 - 1)*c/a^2)*a
```

Fricas [A]
time = 0.43, size = 140, normalized size = 0.80

$$-\frac{9\left(a^{3}cx^{3}-3\,acx\right)\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)^{3}-9\left(a^{2}cx^{2}-7\,c\right)\sqrt{a^{2}x^{2}-1}\,\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)^{2}+6\left(a^{3}cx^{3}-21\,acx\right)\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)-2\left(a^{2}cx^{2}-61\,c\right)\sqrt{a^{2}x^{2}-1}}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 9*(a^2*c*x^
2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 6*(a^3*c*x^3 -
21*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^2*c*x^2 - 61*c)*sqrt(a^2*x^2
- 1))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.32, size = 160, normalized size = 0.91

$$\begin{cases} -\frac{a^{2}cx^{3}\operatorname{acosh}^{3}(ax)}{3}-\frac{2a^{2}cx^{3}\operatorname{acosh}(ax)}{9}+\frac{acx^{2}\sqrt{a^{2}x^{2}-1}\operatorname{acosh}^{2}(ax)}{3}+\frac{2acx^{2}\sqrt{a^{2}x^{2}-1}}{27}+cx\operatorname{acosh}^{3}(ax)+\frac{14cx\operatorname{acosh}(ax)}{3}-\frac{7c\sqrt{a^{2}x^{2}-1}\operatorname{acosh}^{2}(ax)}{3a}-\frac{122c\sqrt{a^{2}x^{2}-1}}{27a} & \text{for}\: a\neq 0 \\ -\frac{i\pi^{3}cx}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**3/3 - 2*a**2*c*x**3*acosh(a*x)/9 + a*c*
x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/3 + 2*a*c*x**2*sqrt(a**2*x**2 - 1)/2
7 + c*x*acosh(a*x)**3 + 14*c*x*acosh(a*x)/3 - 7*c*sqrt(a**2*x**2 - 1)*acosh
(a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 - 1)/(27*a), Ne(a, 0)), (-I*pi**3*c*x
/8, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2),x)
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2), x)
```

3.243 $\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=144

$$\frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}}\right.}{ac}$$

```
[Out] 2*arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+3*arccosh(a*x
)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-3*arccosh(a*x)^2*polylo
g(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*arccosh(a*x)*polylog(3,-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*
x+1)^(1/2))/a/c+6*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*polylog
(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5903, 4267, 2611, 6744, 2320, 6724}

$$\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\cosh^{-1}(ax)\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{6\cosh^{-1}(ax)\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{6\text{Li}_4\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\text{Li}_4\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (3*ArcCosh[a*x]^2*PolyLo
g[2, -E^ArcCosh[a*x]])/(a*c) - (3*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]]
)/(a*c) - (6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (6*ArcCosh[a
*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c) + (6*PolyLog[4, -E^ArcCosh[a*x]])/(a*
c) - (6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{3\text{S}}{}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\text{L}}{a}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\text{L}}{a}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\text{L}}{a}$$
$$= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{3\cosh^{-1}(ax)^2\text{L}}{a}$$

Mathematica [A]
time = 0.07, size = 129, normalized size = 0.90

$$\frac{-\cosh^{-1}(ax)^3\log\left(1-e^{\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)^3\log\left(1+e^{\cosh^{-1}(ax)}\right)+3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)-3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)-6\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)+6\cosh^{-1}(ax)\text{PolyLog}\left(3,e^{\cosh^{-1}(ax)}\right)+6\text{PolyLog}\left(4,-e^{\cosh^{-1}(ax)}\right)-6\text{PolyLog}\left(4,e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^3*Log[1 + E^ArcCo
sh[a*x]] + 3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*
PolyLog[2, E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]] + 6
*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 6*PolyLog[4, -E^ArcCosh[a*x]] -
6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

Maple [A]
time = 1.52, size = 253, normalized size = 1.76

method	result
derivativedivides	$\frac{\frac{\text{arccosh}(ax)^3\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}+\frac{3\text{arccosh}(ax)^2\,\text{polylog}\left(2,-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}-\frac{6\,\text{arccosh}(a}{}}{}$
default	$\frac{\frac{\text{arccosh}(ax)^3\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}+\frac{3\text{arccosh}(ax)^2\,\text{polylog}\left(2,-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{c}-\frac{6\,\text{arccosh}(a}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/c*arccosh(a*x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/c*arccosh(a
*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-6/c*arccosh(a*x)*polylog(
3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+6/c*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1
)^(1/2))-1/c*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/c*arcco
sh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+6/c*arccosh(a*x)*polyl
og(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-6/c*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/
(a*c) - integrate(3/2*((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*
sqrt(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)
*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^{3}{\left(ax \right)}}{a^{2}x^{2}-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**2*x**2 - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^3}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2), x)
```

3.244 $\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)}-\frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

```
[Out] 1/2*x*arccosh(a*x)^3/c^2/(-a^2*x^2+1)-6*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))/a/c^2+arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))/a/c^2-3*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3/2*arccosh
(a*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3*polylog(2,a*x+(
a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/2*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))/a/c^2-3*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x
+1)^(1/2))/a/c^2+3*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/
a/c^2+3*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*polylog(4,a*x+(
a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/2*arccosh(a*x)^2/a/c^2/(a*x-1)^(1/2)/(a
*x+1)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5901, 5903, 4267, 2611, 6744, 2320, 6724, 5915, 5889, 2317, 2438}

$$\frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)}+\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{3\cosh^{-1}(ax)\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{3\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{3\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{3\text{Li}_4\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{3\text{Li}_4\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-3*ArcCosh[a*x]^2)/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x
]^3)/(2*c^2*(1 - a^2*x^2)) - (6*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^
2) + (ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[2, -E^Ar
cCosh[a*x]])/(a*c^2) + (3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*
c^2) + (3*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - (3*ArcCosh[a*x]^2*PolyLog[2
, E^ArcCosh[a*x]])/(2*a*c^2) - (3*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])
/(a*c^2) + (3*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c^2) + (3*PolyLog
[4, -E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[4, E^ArcCosh[a*x]])/(a*c^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^2}\,dx = \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} + \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} + \frac{3\int \frac{\cosh^{-1}(ax)}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^3\text{csch}(x)\,d\right.}{2ac}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{3\text{S}}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{co}}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{co}}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{co}}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2(1-a^2x^2)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{co}}{}$$

Mathematica [A]
time = 1.43, size = 276, normalized size = 1.06

$$\frac{-\pi^4+2\cosh^{-1}(ax)^4-12\cosh^{-1}(ax)^2\coth\left(\tfrac{1}{2}\cosh^{-1}(ax)\right)-2\cosh^{-1}(ax)^3\text{csch}^2\left(\tfrac{1}{2}\cosh^{-1}(ax)\right)+48\cosh^{-1}(ax)\log\left(1-e^{-\cosh^{-1}(ax)}\right)-48\cosh^{-1}(ax)\log\left(1+e^{-\cosh^{-1}(ax)}\right)+8\cosh^{-1}(ax)^3\log\left(1+e^{-\cosh^{-1}(ax)}\right)-8\cosh^{-1}(ax)^3\log\left(1-e^{\cosh^{-1}(ax)}\right)-24\left(-2+\cosh^{-1}(ax)^2\right)\text{PolyLog}\left(2,-e^{-\cosh^{-1}(ax)}\right)-48\text{PolyLog}\left(2,e^{-\cosh^{-1}(ax)}\right)-24\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)-48\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\cosh^{-1}(ax)}\right)+48\cosh^{-1}(ax)\text{PolyLog}\left(3,e^{\cosh^{-1}(ax)}\right)-48\text{PolyLog}\left(4,-e^{-\cosh^{-1}(ax)}\right)-48\text{PolyLog}\left(4,e^{\cosh^{-1}(ax)}\right)-2\cosh^{-1}(ax)^3\text{sech}^2\left(\tfrac{1}{2}\cosh^{-1}(ax)\right)+12\cosh^{-1}(ax)^2\tanh\left(\tfrac{1}{2}\cosh^{-1}(ax)\right)}{16ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-Pi^4 + 2*ArcCosh[a*x]^4 - 12*ArcCosh[a*x]^2*Coth[ArcCosh[a*x]/2] - 2*ArcC
osh[a*x]^3*Csch[ArcCosh[a*x]/2]^2 + 48*ArcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x
])] - 48*ArcCosh[a*x]*Log[1 + E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]^3*Log[1 +
 E^(-ArcCosh[a*x])] - 8*ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]] - 24*(-2 + A
rcCosh[a*x]^2)*PolyLog[2, -E^(-ArcCosh[a*x])] - 48*PolyLog[2, E^(-ArcCosh[a
*x])] - 24*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]] - 48*ArcCosh[a*x]*Poly
Log[3, -E^(-ArcCosh[a*x])] + 48*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] - 4
8*PolyLog[4, -E^(-ArcCosh[a*x])] - 48*PolyLog[4, E^ArcCosh[a*x]] - 2*ArcCos
h[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + 12*ArcCosh[a*x]^2*Tanh[ArcCosh[a*x]/2])/(
16*a*c^2)
```

Maple [A]
time = 2.91, size = 416, normalized size = 1.60

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(ax)^2\left(ax\operatorname{arccosh}(ax)+3\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2\left(a^2x^2-1\right)c^2}+\frac{\operatorname{arccosh}(ax)^3\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}+$ 3arcc
default	$-\frac{\operatorname{arccosh}(ax)^2\left(ax\operatorname{arccosh}(ax)+3\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2\left(a^2x^2-1\right)c^2}+\frac{\operatorname{arccosh}(ax)^3\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2c^2}+$ 3arcc

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2/(a^2*x^2-1)*arccosh(a*x)^2*(a*x*arccosh(a*x)+3*(a*x-1)^(1/2)*(a*x
+1)^(1/2))/c^2+1/2/c^2*arccosh(a*x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
+3/2/c^2*arccosh(a*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/c^2*a
rccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/c^2*polylog(4,-a*
x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-1/2/c^2*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2
)*(a*x+1)^(1/2))-3/2/c^2*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))+3/c^2*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/c^2
*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/c^2*arccosh(a*x)*ln(1+a*x+(a*
x-1)^(1/2)*(a*x+1)^(1/2))-3/c^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
+3/c^2*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/c^2*polylog(2,a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(a^3*c^2*x^2 - a*c^2) - integrate(-3/4
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))^2/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}^3(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**3/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(a^2*c*x^2 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^2, x)
```

3.245 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{x\cosh^{-1}(ax)}{4c^3\,(1-a^2x^2)}+\frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}}-\frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{x\cosh}{4c^3\,(}$$

```
[Out] -1/4*x*arccosh(a*x)/c^3/(-a^2*x^2+1)+1/4*arccosh(a*x)^2/a/c^3/(a*x-1)^(3/2)
/(a*x+1)^(3/2)+1/4*x*arccosh(a*x)^3/c^3/(-a^2*x^2+1)^2+3/8*x*arccosh(a*x)^3
/c^3/(-a^2*x^2+1)-5*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a
/c^3+3/4*arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/2*
polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/8*arccosh(a*x)^2*polylo
g(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+5/2*polylog(2,a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2))/a/c^3-9/8*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a/c^3-9/4*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
/a/c^3+9/4*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/
4*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-9/4*polylog(4,a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2))/a/c^3+1/4/a/c^3/(a*x-1)^(1/2)/(a*x+1)^(1/2)-9/8*arc
cosh(a*x)^2/a/c^3/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5901, 5903, 4267, 2611, 6744, 2320, 6724, 5915, 5889, 2317, 2438, 75}

$$\frac{3x\cosh^{-1}(ax)^3}{8c^3\,(1-a^2x^2)}+\frac{x\cosh^{-1}(ax)^3}{4c^3\,(1-a^2x^2)^2}-\frac{x\cosh^{-1}(ax)}{4c^3\,(1-a^2x^2)}+\frac{9\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)\mathrm{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\frac{9\cosh^{-1}(ax)\mathrm{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{5\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^3}+\frac{5\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{2ac^3}+\frac{9\mathrm{Li}_4\left(-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{9\mathrm{Li}_4\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\frac{1}{4ac^3\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)^2}{4ac^3(ax-1)^{3/2}(ax+1)^{3/2}}+\frac{3\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{5\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (x*ArcCosh[a*x])/(4*c^3*(1 - a^2
*x^2)) + ArcCosh[a*x]^2/(4*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - (9*Arc
Cosh[a*x]^2)/(8*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x]^3)/(4
*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^3)/(8*c^3*(1 - a^2*x^2)) - (5*Arc
Cosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^3) + (3*ArcCosh[a*x]^3*ArcTanh[E^Ar
cCosh[a*x]])/(4*a*c^3) - (5*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*c^3) + (9*Arc
Cosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) + (5*PolyLog[2, E^ArcCos
h[a*x]])/(2*a*c^3) - (9*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]])/(8*a*c^3
) - (9*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(4*a*c^3) + (9*ArcCosh[a*x
]*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3) + (9*PolyLog[4, -E^ArcCosh[a*x]])/(
4*a*c^3) - (9*PolyLog[4, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5903

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^3}\,dx = \frac{x\cosh^{-1}(ax)^3}{4c^3(1-a^2x^2)^2} - \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^3}{4c^3(1-a^2x^2)^2} + \frac{3x\cosh^{-1}(ax)^3}{8c^3(1-a^2x^2)} - \frac{\int \frac{\cosh^{-1}(ax)}{(-1+a^2x^2)^2}\,dx}{2c^3} + \cdots$$

$$= -\frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh\cdots}{4c^3(1\cdots}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\,\mathrm{c}\cdots}{8ac^3\sqrt{-\cdots}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\,\mathrm{c}\cdots}{8ac^3\sqrt{-\cdots}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\,\mathrm{c}\cdots}{8ac^3\sqrt{-\cdots}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\,\mathrm{c}\cdots}{8ac^3\sqrt{-\cdots}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\,\mathrm{c}\cdots}{8ac^3\sqrt{-\cdots}}$$

Mathematica [A]
time = 7.31, size = 455, normalized size = 1.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/64*(3*Pi^4 - 6*ArcCosh[a*x]^4 - 8*Coth[ArcCosh[a*x]/2] + 40*ArcCosh[a*x]
^2*Coth[ArcCosh[a*x]/2] - 4*ArcCosh[a*x]*Csch[ArcCosh[a*x]/2]^2 + 6*ArcCosh
[a*x]^3*Csch[ArcCosh[a*x]/2]^2 - Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCo
sh[a*x]^2*Csch[ArcCosh[a*x]/2]^4 - ArcCosh[a*x]^3*Csch[ArcCosh[a*x]/2]^4 -
160*ArcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x])] + 160*ArcCosh[a*x]*Log[1 + E^(-
ArcCosh[a*x])] - 24*ArcCosh[a*x]^3*Log[1 + E^(-ArcCosh[a*x])] + 24*ArcCosh[
a*x]^3*Log[1 - E^ArcCosh[a*x]] + 8*(-20 + 9*ArcCosh[a*x]^2)*PolyLog[2, -E^(
-ArcCosh[a*x])] + 160*PolyLog[2, E^(-ArcCosh[a*x])] + 72*ArcCosh[a*x]^2*Pol
yLog[2, E^ArcCosh[a*x]] + 144*ArcCosh[a*x]*PolyLog[3, -E^(-ArcCosh[a*x])] -
 144*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 144*PolyLog[4, -E^(-ArcCosh[
```

```
a*x])] + 144*PolyLog[4, E^ArcCosh[a*x]] - 4*ArcCosh[a*x]*Sech[ArcCosh[a*x]/
2]^2 + 6*ArcCosh[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + ArcCosh[a*x]^3*Sech[ArcCos
h[a*x]/2]^4 - (16*ArcCosh[a*x]^2*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 +
a*x))^(3/2)*(1 + a*x)^3) + 8*Tanh[ArcCosh[a*x]/2] - 40*ArcCosh[a*x]^2*Tanh[
ArcCosh[a*x]/2])/(a*c^3)
```

Maple [A]
time = 4.28, size = 527, normalized size = 1.36

method	result
derivativedivides	$-\frac{\frac{9x^2a^2\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}+3x^3a^3\operatorname{arccosh}(ax)^3-11\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}-2\sqrt{ax+}}{8\left(a^4x^4-2a^2x^2+1\right)}}{}$
default	$-\frac{\frac{9x^2a^2\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}+3x^3a^3\operatorname{arccosh}(ax)^3-11\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}-2\sqrt{ax+}}{8\left(a^4x^4-2a^2x^2+1\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/8*(9*x^2*a^2*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*x^3*a^3*a
rccosh(a*x)^3-11*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*a^2*x^2-5*arccosh(a*x)^3*x*a-2*arccosh(a*x)*x^3*a^3+2*(a*x-1
)^(1/2)*(a*x+1)^(1/2)+2*a*x*arccosh(a*x))/(a^4*x^4-2*a^2*x^2+1)/c^3-5/2/c^3
*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-5/2/c^3*polylog(2,-a*x-
(a*x-1)^(1/2)*(a*x+1)^(1/2))+5/2/c^3*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a
*x+1)^(1/2))+5/2/c^3*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/8/c^3*arc
cosh(a*x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+9/8/c^3*arccosh(a*x)^2*po
lylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-9/4/c^3*arccosh(a*x)*polylog(3,-a
*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+9/4/c^3*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1
)^(1/2))-3/8/c^3*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-9/8/c
^3*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+9/4/c^3*arccos
h(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-9/4/c^3*polylog(4,a*x+(a*
x-1)^(1/2)*(a*x+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
```

```
)^3/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-3/16*(6*a^5*x^5 - 16
*a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x
+ 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
 1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sq
rt(a*x - 1))^2/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^
6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1
)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^{3}{\left(a x \right)}}{a^{6} x^{6} - 3 a^{4} x^{4} + 3 a^{2} x^{2} - 1}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{acosh}\left(a\,x\right)}^3}{{\left(c-a^2\,c\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^3,x)
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^3, x)
```

3.246 $\int (c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=605

$$-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{c^2(1-a^2x^2)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{245}{384}c^2x\sqrt{c-a^2cx^2}\ \mathrm{co}$$

```
[Out] 5/24*c*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3+1/6*x*(-a^2*c*x^2+c)^(5/2)*arc
cosh(a*x)^3+245/384*c^2*x*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(-
a*x+1)*(a*x+1)*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+1/36*c^2*x*(-a*x+1)^2*(a*x
+1)^2*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+5/16*c^2*x*arccosh(a*x)^3*(-a^2*c*x
^2+c)^(1/2)-865/2304*a*c^2*x^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(
1/2)+65/2304*a^3*c^2*x^4*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1
/216*c^2*(-a^2*x^2+1)^3*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+
115/768*c^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/
2)-15/32*a*c^2*x^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1
)^(1/2)+5/32*c^2*(-a^2*x^2+1)^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)+1/12*c^2*(-a^2*x^2+1)^3*arccosh(a*x)^2*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/64*c^2*arccosh(a*x)^4*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5897, 5895, 5893, 5883, 5939, 30, 5912, 5914, 5898, 5896, 74, 14, 267}

$$-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{c^2(1-a^2x^2)^3\sqrt{c-a^2cx^2}}{216a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{15ac^2x^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{5}{16}c^2x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3+\frac{245}{384}c^2x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{1}{36}c^2x(1-ax)^2(ax+1)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{65}{576}c^2x(1-ax)(ax+1)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)-\frac{5c^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{c^2(1-a^2x^2)^3\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{12a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{5c^2(1-a^2x^2)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{115c^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{768a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{6}x(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^3+\frac{5}{24}cx(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^3+\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(65*a^3*c^2*x^4*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2])/(216*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) + (245*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/384 + (65*c^2*x*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/576 + (c^2*x*(1 - a*x)^2*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/36 + (115*c^2*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^2)/(768*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c^2*x^2*Sqrt[
c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*(
1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/16 + (5*c*x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3)/24 + (x*(c - a^2*c*x^
2)^(5/2)*ArcCosh[a*x]^3)/6 - (5*c^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(64
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
```

```
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^3\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{6}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3 - \frac{\left(5c^2\sqrt{c-a^2cx^2}\right)}{6}$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{12a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{5}{24}c^2x(1-ax)(1+ax)\sqrt{c-}$$

$$= \frac{1}{36}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) + \frac{5c^2\left(1-a^2x^2\right)^2\sqrt{c-}}{32a\sqrt{-1+}}$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{65}{576}c^2x(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{245}{384}c^2x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) + \frac{65}{576}c^2x$$

$$= -\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{c^2(1-a^2x}{216a\sqrt{-1}}$$

Mathematica [A]

time = 0.84, size = 189, normalized size = 0.31

$$\frac{c^2\sqrt{c-a^2cx^2}\left(-4320\cosh^{-1}(ax)^4-9720\cosh\left(2\cosh^{-1}(ax)\right)+243\cosh\left(4\cosh^{-1}(ax)\right)-8\cosh\left(6\cosh^{-1}(ax)\right)-72\cosh^{-1}(ax)^2\left(270\cosh\left(2\cosh^{-1}(ax)\right)-27\cosh\left(4\cosh^{-1}(ax)\right)+2\cosh\left(6\cosh^{-1}(ax)\right)\right)+288\cosh^{-1}(ax)^3\left(45\sinh\left(2\cosh^{-1}(ax)\right)-9\sinh\left(4\cosh^{-1}(ax)\right)+\sinh\left(6\cosh^{-1}(ax)\right)\right)+12\cosh^{-1}(ax)\left(1620\sinh\left(2\cosh^{-1}(ax)\right)-81\sinh\left(4\cosh^{-1}(ax)\right)+4\sinh\left(6\cosh^{-1}(ax)\right)\right)\right)}{55296a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-4320*ArcCosh[a*x]^4 - 9720*Cosh[2*ArcCosh[a*x]]
+ 243*Cosh[4*ArcCosh[a*x]] - 8*Cosh[6*ArcCosh[a*x]] - 72*ArcCosh[a*x]^2*(27
0*Cosh[2*ArcCosh[a*x]] - 27*Cosh[4*ArcCosh[a*x]] + 2*Cosh[6*ArcCosh[a*x]])
+ 288*ArcCosh[a*x]^3*(45*Sinh[2*ArcCosh[a*x]] - 9*Sinh[4*ArcCosh[a*x]] + Si
nh[6*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(1620*Sinh[2*ArcCosh[a*x]] - 81*Sinh[
4*ArcCosh[a*x]] + 4*Sinh[6*ArcCosh[a*x]])))/(55296*a*Sqrt[(-1 + a*x)/(1 + a
*x)]*(1 + a*x))
```

Maple [A]
time = 2.96, size = 887, normalized size = 1.47

method	result
default	$-\frac{5\sqrt{-c\left(a^2x^2-1\right)}\,\operatorname{arccosh}(ax)^4c^2}{64\sqrt{ax-1}\,\sqrt{ax+1}\,a}+\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(32a^7x^7-64a^5x^5+32\sqrt{ax+1}\,\sqrt{ax-1}\,a^6x^6+38a^3x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -5/64*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
^2+1/13824*(-c*(a^2*x^2-1))^(1/2)*(32*a^7*x^7-64*a^5*x^5+32*(a*x+1)^(1/2)*(
a*x-1)^(1/2)*a^6*x^6+38*a^3*x^3-48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^4*x^4-6*a*
x+18*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(36*a
rccosh(a*x)^3-18*arccosh(a*x)^2+6*arccosh(a*x)-1)*c^2/(a*x-1)/(a*x+1)/a-3/4
096*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*a^4*x^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+
1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c^2/(a*x-
1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c^2/(a*x-1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))
^(1/2)*(2*a^3*x^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c^2/(a
*x-1)/(a*x+1)/a-3/4096*(-c*(a^2*x^2-1))^(1/2)*(-8*(a*x+1)^(1/2)*(a*x-1)^(1/
2)*a^4*x^4+8*a^5*x^5+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-12*a^3*x^3-(a*x-
1)^(1/2)*(a*x+1)^(1/2)+4*a*x)*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arcco
sh(a*x)+3)*c^2/(a*x-1)/(a*x+1)/a+1/13824*(-c*(a^2*x^2-1))^(1/2)*(-32*(a*x+1
)^(1/2)*(a*x-1)^(1/2)*a^6*x^6+32*a^7*x^7+48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^4
*x^4-64*a^5*x^5-18*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+38*a^3*x^3+(a*x-1)^(
```

```
1/2)*(a*x+1)^(1/2)-6*a*x)*(36*arccosh(a*x)^3+18*arccosh(a*x)^2+6*arccosh(a*
x)+1)*c^2/(a*x-1)/(a*x+1)/a
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(-a^2*c*x^2 + c)*arccosh(a
*x)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*acosh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(5/2),x)
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(5/2), x)
```

3.247 $\int (c-a^2cx^2)^{3/2} \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=402

$$-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}}+\frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}}+\frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{3}{32}cx(1-ax)(1+ax)$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3+45/64*c*x*arccosh(a*x)*(-a^2*c*x^
2+c)^(1/2)+3/32*c*x*(-a*x+1)*(a*x+1)*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+3/8*
c*x*arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(-a^2*c*x^2+c)^(1/2)
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*a^3*c*x^4*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(
1/2)/(a*x+1)^(1/2)+27/128*c*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(
1/2)/(a*x+1)^(1/2)-9/16*a*c*x^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)
^(1/2)/(a*x+1)^(1/2)+3/16*c*(-a^2*x^2+1)^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1
/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/32*c*arccosh(a*x)^4*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5897, 5895, 5893, 5883, 5939, 30, 5912, 5914, 5898, 5896, 74, 14}

$$-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3+\frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)+\frac{3}{32}cx(1-ax)(ax+1)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)-\frac{3c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{32a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{27c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{128a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*a
^3*c*x^4*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (45*c*x*
Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/64 + (3*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x])/32 + (27*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12
8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^2)/(16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a
^2*c*x^2]*ArcCosh[a*x]^2)/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x]^3)/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3
)/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(32*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
```

```
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
```

```
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3 + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-}}{4\sqrt{-1}}$$

$$= \frac{3c(1-a^2x^2)^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3 +$$

$$= \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) - \frac{9acx^2\sqrt{c-a^2cx^2}\,\cosh^{-}}{16\sqrt{-1+ax}\,\sqrt{1+}}$$

$$= \frac{45}{64}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) + \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-}$$

$$= -\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{45}{64}cx\sqrt{c-a^2c}$$

Mathematica [A]
time = 0.33, size = 148, normalized size = 0.37

$$-\frac{c\sqrt{c-a^2cx^2}\left(96\cosh^{-1}(ax)^4-3\left(-64\cosh\left(2\cosh^{-1}(ax)\right)+\cosh\left(4\cosh^{-1}(ax)\right)\right)-24\cosh^{-1}(ax)^2\left(-16\cosh\left(2\cosh^{-1}(ax)\right)+\cosh\left(4\cosh^{-1}(ax)\right)\right)+12\cosh^{-1}(ax)\left(-32\sinh\left(2\cosh^{-1}(ax)\right)+\sinh\left(4\cosh^{-1}(ax)\right)\right)+32\cosh^{-1}(ax)^3\left(-8\sinh\left(2\cosh^{-1}(ax)\right)+\sinh\left(4\cosh^{-1}(ax)\right)\right)\right)}{1024a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] -1/1024*(c*Sqrt[c - a^2*c*x^2]*(96*ArcCosh[a*x]^4 - 3*(-64*Cosh[2*ArcCosh[a
*x]] + Cosh[4*ArcCosh[a*x]]) - 24*ArcCosh[a*x]^2*(-16*Cosh[2*ArcCosh[a*x]]
+ Cosh[4*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(-32*Sinh[2*ArcCosh[a*x]] + Sinh[
4*ArcCosh[a*x]]) + 32*ArcCosh[a*x]^3*(-8*Sinh[2*ArcCosh[a*x]] + Sinh[4*ArcC
osh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A]
time = 2.55, size = 536, normalized size = 1.33

method	result
default	$-\frac{3\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\operatorname{arccosh}(ax)^{4}c}{32\sqrt{ax-1}\,\sqrt{ax+1}\,a}-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(8a^{5}x^{5}-12a^{3}x^{3}+8\sqrt{ax+1}\,\sqrt{ax-1}\,a^{4}x^{4}+4ax-8\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x,method=_RETURNVERBOSE)
[Out] -3/32*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x+1)^(1/2)*(a*x-1
)^(1/2)*a^4*x^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(
a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c/(a*
x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/
2)*(2*a^3*x^3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*
x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c/(a*x-1)/
(a*x+1)/a-1/2048*(-c*(a^2*x^2-1))^(1/2)*(-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^4
*x^4+8*a^5*x^5+8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-12*a^3*x^3-(a*x-1)^(1/
2)*(a*x+1)^(1/2)+4*a*x)*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x
)+3)*c/(a*x-1)/(a*x+1)/a
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{acosh}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**3,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*acosh(a*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(a\,x)^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(3/2), x)
```

3.248 $\int \sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=231

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{3}{4}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)+\frac{3\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{3ax^2\sqrt{c-a^2cx^2}\,\cosh^{-}}{4\sqrt{-1+ax}\,\sqrt{1+}}$$

```
[Out] 3/4*x*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+1/2*x*arccosh(a*x)^3*(-a^2*c*x^2+c)
^(1/2)-3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/8*arcco
sh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/4*a*x^2*arcc
osh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/8*arccosh(a*x
)^4*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5895, 5893, 5883, 5939, 30}

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^4}{8a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3-\frac{3ax^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{4\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{8a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3}{4}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c - a^2*c*x^2])/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x])/4 + (3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(8
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x
]^2)/(4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2c\cdots}}{8a\sqrt{-1\cdots}}$$

$$= \frac{3}{4}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) - \frac{3ax^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2c\cdots}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{4}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax) + \frac{3\sqrt{c-a^2cx^2}\,\cos\cdots}{8a\sqrt{-1+ax}\,\sqrt{\cdots}}$$

Mathematica [A]
time = 0.15, size = 98, normalized size = 0.42

$$-\frac{\sqrt{-c(-1+ax)(1+ax)}\left(2\cosh^{-1}(ax)^4+(3+6\cosh^{-1}(ax)^2)\cosh\left(2\cosh^{-1}(ax)\right)-2\cosh^{-1}(ax)\left(3+2\cosh^{-1}(ax)^2\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)}{16a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```

```
[Out] -1/16*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(2*ArcCosh[a*x]^4 + (3 + 6*ArcCosh[a
*x]^2)*Cosh[2*ArcCosh[a*x]] - 2*ArcCosh[a*x]*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*
ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A]
time = 2.77, size = 256, normalized size = 1.11

method	result
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\mathrm{arccosh}(ax)^{4}}{8\sqrt{ax-1}\,\sqrt{ax+1}\,a}+\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(2a^{3}x^{3}-2ax+2\sqrt{ax+1}\,\sqrt{ax-1}\,a^{2}x^{2}-\sqrt{ax-1}\right.}{32(ax-1)(ax+1)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4+1/
32*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^
2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2+6*arc
cosh(a*x)-3)/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x
-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/(a*x-1)/(a*x+1)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^3\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(1/2), x)
```

3.249 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

```
[Out] 1/4*arccosh(a*x)^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5892}

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 1.37, size = 55, normalized size = 1.20

method	result	size
default	$-\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^{4}}{4ac(a^{2}x^{2}-1)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(a^2*x^2-1)
*arccosh(a*x)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}(ax)}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(acosh(a*x)**3/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}(a\,x)^3}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(1/2), x)
```

3.250 $\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}}+\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}}-\frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(1/2)+arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1
)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)-3*arccosh(a*x)^2*ln(1-(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)-3*arc
cosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x
+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)+3/2*polylog(3,(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5899, 5913, 3797, 2221, 2611, 2320, 6724}

$$-\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}}-\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x]^3)/(a*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c*Sqrt[c - a^2*c*x^2]) - (3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/
(a*c*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2
*ArcCosh[a*x])])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\coth(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} + \frac{\left(6\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subs}}{ac\sqrt{c-}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{ac\sqrt{c-a}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{ac\sqrt{c-a}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{ac\sqrt{c-a}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{ac\sqrt{c-a}}$$

Mathematica [A]
time = 0.18, size = 145, normalized size = 0.60

$$\frac{x\cosh^{-1}(ax)^3 + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(\cosh^{-1}(ax)^3 - 3\cosh^{-1}(ax)^2\log\left(1-e^{\cosh^{-1}(ax)}\right) - 3\cosh^{-1}(ax)^2\log\left(1+e^{\cosh^{-1}(ax)}\right) - 6\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right) - 6\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right) + 6\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right) + 6\text{PolyLog}\left(3,e^{\cosh^{-1}(ax)}\right)\right)}{a}}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^3 + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(ArcCosh[a*x]^3 - 3*ArcCo
sh[a*x]^2*Log[1 - E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*Log[1 + E^ArcCosh[a*x]
] - 6*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[2,
E^ArcCosh[a*x]] + 6*PolyLog[3, -E^ArcCosh[a*x]] + 6*PolyLog[3, E^ArcCosh[a*
x]]))/a)/(c*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(252) = 504$.
time = 2.53, size = 548, normalized size = 2.27

method	result
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(-\sqrt{ax-1}\,\sqrt{ax+1}+ax\right)\operatorname{arccosh}\left(ax\right)^{3}}{c^{2}a\left(a^{2}x^{2}-1\right)}-\frac{2\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\sqrt{ax-1}\,\sqrt{ax+1}}{c^{2}a\left(a^{2}x^{2}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-(a*x-1)^(1/2)*(a*x+1)^(1/2)+a*x)*arccosh(a*x)^3/c
^2/a/(a^2*x^2-1)-2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a
/(a^2*x^2-1)*arccosh(a*x)^3+3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)
)+6*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*ar
ccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(-c*(a^2*x^2-1))^(1
/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))+3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/
a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+6*(-c*(a
^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)
*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(-c*(a^2*x^2-1))^(1/2)*(a*x-
1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+
1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}\left(ax\right)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}(a\,x)^3}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(3/2), x)
```

3.251 $\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=413

$$-\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{-1+ax}\,\sqrt{1}}{3ac^2\sqrt{}}$$

```
[Out] 1/3*x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(3/2)-x*arccosh(a*x)/c^2/(-a^2*c*x^2+
c)^(1/2)+2/3*x*arccosh(a*x)^3/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*arccosh(a*x)^2*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)+2/3*arcc
osh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*arccosh
(a*x)^2*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*ln(-a^2*x^2+1)*(a*x-1)^(1/2)*(a*x+1)^(1/2
)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*arccosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*c*x^2+c)^(1/2)+poly
log(3,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^
2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5901, 5899, 5913, 3797, 2221, 2611, 2320, 6724, 5912, 5914, 5900, 266}

$$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\log\left(1-a^2x^2\right)}{2ac^2\sqrt{c-a^2cx^2}}+\frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{3ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}-\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcCosh[a*x])/(c^2*Sqrt[c - a^2*c*x^2])) + (Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*ArcCosh[a*x]^2)/(2*a*c^2*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh
[a*x]^3)/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcCosh[a*x]^3)/(3*c^2*Sqrt[c -
 a^2*c*x^2]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*c^2*Sqr
t[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 -
E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Log[1 - a^2*x^2])/(2*a*c^2*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a
^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a*x])])/
(a*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5900

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(
d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])), x] + Dist[b*c*(n/(d1*d2))*Simp[Sqr
t[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[x*((a
```

```
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2c}}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{}{3c^2(1-a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.71, size = 258, normalized size = 0.62

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(-i\pi^3-\frac{12ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)}{-1+ax}+\frac{6\cosh^{-1}(ax)^2}{1-a^2x^2}+8\cosh^{-1}(ax)^3+\frac{8ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)^3}{-1+ax}-\frac{4ax\left(\frac{-1+ax}{1+ax}\right)^{3/2}\cosh^{-1}(ax)^3}{(-1+ax)^3}-24\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)+12\log\left(\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\right)-24\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)+12\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)\right)}{12ac^2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((-I)*Pi^3 - (12*a*x*Sqrt[(-1 + a*x)/
(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) + (6*ArcCosh[a*x]^2)/(1 - a^2*x^2) + 8*
ArcCosh[a*x]^3 + (8*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*
x) - (4*a*x*((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - 24*
ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])] + 12*Log[Sqrt[(-1 + a*x)/(1 + a*
```

```
x)]*(1 + a*x)] - 24*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] + 12*PolyLo
g[3, E^(2*ArcCosh[a*x])]))/(12*a*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(400) = 800$.
time = 3.07, size = 955, normalized size = 2.31

method	result
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(2a^{3}x^{3}-3ax-2\sqrt{ax+1}\,\sqrt{ax-1}\,a^{2}x^{2}+2\sqrt{ax-1}\,\sqrt{ax+1}\right)\operatorname{arccosh}(ax)\left(6\operatorname{arccosh}(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-3*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*
a^2*x^2+2*(a*x-1)^(1/2)*(a*x+1)^(1/2))*arccosh(a*x)*(6*arccosh(a*x)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)*a^3*x^3+6*arccosh(a*x)*a^4*x^4+6*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*a^3*x^3+6*a^4*x^4+6*arccosh(a*x)^2*a^2*x^2-9*a*x*arccosh(a*x)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)-12*arccosh(a*x)*a^2*x^2-6*(a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x-18*a^2*x^2-8*arccosh(a*x)^2+6*arccosh(a*x)+12)/(3*a^6*x^6-10*a^4*x^4+1
1*a^2*x^2-4)/a/c^3+2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3
/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))-(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1
)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)-1)-4/3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^
(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^3+2*(-c*(a^2*x^2-1))^(1/
2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a
*x-1)^(1/2)*(a*x+1)^(1/2))+4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2))-4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*
polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))+4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a
/(a^2*x^2-1)*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-4*(-c
*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*polylog(3
,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{2,
[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1
,[4,2,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{acosh}\left(a\,x\right)}^3}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(5/2),x)
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(5/2), x)
```

3.252 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}-\frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}}-\frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{-1+ax}\,\sqrt{1+a\ldots}}{20ac^3\left(1-a^2x^2\right)^2\ldots}$$

```
[Out] 1/5*x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)^3/c^2/(-a^2
*c*x^2+c)^(3/2)-x*arccosh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2)-1/10*x*arccosh(a*x)
/c^3/(-a*x+1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+8/15*x*arccosh(a*x)^3/c^3/(-a^2*
c*x^2+c)^(1/2)-1/20*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^2*c*
x^2+c)^(1/2)+3/20*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^
2+1)^2/(-a^2*c*x^2+c)^(1/2)+2/5*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
a/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)+8/15*arccosh(a*x)^3*(a*x-1)^(1/2)*(
a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/5*arccosh(a*x)^2*ln(1-(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^
(1/2)+1/2*ln(-a^2*x^2+1)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(
1/2)-8/5*arccosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)+4/5*polylog(3,(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5901, 5899, 5913, 3797, 2221, 2611, 2320, 6724, 5912, 5914, 5900, 266, 5902, 74, 267}

$$-\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\log\left(1-a^2x^2\right)}{2ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^3}{15c^3\sqrt{c-a^2cx^2}}+\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{15ac^3\sqrt{c-a^2cx^2}}+\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{5ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}-\frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}}-\frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}-\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4x\cosh^{-1}(ax)^3}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x\cosh^{-1}(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/20*(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^
2]) - (x*ArcCosh[a*x])/(c^3*Sqrt[c - a^2*c*x^2]) - (x*ArcCosh[a*x])/(10*c^3
*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*ArcCosh[a*x]^2)/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*
c*x^2]) + (x*ArcCosh[a*x]^3)/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*x*ArcCosh[a*x
]^3)/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcCosh[a*x]^3)/(15*c^3*Sqrt[c -
 a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(15*a*c^3*Sq
rt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 -
 E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[
```

```
1 + a*x]*Log[1 - a^2*x^2])/(2*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x
]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[
c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a
*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5899

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Dist
[b*c*(n/d)*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], Int[x*((a
+ b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 5900

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[x*((a + b*ArcCosh[c*x])^n/(
d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])), x] + Dist[b*c*(n/(d1*d2))*Simp[Sqr
t[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[x*((a
 + b*ArcCosh[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5901

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d*(p +
 1))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/((1 +
c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5902

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(-x)*(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*d1*d2*(p + 1))), x] + (Dist[(2*p +
 3)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*A
rcCosh[c*x])^n, x], x] - Dist[b*c*(n/(2*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c
*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[x*(1 + c*x)^(p + 1/2)*(-1 + c*
x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e
1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5913

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}} \\
&= \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,\cdots}{5c^3\sqrt{c-a^2cx^2}} \\
&= \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\cdots}{15c^3(1\cdots} \\
&= -\frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1\cdots}}{5a\cdots} \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots \\
&= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}}\cdots
\end{aligned}
$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 1.33, size = 363, normalized size = 0.60

$$
-\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(4i\pi^3+\frac{3}{1-a^2x^2}+\frac{60ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)}{-1+ax}-\frac{6ax\left(\frac{-1+ax}{1+ax}\right)^{3/2}\cosh^{-1}(ax)}{(-1+ax)^3}-\frac{9\cosh^{-1}(ax)^2}{(-1+a^2x^2)^2}+\frac{24\cosh^{-1}(ax)^2}{-1+a^2x^2}-32\cosh^{-1}(ax)^3-\frac{32ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)^3}{-1+ax}+\frac{16ax\left(\frac{-1+ax}{1+ax}\right)^{3/2}\cosh^{-1}(ax)^3}{(-1+ax)^3}-\frac{12ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)^3}{(-1+ax)^3(1+ax)^2}+96\cosh^{-1}(ax)^2\log\left(1-e^{2\cosh^{-1}(ax)}\right)-60\log\left(\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\right)+96\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)-48\mathrm{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)\right)}{60ac^3\sqrt{c-a^2cx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/60*(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((4*I)*Pi^3 + 3/(1 - a^2*x^2) +
 (60*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) - (6*a*x*((-1
+ a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x])/(-1 + a*x)^3 - (9*ArcCosh[a*x]^2)/(-1
```

```
 + a^2*x^2)^2 + (24*ArcCosh[a*x]^2)/(-1 + a^2*x^2) - 32*ArcCosh[a*x]^3 - (3
2*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*x) + (16*a*x*((-1
+ a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - (12*a*x*Sqrt[(-1 + a
*x)/(1 + a*x)]*ArcCosh[a*x]^3)/((-1 + a*x)^3*(1 + a*x)^2) + 96*ArcCosh[a*x]
^2*Log[1 - E^(2*ArcCosh[a*x])] - 60*Log[Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x
)] + 96*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] - 48*PolyLog[3, E^(2*Ar
cCosh[a*x])]))/(a*c^3*Sqrt[c - a^2*c*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(564) = 1128$.
time = 2.85, size = 1319, normalized size = 2.17

method	result	size
default	Expression too large to display	1319

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-20*a^3*x^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*a^4*x^4+15*a*x+16*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(24+24*a^8*x^8-96*a^6*x^6+256*arccosh(a*x)^3-192*arccosh(a*
x)*a^8*x^8+852*arccosh(a*x)*a^6*x^6-1368*arccosh(a*x)^2*a^4*x^4+372*a*x*arc
cosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-192*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1
)^(1/2)*a^7*x^7+756*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5-936*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3+144*a^4*x^4-264*arccosh(a*x)
^2-480*arccosh(a*x)-96*a^2*x^2+24*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^7*x^7-84*(a
*x+1)^(1/2)*(a*x-1)^(1/2)*a^5*x^5+105*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3-4
5*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-192*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^
(1/2)*a^7*x^7+744*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5-1020*a
rccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3+495*arccosh(a*x)^2*(a*x-1
)^(1/2)*(a*x+1)^(1/2)*a*x-1590*arccosh(a*x)*a^4*x^4+984*arccosh(a*x)^2*a^2*
x^2+1410*arccosh(a*x)*a^2*x^2-192*arccosh(a*x)^2*a^8*x^8+840*arccosh(a*x)^2
*a^6*x^6+160*arccosh(a*x)^3*a^4*x^4-380*arccosh(a*x)^3*a^2*x^2)/(40*a^10*x^
10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4-(-c*(a^2*x^2-1
))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2)-1)-(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/
(a^2*x^2-1)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+2*(-c*(a^2*x^2-1))^(1/2)*
(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^
(1/2))-16/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*
x^2-1)*arccosh(a*x)^3+8/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2
)/c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+16
/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*arc
cosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(-c*(a^2*x^2-1))^
(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))+8/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
```

```
c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/5
*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*arcco
sh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(-c*(a^2*x^2-1))^(
1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(
1/2)*(a*x+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}(ax)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(7/2),x)
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,%%%{2,
[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1
,[4,2,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(7/2), x)
```

3.253 $\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=315

$$-\frac{45x^2\sqrt{-1+ax}}{128a^3\sqrt{1-ax}}-\frac{3x^4\sqrt{-1+ax}}{128a\sqrt{1-ax}}-\frac{45x\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{64a^4}-\frac{3x^3\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(a}{32a^2}$$

```
[Out] -45/128*x^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/128*x^4*(a*x-1)^(1/2)/a/(-a*
x+1)^(1/2)+45/128*arccosh(a*x)^2*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-9/16*x^2*
arccosh(a*x)^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/16*x^4*arccosh(a*x)^2*(a*
x-1)^(1/2)/a/(-a*x+1)^(1/2)+3/32*arccosh(a*x)^4*(a*x-1)^(1/2)/a^5/(-a*x+1)^
(1/2)-45/64*x*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-3/32*x^3*arccos
h(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-3/8*x*arccosh(a*x)^3*(-a^2*x^2+1)^(
1/2)/a^4-1/4*x^3*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.62, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5938, 5892, 5883, 5939, 5893, 30}

$$\frac{3\sqrt{ax-1}\,\cosh^{-1}(ax)^4}{32a^5\sqrt{1-ax}}+\frac{45\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{128a^5\sqrt{1-ax}}-\frac{45x\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{64a^4}-\frac{45x^2\sqrt{ax-1}}{128a^3\sqrt{1-ax}}-\frac{9x^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{16a^3\sqrt{1-ax}}-\frac{3x^3\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{32a^2}-\frac{x^3\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{4a^2}-\frac{3x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{8a^4}-\frac{3x^4\sqrt{ax-1}}{128a\sqrt{1-ax}}-\frac{3x^4\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{16a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-45*x^2*Sqrt[-1 + a*x])/(128*a^3*Sqrt[1 - a*x]) - (3*x^4*Sqrt[-1 + a*x])/(
128*a*Sqrt[1 - a*x]) - (45*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(64*
a^4) - (3*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(32*a^2) + (45*Sqrt
[-1 + a*x]*ArcCosh[a*x]^2)/(128*a^5*Sqrt[1 - a*x]) - (9*x^2*Sqrt[-1 + a*x]*
ArcCosh[a*x]^2)/(16*a^3*Sqrt[1 - a*x]) - (3*x^4*Sqrt[-1 + a*x]*ArcCosh[a*x]
^2)/(16*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(8*a^4) -
 (x^3*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh
[a*x]^4)/(32*a^5*Sqrt[1 - a*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^3}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ \cdots}}}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1 \cdots}{\cdots}$$

$$= -\frac{3x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{9x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{16a^3\sqrt{1-a^2x^2}} - \frac{3x^4 \cdots}{\cdots}$$

$$= -\frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cosh^{-1}(ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{3x^3(1-ax)(1+ \cdots}{32a^2\sqrt{1- \cdots}}$$

$$= -\frac{45x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{128a^3\sqrt{1-a^2x^2}} - \frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cos \cdots}{64a^4\sqrt{1-a^2x^2}}$$

<u>**Mathematica**</u> [**A**]
time = 0.32, size = 136, normalized size = 0.43

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(-192\left(1+2\cosh^{-1}(ax)^2\right)\cosh\left(2\cosh^{-1}(ax)\right)-3\left(1+8\cosh^{-1}(ax)^2\right)\cosh\left(4\cosh^{-1}(ax)\right)+4\cosh^{-1}(ax)\left(24\cosh^{-1}(ax)^3+32\left(3+2\cosh^{-1}(ax)^2\right)\sinh\left(2\cosh^{-1}(ax)\right)+\left(3+8\cosh^{-1}(ax)^2\right)\sinh\left(4\cosh^{-1}(ax)\right)\right)\right)}{1024a^5\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-192*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*A
rcCosh[a*x]] - 3*(1 + 8*ArcCosh[a*x]^2)*Cosh[4*ArcCosh[a*x]] + 4*ArcCosh[a*
x]*(24*ArcCosh[a*x]^3 + 32*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]] + (3
 + 8*ArcCosh[a*x]^2)*Sinh[4*ArcCosh[a*x]])))/(1024*a^5*Sqrt[-((-1 + a*x)*(1
 + a*x))])
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(259) = 518.
time = 5.08, size = 520, normalized size = 1.65

method	result
default	$-\frac{3\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{arccosh}(ax)^4}{32a^5(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\left(8a^5x^5-12a^3x^3+8\sqrt{ax+1}\,\sqrt{ax-1} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/32*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^4-1/2048*(-a^2*x^2+1)^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*(a*x+1)^(1/2)*(a
*x-1)^(1/2)*a^4*x^4+4*a*x-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/
2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)/a
^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh
(a*x)^2+6*arccosh(a*x)-3)/a^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^
3-2*a*x-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*
(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^5/(a^2*x^2-1)-1/2048
*(-a^2*x^2+1)^(1/2)*(-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^4*x^4+8*a^5*x^5+8*(a*
x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2-12*a^3*x^3-(a*x-1)^(1/2)*(a*x+1)^(1/2)+4*a
*x)*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)+3)/a^5/(a^2*x^2-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.254 $\int \frac{x^3 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=243

$$-\frac{40x\sqrt{-1+ax}}{9a^3\sqrt{1-ax}}-\frac{2x^3\sqrt{-1+ax}}{27a\sqrt{1-ax}}-\frac{40\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{9a^4}-\frac{2x^2\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{9a^2}-$$

```
[Out] -40/9*x*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-2/27*x^3*(a*x-1)^(1/2)/a/(-a*x+1)^
(1/2)-2*x*arccosh(a*x)^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/3*x^3*arccosh(a
*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-40/9*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+
1)^(1/2)/a^4-2/9*x^2*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-2/3*arcc
osh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)
/a^2
```

Rubi [A]

time = 0.41, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5938, 5914, 5879, 5915, 8, 5883, 5939, 30}

$$-\frac{40\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{9a^4}-\frac{40x\sqrt{ax-1}}{9a^3\sqrt{1-ax}}-\frac{2x\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{a^3\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{3a^2}-\frac{2x^2\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{9a^2}-\frac{2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{3a^4}-\frac{2x^3\sqrt{ax-1}}{27a\sqrt{1-ax}}-\frac{x^3\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*x*Sqrt[-1 + a*x])/(9*a^3*Sqrt[1 - a*x]) - (2*x^3*Sqrt[-1 + a*x])/(27*a
*Sqrt[1 - a*x]) - (40*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^4) - (
2*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^2) - (2*x*Sqrt[-1 + a*
x]*ArcCosh[a*x]^2)/(a^3*Sqrt[1 - a*x]) - (x^3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2
)/(3*a*Sqrt[1 - a*x]) - (2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(3*a^4) - (x^2
*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
```

```
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+a}}}{3a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{3a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-a}{3}$$

$$= -\frac{2x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1}}{}$$

$$= -\frac{2x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{9a^2\sqrt{1-a^2}}$$

$$= -\frac{40x\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}}{9a^4\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.12, size = 140, normalized size = 0.58

$$\frac{\sqrt{1-a^2x^2}\left(2ax(60+a^2x^2)-6\sqrt{-1+ax}\,\sqrt{1+ax}\,(20+a^2x^2)\cosh^{-1}(ax)+9ax(6+a^2x^2)\cosh^{-1}(ax)^2-9\sqrt{-1+ax}\,\sqrt{1+ax}\,(2+a^2x^2)\cosh^{-1}(ax)^3\right)}{27a^4\sqrt{-1+ax}\,\sqrt{1+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*a*x*(60 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(
20 + a^2*x^2)*ArcCosh[a*x] + 9*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 9*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a^4*Sqrt[-1 + a*x]
*Sqrt[1 + a*x])
```

Maple [A]
time = 3.42, size = 375, normalized size = 1.54

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(4a^4x^4-5a^2x^2+4\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+1\right)\left(9\mathrm{arccosh}(ax)^3-9\mathrm{arc}\right.}{216a^4(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2+4*(a*x+1)^(1/2)*(a*x-1)^(1/2
)*a^3*x^3-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3-9*arccosh(
a*x)^2+6*arccosh(a*x)-2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccosh(a*x)^3-3*arccosh(a*x)^2+6*arccosh
(a*x)-6)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x
-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arccosh(a*x)^2+6*arccosh(a*x)+6)/a^4/(a^
2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*a^4*x^4-5*a^2*x^2-4*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*a^3*x^3+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3+9*
arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 131, normalized size = 0.54

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^3+\frac{2}{27}a\left(\frac{3\left(-i\sqrt{a^2x^2-1}\,x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arcosh}(ax)}{a^3}+\frac{i\,a^2x^3+60i\,x}{a^4}\right)+\frac{(i\,a^2x^3+6i\,x)\operatorname{arcosh}(ax)^2}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^3
 + 2/27*a*(3*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)*arccos
h(a*x)/a^3 + (I*a^2*x^3 + 60*I*x)/a^4) + 1/3*(I*a^2*x^3 + 6*I*x)*arccosh(a*
x)^2/a^3
```

Fricas [A]
time = 0.40, size = 205, normalized size = 0.84

$$-\frac{9\,(a^4x^4+a^2x^2-2)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^3-9\,(a^3x^3+6\,ax)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2+6\,(a^4x^4+19\,a^2x^2-20)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)-2\,(a^3x^3+60\,ax)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}}{27\,(a^6x^2-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^3 - 9*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1))^2 + 6*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^3*x^3 + 60*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-
a^2*x^2 + 1))/(a^6*x^2 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.255 $\int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=188

$$-\frac{3x^2\sqrt{-1+ax}}{8a\sqrt{1-ax}}-\frac{3x\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{4a^2}+\frac{3\sqrt{-1+ax}\,\cosh^{-1}(ax)^2}{8a^3\sqrt{1-ax}}-\frac{3x^2\sqrt{-1+ax}\,\cosh^{-1}(a}{4a\sqrt{1-ax}}$$

```
[Out] -3/8*x^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+3/8*arccosh(a*x)^2*(a*x-1)^(1/2)/a^
3/(-a*x+1)^(1/2)-3/4*x^2*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+1/8*
arccosh(a*x)^4*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/4*x*arccosh(a*x)*(-a*x+1)
^(1/2)*(a*x+1)^(1/2)/a^2-1/2*x*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5938, 5892, 5883, 5939, 5893, 30}

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^4}{8a^3\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{2a^2}-\frac{3x\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{4a^2}-\frac{3x^2\sqrt{ax-1}}{8a\sqrt{1-ax}}-\frac{3x^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x])/(8*a*Sqrt[1 - a*x]) - (3*x*Sqrt[1 - a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x])/(4*a^2) + (3*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(8*a^3*Sqrt[1
 - a*x]) - (3*x^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(4*a*Sqrt[1 - a*x]) - (x*S
qrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/(2*a^2) + (Sqrt[-1 + a*x]*ArcCosh[a*x]^4)/
(8*a^3*Sqrt[1 - a*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
```

```
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5938

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcCosh[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2
*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] -
 Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^
p)], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcC
osh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1}}{}$$

$$= -\frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-}{}$$

$$= -\frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+}}{8a^3\sqrt{1-}}$$

Mathematica [A]
time = 0.17, size = 98, normalized size = 0.52

$$-\frac{\sqrt{-((-1+ax)(1+ax))}\left(-3\left(1+2\cosh^{-1}(ax)^2\right)\cosh\left(2\cosh^{-1}(ax)\right)+2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)^3+\left(3+2\cosh^{-1}(ax)^2\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)\right)}{16a^3\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/16*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-3*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*ArcC
osh[a*x]] + 2*ArcCosh[a*x]*(ArcCosh[a*x]^3 + (3 + 2*ArcCosh[a*x]^2)*Sinh[2*
ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [A]
time = 4.38, size = 255, normalized size = 1.36

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^4}{8a^3(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\,\left(2a^3x^3-2ax+2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^4-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x+2*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*a^2*x^2-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2
```

```
+6*arccosh(a*x)-3)/a^3/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-2*a*x
-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*x^2+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^3/(a^2*x^2-1)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.256 $\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{6x\sqrt{-1+ax}}{a\sqrt{1-ax}}-\frac{6\sqrt{1-ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a^2}-\frac{3x\sqrt{-1+ax}\,\cosh^{-1}(ax)^2}{a\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{a^2}$$

```
[Out] -6*x*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-3*x*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*
x+1)^(1/2)-6*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-arccosh(a*x)^3*(
-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5914, 5879, 5915, 8}

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{a^2}-\frac{6\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{a^2}-\frac{6x\sqrt{ax-1}}{a\sqrt{1-ax}}-\frac{3x\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-6*x*Sqrt[-1 + a*x])/(a*Sqrt[1 - a*x]) - (6*Sqrt[1 - a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x])/a^2 - (3*x*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(a*Sqrt[1 - a*x]) - (
Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3)/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \cosh^{-1}(ax)^2\,dx}{a\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} + \frac{\left(6\sqrt{-1+\right.}{}$$
$$= -\frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(}{a}$$
$$= -\frac{6x\sqrt{-1+ax}\,\sqrt{1+ax}}{a\sqrt{1-a^2x^2}} - \frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\,\sqrt{1+a}}{a\sqrt{1-a^2}}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 0.92

$$\frac{\sqrt{1-a^2x^2}\left(6ax-6\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)+3ax\cosh^{-1}(ax)^2-\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\right)}{a^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*a*x - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + 3
*a*x*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3))/(a^2*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])
```

Maple [A]
time = 2.45, size = 155, normalized size = 1.41

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)\left(\operatorname{arccosh}(ax)^3-3\operatorname{arccosh}(ax)^2+6\operatorname{arccosh}(ax)-6\right)}{2a^2(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^3-3*arccosh(a*x)^2+6*arccosh(a*x)-6)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1
)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arcco
sh(a*x)^2+6*arccosh(a*x)+6)/a^2/(a^2*x^2-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 65, normalized size = 0.59

$$\frac{3i\,x\operatorname{arcosh}\left(ax\right)^2}{a}-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}\left(ax\right)^3}{a^2}+\frac{6\left(i\,x-\frac{i\sqrt{a^2x^2-1}\,\operatorname{arcosh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*I*x*arccosh(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/a^2 + 6*(I*x - I
*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a)/a
```

Fricas [A]
time = 0.37, size = 159, normalized size = 1.45

$$\frac{3\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2+(-a^2x^2+1)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^3+6\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax-6\,(a^2x^2-1)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (3*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2
+ (-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 6*sqrt(a^2*x^2 - 1)
*sqrt(-a^2*x^2 + 1)*a*x - 6*(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\operatorname{acosh}^3\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.48, size = 103, normalized size = 0.94

$$-\frac{\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^2}-\frac{3i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2-1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^3/a^2 - 3*I*(x*log(a*x + s
qrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
 1))/a^2))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

3.257 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{-1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-ax}}$$

[Out] 1/4*arccosh(a*x)^4*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5892}

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]

[Out] (Sqrt[-1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a*x])

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.93, size = 51, normalized size = 1.59

method	result	size
default	$-\frac{\sqrt{-\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a x -1}\, \sqrt{a x +1}\, \operatorname{arccosh}\left(a x \right)^{4}}{4 a \left(a^{2} x^{2}-1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}\left(a x\right)}{\sqrt{-\left(a x - 1\right)\left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

3.258 $\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=265

$$\frac{2\sqrt{-1+ax}\,\cosh^{-1}(ax)^3\mathrm{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3i\sqrt{-1+ax}\,\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3i\sqrt{-}}{}$$

```
[Out] 2*arccosh(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*
x+1)^(1/2)-3*I*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)
))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3*I*arccosh(a*x)^2*polylog(2,I*(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+6*I*arccosh(a*x)*polylo
g(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-6*I*
arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(
-a*x+1)^(1/2)-6*I*polylog(4,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(
1/2)/(-a*x+1)^(1/2)+6*I*polylog(4,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x
-1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5946, 4265, 2611, 6744, 2320, 6724}

$$\frac{2\sqrt{ax-1}\,\cosh^{-1}(ax)^3\mathrm{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3i\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3i\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{6i\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{6i\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_3\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{6i\sqrt{ax-1}\,\mathrm{Li}_4\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{6i\sqrt{ax-1}\,\mathrm{Li}_4\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
3*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1
- a*x] + ((3*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])
/Sqrt[1 - a*x] + ((6*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCo
sh[a*x]])/Sqrt[1 - a*x] - ((6*I)*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, I*E
^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((6*I)*Sqrt[-1 + a*x]*PolyLog[4, (-I)*E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + ((6*I)*Sqrt[-1 + a*x]*PolyLog[4, I*E^ArcCosh[a*
x]])/Sqrt[1 - a*x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(3i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{S}}{}$$

$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{\sqrt{1-}}$$

$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{\sqrt{1-}}$$

$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{\sqrt{1-}}$$

$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{\sqrt{1-}}$$

Mathematica [**A**]
time = 0.46, size = 488, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/64)*Sqrt[-((-1 + a*x)*(1 + a*x))]*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[a*x] + 2
4*Pi^2*ArcCosh[a*x]^2 - (32*I)*Pi*ArcCosh[a*x]^3 - 16*ArcCosh[a*x]^4 + (8*I
)*Pi^3*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] - (96*I)*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*ArcCosh[a*
x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a
*x]] + (96*I)*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*Pi^3*Log[
1 + I*E^ArcCosh[a*x]] + 64*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)
*Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*(Pi - (2*I)*ArcCosh[a*x])^
2*PolyLog[2, (-I)/E^ArcCosh[a*x]] + 192*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^Ar
cCosh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*ArcCosh[a*x
]*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]]
 + 384*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] - 384*ArcCosh[a*x]*Poly
Log[3, (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcCosh[a*x]] + 384
```

```
*PolyLog[4, (-I)/E^ArcCosh[a*x]] + 384*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/(S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
[Out] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.259 $\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=166

$$\frac{a\sqrt{-1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{x}-\frac{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^2\log\left(1+e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3a\sqrt{}}{}$$

```
[Out] a*arccosh(a*x)^3*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*a*arccosh(a*x)^2*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*a*arccosh(a*
x)*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(
1/2)+3/2*a*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)-arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5917, 5882, 3799, 2221, 2611, 2320, 6724}

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{x}-\frac{3a\sqrt{ax-1}\,\cosh^{-1}(ax)\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3a\sqrt{ax-1}\,\text{Li}_3\left(-e^{2\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{a\sqrt{ax-1}\,\cosh^{-1}(ax)^3}{\sqrt{1-ax}}-\frac{3a\sqrt{ax-1}\,\cosh^{-1}(ax)^2\log\left(e^{2\cosh^{-1}(ax)}+1\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*ArcCosh[a*x]^3)/Sqrt[1 - a*x] - (Sqrt[1 - a^2*x^2]*ArcCos
h[a*x]^3)/x - (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])
])/Sqrt[1 - a*x] - (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCos
h[a*x])])/Sqrt[1 - a*x] + (3*a*Sqrt[-1 + a*x]*PolyLog[3, -E^(2*ArcCosh[a*x]
)])/(2*Sqrt[1 - a*x])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5917

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d +
e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)
*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3,
 0] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx,\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(6a\sqrt{-1+ax}\right.}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}}{}$$

Mathematica [A]
time = 0.37, size = 137, normalized size = 0.83

$$\frac{a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(2\cosh^{-1}(ax)^2\left(-\cosh^{-1}(ax)+\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\cosh^{-1}(ax)}{ax}-3\log\left(1+e^{-2\cosh^{-1}(ax)}\right)\right)+6\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax)}\right)+3\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(ax)}\right)\right)}{2\sqrt{-((-1+ax)(1+ax))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(2*ArcCosh[a*x]^2*(-ArcCosh[a*x] +
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 3*Log[1 + E^(-2
*ArcCosh[a*x])]) + 6*ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] + 3*Poly
Log[3, -E^(-2*ArcCosh[a*x])]))/(2*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [A]
time = 2.67, size = 313, normalized size = 1.89

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-1\right)\operatorname{arccosh}(ax)^3}{x(a^2x^2-1)}-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{arc}}{a^2x^2-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^3/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^3*a+3*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+3*(-a^2*x
^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*polylog(2,
-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a-3/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2
)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(3*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/((sqrt(a*x + 1)*a*x^2 + (a*x + 1
)*sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}(ax)}{x^{2}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

3.260 $\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=460

$$\frac{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{2x^2}-\frac{6a^2\sqrt{-1+ax}\,\cosh^{-1}(ax)\mathrm{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{a^2\cdots}{\cdots}$$

```
[Out] 3/2*a*arccosh(a*x)^2*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)-6*a^2*arccosh(a*x)*arct
an(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+a^2*arccos
h(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/
2)+3*I*a^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)-3/2*I*a^2*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*polylog(2,I*(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3/2*I*a^2*arccosh(a*x)^2*p
olylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+
3*I*a^2*arccosh(a*x)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1
)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*polylog(4,-I*(a*x+(a*x
-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3*I*a^2*polylog(4,I*
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*arccosh
(a*x)^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.37, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5932, 5946, 4265, 2611, 6744, 2320, 6724, 5883, 5947, 2317, 2438}

$$\frac{a^2\sqrt{ax-1}\,\cosh^{-1}(ax)^3\mathrm{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{6a^2\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{ArcTan}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_3\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3ia^2\sqrt{ax-1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{3ia^2\sqrt{ax-1}\,\mathrm{Li}_4\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\mathrm{Li}_4\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{2x^2}+\frac{3a\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (3*a*Sqrt[-1 + a*x]*ArcCosh[a*x]^2)/(2*x*Sqrt[1 - a*x]) - (Sqrt[1 - a^2*x^2
]*ArcCosh[a*x]^3)/(2*x^2) - (6*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*ArcTan[E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + (a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] + ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[2, (-I)*E^Arc
Cosh[a*x]])/Sqrt[1 - a*x] - (((3*I)/2)*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]^2*Po
lyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((3*I)*a^2*Sqrt[-1 + a*x]*Po
lyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + (((3*I)/2)*a^2*Sqrt[-1 + a*x]*A
rcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a*x] + ((3*I)*a^2*Sqrt
[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a*x] - ((
3*I)*a^2*Sqrt[-1 + a*x]*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 -
 a*x] - ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
```

```
a*x] + ((3*I)*a^2*Sqrt[-1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a*
x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcCosh[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1
))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[
b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && ILtQ[m, -1]
```

Rule 5946

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x
]/Sqrt[d + e*x^2])], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && Inte
gerQ[m]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{\right.}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+a\right.}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+a}}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+a}}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+a}}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+a}}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1051 vs. $2(460) = 920$.
time = 4.03, size = 1051, normalized size = 2.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-1/128*I)*a^2*(1 + a*x)*(7*Pi^4*Sqrt[(-1 + a*x)/(1 + a*x)] + (8*I)*Pi^3*S
qrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + 24*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]
*ArcCosh[a*x]^2 + ((192*I)*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2)/(a*x)
 + ((64*I)*(-1 + a*x)*ArcCosh[a*x]^3)/(a^2*x^2) - (32*I)*Pi*Sqrt[(-1 + a*x)
/(1 + a*x)]*ArcCosh[a*x]^3 - 16*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^4 -
 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] + (8
*I)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I/E^ArcCosh[a*x]] + 384*Sqrt[(-
1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*Sqrt[(
-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - (96*I)*Pi*Sqr
```

```
t[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*Sqrt[
(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*Sq
rt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a*x]] + (96*I)*Pi
*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I
)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I*E^ArcCosh[a*x]] + 64*Sqrt[(-1 +
 a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)*Pi^3*Sqrt
[(-1 + a*x)/(1 + a*x)]*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*Sqrt[(-1
+ a*x)/(1 + a*x)]*(8 + Pi^2 - (4*I)*Pi*ArcCosh[a*x] - 4*ArcCosh[a*x]^2)*Pol
yLog[2, (-I)/E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/
E^ArcCosh[a*x]] + 192*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*PolyLog[2,
(-I)*E^ArcCosh[a*x]] - 48*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I*E^Ar
cCosh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[2,
 I*E^ArcCosh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, (-I)/
E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3, (-
I)/E^ArcCosh[a*x]] - 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3,
 (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, I*
E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)/E^ArcCosh[
a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/Sq
rt[1 - a^2*x^2]
```

Maple [**F**]
time = 3.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
[Out] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

3.261 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=33

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 2.96, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^3}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*
x) + a^3)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\operatorname{acosh}(cx)\right)^3}{\sqrt{-(cx-1)(cx+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**3/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**3/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^3\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.262 $\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=67

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

```
[Out] 35/64*c^3*Shi(arccosh(a*x))/a-21/64*c^3*Shi(3*arccosh(a*x))/a+7/64*c^3*Shi(
5*arccosh(a*x))/a-1/64*c^3*Shi(7*arccosh(a*x))/a
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5906, 3393, 3379}

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (35*c^3*SinhIntegral[ArcCosh[a*x]])/(64*a) - (21*c^3*SinhIntegral[3*ArcCosh
[a*x]])/(64*a) + (7*c^3*SinhIntegral[5*ArcCosh[a*x]])/(64*a) - (c^3*SinhInt
egral[7*ArcCosh[a*x]])/(64*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)}\,dx = -\frac{c^3\text{Subst}\left(\int \frac{\sinh^7(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{(ic^3)\,\text{Subst}\left(\int \left(\frac{35i\sinh(x)}{64x} - \frac{21i\sinh(3x)}{64x} + \frac{7i\sinh(5x)}{64x} - \frac{i\sinh(7x)}{64x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^3\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a} + \frac{(7c^3)\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a} -$$

$$= \frac{35c^3\text{Shi}(\cosh^{-1}(ax))}{64a} - \frac{21c^3\text{Shi}(3\cosh^{-1}(ax))}{64a} + \frac{7c^3\text{Shi}(5\cosh^{-1}(ax))}{64a} - \frac{c^3\text{Shi}(7\cos}{64}$$

Mathematica [**A**]
time = 0.21, size = 45, normalized size = 0.67

$$\frac{c^3\left(35\text{Shi}(\cosh^{-1}(ax)) - 21\text{Shi}(3\cosh^{-1}(ax)) + 7\text{Shi}(5\cosh^{-1}(ax)) - \text{Shi}(7\cosh^{-1}(ax))\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (c^3*(35*SinhIntegral[ArcCosh[a*x]] - 21*SinhIntegral[3*ArcCosh[a*x]] + 7*S
inhIntegral[5*ArcCosh[a*x]] - SinhIntegral[7*ArcCosh[a*x]]))/(64*a)
```

Maple [**A**]
time = 2.43, size = 42, normalized size = 0.63

method	result
derivativedivides	$-\frac{c^3(-35\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))+21\,\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax))-7\,\text{hyperbolicSineIntegral}(5}{64a}$
default	$-\frac{c^3(-35\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))+21\,\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax))-7\,\text{hyperbolicSineIntegral}(5}{64a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/a*c^3*(-35*Shi(arccosh(a*x))+21*Shi(3*arccosh(a*x))-7*Shi(5*arccosh(a
*x))+Shi(7*arccosh(a*x)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{acosh}\left(ax\right)}\,dx+\int \left(-\frac{3a^{4}x^{4}}{\operatorname{acosh}\left(ax\right)}\right)dx+\int \frac{a^{6}x^{6}}{\operatorname{acosh}\left(ax\right)}\,dx+\int \left(-\frac{1}{\operatorname{acosh}\left(ax\right)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x), x) + Integral(-3*a**4*x**4/acosh(a*
x), x) + Integral(a**6*x**6/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^3}{\operatorname{acosh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/acosh(a*x),x)
```

```
[Out] int((c - a^2*c*x^2)^3/acosh(a*x), x)
```

3.263 $\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

```
[Out] 5/8*c^2*Shi(arccosh(a*x))/a-5/16*c^2*Shi(3*arccosh(a*x))/a+1/16*c^2*Shi(5*a
rccosh(a*x))/a
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5906, 3393, 3379}

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
```

```
[Out] (5*c^2*SinhIntegral[ArcCosh[a*x]])/(8*a) - (5*c^2*SinhIntegral[3*ArcCosh[a*
x]])/(16*a) + (c^2*SinhIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)}\,dx = \frac{c^2\text{Subst}\left(\int \frac{\sinh^5(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8x} - \frac{5i\sinh(3x)}{16x} + \frac{i\sinh(5x)}{16x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a} - \frac{(5c^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a} + \frac{(5c}{}$$
$$= \frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a} - \frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a} + \frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Mathematica [A]
time = 0.12, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\text{Shi}\left(\cosh^{-1}(ax)\right) - 5\text{Shi}\left(3\cosh^{-1}(ax)\right) + \text{Shi}\left(5\cosh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
[Out] (c^2*(10*SinhIntegral[ArcCosh[a*x]] - 5*SinhIntegral[3*ArcCosh[a*x]] + Sinh
Integral[5*ArcCosh[a*x]]))/(16*a)
```

Maple [A]
time = 2.77, size = 35, normalized size = 0.70

method	result
derivativedivides	$-\frac{c^2(-10\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))+5\,\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax))-\text{hyperbolicSineIntegral}(5\,\text{arcc}}{16a}$
default	$-\frac{c^2(-10\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))+5\,\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax))-\text{hyperbolicSineIntegral}(5\,\text{arcc}}{16a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x),x,method=_RETURNVERBOSE)
[Out] -1/16/a*c^2*(-10*Shi(arccosh(a*x))+5*Shi(3*arccosh(a*x))-Shi(5*arccosh(a*x)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\operatorname{acosh}(ax)}\right)dx+\int\frac{a^4x^4}{\operatorname{acosh}(ax)}\,dx+\int\frac{1}{\operatorname{acosh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x), x) + Integral(a**4*x**4/acosh(a*x),
 x) + Integral(1/acosh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(c-a^2\,c\,x^2\right)}^2}{\operatorname{acosh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/acosh(a*x),x)
```

```
[Out] int((c - a^2*c*x^2)^2/acosh(a*x), x)
```

3.264 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*c*Shi(arccosh(a*x))/a-1/4*c*Shi(3*arccosh(a*x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5906, 3393, 3379}

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x],x]
```

```
[Out] (3*c*SinhIntegral[ArcCosh[a*x]])/(4*a) - (c*SinhIntegral[3*ArcCosh[a*x]])/(
4*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx &= -\frac{c\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{a}\\ &= -\frac{(ic)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)\,dx,x,\cosh^{-1}(ax)\right)}{a}\\ &= -\frac{c\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{4a}+\frac{(3c)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{4a}\\ &= \frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 25, normalized size = 0.86

$$\frac{c\left(3\text{Shi}\left(\cosh^{-1}(ax)\right)-\text{Shi}\left(3\cosh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x],x]
[Out] (c*(3*SinhIntegral[ArcCosh[a*x]] - SinhIntegral[3*ArcCosh[a*x]]))/(4*a)
```

Maple [A]
time = 1.74, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{c(3\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))-\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax)))}{4a}$	24
default	$\frac{c(3\,\text{hyperbolicSineIntegral}(\text{arccosh}(ax))-\text{hyperbolicSineIntegral}(3\,\text{arccosh}(ax)))}{4a}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/4/a*c*(3*Shi(arccosh(a*x))-Shi(3*arccosh(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)/arccosh(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\operatorname{acosh}(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acosh}(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x),x)
[Out] -c*(Integral(a**2*x**2/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c-a^2\,c\,x^2}{\operatorname{acosh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/acosh(a*x),x)
[Out] int((c - a^2*c*x^2)/acosh(a*x), x)
```

3.265 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
[Out] Defer[Int][1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx=\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2c\,x^2+c)\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
[Out] -integrate(1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2}x^{2}\operatorname{acosh}{\left(ax \right)}-\operatorname{acosh}{\left(ax \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x),x)
[Out] -Integral(1/(a**2*x**2*acosh(a*x) - acosh(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)\,\left(c-a^2\,c\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(c - a^2*c*x^2)),x)
[Out] int(1/(acosh(a*x)*(c - a^2*c*x^2)), x)
```

3.266 $\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
```

```
[Out] Defer[Int][1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)}\,dx=\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)}\,dx$$

Mathematica [A]
time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^2\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Maple [A]
time = 2.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^2\operatorname{arccosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4 \operatorname{acosh}(ax) - 2a^2x^2 \operatorname{acosh}(ax) + \operatorname{acosh}(ax)}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x) - 2*a**2*x**2*acosh(a*x) + acosh(a*x)), x)
/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{acosh}(a\,x)\,\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(c - a^2*c*x^2)^2),x)
[Out] int(1/(acosh(a*x)*(c - a^2*c*x^2)^2), x)
```

3.267 $\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^5\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)}{32bc^5\sqrt{}}$$

```
[Out] -1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*x-1)^
(1/2)+1/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*
x-1)^(1/2)+1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^
5/(c*x-1)^(1/2)-1/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^5/(c*x-1)^(1/2
)+1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*x-1)
^(1/2)-1/16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^5/(c
*x-1)^(1/2)-1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c
^5/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^5\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^5\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/32*(Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]
)/(b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a
+ b*ArcCosh[c*x]))/b])/(16*b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(6*a
)/b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*Sqrt[-1 + c*x]) -
(Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]) + (Sqrt[1
 - c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*S
qrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCos
h[c*x]))/b])/(16*b*c^5*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhI
ntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^5*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)\,dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{32c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\right.}{32c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\right.}{16bc^5\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [**A**]
time = 0.38, size = 188, normalized size = 0.55

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\log\left(a+b\cosh^{-1}(cx)\right)+\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{32c^5\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]) +
 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2*a)/b]
*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(3
2*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(297) = 594.
time = 4.58, size = 597, normalized size = 1.76

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,6\,\text{arccosh}(cx)+\frac{6a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+6a}{b}}}{64(cx+1)c^5(cx-1)b}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/64
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*ar
ccosh(c*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/16*
(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*ln(a+b*arccosh(c*x))/b+1
/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*
arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*
(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arcc
osh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh
(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/32*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(
c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^5/(c*x-1)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\operatorname{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{1-c^2\,x^2}}{a+b\,\operatorname{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

3.268 $\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^4\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Chi}}{16bc^4\sqrt{-1}}$$

```
[Out] -1/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
+1/16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^
(1/2)+1/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*
x-1)^(1/2)+1/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*
x-1)^(1/2)-1/16*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^
4/(c*x-1)^(1/2)-1/16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)
/b/c^4/(c*x-1)^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*c^4*
Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(5*a)/b]*Cosh
Integral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 -
 c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^4*Sqrt[-1 + c*
x]) - (Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b]
)/(16*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*
(a + b*ArcCosh[c*x]))/b])/(16*b*c^4*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)}+\frac{\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x\right.}{16c^4\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\right.}{}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cos\right.}{16bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.32, size = 171, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{16c^4\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral
[5*(a/b + ArcCosh[c*x])]))/(16*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(261) = 522$.
time = 3.35, size = 547, normalized size = 1.84

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,5\,\text{arccosh}(cx)+\frac{5a}{b}\right)\mathrm{e}^{\frac{b\,\text{arccosh}(cx)+5a}{b}}}{32(cx+1)c^4(cx-1)b}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5
*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arc
cosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccos
h(c*x)-3*a/b)*exp(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh
(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x
)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*ex
p(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\operatorname{acosh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

3.269 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc^3\sqrt{-1+cx}}$$

```
[Out] 1/8*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1
/2)-1/8*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)-1/8*Shi(4*(
a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5952, 5556, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b
*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(8*b*c^3*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[
c*x]))/b])/(8*b*c^3*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{8c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\right.}{8c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.22, size = 103, normalized size = 0.74

$$-\frac{\sqrt{-((-1+cx)(1+cx))}\left(-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\cosh^{-1}(cx)\right)+\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{8bc^3\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
[Out] -1/8*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(-(Cosh[(4*a)/b]*CoshIntegral[4*(a/b +
ArcCosh[c*x])]) + Log[a + b*ArcCosh[c*x]] + Sinh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcCosh[c*x])]))/(b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A]
time = 5.30, size = 229, normalized size = 1.65

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,4\,\text{arccosh}(cx)+\frac{4a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+4a}{b}}}{16(cx+1)c^3(cx-1)b}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4
*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/16
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*ar
ccosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/8*(
-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\text{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

3.270 $\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^2\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\right.}{4bc^2\sqrt{-1+}}$$

```
[Out] -1/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
+1/4*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(
1/2)+1/4*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(
1/2)-1/4*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-
1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/4*(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*c^2*
Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(4*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[a/b]*SinhInteg
ral[(a + b*ArcCosh[c*x])/b])/(4*b*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh
[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*c^2*Sqrt[-1 + c*x]
)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x\right.}{4c^2\sqrt{-1+cx}\sqrt{1+}}\\
&= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\right.}{}\\
&= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cos\right.}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 127, normalized size = 0.64

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{4c^2\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[a/b]*SinhIntegral[a/b +
ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*c^2
*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(173) = 346$.
time = 2.29, size = 363, normalized size = 1.84

method	result
default	$\frac{\sqrt{-c^{2} x^{2}+1}\, \left(-\sqrt{c x +1}\, \sqrt{c x -1}\, x c +c^{2} x^{2}-1\right) \operatorname{expIntegral}\left(1, 3 \operatorname{arccosh}\left(c x \right)+\frac{3 a}{b}\right) {\mathrm e}^{\frac{b \operatorname{arccosh}\left(c x \right)+3 a}{b}}}{8 \left(c x +1\right) c^{2} \left(c x -1\right) b}+\frac{\sqrt{-c^{2} x^{2}+\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*
arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/8*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arcc
osh(c*x)-3*a/b)*exp(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c
*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*e
xp(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

3.271 $\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\,\text{c}}{}\right.}{2bc\sqrt{-1+cx}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2
)-1/2*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-1/2*Shi(2*(a+b*
arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5906, 3393, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*b
*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*c*Sqrt[-1
 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]
))/b])/(2*b*c*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\cosh^{-1}(cx\right.}{2c\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right.}{a+bx}\right.}{2c\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 105, normalized size = 0.76

$$\frac{\sqrt{-((-1+cx)(1+cx))}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\log\left(a+b\cosh^{-1}(cx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{2bc\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]

[Out] (Sqrt[-((-1 + c*x)*(1 + c*x))]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] - Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(2*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))

Maple [A]
time = 3.24, size = 229, normalized size = 1.65

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,2\,\mathrm{arccosh}(cx)+\frac{2a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+2a}{b}}}{4(cx+1)(cx-1)cb}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b-1/2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)), x)
```

3.272 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{-1+cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b\sqrt{1-cx}}+\mathrm{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\,(a+$$

```
[Out] -Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Shi((a+
b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/
x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*Sqrt[1
 - c*x])) + (Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(b*Sqrt[1 - c*x]) + Defer[Int][1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])
), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(-\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}+\frac{c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\,(a+\ldots}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{1-c^2x^2}\right)\int \ldots}{\sqrt{-\ldots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \ldots\right.}{\sqrt{-1+c\ldots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\ldots\right.}{\sqrt{\ldots}}$$

$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh\ldots\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x)),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))), x)
```

3.273 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=66

$$-\frac{c\sqrt{-1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{1-cx}}+\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] -c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/x^2/(
a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((c*Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*Sqrt[1 - c*x])) + Defer[Int
][1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{1-c^2x^2}\int \left(\frac{c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(\ldots}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(c^2\sqrt{1-c^2x^2}\right)\int \ldots}{\ldots}$$

$$=\frac{c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(\ldots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^2\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))), x)
```

3.274 $\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^3(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^3\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))), x)
```

3.275 $\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) + a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^{4}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x)),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))), x)
```

3.276 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^4\sqrt{-1+cx}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Ch}}{64bc^4\sqrt{-1}}$$

```
[Out] -3/64*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2
)+3/64*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)
^(1/2)+1/64*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^4/(c
*x-1)^(1/2)-1/64*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c
^4/(c*x-1)^(1/2)+3/64*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/
c^4/(c*x-1)^(1/2)-3/64*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/
2)/b/c^4/(c*x-1)^(1/2)-1/64*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1
)^(1/2)/b/c^4/(c*x-1)^(1/2)+1/64*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(64*b*c^4
*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(5*a)/b]*C
oshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[
1 - c*x]*Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*
Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*
x])/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhInte
gral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x
]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^4*Sqrt[-1
 + c*x]) + (Sqrt[1 - c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]
))/b])/(64*b*c^4*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{64(a+bx)} - \frac{3\cosh(3x)}{64(a+bx)} - \frac{\cosh(5x)}{64(a+bx)} + \frac{\cosh(7x)}{64(a+bx)}\right) dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x\right.}{64c^4\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3\sqrt{1-c^2x^2}\,\text{c}\right.}{}$$

$$= -\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\right.}{64bc^4\sqrt{-1+cx}\,\sqrt{1+c}}$$

Mathematica [A]
time = 0.68, size = 215, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{64c^4\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] - Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x])
] + 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[
c*x])] + Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^4*Sqrt[
(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(349) = 698.
time = 3.30, size = 731, normalized size = 1.84

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,7\,\text{arccosh}(cx)+\frac{7a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+7a}{b}}}{128(cx+1)c^4(cx-1)b} - \frac{\sqrt{-c^2x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1
,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/
128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7
*arccosh(c*x)-7*a/b)*exp(-(-b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/
128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*
arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arc
cosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(
c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-c^2*x^2+1
)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b
)*exp(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b)*
exp(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/128*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/b)*
exp(-(-b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

3.277 $\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{C}}{32bc^3\sqrt{-}}$$

```
[Out] 1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(
1/2)+1/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x
-1)^(1/2)-1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^3
/(c*x-1)^(1/2)-1/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)
-1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^
(1/2)-1/16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*
x-1)^(1/2)+1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c^
3/(c*x-1)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*
b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b])/(16*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(6*a)/b
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sq
rt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 -
c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt
[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c
*x]))/b])/(16*b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhInte
gral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} - \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right) dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\right.}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\,\mathrm{c}\right.}{16bc^3\sqrt{-1+cx}\,\sqrt{1+ca}}$$

Mathematica [A]
time = 0.54, size = 188, normalized size = 0.55

$$-\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\log\left(a+b\cosh^{-1}(cx)\right)+\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{32c^3\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/32*(Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x
])]) - 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]
*CoshIntegral[6*(a/b + ArcCosh[c*x])] + 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2
*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[
4*(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])
]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(297) = 594.
time = 4.00, size = 597, normalized size = 1.76

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,6\,\mathrm{arccosh}(cx)+\frac{6a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+6a}{b}}}{64(cx+1)c^3(cx-1)b}-\frac{\sqrt{-c^2x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/6
4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*a
rccosh(c*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/16
*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b+
1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4
*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arc
cosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccos
h(c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh
(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

3.278 $\int \frac{x(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^2\sqrt{-1+cx}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{C}}{16bc^2\sqrt{-}}$$

```
[Out] -1/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
+3/16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^
(1/2)-1/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*
x-1)^(1/2)+1/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*
x-1)^(1/2)-3/16*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^
2/(c*x-1)^(1/2)+1/16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)
/b/c^2/(c*x-1)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^2\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*c^2*
Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(5*a)/b]*Co
shIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1
 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*c^2*Sqrt[-1 +
c*x]) - (3*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x])
)/b])/(16*b*c^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[(5*a)/b]*SinhIntegral
[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c^2*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{16c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x\right.}{8c^2\sqrt{-1+cx}\,\sqrt{1+}} \\
&= -\frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(3\sqrt{1-c^2x^2}\,\cos\right.}{} \\
&= -\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{c}\right.}{16bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 0.51, size = 172, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{16c^2\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*
Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcCosh[c*x])]))/(16*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*
x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(261) = 522$.
time = 2.92, size = 547, normalized size = 1.84

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,5\,\mathrm{arccosh}(cx)+\frac{5a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+5a}{b}}}{32(cx+1)c^2(cx-1)b}-\frac{\sqrt{-c^2x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/3
2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*a
rccosh(c*x)-5*a/b)*exp(-(-b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arc
cosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c
*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*
exp(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32*(-c^2*x^2+1)^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b)*exp
(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

3.279 $\int \frac{(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc\sqrt{-1+cx}}-\frac{3\sqrt{1-cx}\,\log\left(a+b\right.}{8bc\sqrt{-1+c}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2
)-1/8*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)-3/8*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-1/2*Shi(2*(a+
b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)+1/8*Shi(4*(
a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5906, 3393, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8bc\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*b
*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*Ar
cCosh[c*x]))/b])/(8*b*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Log[a + b*ArcCos
h[c*x]])/(8*b*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral
[(2*(a + b*ArcCosh[c*x]))/b])/(2*b*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b*c*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{3\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{8c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{3\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2\right.}{a+bx}\right.}{2c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\,\text{c}\right.}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 147, normalized size = 0.62

$$-\frac{\sqrt{1-c^2x^2}\left(-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3\log\left(a+b\cosh^{-1}(cx)\right)+4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{8bc\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(Sqrt[1 - c^2*x^2]*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x
])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + 3*Log[a + b*ArcC
osh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Sinh[(4*
a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(b*c*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x))
```

Maple [A]
time = 3.24, size = 413, normalized size = 1.73

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,4\,\mathrm{arccosh}(cx)+\frac{4a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+4a}{b}}}{16(cx+1)(cx-1)cb}-\frac{\sqrt{-c^2x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b-1/16*
(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arc
cosh(c*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b-3/8*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b+1/4*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh
(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c^2*x^2+
1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-
2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)),x)
[Out] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)), x)
```

3.280 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=216

$$-\frac{5\sqrt{-1+cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b\sqrt{1-cx}}+\frac{5\sqrt{-1+cx}\,\sinh}{4b\sqrt{}}$$

```
[Out] -5/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/4
*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+5/4
*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/4*Shi
(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Uninteg
rable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (-5*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*Sqr
t[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b])/(4*b*Sqrt[1 - c*x]) + (5*Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(
a + b*ArcCosh[c*x])/b])/(4*b*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[(3*a)/b]
*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*Sqrt[1 - c*x]) + Defer[Int]
[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \left(\frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))} - \frac{2c^2x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+\cdots}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int \cdots}{\sqrt{-\cdots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \cdots\right.}{\sqrt{-1+c\cdots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\cdots\right.\right.}{\sqrt{\cdots}}$$

$$= \frac{2\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{co}\cdots\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{co}\cdots\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{5\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cdots\right.}{4b\sqrt{-1+cx}\,\sqrt{1+c\cdots}}$$

Mathematica [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\,\mathrm{arccosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x*arccosh(c*x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))), x)
```

3.281 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=164

$$\frac{c\sqrt{-1+cx}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2b\sqrt{1-cx}}-\frac{3c\sqrt{-1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2b\sqrt{1-cx}}-\frac{c\sqrt{-1+cx}\,\sinh\left(\frac{2a}{b}\right)\mathrm{S}}{2b\sqrt{1-}}$$

```
[Out] 1/2*c*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2
)-3/2*c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/2*c*Shi(2*(a+
b*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(
1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (c*Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
2*b*Sqrt[1 - c*x]) - (3*c*Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*Sqrt
[1 - c*x]) - (c*Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(2*b*Sqrt[1 - c*x]) + Defer[Int][1/(x^2*Sqrt[1 - c^2*x^2]*(a +
b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \left(-\frac{2c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))} + \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int \ldots}{\sqrt{\ldots}}$$

$$= \frac{2c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a\ldots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a\ldots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a\ldots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a\ldots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\,\text{co}\ldots\right.}{2b\sqrt{-1+cx}\,\sqrt{1+\ldots}}$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^2\left(a+b\,\text{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{2}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))), x)
```

3.282 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^3\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{3}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(1-c^2\,x^2)^{3/2}}{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))), x)
```

3.283 $\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\cosh^{-1}(cx))} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4(a+b\cosh^{-1}(cx))} dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^4\left(a+b\operatorname{arccosh}(cx)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^4*arccosh(c*x) + a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^4(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))), x)
```

3.284 $\int \frac{x^3\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128bc^4\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^4\sqrt{-1+cx}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{7a}{b}\right)\text{C}}{256bc^4\sqrt{-}}$$

```
[Out] -3/128*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/
2)+1/32*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1
)^(1/2)-3/256*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c^4/
(c*x-1)^(1/2)+1/256*Chi(9*(a+b*arccosh(c*x))/b)*cosh(9*a/b)*(-c*x+1)^(1/2)/
b/c^4/(c*x-1)^(1/2)+3/128*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2
)/b/c^4/(c*x-1)^(1/2)-1/32*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)
^(1/2)/b/c^4/(c*x-1)^(1/2)+3/256*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)-1/256*Shi(9*(a+b*arccosh(c*x))/b)*sinh(9*a
/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{256bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{256bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{256bc^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(\frac{9\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{256bc^4\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(128*b*c^
4*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b])/(32*b*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(7*a)/b]*
CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(256*b*c^4*Sqrt[-1 + c*x]) + (Sqr
t[1 - c*x]*Cosh[(9*a)/b]*CoshIntegral[(9*(a + b*ArcCosh[c*x]))/b])/(256*b*c
^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh
[c*x])/b])/(128*b*c^4*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIn
tegral[(3*(a + b*ArcCosh[c*x]))/b])/(32*b*c^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
 c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(256*b*c^4*Sq
rt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(9*a)/b]*SinhIntegral[(9*(a + b*ArcCosh
[c*x]))/b])/(256*b*c^4*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^6(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{3\cosh(x)}{128(a+bx)}+\frac{\cosh(3x)}{32(a+bx)}-\frac{3\cosh(7x)}{256(a+bx)}+\frac{\cosh(9x)}{256(a+bx)}\right)\,dx,x,\cosh^{-1}(\right.}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(9x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{256c^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\right.}{256c^4\sqrt{-1+cx}\,\sqrt{}} \\
&= -\frac{\left(3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{128c^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\text{c}\right.}{} \\
&= -\frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{128bc^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\right.}{32bc^4\sqrt{-1+cx}\,\sqrt{1+}}
\end{aligned}$$

Mathematica [A]
time = 0.95, size = 216, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-6\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+8\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{9a}{b}\right)\text{Chi}\left(9\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+6\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-8\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{9a}{b}\right)\text{Shi}\left(9\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{256c^4\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-6*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 8*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Cosh[(7*a)/b]*CoshIntegra
l[7*(a/b + ArcCosh[c*x])] + Cosh[(9*a)/b]*CoshIntegral[9*(a/b + ArcCosh[c*x
])] + 6*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 8*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcC
osh[c*x])] - Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcCosh[c*x])]))/(256*c^4*
Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(349) = 698$.
time = 4.29, size = 731, normalized size = 1.84

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,9\,\text{arccosh}(cx)+\frac{9a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+9a}{b}}}{512(cx+1)c^4(cx-1)b}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
9*arccosh(c*x)+9*a/b)*exp((b*arccosh(c*x)+9*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/5
12*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-9*
arccosh(c*x)-9*a/b)*exp(-(-b*arccosh(c*x)+9*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/5
12*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*a
rccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcco
sh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*
x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*
exp(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b)*exp
(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/512*(-c^2*x^2+1)^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*arccosh(c*x)-7*a/b)*exp
(-(-b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(1-c^2x^2\right)^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

3.285 $\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=439

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}}{32bc^3\sqrt{-1}}$$

```
[Out] 1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(
1/2)+1/32*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x
-1)^(1/2)-1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^3
/(c*x-1)^(1/2)+1/128*Chi(8*(a+b*arccosh(c*x))/b)*cosh(8*a/b)*(-c*x+1)^(1/2)
/b/c^3/(c*x-1)^(1/2)-5/128*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1
)^(1/2)-1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(
c*x-1)^(1/2)-1/32*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/
c^3/(c*x-1)^(1/2)+1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/
2)/b/c^3/(c*x-1)^(1/2)-1/128*Shi(8*(a+b*arccosh(c*x))/b)*sinh(8*a/b)*(-c*x+
1)^(1/2)/b/c^3/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128bc^3\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*
b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[(6*a)/b
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) + (Sq
rt[1 - c*x]*Cosh[(8*a)/b]*CoshIntegral[(8*(a + b*ArcCosh[c*x]))/b])/(128*b*
c^3*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(128*b*c^3*
Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCo
sh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(4*a)/b]*Sinh
Integral[(4*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 -
 c*x]*Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c^3*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcCosh[
c*x]))/b])/(128*b*c^3*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)}+\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{32(a+bx)}-\frac{\cosh(6x)}{32(a+bx)}+\frac{\cosh(8x)}{128(a+bx)}\right)\,dx, x,\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{128c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2\right.}{a+bx}\right.}{32c^3\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\,\text{c}\right.}{32bc^3\sqrt{-1+cx}\,\sqrt{1+c}}$$

Mathematica [A]
time = 0.88, size = 233, normalized size = 0.53

$$\frac{\sqrt{1-c^2x^2}\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{8a}{b}\right)\text{Chi}\left(8\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5\log\left(a+b\cosh^{-1}(cx)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{8a}{b}\right)\text{Shi}\left(8\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{128c^3\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] +
4*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] - 4*Cosh[(6*a)/b]*Cosh
Integral[6*(a/b + ArcCosh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcC
osh[c*x])] - 5*Log[a + b*ArcCosh[c*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcCosh[c*x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] +
 4*Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - Sinh[(8*a)/b]*SinhI
ntegral[8*(a/b + ArcCosh[c*x])]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(385) = 770$.
time = 3.92, size = 781, normalized size = 1.78

method	result

default	$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,8\,\mathrm{arccosh}(cx)+\frac{8a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+8a}{b}}}{256(cx+1)c^3(cx-1)b}+\frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/2
56*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-8*
arccosh(c*x)-8*a/b)*exp(-(-b*arccosh(c*x)+8*a)/b)/(c*x+1)/c^3/(c*x-1)/b-5/1
28*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/
b-1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1
,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/
64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*a
rccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(
c*x)-2*a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c
*x)-4*a/b)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/64*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arccosh(c*
x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(-(cx-1)\,(cx+1))^{\frac{5}{2}}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
[Out] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

3.286 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^2\sqrt{-1+cx}}+\frac{9\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{-1+cx}}-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)}{64bc^2\sqrt{-}}$$

```
[Out] -5/64*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2
)+9/64*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)
^(1/2)-5/64*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^2/(c
*x-1)^(1/2)+1/64*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c
^2/(c*x-1)^(1/2)+5/64*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/
c^2/(c*x-1)^(1/2)-9/64*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/
2)/b/c^2/(c*x-1)^(1/2)+5/64*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1
)^(1/2)/b/c^2/(c*x-1)^(1/2)-1/64*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5952, 5556, 3384, 3379, 3382}

$$-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{9\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^2\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(\frac{7(a+b\cosh^{-1}(cx))}{b}\right)}{64bc^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*Sqrt[1 - c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(64*b*c^2
*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[(5*a)/b]
*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) + (Sqr
t[1 - c*x]*Cosh[(7*a)/b]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b*c^
2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[
c*x])/b])/(64*b*c^2*Sqrt[-1 + c*x]) - (9*Sqrt[1 - c*x]*Sinh[(3*a)/b]*SinhIn
tegral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 -
 c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(64*b*c^2*Sqr
t[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcCosh[
c*x]))/b])/(64*b*c^2*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64(a+bx)}+\frac{9\cosh(3x)}{64(a+bx)}-\frac{5\cosh(5x)}{64(a+bx)}+\frac{\cosh(7x)}{64(a+bx)}\right)dx, x, \cosh^{-1}(cx\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\right.}{64c^2\sqrt{-1+cx}\,\sqrt{1}} \\
&= -\frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(9\sqrt{1-c^2x^2}\right.}{} \\
&= -\frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\right.}{64bc^2\sqrt{-1+cx}\,\sqrt{1+}}
\end{aligned}
$$

Mathematica [A]
time = 0.79, size = 216, normalized size = 0.54

$$
\frac{\sqrt{1-c^2x^2}\left(-5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-9\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{7a}{b}\right)\text{Shi}\left(7\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{64c^2\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-5*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 9*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegra
l[5*(a/b + ArcCosh[c*x])] + Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x
])] + 5*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 9*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcC
osh[c*x])] - Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^2*S
qrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(349) = 698$.
time = 2.14, size = 731, normalized size = 1.84

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,7\operatorname{arccosh}(cx)+\frac{7a}{b}\right)e^{\frac{b\operatorname{arccosh}(cx)+7a}{b}}}{128(cx+1)c^2(cx-1)b} + \frac{\sqrt{-c^2x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*
arccosh(c*x)-7*a/b)*exp(-(-b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*a
rccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*
(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcc
osh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c
*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)
*exp(-(-b*arccosh(c*x)+a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*(-c^2*x^2+1)^(1/2)
*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b)*e
xp(-(-b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1/2)
*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/b)*e
xp(-(-b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

3.287 $\int \frac{(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{15\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{-1+cx}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)}{32bc\sqrt{-}}$$

```
[Out] 15/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)-3/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)
^(1/2)+1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c/(c*x
-1)^(1/2)-5/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-15/32*
Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)+3/
16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
-1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)
```

Rubi [A]

time = 0.22, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5906, 3393, 3384, 3379, 3382}

$$\frac{15\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}}-\frac{15\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{16bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*Sqrt[1 - c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
32*b*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a
+ b*ArcCosh[c*x]))/b])/(16*b*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(6*a)/
b]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt[-1 + c*x]) - (5*S
qrt[1 - c*x]*Log[a + b*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]) - (15*Sqrt[1
- c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt
[-1 + c*x]) + (3*Sqrt[1 - c*x]*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh
[c*x]))/b])/(16*b*c*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Sinh[(6*a)/b]*SinhInte
gral[(6*(a + b*ArcCosh[c*x]))/b])/(32*b*c*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{5}{16(a+bx)} - \frac{15\cosh(2x)}{32(a+bx)} + \frac{3\cosh(4x)}{16(a+bx)} - \frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(c\right.}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \cosh^{-1}\right.}{32c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(15\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh(}{a+}\right.}{32c\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{15\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}\right.}{16bc\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [**A**]

time = 0.55, size = 191, normalized size = 0.56

$$\frac{\sqrt{1-c^2x^2}\left(15\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-6\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-10\log\left(a+b\cosh^{-1}(cx)\right)-15\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+6\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{32bc\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] -
 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 10*Log[a + b*ArcCosh[c*x]] - 15*Sinh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*Sinh[(4*a)/b]*SinhIntegral[4*
(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])])
)/(32*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(297) = 594$.
time = 4.40, size = 597, normalized size = 1.76

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,6\,\mathrm{arccosh}(cx)+\frac{6a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+6a}{b}}}{64(cx+1)(cx-1)cb}+\frac{\sqrt{-c^2x^2+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arcc
osh(c*x)-6*a/b)*exp(-(-b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c/b-5/16*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b-3/32*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccos
h(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x
)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*
a/b)*exp(-(-b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b-3/32*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-4*a/b
)*exp(-(-b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{a+b\operatorname{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)),x)
[Out] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)), x)
```

3.288 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\cosh^{-1}(cx))} dx$

Optimal. Leaf size=310

$$-\frac{11\sqrt{-1+cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b\sqrt{1-cx}}+\frac{7\sqrt{-1+cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16b\sqrt{1-cx}}-\frac{\sqrt{-1+cx}\cosh}{16}$$

```
[Out] -11/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+7/
16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1
/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+
11/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-7/1
6*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/
16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+U
nintegrable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (-11*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*Sq
rt[1 - c*x]) + (7*Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Cosh[(5*a)/b]*CoshInte
gral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) + (11*Sqrt[-1 + c*x]
*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b*Sqrt[1 - c*x]) - (7*S
qrt[-1 + c*x]*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16*b
*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcC
osh[c*x]))/b])/(16*b*Sqrt[1 - c*x]) + Defer[Int][1/(x*Sqrt[1 - c^2*x^2]*(a
+ b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(-\frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))} + \frac{3c^2x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(\ldots}\right.}{\sqrt{-\ldots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2\sqrt{1-c^2x^2}\right)\int \ldots}{\sqrt{\ldots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \ldots\right.}{\sqrt{-1+\ldots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\ldots\right.}{b\sqrt{-1+cx}\,\sqrt{1+c\ldots}}$$

$$= \frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\ldots\right.}{b\sqrt{-1+cx}\,\sqrt{1+c\ldots}}$$

$$= \frac{11\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{7\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}\ldots\right.}{16b\sqrt{-1+cx}\,\sqrt{1\ldots}}$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a
*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))), x)
```

3.289 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=255

$$\frac{c\sqrt{-1+cx}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b\sqrt{1-cx}}-\frac{c\sqrt{-1+cx}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{8b\sqrt{1-cx}}-\frac{15c\sqrt{-1+cx}\,\mathrm{lo}}{8b\sqrt{}}$$

```
[Out] c*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/
8*c*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-
15/8*c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-c*Shi(2*(a+b*arc
cosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/8*c*Shi(4*(a+b*a
rccosh(c*x))/b)*sinh(4*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/x
^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (c*Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(
b*Sqrt[1 - c*x]) - (c*Sqrt[-1 + c*x]*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*A
rcCosh[c*x]))/b])/(8*b*Sqrt[1 - c*x]) - (15*c*Sqrt[-1 + c*x]*Log[a + b*ArcC
osh[c*x]])/(8*b*Sqrt[1 - c*x]) - (c*Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegr
al[(2*(a + b*ArcCosh[c*x]))/b])/(b*Sqrt[1 - c*x]) + (c*Sqrt[-1 + c*x]*Sinh[
(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b*Sqrt[1 - c*x]) + De
fer[Int][1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\,\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\int \left(\frac{3c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))} - \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(\cdots}\right.}{\sqrt{\cdots}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2\sqrt{1-c^2x^2}\right)\cdots}{\cdots}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cdots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cdots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\cdots}}{\sqrt{-1+cx}\,\sqrt{1+c\cdots}}$$

$$= \frac{15c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\cdots}}{\sqrt{-1+cx}\,\sqrt{1+c\cdots}}$$

$$= -\frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Ch}\cdots}{8b\sqrt{-1+cx}\cdots}$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^2\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) +
 a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^2(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1 - c^2\,x^2\right)^{5/2}}{x^2\,\left(a + b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))), x)
```

3.290 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) +
 a*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^3(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))), x)
```

3.291 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\cosh^{-1}(cx))} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4(a+b\cosh^{-1}(cx))} dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\,\mathrm{arccosh}\,(cx)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) +
 a*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x^{4}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**4*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))), x)
```

3.292 $\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{-1+ax}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-ax}}+\frac{\sqrt{-1+ax}\,\mathrm{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}+\frac{3\sqrt{-1+ax}\,\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}$$

```
[Out] 1/2*Chi(2*arccosh(a*x))*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)+1/8*Chi(4*arccosh(
a*x))*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)+3/8*ln(arccosh(a*x))*(a*x-1)^(1/2)/a
^5/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5952, 3393, 3382}

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\,\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^5*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*CoshIntegral[4*ArcCosh[a*x]])/(8*a^5*Sqrt[1 - a*x]) + (3*Sqrt[-1
 + a*x]*Log[ArcCosh[a*x]])/(8*a^5*Sqrt[1 - a*x])
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^5\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \cosh^{-1}(a\right.}{a^5\sqrt{1-a^2x^2}}$$
$$= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\right.}{8a^5\sqrt{1-}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(4\cosh\right.}{8a^5\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.70

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\left(4\text{Chi}\left(2\cosh^{-1}(ax)\right)+\text{Chi}\left(4\cosh^{-1}(ax)\right)+3\log\left(\cosh^{-1}(ax)\right)\right)}{8a^5\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(4*CoshIntegral[2*ArcCosh[a*x]] + Cos
hIntegral[4*ArcCosh[a*x]] + 3*Log[ArcCosh[a*x]]))/(8*a^5*Sqrt[-((-1 + a*x)*
(1 + a*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(80) = 160$.
time = 4.81, size = 249, normalized size = 2.54

method	result
default	$\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,\text{expIntegral}(1,4\,\text{arccosh}(ax))}{16a^5(a^2x^2-1)} + \frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,\text{expIn}}{16a^5(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,4*
arccosh(a*x))+1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*
x^2-1)*Ei(1,-4*arccosh(a*x))-3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)/a^5/(a^2*x^2-1)*ln(arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,2*arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,-2*arccosh(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(ax-1)(ax+1)}\ \operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.293 $\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{3\sqrt{-1+ax}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}+\frac{\sqrt{-1+ax}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}$$

```
[Out] 3/4*Chi(arccosh(a*x))*(a*x-1)^(1/2)/a^4/(-a*x+1)^(1/2)+1/4*Chi(3*arccosh(a*
x))*(a*x-1)^(1/2)/a^4/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {5952, 3393, 3382}

$$\frac{3\sqrt{ax-1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (3*Sqrt[-1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(4*a^4*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*CoshIntegral[3*ArcCosh[a*x]])/(4*a^4*Sqrt[1 - a*x])
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+a\right.}{}$$
$$= \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{Chi}\left(3\cosh\right.}{4a^4\sqrt{1-a^2x^2}}$$

Mathematica [**A**]
time = 0.07, size = 60, normalized size = 0.92

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\left(3\text{Chi}\left(\cosh^{-1}(ax)\right)+\text{Chi}\left(3\cosh^{-1}(ax)\right)\right)}{4a^4\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(3*CoshIntegral[ArcCosh[a*x]] + CoshI
ntegral[3*ArcCosh[a*x]]))/(4*a^4*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(53) = 106$.
time = 5.56, size = 200, normalized size = 3.08

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{expIntegral}(1,3\,\text{arccosh}(ax))}{8a^4(a^2x^2-1)} + \frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{expIn}}{8a^4(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,3*a
rccosh(a*x))+1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^
2-1)*Ei(1,-3*arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
```

```
2)/a^4/(a^2*x^2-1)*Ei(1,arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,-arccosh(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.294 $\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{-1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}+\frac{\sqrt{-1+ax}\,\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}$$

```
[Out] 1/2*Chi(2*arccosh(a*x))*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)+1/2*ln(arccosh(a*x
))*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5952, 3393, 3382}

$$\frac{\sqrt{ax-1}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^3*Sqrt[1 - a*x]) + (Sqrt
[-1 + a*x]*Log[ArcCosh[a*x]])/(2*a^3*Sqrt[1 - a*x])
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)\,dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int\right.}{2a^3\sqrt{1-a}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh\right.}{2a^3\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.92

$$-\frac{\sqrt{-((-1+ax)(1+ax))}\left(\text{Chi}\left(2\cosh^{-1}(ax)\right)+\log\left(\cosh^{-1}(ax)\right)\right)}{2a^3\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -1/2*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(CoshIntegral[2*ArcCosh[a*x]] + Log[Arc
Cosh[a*x]]))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(53) = 106$.
time = 4.31, size = 149, normalized size = 2.29

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{expIntegral}(1,2\,\text{arccosh}(ax))}{4a^3(a^2x^2-1)} + \frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{expIn}}{4a^3(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*Ei(1,2*a
rccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^
```

```
2-1)*Ei(1,-2*arccosh(a*x))-1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a^3/(a^2*x^2-1)*ln(arccosh(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.295 $\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{-1+ax}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-ax}}$$

```
[Out] Chi(arccosh(a*x))*(a*x-1)^(1/2)/a^2/(-a*x+1)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5952, 3382}

$$\frac{\sqrt{ax-1}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[1 - a*x])
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x}{\sqrt{-1+ax}\,\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.79

$$-\frac{\sqrt{-((-1+ax)(1+ax))}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -((Sqrt[-((-1 + a*x)*(1 + a*x))]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[(-1
+ a*x)/(1 + a*x)]*(1 + a*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(24) = 48.
time = 3.01, size = 100, normalized size = 3.57

method	result
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\ \mathrm{expIntegral}(1,\mathrm{arccosh}(ax))}{2a^2(a^2x^2-1)}+\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\ \mathrm{expInte}}{2a^2(a^2x^2-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-1)*Ei(1,arc
cosh(a*x))+1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-
1)*Ei(1,-arccosh(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{acosh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.296 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{-1+ax}\,\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-ax}}$$

```
[Out] ln(arccosh(a*x))*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5890}

$$\frac{\sqrt{ax-1}\,\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Log[ArcCosh[a*x]])/(a*Sqrt[1 - a*x])
```

Rule 5890

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])
]*Log[a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\,\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.68

$$\frac{\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{-((-1+ax)(1+ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Log[ArcCosh[a*x]])/(a*Sqrt[-((-1 + a*
x)*(1 + a*x))])
```

Maple [A]
time = 3.64, size = 48, normalized size = 1.71

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{a x -1}\, \sqrt{a x +1}\, \ln \left(\operatorname{arccosh}\left(a x \right)\right)}{a \left(a^{2} x^{2}-1\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*ln(arccosh(a*
x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.
time = 0.36, size = 55, normalized size = 1.96

$$-\frac{\sqrt{a^{2} x^{2} - 1} \sqrt{-a^{2} x^{2} + 1} \log\left(\log\left(a x + \sqrt{a^{2} x^{2} - 1}\right)\right)}{a^{3} x^{2} - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(log(a*x + sqrt(a^2*x^2 - 1)))/(a^
3*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)} \operatorname{acosh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.297 $\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
[Out] Defer[Int][1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
[Out] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{arccosh}\,(ax)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arccosh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{acosh}(a x) \sqrt{1-a^2 x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.298 $\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
[Out] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

Maple [A]
time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,\mathrm{arccosh}\,(ax)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arccosh(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

3.299 $\int \frac{x^3}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=197

$$\frac{3\sqrt{-1+cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4bc^4\sqrt{1-cx}}-\frac{3\sqrt{-1+cx}\,\sinh\left(\frac{a}{b}\right)}{4bc^4\sqrt{1}}$$

```
[Out] 3/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1/2)+
1/4*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1
/2)-3/4*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1
/2)-1/4*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1
)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 3393, 3384, 3379, 3382}

$$\frac{3\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4bc^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4bc^4\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (3*Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(4*b*c^4*
Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCo
sh[c*x]))/b])/(4*b*c^4*Sqrt[1 - c*x]) - (3*Sqrt[-1 + c*x]*Sinh[a/b]*SinhInt
egral[(a + b*ArcCosh[c*x])/b])/(4*b*c^4*Sqrt[1 - c*x]) - (Sqrt[-1 + c*x]*Si
nh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b*c^4*Sqrt[1 - c*x
])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)\,dx,x,\cosh^{-1}(\right.}{c^4\sqrt{1-c^2x^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}}+\frac{\left(3\sqrt{\right.}{}\\
&= \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{4c^4\sqrt{1-c^2x^2}}\\
&= \frac{3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}}+\frac{\sqrt{-1+cx}}{}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 130, normalized size = 0.66

$$\frac{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{4bc^4\sqrt{-((-1+cx)(1+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Cosh[c*x])]))/(4*b*c^4*Sqrt[-((-1 + c*x)*(1 + c*x))])
```

Maple [A]
time = 4.22, size = 347, normalized size = 1.76

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,3\,\mathrm{arccosh}(cx)+\frac{3a}{b}\right)\mathrm{e}^{\frac{-b\,\mathrm{arccosh}(cx)+3a}{b}}}{8b(c^2x^2-1)c^4}+\frac{\sqrt{-c^2x^2+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*a
rccosh(c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)/b)/b/(c^2*x^2-1)/c^4+1/8*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c
*x)-3*a/b)*exp(-(b*arccosh(c*x)+3*a)/b)/b/(c^2*x^2-1)/c^4+3/8*(-c^2*x^2+1)^
(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*ex
p((-b*arccosh(c*x)+a)/b)/b/(c^2*x^2-1)/c^4+3/8*(-c^2*x^2+1)^(1/2)*((c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*exp(-(a+b*arccos
h(c*x))/b)/b/(c^2*x^2-1)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a+b\operatorname{acosh}(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.300 $\int \frac{x^2}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{-1+cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc^3\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-cx}}-\frac{\sqrt{-1+cx}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(}{}\right.}{2bc^3\sqrt{1-cx}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1
/2)+1/2*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1/2)-1/2*Shi(2*(
a+b*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5952, 3393, 3384, 3379, 3382}

$$\frac{\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc^3\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2bc^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(2*
b*c^3*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(2*b*c^3*Sq
rt[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(2*b*c^3*Sqrt[1 - c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)\,dx, x, \cosh^{-1}(\ldots\right.}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+c\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+c\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+c\ldots}}{\ldots}$$

Mathematica [A]
time = 0.21, size = 99, normalized size = 0.71

$$-\frac{\sqrt{1-c^2x^2}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\cosh^{-1}(cx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{2c^3\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -1/2*(Sqrt[1 - c^2*x^2]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]
 + Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*
x])]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

Maple [A]
time = 5.13, size = 232, normalized size = 1.67

method	result
default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{expIntegral}\left(1,2\,\mathrm{arccosh}(cx)+\frac{2a}{b}\right)\mathrm{e}^{\frac{-b\,\mathrm{arccosh}(cx)+2a}{b}}}{4b(c^2x^2-1)c^3}+\frac{\sqrt{-c^2x^2+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*a
rccosh(c*x)+2*a/b)*exp((-b*arccosh(c*x)+2*a)/b)/b/(c^2*x^2-1)/c^3+1/4*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c
*x)-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b)/b/(c^2*x^2-1)/c^3-1/2*(-c^2*x^2+1)^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3*ln(a+b*arccosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.301 $\int \frac{x}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{-1+cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}-\frac{\sqrt{-1+cx}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}$$

```
[Out] Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/c^2/(-c*x+1)^(1/2)-Shi(
(a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/c^2/(-c*x+1)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5952, 3384, 3379, 3382}

$$\frac{\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c*x])/b])/(b*c^2*Sqrt
[1 - c*x]) - (Sqrt[-1 + c*x]*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b]
)/(b*c^2*Sqrt[1 - c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}} - \frac{\sqrt{-1+cx}\,\sqrt{}}{}$$

Mathematica [**A**]
time = 0.16, size = 81, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{c^2\sqrt{\frac{-1+cx}{1+cx}}\,(b+bcx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Sinh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]]))/(c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
b*c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(84) = 168.
time = 3.55, size = 171, normalized size = 1.86

method	result

default	$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)e^{\frac{-b\,\text{arccosh}(cx)+a}{b}}}{2b(c^2x^2-1)c^2}+\frac{\sqrt{-c^2x^2+1}\,(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arc
cosh(c*x)+a/b)*exp((-b*arccosh(c*x)+a)/b)/b/(c^2*x^2-1)/c^2+1/2*(-c^2*x^2+1
)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)
*exp(-(a+b*arccosh(c*x))/b)/b/(c^2*x^2-1)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x) -
a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.302 $\int \frac{1}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{-1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-cx}}$$

```
[Out] ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5890}

$$\frac{\sqrt{cx-1}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[1 - c*x])
```

Rule 5890

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])
]*Log[a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 1.54

$$\frac{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{-((-1+cx)(1+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[-(
(-1 + c*x)*(1 + c*x))])
```

Maple [A]
time = 2.57, size = 55, normalized size = 1.57

method	result	size
default	$-\frac{\sqrt{-c^{2} x^{2}+1}\,\sqrt{c x -1}\,\sqrt{c x +1}\,\ln \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}{c \left(c^{2} x^{2}-1\right) b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)*ln(a+b*arccos
h(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.
time = 0.38, size = 65, normalized size = 1.86

$$-\frac{\sqrt{c^{2} x^{2}-1}\,\sqrt{-c^{2} x^{2}+1}\,\log\left(\frac{b \log\left(c x+\sqrt{c^{2} x^{2}-1}\right)+a}{b}\right)}{b c^{3} x^{2}-b c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*log((b*log(c*x + sqrt(c^2*x^2 - 1)) +
 a)/b)/(b*c^3*x^2 - b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(c x-1\right)\left(c x+1\right)}\,\left(a+b \operatorname{acosh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,\mathrm{acosh}\left(c\,x\right))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.303 $\int \frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arccosh(c
*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
[Out] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.304 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,\text{arccosh}\left(cx\right)\right)\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcco
sh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)

[Out] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)

3.305 $\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^2}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 3.39, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(a + b\,\text{acosh}\,(c\,x))\,\,(1 - c^2\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.306 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 5.13, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c
^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.307 $\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2
*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.308 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]

[Out] Defer[Int][1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]

[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 - 2
*b*c^2*x^3 + b*x)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.309 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]

[Out] Defer[Int][1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]

[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 -
 2*b*c^2*x^4 + b*x^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}} \left(a+b\operatorname{acosh}\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.310 $\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^m(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(-c^2x^2+1)^{\frac{3}{2}}}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccosh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(1-c^2\,x^2\right)^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

3.311 $\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^m\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-c^2x^2+1}}{a+b\operatorname{arccosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-\left(cx - 1\right)\left(cx + 1\right)}}{a + b \operatorname{acosh}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \sqrt{1-c^2 x^2}}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

3.312 $\int \frac{x^m}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}\,\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a + b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.313 $\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}\,(a+b\,\mathrm{arccosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**m/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

3.314 $\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]

[Out] Defer[Int][x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]

Rubi steps

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]

[Out] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (
b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccosh(c*x) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

3.315 $\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=98

$$\frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)}+\frac{35c^3\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\mathrm{C}}{}$$

```
[Out] c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)/a/arccosh(a*x)+35/64*c^3*Chi(arccosh(a*x))/
a-63/64*c^3*Chi(3*arccosh(a*x))/a+35/64*c^3*Chi(5*arccosh(a*x))/a-7/64*c^3*
Chi(7*arccosh(a*x))/a
```

Rubi [A]
time = 0.22, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5904, 5953, 5556, 3382}

$$\frac{35c^3\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\mathrm{Chi}\left(7\cosh^{-1}(ax)\right)}{64a}+\frac{c^3(ax-1)^{7/2}(ax+1)^{7/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2))/(a*ArcCosh[a*x]) + (35*c^3*CoshInteg
ral[ArcCosh[a*x]])/(64*a) - (63*c^3*CoshIntegral[3*ArcCosh[a*x]])/(64*a) +
(35*c^3*CoshIntegral[5*ArcCosh[a*x]])/(64*a) - (7*c^3*CoshIntegral[7*ArcCos
h[a*x]])/(64*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
```

```
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)^2}\,dx = \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \left(7ac^3\right)\int \frac{x(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)}\,dx$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\mathrm{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x}+\frac{9\cosh(3x)}{64x}-\frac{5\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)\right.}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\mathrm{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a} + \frac{\left(35c^3\right)\mathrm{Subst}\left(\right.}{}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} + \frac{35c^3\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{64a} - \frac{63c^3\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{64a} + \frac{35c^3\mathrm{C}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(98) = 196.
time = 0.27, size = 257, normalized size = 2.62

$$\frac{c^3\left(-64\sqrt{\frac{-1+ax}{1+ax}}-64ax\sqrt{\frac{-1+ax}{1+ax}}+192a^2x^2\sqrt{\frac{-1+ax}{1+ax}}+192a^3x^3\sqrt{\frac{-1+ax}{1+ax}}-192a^4x^4\sqrt{\frac{-1+ax}{1+ax}}-192a^5x^5\sqrt{\frac{-1+ax}{1+ax}}+64a^6x^6\sqrt{\frac{-1+ax}{1+ax}}+64a^7x^7\sqrt{\frac{-1+ax}{1+ax}}+35\cosh^{-1}(ax)\mathrm{Chi}\left(\cosh^{-1}(ax)\right)-63\cosh^{-1}(ax)\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)+35\cosh^{-1}(ax)\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)-7\cosh^{-1}(ax)\mathrm{Chi}\left(7\cosh^{-1}(ax)\right)\right)}{64a\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(-64*Sqrt[(-1 + a*x)/(1 + a*x)] - 64*a*x*Sqrt[(-1 + a*x)/(1 + a*x)] +
192*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)] + 192*a^3*x^3*Sqrt[(-1 + a*x)/(1 + a
*x)] - 192*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] - 192*a^5*x^5*Sqrt[(-1 + a*x)
/(1 + a*x)] + 64*a^6*x^6*Sqrt[(-1 + a*x)/(1 + a*x)] + 64*a^7*x^7*Sqrt[(-1 +
 a*x)/(1 + a*x)] + 35*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] - 63*ArcCosh[
```

a*x]*CoshIntegral[3*ArcCosh[a*x]] + 35*ArcCosh[a*x]*CoshIntegral[5*ArcCosh[
a*x]] - 7*ArcCosh[a*x]*CoshIntegral[7*ArcCosh[a*x]]))/(64*a*ArcCosh[a*x])

Maple [A]
time = 4.33, size = 109, normalized size = 1.11

method	result
derivativedivides	$-\frac{c^3\left(7\,\mathrm{hyperbolicCosineIntegral}(7\,\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-35\,\mathrm{hyperbolicCosineIntegral}(\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)+\right.}{}$
default	$-\frac{c^3\left(7\,\mathrm{hyperbolicCosineIntegral}(7\,\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-35\,\mathrm{hyperbolicCosineIntegral}(\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/a*c^3*(7*Chi(7*arccosh(a*x))*arccosh(a*x)-35*Chi(arccosh(a*x))*arccos
h(a*x)+63*Chi(3*arccosh(a*x))*arccosh(a*x)-35*Chi(5*arccosh(a*x))*arccosh(a
*x)+35*(a*x-1)^(1/2)*(a*x+1)^(1/2)-21*sinh(3*arccosh(a*x))+7*sinh(5*arccosh
(a*x))-sinh(7*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^9*c^3*x^9 - 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 4*a^3*c^3*x^3 + a*c^3*x + (a
^8*c^3*x^8 - 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 4*a^2*c^3*x^2 + c^3)*sqrt(a*x
+ 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((7*a^10*c^3*x^10 - 29*a^8*c
^3*x^8 + 46*a^6*c^3*x^6 - 34*a^4*c^3*x^4 + 11*a^2*c^3*x^2 + (7*a^8*c^3*x^8
- 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 - 4*a^2*c^3*x^2 - c^3)*(a*x + 1)*(a*x - 1
) - c^3 + 7*(2*a^9*c^3*x^9 - 7*a^7*c^3*x^7 + 9*a^5*c^3*x^5 - 5*a^3*c^3*x^3
+ a*c^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2
*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x)^
2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3a^2x^2}{\operatorname{acosh}^2(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acosh}^2(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acosh}^2(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acosh}^2(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x)**2,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x)**2, x) + Integral(-3*a**4*x**4/acosh
(a*x)**2, x) + Integral(a**6*x**6/acosh(a*x)**2, x) + Integral(-1/acosh(a*x
)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^3}{{\mathrm{acosh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)^3/acosh(a*x)^2, x)
```

3.316 $\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=82

$$-\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{5c^2\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

```
[Out] -c^2*(a*x-1)^(5/2)*(a*x+1)^(5/2)/a/arccosh(a*x)+5/8*c^2*Chi(arccosh(a*x))/a
-15/16*c^2*Chi(3*arccosh(a*x))/a+5/16*c^2*Chi(5*arccosh(a*x))/a
```

Rubi [A]
time = 0.20, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5904, 5953, 5556, 3382}

$$\frac{5c^2\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}-\frac{c^2(ax-1)^{5/2}(ax+1)^{5/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] -((c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2))/(a*ArcCosh[a*x])) + (5*c^2*CoshInt
egral[ArcCosh[a*x]])/(8*a) - (15*c^2*CoshIntegral[3*ArcCosh[a*x]])/(16*a) +
 (5*c^2*CoshIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c-a^2cx^2\right)^2}{\cosh^{-1}(ax)^2}\,dx &= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\left(5ac^2\right)\int \frac{x(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)}\,dx\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{a}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x}-\frac{3\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)\,dx,x,\cosh\right.}{a}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subst}}{}\\
&= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)}+\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Ch}}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(82) = 164.
time = 0.21, size = 194, normalized size = 2.37

$$-\frac{c^2\left(16\sqrt{\frac{-1+ax}{1+ax}}+16ax\sqrt{\frac{-1+ax}{1+ax}}-32a^2x^2\sqrt{\frac{-1+ax}{1+ax}}-32a^3x^3\sqrt{\frac{-1+ax}{1+ax}}+16a^4x^4\sqrt{\frac{-1+ax}{1+ax}}+16a^5x^5\sqrt{\frac{-1+ax}{1+ax}}-10\cosh^{-1}(ax)\text{Chi}\left(\cosh^{-1}(ax)\right)+15\cosh^{-1}(ax)\text{Chi}\left(3\cosh^{-1}(ax)\right)-5\cosh^{-1}(ax)\text{Chi}\left(5\cosh^{-1}(ax)\right)\right)}{16a\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] -1/16*(c^2*(16*Sqrt[(-1 + a*x)/(1 + a*x)] + 16*a*x*Sqrt[(-1 + a*x)/(1 + a*x
)] - 32*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)] - 32*a^3*x^3*Sqrt[(-1 + a*x)/(1
+ a*x)] + 16*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] + 16*a^5*x^5*Sqrt[(-1 + a*x
)/(1 + a*x)] - 10*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] + 15*ArcCosh[a*x]
*CoshIntegral[3*ArcCosh[a*x]] - 5*ArcCosh[a*x]*CoshIntegral[5*ArcCosh[a*x]]
))/(a*ArcCosh[a*x])
```

Maple [A]
time = 2.48, size = 85, normalized size = 1.04

method	result
derivativedivides	$-\frac{c^2\left(10\sqrt{ax-1}\,\sqrt{ax+1}+15\,\text{hyperbolicCosineIntegral}(3\,\text{arccosh}(ax))\text{arccosh}(ax)-5\,\text{hyperbolicCosineInteg}\right.}{}$
default	$-\frac{c^2\left(10\sqrt{ax-1}\,\sqrt{ax+1}+15\,\text{hyperbolicCosineIntegral}(3\,\text{arccosh}(ax))\text{arccosh}(ax)-5\,\text{hyperbolicCosineInteg}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/16/a*c^2*(10*(a*x-1)^(1/2)*(a*x+1)^(1/2)+15*Chi(3*arccosh(a*x))*arccosh(
a*x)-5*Chi(5*arccosh(a*x))*arccosh(a*x)-10*Chi(arccosh(a*x))*arccosh(a*x)-5
*sinh(3*arccosh(a*x))+sinh(5*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
[Out] -(a^7*c^2*x^7 - 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x + (a^6*c^2*x^6 - 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
1))) + integrate((5*a^8*c^2*x^8 - 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 - 8*a^2*c
^2*x^2 + (5*a^6*c^2*x^6 - 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*(a*x + 1)*(a
*x - 1) + 5*(2*a^7*c^2*x^7 - 5*a^5*c^2*x^5 + 4*a^3*c^2*x^3 - a*c^2*x)*sqrt(
a*x + 1)*sqrt(a*x - 1) + c^2)/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a
^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a*x + sqrt(
a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-\frac{2a^{2}x^{2}}{\operatorname{acosh}^{2}(ax)}\right)dx+\int\frac{a^{4}x^{4}}{\operatorname{acosh}^{2}(ax)}\,dx+\int\frac{1}{\operatorname{acosh}^{2}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x)**2, x) + Integral(a**4*x**4/acosh(a*
x)**2, x) + Integral(acosh(a*x)**(-2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c-a^{2}\,c\,x^{2}\right)}^{2}}{{\operatorname{acosh}\left(a\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)^2/acosh(a*x)^2, x)
```

3.317 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=58

$$\frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)}+\frac{3c\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

```
[Out] c*(a*x-1)^(3/2)*(a*x+1)^(3/2)/a/arccosh(a*x)+3/4*c*Chi(arccosh(a*x))/a-3/4*
c*Chi(3*arccosh(a*x))/a
```

Rubi [A]
time = 0.15, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5904, 5953, 5556, 3382}

$$\frac{3c\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2))/(a*ArcCosh[a*x]) + (3*c*CoshIntegral[A
rcCosh[a*x]])/(4*a) - (3*c*CoshIntegral[3*ArcCosh[a*x]])/(4*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2}\,dx &= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - (3ac)\int \frac{x\sqrt{-1+ax}\,\sqrt{1+ax}}{\cosh^{-1}(ax)}\,dx \\
&= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)\,dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{(3c)\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a} - \frac{(3c)\mathrm{Subst}\left(\int \frac{\cosh}{x}\right.}{} \\
&= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{3c\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a} - \frac{3c\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(58) = 116.
time = 1.29, size = 217, normalized size = 3.74

$$
\frac{c\sqrt{-1+ax}\left(4\left(\frac{-1+ax}{1+ax}\right)^{5/2}(1+ax)^5-3(-1+a^2x^2)\cosh^{-1}(ax)\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)-(-1+ax)\cosh^{-1}(ax)\mathrm{Chi}\left(\cosh^{-1}(ax)\right)\left(1+ax-4\sqrt{-1+ax}\,\sqrt{1+ax}\,\coth\left(\frac{1}{2}\cosh^{-1}(ax)\right)\right)+4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\left(\sqrt{\frac{-1+ax}{1+ax}}(1+ax)+(1-ax)\coth\left(\frac{1}{2}\cosh^{-1}(ax)\right)\right)\log\left(\cosh^{-1}(ax)\right)\right)}{4a\left(\frac{-1+ax}{1+ax}\right)^{3/2}(1+ax)^{5/2}\cosh^{-1}(ax)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*Sqrt[-1 + a*x]*(4*((-1 + a*x)/(1 + a*x))^(5/2)*(1 + a*x)^5 - 3*(-1 + a^2
*x^2)*ArcCosh[a*x]*CoshIntegral[3*ArcCosh[a*x]] - (-1 + a*x)*ArcCosh[a*x]*C
oshIntegral[ArcCosh[a*x]]*(1 + a*x - 4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Coth[Ar
cCosh[a*x]/2]) + 4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*(Sqrt[(-1 + a*
x)/(1 + a*x)]*(1 + a*x) + (1 - a*x)*Coth[ArcCosh[a*x]/2])*Log[ArcCosh[a*x]]
))/(4*a*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])
```

Maple [A]
time = 2.12, size = 61, normalized size = 1.05

method	result
derivativedivides	$\frac{c\left(3\,\mathrm{hyperbolicCosineIntegral}(\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-3\,\mathrm{hyperbolicCosineIntegral}(3\,\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-3\sqrt{a}\right.}{4a\,\mathrm{arccosh}(ax)}$
default	$\frac{c\left(3\,\mathrm{hyperbolicCosineIntegral}(\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-3\,\mathrm{hyperbolicCosineIntegral}(3\,\mathrm{arccosh}(ax))\mathrm{arccosh}(ax)-3\sqrt{a}\right.}{4a\,\mathrm{arccosh}(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*c*(3*Chi(arccosh(a*x))*arccosh(a*x)-3*Chi(3*arccosh(a*x))*arccosh(a*x
)-3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+sinh(3*arccosh(a*x)))/arccosh(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 - 2*a^2*c*x^2 + c)*sqrt(a*x +
 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((3*a^6*c*x^6 - 7*a^4*c*x^4 +
 5*a^2*c*x^2 + (3*a^4*c*x^4 - 2*a^2*c*x^2 - c)*(a*x + 1)*(a*x - 1) + 3*(2*a
^5*c*x^5 - 3*a^3*c*x^3 + a*c*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - c)/((a^4*x^4
+ (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\mathrm{acosh}^2\,(ax)}\,dx+\int\left(-\frac{1}{\mathrm{acosh}^2\,(ax)}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x)**2,x)
```

```
[Out] -c*(Integral(a**2*x**2/acosh(a*x)**2, x) + Integral(-1/acosh(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c-a^2\,c\,x^2}{{\mathrm{acosh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)/acosh(a*x)^2, x)
```

3.318 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{ac\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}+\frac{a\mathrm{Int}\left(\frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)},x\right)}{c}$$

```
[Out] 1/a/c/arccosh(a*x)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+a*Unintegrable(x/(a*x-1)^(3/
2)/(a*x+1)^(3/2)/arccosh(a*x),x)/c
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] 1/(a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]) + (a*Defer[Int][x/((-1 +
a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx=\frac{1}{ac\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}+\frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}\,dx}{c}$$

Mathematica [A]
time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2c\,x^2+c)\,\mathrm{arccosh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*c*x^2 + sqrt(a*x + 1)*sqrt(a*x -
1)*a^2*c*x - a*c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) + integrate((a^4*
x^4 + (a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + (2*a^3*x^3 - a*x)*sqrt(a*x + 1)*s
qrt(a*x - 1) - 1)/((a^6*c*x^6 - 3*a^4*c*x^4 + 3*a^2*c*x^2 + (a^4*c*x^4 - a^
2*c*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x)*sqrt(a*x
 + 1)*sqrt(a*x - 1) - c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2}x^{2}\operatorname{acosh}^{2}{\left(ax \right)} - \operatorname{acosh}^{2}{\left(ax \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*acosh(a*x)**2 - acosh(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^2\,\left(c-a^2\,c\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)),x)
[Out] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)), x)
```

3.319 $\int \frac{1}{(c-a^2cx^2)^2\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{ac^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}-\frac{3a\text{Int}\left(\frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)},x\right)}{c^2}$$

```
[Out] -1/a/c^2/(a*x-1)^(3/2)/(a*x+1)^(3/2)/arccosh(a*x)-3*a*Unintegrable(x/(a*x-1
)^(5/2)/(a*x+1)^(5/2)/arccosh(a*x),x)/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^2\cosh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
[Out] -(1/(a*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])) - (3*a*Defer[Int
][x/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^2\cosh^{-1}(ax)^2}\,dx=-\frac{1}{ac^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}-\frac{(3a)\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}$$

Mathematica [A]
time = 9.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^2\cosh^{-1}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2), x]
```

Maple [A]
time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2c\,x^2+c)^2\operatorname{arccosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^5*c^2*x^4 - 2*a^3*c^2*x^2 + a*c^2
+ (a^4*c^2*x^3 - a^2*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))) - integrate((3*a^4*x^4 - 2*a^2*x^2 + (3*a^2*x^2 - 1)*
(a*x + 1)*(a*x - 1) + 3*(2*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - 1)/
((a^8*c^2*x^8 - 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 4*a^2*c^2*x^2 + (a^6*c^2*x^
6 - 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^7*c^2*x^7 - 3*a
^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + c^2)*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{acosh}^2(ax)-2a^2x^2\operatorname{acosh}^2(ax)+\operatorname{acosh}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x)**2 - 2*a**2*x**2*acosh(a*x)**2 + acosh(a*x
)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^2\,{\left(c-a^2\,c\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)^2),x)
```

```
[Out] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)^2), x)
```

3.320 $\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^4\sqrt{-1+cx}}-\frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{-1+cx}}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^4/(c*x-1)^(1/
2)+3/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^4/(c*x
-1)^(1/2)+5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
^4/(c*x-1)^(1/2)+1/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2
/c^4/(c*x-1)^(1/2)-3/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1
/2)/b^2/c^4/(c*x-1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*
x+1)^(1/2)/b^2/c^4/(c*x-1)^(1/2)-x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.47, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5942, 5887, 5556, 3384, 3379, 3382}

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}-\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[
c*x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8
*b^2*c^4*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cos
hIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^4*Sqrt[-1 +
c*x]) - (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
^2*c^4*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c*x]))/b])/(16*b^2*c^4*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cosh[
(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^4*Sqrt[-1 + c*
x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(5c\sqrt{\right.}}{}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x\right.}{bc^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right.\right.}{bc^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh\right.}{16bc^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\right.}{8bc^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.56, size = 322, normalized size = 0.92

$$\frac{\sqrt{1-c^2x^2}\left(16bc^3x^3-16bc^5x^5+2\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-3\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-5a\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-5b\cosh^{-1}(cx)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-2a\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-2b\cosh^{-1}(cx)\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3a\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3b\cosh^{-1}(cx)\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5a\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5b\cosh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{16b^2c^4\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(16*b*c^3*x^3 - 16*b*c^5*x^5 + 2*(a + b*ArcCosh[c*x])*Co
shIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshInteg
ral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*CoshIntegral[5*(a/b + ArcCo
sh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*
x])]*Sinh[(5*a)/b] - 2*a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 2*b*A
rcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cosh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + A
rcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCos
h[c*x])]))/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(308) = 616$.
time = 5.94, size = 1029, normalized size = 2.94

method	result
default	$\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-16\sqrt{cx+1}\,\sqrt{cx-1}\,x^{5}c^{5}+16x^{6}c^{6}+20\sqrt{cx+1}\,\sqrt{cx-1}\,x^{3}c^{3}-28c^{4}x^{4}-5\sqrt{cx+1}\,\sqrt{cx-}\right.}{32(cx+1)c^{4}(cx-1)(a+b\,\mathrm{arccosh}(cx))b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6
+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^4/(c*x-1)/(a+b*arccosh(c*x))/b-5/32*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c
*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2+1/32*(-c^2*x
^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^4/(c*x-1)/(a+b*arccosh(c*x))/b-
3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3
*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2-1/
32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*b*c^2*x^2+4*b*c^3*x^3+3*Ei(1,-3*arccosh(c*x)-3*a/b)*arccosh(c*x)*exp(
-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*b-3*b*c*x)/c^4/b^2/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^4*x^4+16*b*c^5*x^
5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2-20*b*c^3*x^3+5*arccosh(c*x)*Ei(1
,-5*arccosh(c*x)-5*a/b)*exp(-5*a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*Ei(1,
-5*arccosh(c*x)-5*a/b)*exp(-5*a/b)*a+5*b*c*x)/c^4/b^2/(a+b*arccosh(c*x))-1/
16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/
c^4/(c*x-1)/(a+b*arccosh(c*x))/b+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)
/(c*x+1)/c^4/(c*x-1)/b^2+1/16*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(
1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+b*c*x)/c^4/b^2/(a+b*arccosh(c*x)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^5 - x^3)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^6 - c*x^4)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
```

```
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((5*c^3*x^5 - 2*c*x^3)*(c*x + 1
)^(3/2)*(c*x - 1) + (10*c^4*x^6 - 11*c^2*x^4 + 3*x^2)*(c*x + 1)*sqrt(c*x -
1) + (5*c^5*x^7 - 9*c^3*x^5 + 4*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c
^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
[Out] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

3.321 $\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(}{}\right.}{2b^2c^3\sqrt{-1+c}}$$

```
[Out] 1/2*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^
(1/2)-1/2*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c
*x-1)^(1/2)-x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arc
cosh(c*x))
```

Rubi [A]

time = 0.33, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5942, 5887, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[
c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sinh[(4*a
)/b])/(2*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*SinhIntegra
l[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2*c^3*Sqrt[-1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(4c\sqrt{}}{}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4\right.}{a+bx}\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)\sinh\left(\frac{4}{b}\right.}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.35, size = 130, normalized size = 0.84

$$\frac{\sqrt{1-c^2x^2}\left(-2bc^2x^2(-1+c^2x^2)-\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)+\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*b*c^2*x^2*(-1 + c^2*x^2) - (a + b*ArcCosh[c*x])*Cosh
Integral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + (a + b*ArcCosh[c*x])*Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(136) = 272.
time = 4.29, size = 422, normalized size = 2.74

method	result

default	$\frac{\sqrt{-c^2x^2+1}\,\left(-8\sqrt{cx+1}\,\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\,\sqrt{cx+1}\right.}{16(cx+1)(cx-1)c^3(a+b\,\mathrm{arccosh}(cx))b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/4*(-c^2*x^2+1)^(1/2)*(-(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b
*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/16*(-c^2*x^2+1)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4
-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)
*Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+
b)/c^3/b^2/(a+b*arccosh(c*x))+1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/c^3/(a+b*arccosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^5 - c*x^3)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((4*c^3*x^4 - c*x^2)*(c*x + 1)^
(3/2)*(c*x - 1) + 2*(4*c^4*x^5 - 4*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (
4*c^5*x^6 - 7*c^3*x^4 + 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3
 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x -
 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

3.322 $\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^2\sqrt{-1+cx}}-\frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^2\sqrt{-1+cx}}$$

```
[Out] -1/4*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/
2)+3/4*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-
1)^(1/2)+1/4*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*
x-1)^(1/2)-3/4*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/c
^2/(c*x-1)^(1/2)-x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*
arccosh(c*x))
```

Rubi [A]
time = 0.27, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5942, 5881, 3384, 3379, 3382, 5887, 5556}

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c*
x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b
^2*c^2*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*
x]))/b]*Sinh[(3*a)/b])/(4*b^2*c^2*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*Cosh[a/b
]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2*c^2*Sqrt[-1 + c*x]) + (3*Sqr
t[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c
^2*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rubi steps

$$\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c\sqrt{1-c^2\cdots\right.}{b\sqrt{-\cdots}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\,\text{co}\cdots\right.}{b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)\cdots\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \cdots$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \cdots$$

$$= \frac{x(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}} \cdots$$

Mathematica [**A**]
time = 0.32, size = 217, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(4bcx-4bc^3x^3+\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-3\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-a\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-b\cosh^{-1}(cx)\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3a\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3b\cosh^{-1}(cx)\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{4b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(4*b*c*x - 4*b*c^3*x^3 + (a + b*ArcCosh[c*x])*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a
/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]] - b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(218) = 436.
time = 2.67, size = 622, normalized size = 2.51

method	result
default	$\frac{\sqrt{-c^2x^2+1}\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3+4c^4x^4+3\sqrt{cx+1}\,\sqrt{cx-1}\,xc-5c^2x^2+1\right)}{8(cx+1)c^2(cx-1)(a+b\,\mathrm{arccosh}(cx))b}-\frac{3\sqrt{-c^2x^2+1}\,(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/(a+b*arcco
sh(c*x))/b-3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x
-1)/b^2-1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*b*c^2*x^2+4*b*c^3*x^3+3*Ei(1,-3*arccosh(c*x)-3*a/b)*arccosh(
c*x)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2
)*(c*x-1)^(1/2)*b-3*b*c*x)/c^2/b^2/(a+b*arccosh(c*x))-1/8*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arc
cosh(c*x))/b+1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x
^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/
b^2+1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a
/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,
-arccosh(c*x)-a/b)*a+b*c*x)/c^2/b^2/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^4 - c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c
 + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x - 1)*c^3*
x^3 + (6*c^4*x^4 - 5*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (3*c^5*x^5 - 5*
c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sq
rt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
- 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

3.323 $\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+}{}\right.}{b^2c\sqrt{-1+cx}}$$

```
[Out] cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]

time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5904, 5887, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCosh[c*x]
))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b]
)/(b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a
+ b*ArcCosh[c*x]))/b])/(b^2*c*Sqrt[-1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2c\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2\right.}{a+bx}\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\right.}{b^2c\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.16, size = 121, normalized size = 0.83

$$-\frac{\sqrt{1-c^2x^2}\left(b(-1+c^2x^2)+\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{b^2c\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*(b*(-1 + c^2*x^2) + (a + b*ArcCosh[c*x])*CoshIntegral[
2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*
SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(132) = 264.
time = 4.74, size = 361, normalized size = 2.47

method	result

default	$\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-2\sqrt{cx+1}\,\sqrt{cx-1}\,x^{2}c^{2}+2c^{3}x^{3}+\sqrt{cx-1}\,\sqrt{cx+1}-2cx\right)}{4(cx+1)(cx-1)c(a+b\,\mathrm{arccosh}(cx))b}-\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c
*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/(a+b*arccosh(c*x))/b-1/2
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arc
cosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b^2-1/4*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b
*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2
*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+1/2*
(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c/(a+b*arccosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
 (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))) + integrate(((2*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c
*x - 1) + 2*(2*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1) + (2*c^4*x^4 - 3*c^2*
x^2 + 1)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c
*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*
x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2, x)
```

3.324 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c}{b}\right.}{b^2\sqrt{-1+cx}}$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-Chi((a
+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-(c*x-1)^(1/2
)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/x/(a+b*arccosh(c*x))+(-c*x+1)^(1/2)*
Unintegrable(1/x^2/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x*(a + b*ArcCosh[c*
x]))) - (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2
*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x
])/b])/(b^2*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Defer[Int][1/(x^2*(a + b*ArcCo
sh[c*x])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(c\sqrt{}}{}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+\right.}{b^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(\sqrt{}}{}$$

$$= \frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [**A**]
time = 25.46, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [**A**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c
*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*x^3 + 2*c*x)*(c*x + 1
)^(3/2)*(c*x - 1) + (2*c^4*x^4 + c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^
5*x^5 - c^3*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x
^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^
3*x^4 - 2*b^2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x) +
a^2*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2), x)
```

3.325 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx^2\left(a+b\cosh^{-1}(cx)\right)}+\frac{2\sqrt{1-cx}\,\mathrm{Int}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{-1+cx}}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))+
2*(-c*x+1)^(1/2)*Unintegrable(1/x^3/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x^2*(a + b*ArcCosh[
c*x]))) + (2*Sqrt[1 - c*x]*Defer[Int][1/(x^3*(a + b*ArcCosh[c*x])), x])/(b*
c*Sqrt[-1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}\end{aligned}$$

Mathematica [A]
time = 8.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c
*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x
 - 1)*c*x + 2*(2*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqr
t(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^5 -
 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^2*c^3*x^
5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x
) + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^2\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

3.326 $\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 129.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^3\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c
*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((c^3*x^3 - 4*c*x)*(c*x
 + 1)^(3/2)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1)
 + (c^5*x^5 - 3*c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4
*x^7 - a*b*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^6 - 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^
2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x
) + a^2*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^{3}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x))**2,x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

3.327 $\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c
*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((2*c^3*x^3 - 5*c*x)*(c
*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^4*x^4 - 5*c^2*x^2 + 2)*(c*x + 1)*sqrt(c*x
- 1) + (2*c^5*x^5 - 5*c^3*x^3 + 3*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a
*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x
 + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 -
b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x
) + a^2*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^{4}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x))**2,x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

3.328 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=354

$$-\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{3/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4(a+b\cosh^{-1}}{b}\right.}{4b^2c^3\sqrt{-1+}}$$

```
[Out] 1/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)
^(1/2)+1/4*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(
c*x-1)^(1/2)-3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^3/(c*x-1)^(1/2)-1/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(
1/2)/b^2/c^3/(c*x-1)^(1/2)-1/4*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*
x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)+3/16*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/
b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-x^2*(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]

time = 0.55, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5942, 5912, 5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)^{3/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCos
h[c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2
*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c^3*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3*Sq
rt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(4*a)/b]*Sinh
Integral[(4*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c^3*Sqrt[-1 + c*x]) - (3*Sqrt[
1 - c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^
3*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(6c\right.}{}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh}{8(a}\right.\right.}{bc^3\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cos\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+\right.}{a+bx}\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\right.}{16b^2c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.75, size = 338, normalized size = 0.95

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}\left(16bc^2x^2-32bc^4x^4+16bc^6x^6-\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-4\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)+3a\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)+3b\cosh^{-1}(cx)\text{Chi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{6a}{b}\right)+a\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+b\cosh^{-1}(cx)\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4a\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4b\cosh^{-1}(cx)\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3a\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3b\cosh^{-1}(cx)\cosh\left(\frac{6a}{b}\right)\text{Shi}\left(6\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{16b^2c^3\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/16*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(16*b*c^2*x^2 - 32*b*c^4*x^4 + 16*b*c^6
*x^6 - (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)
/b] - 4*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a
)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcCosh
[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*
SinhIntegral[2*(a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcCosh[c*x])] + 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + Arc
Cosh[c*x])] + 4*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - 3*b*ArcCo
sh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[
1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(312) = 624$.
time = 4.97, size = 1176, normalized size = 3.32

method	result	size
default	Expression too large to display	1176

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^
7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-
1)/c^3/(a+b*arccosh(c*x))/b+3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b
)/(c*x+1)/(c*x-1)/c^3/b^2+1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/
2)*(32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6-32*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*b*c^3*x^3-48*b*c^4*x^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+18
*b*c^2*x^2+6*Ei(1,-6*arccosh(c*x)-6*a/b)*arccosh(c*x)*exp(-6*a/b)*b+6*Ei(1,
-6*arccosh(c*x)-6*a/b)*exp(-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))+1/16*(-c
^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3/(a+b*arccosh(c*x))/b+1/32*(
-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x
)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arcco
sh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(
c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh
(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^
2*x^2+1)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c
*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)
*exp(-2*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4-
4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)*
Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b
)/c^3/b^2/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^6 - 2*c^2*x^4 + x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^7 - 2*c^3*x^5
 + c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c
*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((6*c^5*
x^6 - 7*c^3*x^4 + c*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(6*c^6*x^7 - 11*c^4*
x^5 + 6*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^7*x^8 - 5*c^5*x^6 + 4
*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*s
qrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*ar
ccosh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
[Out] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

3.329 $\int \frac{x\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^{3/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^2\sqrt{-1+cx}}-\frac{9\sqrt{1-cx}\,\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right.}{16b^2c^2\sqrt{-1+cx}}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/
2)+9/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x
-1)^(1/2)-5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
^2/(c*x-1)^(1/2)+1/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2
/c^2/(c*x-1)^(1/2)-9/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1
/2)/b^2/c^2/(c*x-1)^(1/2)+5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*
x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-x*(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]

time = 0.55, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5942, 5889, 5906, 3393, 3384, 3379, 3382, 5912, 5952, 5556}

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^2\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^2\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^2\sqrt{cx-1}}+\frac{9\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^2\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1-c^2x^2\right)^{3/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh[
c*x]))) + (Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8
*b^2*c^2*Sqrt[-1 + c*x]) - (9*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cos
hIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^2*Sqrt[-1 +
c*x]) - (Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
^2*c^2*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcCosh[c*x]))/b])/(16*b^2*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[
(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^2*Sqrt[-1 + c*
x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(1-c^2x^2)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(5c\sqrt{1-\right.}{b\sqrt{}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3}{4(a+b}\right.\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{4bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right.}{a+bx}\right.}{8bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.67, size = 327, normalized size = 0.94

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(-16bcx+32bc^3x^3-16bc^5x^5-2\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+9\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-5a\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-5b\cosh^{-1}(cx)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)+2a\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+2b\cosh^{-1}(cx)\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-9a\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-9b\cosh^{-1}(cx)\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5a\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+5b\cosh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{16b^2c^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c*x + 32*b*c^3*x^3 - 16*b*c^5*x^5 - 2*
(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] + 9*(a + b*
ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*Cosh
Integral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshInteg
ral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 2*a*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x
]] - 9*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 9*b*ArcCosh[c
*x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhI
ntegral[5*(a/b + ArcCosh[c*x])]))/(16*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(306) = 612$.
time = 3.15, size = 1029, normalized size = 2.96

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c^5+16x^6c^6+20\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-28c^4x^4-5\sqrt{cx+1}\,\sqrt{cx}\right.}{32(cx+1)c^2(cx-1)(a+b\,\mathrm{arccosh}(cx))b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+5/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(
c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+1/32/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*
c^4*x^4+16*b*c^5*x^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2-20*b*c^3*x^3+
5*Ei(1,-5*arccosh(c*x)-5*a/b)*arccosh(c*x)*exp(-5*a/b)*b+5*Ei(1,-5*arccosh(
c*x)-5*a/b)*exp(-5*a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*b*c*x)/c^2/b^2/(a
+b*arccosh(c*x))+3/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/
(c*x-1)/(a+b*arccosh(c*x))/b-9/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/
```

```
b)/(c*x+1)/c^2/(c*x-1)/b^2-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+1/16*(-c^2*x^
2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a
/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2+1/16*(-c^2*x^2+1)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a
/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+b*c
*x)/c^2/b^2/(a+b*arccosh(c*x))-3/32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2+4*b*c^3*x^3+3*Ei(1,-3*arcc
osh(c*x)-3*a/b)*arccosh(c*x)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*ex
p(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*b*c*x)/c^2/b^2/(a+b*arccosh(c*x
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] ((c^4*x^5 - 2*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^6 - 2*c^3*x^4 +
 c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x
 - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^
2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate((5*(c^5*x^
5 - c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + (10*c^6*x^6 - 17*c^4*x^4 + 8*c^2*x
^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (5*c^7*x^7 - 12*c^5*x^5 + 9*c^3*x^3 - 2*c
*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^
3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*s
qrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x
^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arcc
osh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

3.330 $\int \frac{(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{3/2}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}}+\frac{\sqrt{1-cx}\,\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx}{b}\right)}{2b^2c\sqrt{-1+cx}}$$

```
[Out] cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
1/2*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1
/2)-Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1
/2)+1/2*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1
)^(1/2)-(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x
))
```

Rubi [A]
time = 0.24, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5904, 5912, 5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)^{3/2}}{bc\,(a+b\cosh^{-1}(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCosh[c*
x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/
b])/(b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*ArcCosh[
c*x]))/b]*Sinh[(4*a)/b])/(2*b^2*c*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*
a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b])/(b^2*c*Sqrt[-1 + c*x]) - (S
qrt[1 - c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2
*c*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4c\sqrt{1-c^2x^2}\right)\int \frac{x(-1+c^2x^2)}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4}{8(a+}\right.\right.}{bc\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2\right.}{a+bx}\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2}{b}\right.}{b^2c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [**A**]
time = 0.43, size = 232, normalized size = 0.94

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(-2b+4bc^2x^2-2bc^4x^4+2\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2a\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2b\cosh^{-1}(cx)\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+a\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+b\cosh^{-1}(cx)\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{2b^2c\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-2*b + 4*b*c^2*x^2 - 2*b*c^4*x^4 + 2*(a + b*
ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*A
rcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] - 2*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Cosh[(2*a)
/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + a*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + A
rcCosh[c*x])]))/(2*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(220) = 440$.
time = 4.68, size = 737, normalized size = 3.00

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-8\sqrt{cx+1}\,\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\,\sqrt{cx+1}\right.}{16(cx+1)(cx-1)cb(a+b\operatorname{arccosh}(cx))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c*x)/(c*x+1)/(c*x-1)/c/b/(a+b*arccosh(c*x))+1/4*(-c^2*x^2+1)^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*
arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b^2+1/16/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*(-c^2*x^2+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4-4*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)*Ei
(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/
c/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
/c/(a+b*arccosh(c*x))/b+1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/
(a+b*arccosh(c*x))/b-1/2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1
)/(c*x-1)/c/b^2-1/4*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x
)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2
/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*
x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((4*c^4*x^4
- 3*c^2*x^2 - 1)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^5*x^5 - 3*c^3*x^3 + c*x
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^6*x^6 - 9*c^4*x^4 + 6*c^2*x^2 - 1)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a
*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b +
(b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3
*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2,x)
[Out] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2, x)
```

3.331 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{3/2}}{bcx\,(a+b\cosh^{-1}(cx))}-\frac{9\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2\sqrt{-1+cx}}+\frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2\sqrt{-1+cx}}$$

```
[Out] 9/4*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-3/
4*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-
9/4*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)+3/
4*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-
(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x/(a+b*arccosh(c*x))-(-c
*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^2/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^
(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x*(a + b*ArcCosh[
c*x]))) - (9*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/
(4*b^2*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*
x]))/b]*Sinh[(3*a)/b])/(4*b^2*Sqrt[-1 + c*x]) + (9*Sqrt[1 - c*x]*Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2*Sqrt[-1 + c*x]) - (3*Sqrt[1 -
c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*Sqrt[-1
 + c*x]) - (Sqrt[1 - c*x]*Defer[Int][(-1 + c^2*x^2)/(x^2*(a + b*ArcCosh[c*x
])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{(3c}{}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, c}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i}{4}\right.\right.}{b\sqrt{-1+cx}\,\sqrt{1-}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, c}{4b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(9\sqrt{}}{}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{9\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh}{4b^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 23.72, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x\,(a+b\,\text{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((3*
c^5*x^5 - c^3*x^3 - 2*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (6*c^6*x^6 - 7*c^4*x
^4 + 1)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a
*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 +
b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2), x)
```

3.332 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{3/2}}{bcx^2\,(a+b\cosh^{-1}(cx))}-\frac{2\sqrt{1-cx}\,\mathrm{Int}\left(\frac{-1+c^2x^2}{x^3(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{-1+cx}}-\frac{2c\sqrt{1-cx}\,\mathrm{Int}\left(\frac{-1+c^2x^2}{x(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{-1+cx}}$$

```
[Out] -(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))-
2*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^3/(a+b*arccosh(c*x)),x)/b/c/(c*
x-1)^(1/2)-2*c*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x/(a+b*arccosh(c*x))
,x)/b/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x^2*(a + b*ArcCos
h[c*x]))) - (2*Sqrt[1 - c*x]*Defer[Int][(-1 + c^2*x^2)/(x^3*(a + b*ArcCosh[
c*x])), x])/(b*c*Sqrt[-1 + c*x]) - (2*c*Sqrt[1 - c*x]*Defer[Int][(-1 + c^2*
x^2)/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-c^2x^2)^{3/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))}-\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^3(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}-\end{aligned}$$

Mathematica [A]
time = 48.72, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((2*c^5*x^5 + c^3*x^3 - 3*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^6*x^6 - c
^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (2*c^7*x^7 - 3*c^5*x^5 +
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^
2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c
*x) + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^2 (a + b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

3.333 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]

time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]

time = 123.14, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^3\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((c^5*x^5 + 3*c^3*x^3 - 4*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (2*c^6*x^6 + 3*c
^4*x^4 - 8*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7 - 3*c^3*x^3 + 2*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*b^
2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c
*x) + a^2*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(cx-1)\,(cx+1)\right)^{\frac{3}{2}}}{x^3\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

3.334 $\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{3/2}}{bcx^4\,(a+b\cosh^{-1}(cx))}-\frac{4\sqrt{1-cx}\,\mathrm{Int}\left(\frac{-1+c^2x^2}{x^5(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{-1+cx}}$$

```
[Out] -(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^4/(a+b*arccosh(c*x))-
4*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^5/(a+b*arccosh(c*x)),x)/b/c/(c*
x-1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(b*c*x^4*(a + b*ArcCos
h[c*x]))) - (4*Sqrt[1 - c*x]*Defer[Int][(-1 + c^2*x^2)/(x^5*(a + b*ArcCosh[
c*x])), x])/(b*c*Sqrt[-1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-c^2x^2)^{3/2}}{x^4\,(a+b\cosh^{-1}(cx))^2}\,dx &= -\frac{\sqrt{1-c^2x^2}\,\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^4\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))}-\frac{\left(4\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^5(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}\end{aligned}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{4} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
(5*(c^3*x^3 - c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^4*x^4 - 3*c^2*x^2 + 1
)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^5*x^5 - 2*c^3*x^3 + c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7
 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) +
(b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5
+ 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c
*x) + a^2*x^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{4}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

3.335 $\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=454

$$-\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^{5/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}}-\frac{\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}\right.}{b}\right.}{8b^2c^3\sqrt{-1+}}$$

```
[Out] 1/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)
^(1/2)+1/8*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(
c*x-1)^(1/2)-3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^3/(c*x-1)^(1/2)+1/16*cosh(8*a/b)*Shi(8*(a+b*arccosh(c*x))/b)*(-c*x+1)^(
1/2)/b^2/c^3/(c*x-1)^(1/2)-1/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c
*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-1/8*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/
b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)+3/16*Chi(6*(a+b*arccosh(c*x))/b)*si
nh(6*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-1/16*Chi(8*(a+b*arccosh(c*x)
)/b)*sinh(8*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-x^2*(-c^2*x^2+1)^(5/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.83, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5942, 5912, 5952, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{8a}{b}\right)\mathrm{Chi}\left(\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8b^2c^3\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{8a}{b}\right)\mathrm{Shi}\left(\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1-c^2x^2\right)^{5/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCos
h[c*x]))) - (Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2
*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(4*(a + b
*ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(8*b^2*c^3*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3*Sq
rt[-1 + c*x]) - (Sqrt[1 - c*x]*CoshIntegral[(8*(a + b*ArcCosh[c*x]))/b]*Sin
h[(8*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(2*a)/b]*Sinh
Integral[(2*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1
 - c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(8*b^2*c^3*
Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*Arc
Cosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x]) + (Sqrt[1 - c*x]*Cosh[(8*a)/b]*
SinhIntegral[(8*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^3*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
```

```
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x(-1+c^2x^2)^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(8c\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx\ldots\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh\ldots}{8(a+\ldots}\right.\right.}{bc^3\sqrt{-1+cx}\ldots}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-1}\ldots\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}\ldots\right.}{a+b\ldots}\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+\ldots}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\ldots\right.}{16b^2c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 1.20, size = 446, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6
+ 16*b*c^8*x^8 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*
Sinh[(2*a)/b] + 2*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]
*Sinh[(4*a)/b] - 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 3
*b*ArcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh
Integral[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] + b*ArcCosh[c*x]*CoshIntegra
```

```
l[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] - a*Cosh[(2*a)/b]*SinhIntegral[2*(a
/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cCosh[c*x])] - 2*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] - 2*b
*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] + 3*a*Cosh
[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(6*a
)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*
(a/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b +
ArcCosh[c*x])]))/(16*b^2*c^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(400) = 800$.
time = 5.31, size = 1676, normalized size = 3.69

method	result	size
default	Expression too large to display	1676

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*
x^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*
c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh
(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*Ei(1,8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/(c*x-1)/c
^3/b^2-1/256/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(128*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*b*c^7*x^7+128*b*c^8*x^8-192*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b
*c^5*x^5-256*b*c^6*x^6+80*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+160*b*c^4*x
^4-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x-32*b*c^2*x^2+8*arccosh(c*x)*exp(-8*a
/b)*Ei(1,-8*arccosh(c*x)-8*a/b)*b+8*exp(-8*a/b)*Ei(1,-8*arccosh(c*x)-8*a/b)
*a+b)/c^3/b^2/(a+b*arccosh(c*x))+5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/c^3/(a+b*arccosh(c*x))/b-1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4
-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)
*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b+3/32*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x
)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2
+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1
)/(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)
+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/
(c*x-1)/c^3/(a+b*arccosh(c*x))/b-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2
```

```
*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1
)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(
1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2
*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4-4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)*Ei(1,-4
*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/c^3/b
^2/(a+b*arccosh(c*x))+1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(
32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6-32*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*b*c^3*x^3-48*b*c^4*x^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+18*b*c
^2*x^2+6*arccosh(c*x)*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-6*a/b)*b+6*Ei(1,-6*a
rccosh(c*x)-6*a/b)*exp(-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^
9 - 3*c^5*x^7 + 3*c^3*x^5 - c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))) + integrate(((8*c^7*x^8 - 17*c^5*x^6 + 10*c^3*x^4 - c*x^2)*(c*x + 1)^(
3/2)*(c*x - 1) + 2*(8*c^8*x^9 - 22*c^6*x^7 + 21*c^4*x^5 - 8*c^2*x^3 + x)*(c
*x + 1)*sqrt(c*x - 1) + (8*c^9*x^10 - 27*c^7*x^8 + 33*c^5*x^6 - 17*c^3*x^4
+ 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)
*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^
2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2
 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{5/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

3.336 $\int \frac{x\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=448

$$-\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^{5/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{5\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{64b^2c^2\sqrt{-1+cx}}-\frac{27\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(}{b}\right.}{64b^2c^2\sqrt{-1+c}}$$

```
[Out] -5/64*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1
/2)+27/64*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c
*x-1)^(1/2)-25/64*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^2/(c*x-1)^(1/2)+7/64*cosh(7*a/b)*Shi(7*(a+b*arccosh(c*x))/b)*(-c*x+1)^(
1/2)/b^2/c^2/(c*x-1)^(1/2)+5/64*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1
)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-27/64*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)
*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)+25/64*Chi(5*(a+b*arccosh(c*x))/b)*sin
h(5*a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-7/64*Chi(7*(a+b*arccosh(c*x))
/b)*sinh(7*a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-x*(-c^2*x^2+1)^(5/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.79, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5942, 5889, 5906, 3393, 3384, 3379, 3382, 5912, 5952, 5556}

$$\frac{5\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{27\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}+\frac{25\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{7\sqrt{1-cx}\,\sinh\left(\frac{7a}{b}\right)\mathrm{Chi}\left(\frac{7(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64b^2c^2\sqrt{cx-1}}+\frac{27\sqrt{1-cx}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{25\sqrt{1-cx}\,\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}+\frac{7\sqrt{1-cx}\,\cosh\left(\frac{7a}{b}\right)\mathrm{Shi}\left(\frac{7(a+b\cosh^{-1}(cx))}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1-c^2x^2\right)^{5/2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh[
c*x]))) + (5*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/
(64*b^2*c^2*Sqrt[-1 + c*x]) - (27*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcC
osh[c*x]))/b]*Sinh[(3*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (25*Sqrt[1 - c*x
]*CoshIntegral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(64*b^2*c^2*Sqrt[
-1 + c*x]) - (7*Sqrt[1 - c*x]*CoshIntegral[(7*(a + b*ArcCosh[c*x]))/b]*Sinh
[(7*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*Cosh[a/b]*SinhInt
egral[(a + b*ArcCosh[c*x])/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (27*Sqrt[1 - c
*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(64*b^2*c^2*Sqr
t[-1 + c*x]) - (25*Sqrt[1 - c*x]*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCo
sh[c*x]))/b])/(64*b^2*c^2*Sqrt[-1 + c*x]) + (7*Sqrt[1 - c*x]*Cosh[(7*a)/b]*
SinhIntegral[(7*(a + b*ArcCosh[c*x]))/b])/(64*b^2*c^2*Sqrt[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(d1*d2 + e1*e2*x^2)^p*(a + b*A
rcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 +
 d1*e2, 0] && IntegerQ[p]
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
```

```
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(1-c^2x^2)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{(-1+c^2x^2)^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(7c\sqrt{1-c\right.}{b\sqrt{-}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3}{16(a+bx}\right.\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{16bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(35\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\right.}{64bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{5\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 1.01, size = 436, normalized size = 0.97

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-64*b*c*x + 192*b*c^3*x^3 - 192*b*c^5*x^5 +
64*b*c^7*x^7 - 5*(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh
[a/b] + 27*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(
3*a)/b] - 25*a*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 25*b*Ar
cCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 7*a*CoshInt
egral[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 7*b*ArcCosh[c*x]*CoshIntegral
[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 5*a*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + 5*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
- 27*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 27*b*ArcCosh[c*
x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 25*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 25*b*ArcCosh[c*x]*Cosh[(5*a)/b]*Sinh
Integral[5*(a/b + ArcCosh[c*x])] - 7*a*Cosh[(7*a)/b]*SinhIntegral[7*(a/b +
```

```
ArcCosh[c*x])] - 7*b*ArcCosh[c*x]*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCo
sh[c*x])]))/(64*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(394) = 788$.
time = 4.50, size = 1499, normalized size = 3.35

method	result	size
default	Expression too large to display	1499

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^7*x^7+64*c^8*x^
8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+
1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b-7/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*ar
ccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2-1/128/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*(-c^2*x^2+1)^(1/2)*(64*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^6*x^6+64*b*c^7*x^7
-80*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^4*x^4-112*b*c^5*x^5+24*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*b*c^2*x^2+56*b*c^3*x^3+7*exp(-7*a/b)*arccosh(c*x)*Ei(1,-7*arcco
sh(c*x)-7*a/b)*b+7*exp(-7*a/b)*Ei(1,-7*arccosh(c*x)-7*a/b)*a-(c*x+1)^(1/2)*
(c*x-1)^(1/2)*b-7*b*c*x)/c^2/b^2/(a+b*arccosh(c*x))-5/128*(-c^2*x^2+1)^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+25/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*ar
ccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+9/128*(-c^2*x^2+1)^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b-27/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a
/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2-5/128*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b
*arccosh(c*x))/b+5/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c
*x-1)/b^2+5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x
)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/
b)*Ei(1,-arccosh(c*x)-a/b)*a+b*c*x)/c^2/b^2/(a+b*arccosh(c*x))-9/128*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c
^2*x^2+4*b*c^3*x^3+3*Ei(1,-3*arccosh(c*x)-3*a/b)*arccosh(c*x)*exp(-3*a/b)*b
+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-
3*b*c*x)/c^2/b^2/(a+b*arccosh(c*x))+5/128/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2
*x^2+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^4*x^4+16*b*c^5*x^5-12*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2-20*b*c^3*x^3+5*Ei(1,-5*arccosh(c*x)-5*a/
```

```
b)*arccosh(c*x)*exp(-5*a/b)*b+5*Ei(1,-5*arccosh(c*x)-5*a/b)*exp(-5*a/b)*a+(
c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*b*c*x)/c^2/b^2/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^8
- 3*c^5*x^6 + 3*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x
+ 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
)) + integrate((7*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1)
 + (14*c^8*x^8 - 37*c^6*x^6 + 33*c^4*x^4 - 11*c^2*x^2 + 1)*(c*x + 1)*sqrt(c
*x - 1) + (7*c^9*x^9 - 23*c^7*x^7 + 27*c^5*x^5 - 13*c^3*x^3 + 2*c*x)*sqrt(c
*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*
a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c
 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-c^2\,x^2\right)^{5/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

3.337 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=351

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{5/2}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{15\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c\sqrt{-1+cx}}+\frac{3\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}}{b}\right.}{4b^2c\sqrt{-1+c}}$$

```
[Out] 15/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^
(1/2)-3/4*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x
-1)^(1/2)+3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
/(c*x-1)^(1/2)-15/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)
/b^2/c/(c*x-1)^(1/2)+3/4*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(
1/2)/b^2/c/(c*x-1)^(1/2)-3/16*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x
+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.35, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5904, 5912, 5952, 5556, 3384, 3379, 3382}

$$-\frac{15\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}+\frac{15\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\mathrm{Shi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)^{5/2}}{bc\,(a+b\cosh^{-1}(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCosh[c*
x]))) - (15*Sqrt[1 - c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*
a)/b])/(16*b^2*c*Sqrt[-1 + c*x]) + (3*Sqrt[1 - c*x]*CoshIntegral[(4*(a + b*
ArcCosh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x
]*CoshIntegral[(6*(a + b*ArcCosh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c*Sqrt[-1
 + c*x]) + (15*Sqrt[1 - c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c
*x]))/b])/(16*b^2*c*Sqrt[-1 + c*x]) - (3*Sqrt[1 - c*x]*Cosh[(4*a)/b]*SinhIn
tegral[(4*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c*Sqrt[-1 + c*x]) + (3*Sqrt[1 -
c*x]*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c*Sqrt
[-1 + c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6c\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx, x,\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh(4x)}{8(a+bx)}\right.\right.}{bc\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-}\right.}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+\right.}{a+bx}\right.}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{15\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\right.}{16b^2c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [**A**]
time = 0.72, size = 343, normalized size = 0.98

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b + 48*b*c^2*x^2 - 48*b*c^4*x^4 + 16*b*c
^6*x^6 + 15*(a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[
(2*a)/b] - 12*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sin
h[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*A
rcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 15*b*ArcCosh[c*x]*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 12*a*Cosh[(4*a)/b]*SinhIntegral
[4*(a/b + ArcCosh[c*x])] + 12*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x]
)] - 3*b*ArcCosh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/
(16*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(309) = 618$.

time = 4.03, size = 1176, normalized size = 3.35

method	result	size
default	Expression too large to display	1176

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7
+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1
)/c/b/(a+b*arccosh(c*x))-3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(
c*x+1)/(c*x-1)/c/b^2-1/64/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(3
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*b*c^5*x^5+32*b*c^6*x^6-32*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*b*c^3*x^3-48*b*c^4*x^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+18*b*c^
2*x^2+6*arccosh(c*x)*Ei(1,-6*arccosh(c*x)-6*a/b)*exp(-6*a/b)*b+6*Ei(1,-6*ar
ccosh(c*x)-6*a/b)*exp(-6*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+5/16*(-c^2*x^2+
1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c/(a+b*arccosh(c*x))/b-3/32*(-c^2*x^2+
1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1)
/(c*x-1)/c/b/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a
)/b)/(c*x+1)/(c*x-1)/c/b^2+15/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-
1)/c/(a+b*arccosh(c*x))/b-15/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)
/(c*x+1)/(c*x-1)/c/b^2-15/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(1,-2*ar
ccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a
-b)/c/b^2/(a+b*arccosh(c*x))+3/32/(c*x+1)^(1/2)/(c*x-1)^(1/2)*(-c^2*x^2+1)^
(1/2)*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4-4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)*Ei(1,-4*arccosh(
c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/c/b^2/(a+b*arc
cosh(c*x))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate(((6*c^6*x^6 - 11*c^4*x^4 + 4*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c*x
 - 1) + 6*(2*c^7*x^7 - 5*c^5*x^5 + 4*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1)
 + (6*c^8*x^8 - 19*c^6*x^6 + 21*c^4*x^4 - 9*c^2*x^2 + 1)*sqrt(c*x + 1))*sqr
t(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2
+ 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^
4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*
c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2,x)
[Out] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2, x)
```

3.338 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=386

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{5/2}}{bcx\,(a+b\cosh^{-1}(cx))}-\frac{25\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2\sqrt{-1+cx}}+\frac{25\sqrt{1-cx}\,\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx}{b}\right.}{16b^2\sqrt{-1+cx}}$$

```
[Out] 25/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-2
5/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/
2)+5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^
(1/2)-25/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(
1/2)+25/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/(c*x-
1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)/b^2/(c
*x-1)^(1/2)-(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x/(a+b*arcco
sh(c*x))+(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x^2/(a+b*arccosh(c*x)),x
)/b/c/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*x*(a + b*ArcCosh[
c*x]))) - (25*Sqrt[1 - c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])
/(8*b^2*Sqrt[-1 + c*x]) + (25*Sqrt[1 - c*x]*CoshIntegral[(3*(a + b*ArcCosh[
c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*Sqrt[-1 + c*x]) - (5*Sqrt[1 - c*x]*CoshInt
egral[(5*(a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*Sqrt[-1 + c*x]) +
(25*Sqrt[1 - c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*Sq
rt[-1 + c*x]) - (25*Sqrt[1 - c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcC
osh[c*x]))/b])/(16*b^2*Sqrt[-1 + c*x]) + (5*Sqrt[1 - c*x]*Cosh[(5*a)/b]*Sin
hIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*Sqrt[-1 + c*x]) + (Sqrt[1 -
c*x]*Defer[Int][(-1 + c^2*x^2)^2/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[
-1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(5c$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, c\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(5i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5}{1}\right.\right.}{b\sqrt{-1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, c\right.}{16b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(25$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{25\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh}{8b^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 5.95, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))) + integrate(((5*c^7*x^7 - 8*c^5*x^5 + c^3*x^3 + 2*c*x)*(c*x + 1)^(
3/2)*(c*x - 1) + (10*c^8*x^8 - 23*c^6*x^6 + 15*c^4*x^4 - c^2*x^2 - 1)*(c*x
+ 1)*sqrt(c*x - 1) + 5*(c^9*x^9 - 3*c^7*x^7 + 3*c^5*x^5 - c^3*x^3)*sqrt(c*x
 + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a*
b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2
 + 2*a*b*x*arccosh(c*x) + a^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-c^2\,x^2)^{5/2}}{x\,(a+b\,\mathrm{acosh}\,(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2), x)
```

3.339 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\cosh^{-1}(cx))^2} dx$

Optimal. Leaf size=162

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^{5/2}}{bcx^2\,(a+b\cosh^{-1}(cx))}+\frac{2\sqrt{1-cx}\,\text{Int}\left(\frac{(-1+c^2x^2)^2}{x^3(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{-1+cx}}+\frac{4c\sqrt{1-cx}\,\text{Int}\left(\frac{(-1+c^2x^2)^2}{x(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{-1+cx}}$$

```
[Out] -(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))+
2*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x^3/(a+b*arccosh(c*x)),x)/b/c/(
c*x-1)^(1/2)+4*c*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x/(a+b*arccosh(c
*x)),x)/b/(c*x-1)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(b*c*x^2*(a + b*ArcCos
h[c*x]))) + (2*Sqrt[1 - c*x]*Defer[Int][(-1 + c^2*x^2)^2/(x^3*(a + b*ArcCos
h[c*x])), x])/(b*c*Sqrt[-1 + c*x]) + (4*c*Sqrt[1 - c*x]*Defer[Int][(-1 + c^
2*x^2)^2/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))}+\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{(-1+c^2x^2)^2}{x^3(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\end{aligned}$$

Mathematica [A]
time = 11.89, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((4*c^7*x^7 - 5*c^5*x^5 - 2*c^3*x^3 + 3*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(4*c^8*x^8 - 8*c^6*x^6 + 3*c^4*x^4 + 2*c^2*x^2 - 1
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^9*x^9 - 11*c^7*x^7 + 9*c^5*x^5 - c^3*x^3 -
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*
c^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*
x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)
^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^2 (a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-c^2\,x^2)^{5/2}}{x^2\,(a+b\operatorname{acosh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

3.340 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 15.71, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^3\,(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((3*c^7*x^7 - 2*c^5*x^5 - 5*c^3*x^3 + 4*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 3*(2*c^8*x^8 - 3*c^6*x^6 - c^4*x^4 + 3*c^2*x^2 - 1)*
(c*x + 1)*sqrt(c*x - 1) + (3*c^9*x^9 - 7*c^7*x^7 + 3*c^5*x^5 + 3*c^3*x^3 -
2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b
*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(
c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)
^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x^{3}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acosh(c*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

3.341 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 15.42, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((2*c^7*x^7 + c^5*x^5 - 8*c^3*x^3 + 5*c*x)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(2*c^8*x^8 - c^6*x^6 - 6*c^4*x^4 + 7*c^2*x^2 - 2)*(c
*x + 1)*sqrt(c*x - 1) + (2*c^9*x^9 - 3*c^7*x^7 - 3*c^5*x^5 + 7*c^3*x^3 - 3*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)
^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

3.342 $\int \frac{x^5}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=337

$$-\frac{x^5\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{5\sqrt{-1+cx}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^6\sqrt{1-cx}}-\frac{15\sqrt{-1+cx}\,\text{Chi}\left(\frac{3(a+b\cosh^{-1}(c}{b}\right.}{16b^2c^6\sqrt{1-cx}}$$

```
[Out] -x^5*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+5/8*cosh(a/b)*Shi(
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)+15/16*cosh(3*a/b
)*Shi(3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)+5/16*cos
h(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)-5
/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)
-15/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+
1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(c*x-1)^(1/2)/b^2/c^6
/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5950, 5887, 5556, 3384, 3379, 3382}

$$-\frac{5\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^6\sqrt{1-cx}}-\frac{15\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}-\frac{5\sqrt{cx-1}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}+\frac{5\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^6\sqrt{1-cx}}+\frac{15\sqrt{cx-1}\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}+\frac{5\sqrt{cx-1}\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}-\frac{x^5\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^5*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (5*Sqrt[
-1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(8*b^2*c^6*Sqrt[1
 - c*x]) - (15*Sqrt[-1 + c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh
[(3*a)/b])/(16*b^2*c^6*Sqrt[1 - c*x]) - (5*Sqrt[-1 + c*x]*CoshIntegral[(5*(
a + b*ArcCosh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^6*Sqrt[1 - c*x]) + (5*Sqrt
[-1 + c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*c^6*Sqrt[
1 - c*x]) + (15*Sqrt[-1 + c*x]*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh
[c*x]))/b])/(16*b^2*c^6*Sqrt[1 - c*x]) + (5*Sqrt[-1 + c*x]*Cosh[(5*a)/b]*Si
nhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b^2*c^6*Sqrt[1 - c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5950

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Di
st[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
, Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\ }{a}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Sub}}{bc^6}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Sub}}{\ }$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Sub}}{16bc^6\sqrt{}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\right.}{8}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{5\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{a}{b}\right.}{8b^2c^6\sqrt{1-}}$$

Mathematica [**A**]
time = 0.56, size = 190, normalized size = 0.56

$$\frac{\sqrt{1-c^2x^2}\left(\frac{16bc^5x^5}{a+b\cosh^{-1}(cx)}+5\left(2\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right)-2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)\right)}{16b^2c^6\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((16*b*c^5*x^5)/(a + b*ArcCosh[c*x]) + 5*(2*CoshIntegral
[a/b + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 2*Cosh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])])))/(16
*b^2*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(297) = 594$.
time = 6.86, size = 1044, normalized size = 3.10

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c^5+16x^6c^6+20\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-28c^4x^4-5\sqrt{cx+1}\,\sqrt{cx}\right.}{32(c^2x^2-1)c^6b(a+b\,\mathrm{arccosh}(cx))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)/(c^2*x^2-1)/c^6/b/(a+b*arccosh(c*x))-5/32*((c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,5*arccosh(c*x)+
5*a/b)*exp((-b*arccosh(c*x)+5*a)/b)/b^2/(c^2*x^2-1)/c^6+1/32*(-c^2*x^2+1)^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^6*(16*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*b*c^4*x^4+16*b*c^5*x^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2-20*b*
c^3*x^3+5*Ei(1,-5*arccosh(c*x)-5*a/b)*arccosh(c*x)*exp(-5*a/b)*b+5*Ei(1,-5*
arccosh(c*x)-5*a/b)*exp(-5*a/b)*a+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*b*c*x)/b^
2/(a+b*arccosh(c*x))-5/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2
-1)/c^6/b/(a+b*arccosh(c*x))-15/32*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)
/b)/b^2/(c^2*x^2-1)/c^6-5/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x*c+c^2*x^2-1)/(c^2*x^2-1)/c^6/b/(a+b*arccosh(c*x))-5/16*((c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp(
(-b*arccosh(c*x)+a)/b)/b^2/(c^2*x^2-1)/c^6+5/16*(-c^2*x^2+1)^(1/2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^6*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c
*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*
a+b*c*x)/b^2/(a+b*arccosh(c*x))+5/32*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*x^2-1)/c^6*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2+4*b*c^3*x
^3+3*Ei(1,-3*arccosh(c*x)-3*a/b)*arccosh(c*x)*exp(-3*a/b)*b+3*Ei(1,-3*arcco
sh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*b*c*x)/b^2/(a+
b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^8 - c*x^6 + (c^2*x^7 - x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
```

```
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((5
*c^5*x^9 - 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 - 4*c*x^5)*(c*x + 1)*(c*x - 1)
 + 5*(2*c^4*x^8 - 3*c^2*x^6 + x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)
^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*
c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^
4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.343 $\int \frac{x^4}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{x^4\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{-1+cx}\,\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5\sqrt{1-cx}}-\frac{\sqrt{-1+cx}\,\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(c}{b}\right.}{2b^2c^5\sqrt{1-cx}}$$

```
[Out] -x^4*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(2*a/b)*Shi(2*
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)+1/2*cosh(4*a/b)*
Shi(4*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)-Chi(2*(a+b
*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)-1/2*Chi(
4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5950, 5887, 5556, 3384, 3379, 3382}

$$-\frac{\sqrt{cx-1}\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^5\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c^5\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^5\sqrt{1-cx}}+\frac{\sqrt{cx-1}\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c^5\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^4*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1
 + c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c^5*Sq
rt[1 - c*x]) - (Sqrt[-1 + c*x]*CoshIntegral[(4*(a + b*ArcCosh[c*x]))/b]*Sin
h[(4*a)/b])/(2*b^2*c^5*Sqrt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*SinhI
ntegral[(2*(a + b*ArcCosh[c*x]))/b])/(b^2*c^5*Sqrt[1 - c*x]) + (Sqrt[-1 + c
*x]*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c*x]))/b])/(2*b^2*c^5*Sqrt
[1 - c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5950

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Di
st[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
, Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{}{a\cdots}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Sub}\cdots}{bc^5\cdots}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subs}\cdots}{2bc^5\sqrt{1\cdots}}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\text{Chi}\left(\frac{2a}{b}\cdots\right.}{b^2c^5\sqrt{1-\cdots}}$$

Mathematica [A]
time = 0.35, size = 149, normalized size = 0.63

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2bc^4x^4}{a+b\cosh^{-1}(cx)}+2\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{2b^2c^5\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((2*b*c^4*x^4)/(a + b*ArcCosh[c*x]) + 2*CoshIntegral[2*(
a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] + CoshIntegral[4*(a/b + ArcCosh[c*x])]*S
inh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^5*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(212) = 424$.
time = 6.07, size = 758, normalized size = 3.21

method	result

default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-8\sqrt{cx+1}\,\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\,\sqrt{cx+1}\right.}{16(c^2x^2-1)c^5(a+b\,\mathrm{arccosh}(cx))b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((-b*arccosh(c*x)+4*a)/b)/b^2/(c^2*x^2-1)/c^5+1/16*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^5*(8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c^3*x^3+8*b*c^4*x^4-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x-8*b*c^2*x^2+4*arccosh(c*x)*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*b+4*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)*a+b)/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((-b*arccosh(c*x)+2*a)/b)/b^2/(c^2*x^2-1)/c^5+1/4*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^5*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh(c*x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -(c^3*x^7 - c*x^5 + (c^2*x^6 - x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4*c^5*x^8 - 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 - 3*c*x^4)*(c*x + 1)*(c*x - 1) + 4*(2*c^4*x^7 - 3*c^2*x^5 + x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.344 $\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{x^3\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{3\sqrt{-1+cx}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^4\sqrt{1-cx}}-\frac{3\sqrt{-1+cx}\,\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right.}{4b^2c^4\sqrt{1-cx}}$$

```
[Out] -x^3*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+3/4*cosh(a/b)*Shi(
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)+3/4*cosh(3*a/b)*
Shi(3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)-3/4*Chi((a
+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)-3/4*Chi(
3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5950, 5887, 5556, 3384, 3379, 3382}

$$-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^4\sqrt{1-cx}}+\frac{3\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^4\sqrt{1-cx}}+\frac{3\sqrt{cx-1}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^4\sqrt{1-cx}}-\frac{x^3\sqrt{cx-1}}{bc\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^3*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (3*Sqrt[
-1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b^2*c^4*Sqrt[1
 - c*x]) - (3*Sqrt[-1 + c*x]*CoshIntegral[(3*(a + b*ArcCosh[c*x]))/b]*Sinh[
(3*a)/b])/(4*b^2*c^4*Sqrt[1 - c*x]) + (3*Sqrt[-1 + c*x]*Cosh[a/b]*SinhInteg
ral[(a + b*ArcCosh[c*x])/b])/(4*b^2*c^4*Sqrt[1 - c*x]) + (3*Sqrt[-1 + c*x]*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2*c^4*Sqrt[1 -
 c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5950

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Di
st[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
, Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{}{a+}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subs}}{bc^4\sqrt{}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subs}}{b}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subs}}{4bc^4\sqrt{1}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh(\right.}{4b}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{-1+cx}\,\sqrt{1+cx}\,\text{Chi}\left(\frac{a}{b}+\right.}{4b^2c^4\sqrt{1-c}}$$

Mathematica [A]
time = 0.33, size = 144, normalized size = 0.61

$$\frac{\sqrt{1-c^2x^2}\left(\frac{4bc^3x^3}{a+b\cosh^{-1}(cx)}+3\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+3\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{4b^2c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((4*b*c^3*x^3)/(a + b*ArcCosh[c*x]) + 3*CoshIntegral[a/b
 + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3
*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(209) = 418$.
time = 4.91, size = 632, normalized size = 2.67

method	result

default	$-\frac{\sqrt{-c^2x^2+1}\,\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3+4c^4x^4+3\sqrt{cx+1}\,\sqrt{cx-1}\,xc-5c^2x^2+1\right)}{8(c^2x^2-1)c^4b(a+b\,\mathrm{arccosh}(cx))}-\frac{3\left(\sqrt{cx+1}\,\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2-1)/c^4/b/(a+b*arccos
h(c*x))-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*
Ei(1,3*arccosh(c*x)+3*a/b)*exp((-b*arccosh(c*x)+3*a)/b)/b^2/(c^2*x^2-1)/c^4
+1/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*b*c^2*x^2+4*b*c^3*x^3+3*Ei(1,-3*arccosh(c*x)-3*a/b)
*arccosh(c*x)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*
x+1)^(1/2)*(c*x-1)^(1/2)*b-3*b*c*x)/b^2/(a+b*arccosh(c*x))-3/8*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c^2*x^2-1)/c^4/b/(a+b*
arccosh(c*x))-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^
(1/2)*Ei(1,arccosh(c*x)+a/b)*exp((-b*arccosh(c*x)+a)/b)/b^2/(c^2*x^2-1)/c^4
+3/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(arccos
h(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+ex
p(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+b*c*x)/b^2/(a+b*arccosh(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^6 - c*x^4 + (c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3
*c^5*x^7 - 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1)
+ 3*(2*c^4*x^6 - 3*c^2*x^4 + x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c
^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4
 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.345 $\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{x^2\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{-1+cx}\,\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(\cdots}{}\right.}{b^2c^3\sqrt{1-cx}}$$

```
[Out] -x^2*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(2*a/b)*Shi(2*
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^3/(-c*x+1)^(1/2)-Chi(2*(a+b*arcco
sh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b^2/c^3/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5950, 5887, 5556, 12, 3384, 3379, 3382}

$$-\frac{\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1
 + c*x]*CoshIntegral[(2*(a + b*ArcCosh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c^3*Sq
rt[1 - c*x]) + (Sqrt[-1 + c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh
[c*x]))/b])/(b^2*c^3*Sqrt[1 - c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5950

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Di
st[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
, Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{}{a}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\,\mathrm{Sub}}{bc^3}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\,\mathrm{Sub}}{bc^3\sqrt{1}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\,\mathrm{Subs}}{bc^3\sqrt{1}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\right.\right.}{}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{2a}{b}\right.}{b^2c^3\sqrt{1-}}$$

Mathematica [**A**]
time = 0.21, size = 117, normalized size = 0.86

$$\frac{\sqrt{1-c^2x^2}\left(bc^2x^2+\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\right)}{b^2c^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(b*c^2*x^2 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b +
ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*SinhInteg
ral[2*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(124) = 248.
time = 6.35, size = 377, normalized size = 2.77

method	result

default	$-\frac{\sqrt{-c^2x^2+1}\left(-2\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+2c^3x^3+\sqrt{cx-1}\,\sqrt{cx+1}\,-2cx\right)}{4(c^2x^2-1)c^3(a+b\,\mathrm{arccosh}(cx))b}-\frac{\left(\sqrt{cx+1}\,\sqrt{cx-1}\,x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(
c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c^2*x^2-1)/c^3/(a+b*arccosh(c*x))/b-1/2*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp((-b*arccosh(c*x)+2*a)/b)/b^2/(c^2*x^2-1)/c^3+1/4*(-c^2*x
^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3*(2*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*b*c*x+2*b*c^2*x^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*ex
p(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh
(c*x))+1/2*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3/(
a+b*arccosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^5 - c*x^3 + (c^2*x^4 - x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2
*c^5*x^6 - 5*c^3*x^4 + (2*c^3*x^4 - c*x^2)*(c*x + 1)*(c*x - 1) + 3*c*x^2 +
2*(2*c^4*x^5 - 3*c^2*x^3 + x)*sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)^(3/2
)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x
^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.346 $\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{x\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{-1+cx}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{1-cx}}+\frac{\sqrt{-1+cx}\,\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}$$

```
[Out] -x*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(a/b)*Shi((a+b*a
rccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^2/(-c*x+1)^(1/2)-Chi((a+b*arccosh(c*x))
/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^2/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5950, 5881, 3384, 3379, 3382}

$$-\frac{\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1 +
 c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2*c^2*Sqrt[1 - c*x
]) + (Sqrt[-1 + c*x]*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c
^2*Sqrt[1 - c*x])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5950

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(
b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])], x] - Di
st[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d + e*x^2])]
, Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{}{a+}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subs}}{b^2c}$$

$$= -\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\right.\right.}{}$$

$$= -\frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{a+}{}\right.}{b^2c^2\sqrt{1-c^2}}$$

Mathematica [A]
time = 0.16, size = 107, normalized size = 0.82

$$\frac{\sqrt{1-c^2x^2}\,\left(bcx+\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]

[Out] (Sqrt[1 - c^2*x^2]*(b*c*x + (a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - (a + b*ArcCosh[c*x])*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]))/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(118) = 236$.
time = 2.95, size = 281, normalized size = 2.16

method	result
default	$-\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)}{2c^2(c^2x^2-1)b(a+b\,\mathrm{arccosh}(cx))}-\frac{\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\sqrt{-c^2x^2+1}\,\exp}{2b^2(c^2x^2-1)c^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/c^2/(c^2*x^2-1)/b/(a+b*arccosh(c*x))-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp((-b*arccosh(c*x)+a)/b)/b^2/(c^2*x^2-1)/c^2+1/2*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(c^2*x^2-1)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+b*c*x)/b^2/(a+b*arccosh(c*x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -(c^3*x^4 - c*x^2 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5*x^5 + (c*x + 1)*(c*x - 1)*c^3*x^3 - 3*c^3*x^3 + (2*c^4*x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*c*x)/((((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(c
*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.347 $\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{-1+cx}}{bc\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5892}

$$-\frac{\sqrt{cx-1}}{bc\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])))
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.35

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
)))
```

Maple [A]
time = 1.38, size = 57, normalized size = 1.54

method	result	size
default	$\frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}}{c\left(c^{2}x^{2}-1\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)b}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)/(a+b*arc
cosh(c*x))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*
x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(-(c^2*
x^2 - (c*x + 1)*(c*x - 1) - 1)/((((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^2*x^2 + 2*
(b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^4*x^4 - 2*b^2*c^2*
x^2 + b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*(
c*x + 1)*sqrt(c*x - 1) + (a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(c*x + 1))
*sqrt(-c*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.
time = 0.35, size = 75, normalized size = 2.03

$$\frac{\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}}{abc^3x^2-abc+(b^2c^3x^2-b^2c)\log\left(cx+\sqrt{c^2x^2-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 -
b^2*c)*log(c*x + sqrt(c^2*x^2 - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [B]
time = 0.45, size = 59, normalized size = 1.59

$$-\frac{b\sqrt{1-c^2\,x^2}\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c\,\left(a+b\operatorname{acosh}\left(c\,x\right)\right)\,\left(b^2-b^2\,c^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] -(b*(1 - c^2*x^2)^(1/2)*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/(c*(a + b*acosh(c*
x))*(b^2 - b^2*c^2*x^2))
```

3.348 $\int \frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{-1+cx}}{bcx\sqrt{1-cx}\,(a+b\cosh^{-1}(cx))}-\frac{\sqrt{-1+cx}\,\mathrm{Int}\left(\frac{1}{x^2(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{1-cx}}$$

```
[Out] -(c*x-1)^(1/2)/b/c/x/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)-(c*x-1)^(1/2)*Uninte
grable(1/x^2/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*x*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1 + c
*x]*Defer[Int][1/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bcx\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int}{bc\sqrt{1-c^2}}$$

Mathematica [A]
time = 3.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x^2 + (a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrat
e((c^5*x^5 - c^3*x^3 + (c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 -
 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2)*(c*x - 1)*b^
2*c^3*x^4 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^
5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^4 + 2*(
a*b*c^4*x^5 - a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c
^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^3 - a^2*x + (b^2*c^2*x^3 - b^2*x)*a
rccosh(c*x)^2 + 2*(a*b*c^2*x^3 - a*b*x)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.349 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{-1+cx}}{bcx^2\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{2\sqrt{-1+cx}\,\mathrm{Int}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}$$

```
[Out] -(c*x-1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)-2*(c*x-1)^(1/2)*Un
integrable(1/x^3/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*x^2*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) - (2*Sqrt[-1
 + c*x]*Defer[Int][1/(x^3*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bcx^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{bc\sqrt{1-c}}\end{aligned}$$

Mathematica [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2} \sqrt{-c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxim
a")
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x^3 + (a*b*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - inte
grate((2*c^5*x^5 - 3*c^3*x^3 + (2*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*
(2*c^4*x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^5 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*(c*x + 1)*sqrt(
c*x - 1) + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c
*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)
*a*b*c^3*x^5 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b
*c^5*x^7 - 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="frica
s")
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x^
2)*arccosh(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.350 $\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 21.75, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 - 5*c^3*x^5 + 4*
c*x^3 + (c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 - 7*c^2*x^4 +
3*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)
^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*
x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x
^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^
3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a
*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.351 $\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))}+\frac{2\sqrt{-1+cx}\,\text{Int}\left(\frac{x}{(-1+c^2x^2)^2(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{1-cx}}$$

```
[Out] -x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))+
2*(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^2/(a+b*arccosh(c*x)),x)/b/c/(-c*
x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos
h[c*x]))) + (2*Sqrt[-1 + c*x]*Defer[Int][x/((-1 + c^2*x^2)^2*(a + b*ArcCosh
[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi steps

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}+\frac{\left(2\sqrt{-1+\right.}{}$$

Mathematica [A]
time = 4.71, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}} \left(a+b \operatorname{arccosh}\left(cx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^3*x^4 + (c*x + 1)*(c
*x - 1)*c*x^2 - 3*c*x^2 + 2*(2*c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/((
(b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*
b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5
*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x
 - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
 + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqr
t(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.352 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 15.54, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*
b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5*x^5 + (c*x + 1)*(c*x -
 1)*c^3*x^3 + c^3*x^3 + (2*c^4*x^4 + c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x -
1) - 2*c*x)/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^
2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x
^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x +
 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)
*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt
(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 -
 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2
 + a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.353 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))}+\frac{2c\sqrt{-1+cx}\,\text{Int}\left(\frac{x}{(-1+c^2x^2)^2(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{1-cx}}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))+2*c*
(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^2/(a+b*arccosh(c*x)),x)/b/(-c*x+1)
^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*
x]))) + (2*c*Sqrt[-1 + c*x]*Defer[Int][x/((-1 + c^2*x^2)^2*(a + b*ArcCosh[c
*x])), x])/(b*Sqrt[1 - c*x])
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$
$$= -\frac{\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}+\frac{\left(2c\sqrt{-1+\right.}{}$$

Mathematica [A]
time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (
b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)
*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2*c^4*x^4 - c^2*x^2 + (2*c^2*x
^2 - 1)*(c*x + 1)*(c*x - 1) + 2*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x -
1) - 1)/(((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^6*x^6 - 3
*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)^(3/2
)*(c*x - 1) + 2*(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*(c*x + 1)*sqrt(c*x
- 1) + (a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*sqrt(c*x + 1))*s
qrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2
*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 +
 a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.354 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 16.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 +
 (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 + (a*b*c^3*x^3 -
a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^5*x^5 - 3*c^3*x^3
+ (3*c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (6*c^4*x^4 - 5*c^2*x^2 + 1)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^6 - b^2*c^3*x^4)*(c*x + 1)^(3/2)*(c*
x - 1) + 2*(b^2*c^6*x^7 - 2*b^2*c^4*x^5 + b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x -
 1) + (b^2*c^7*x^8 - 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x +
1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^6 -
 a*b*c^3*x^4)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^7 - 2*a*b*c^4*x^5 +
a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^8 - 3*a*b*c^5*x^6 + 3*a*b
*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4
*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arccosh(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2*x
^3 + a*b*x)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.355 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 14.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 +
 (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 + (a*b*c^3*x^4
- a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4*c^5*x^5 - 5*c^3*
x^3 + (4*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*(4*c^4*x^4 - 4*c^2*x^2 +
1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^5*x^7 - b^2*c^3*x^5)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^8 - 2*b^2*c^4*x^6 + b^2*c^2*x^4)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^7*x^9 - 3*b^2*c^5*x^7 + 3*b^2*c^3*x^5 - b^2*c*x^3)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^5*x^7 - a*b*c^3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^8 - 2*a*
b*c^4*x^6 + a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^9 - 3*a*b*c^5
*x^7 + 3*a*b*c^3*x^5 - a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c
^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b*c
^2*x^4 + a*b*x^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(- \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.356 $\int \frac{x^4}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))}-\frac{4\sqrt{-1+cx}\,\text{Int}\left(\frac{x^3}{(-1+c^2x^2)^3(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{1-cx}}$$

```
[Out] -x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))-
4*(c*x-1)^(1/2)*Unintegrable(x^3/(c^2*x^2-1)^3/(a+b*arccosh(c*x)),x)/b/c/(-
c*x+1)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos
h[c*x]))) - (4*Sqrt[-1 + c*x]*Defer[Int][x^3/((-1 + c^2*x^2)^3*(a + b*ArcCo
sh[c*x])), x])/(b*c*Sqrt[1 - c*x])
```

Rubi steps

$$\int \frac{x^4}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))} - \frac{\left(4\sqrt{-\right.}{}$$

Mathematica [A]
time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^4)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^3*x^6 + 3*(c*x + 1)*(c*x - 1)*
c*x^4 - 5*c*x^4 + 4*(2*c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c
^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c
^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^
8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**4/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.357 $\int \frac{x^3}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 35.32, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
[Out] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
[Out] -(c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 + 3*c^3*x^5 - 4*c*x^3 + (c
^3*x^5 + 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 + 5*c^2*x^4 - 3*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b
^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^
5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a
*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^
5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.358 $\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^2}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 5.25, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/((((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((2*c^5*x^6 + c^3*x^4 + (2*c^3*x^4 +
 c*x^2)*(c*x + 1)*(c*x - 1) - 3*c*x^2 + 2*(2*c^4*x^5 + c^2*x^3 - x)*sqrt(c*
x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^
4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,(1-c^2\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.359 $\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 31.67, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)) - integrate((3*c^5*x^5 + 3*(c*x + 1)*(c*x - 1)*c^
3*x^3 - c^3*x^3 + (6*c^4*x^4 - c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2
*c*x)/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^
2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^
2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2
 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.360 $\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))}-\frac{4c\sqrt{-1+cx}\,\mathrm{Int}\left(\frac{x}{(-1+c^2x^2)^3(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{1-cx}}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))-4*c*
(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^3/(a+b*arccosh(c*x)),x)/b/(-c*x+1)
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*
x]))) - (4*c*Sqrt[-1 + c*x]*Defer[Int][x/((-1 + c^2*x^2)^3*(a + b*ArcCosh[c
*x])), x])/(b*Sqrt[1 - c*x])
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))} - \frac{(4c\sqrt{-}$$

Mathematica [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-
c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c^2*x
)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x
+ 1))*sqrt(-c*x + 1)) - integrate((4*c^4*x^4 - 3*c^2*x^2 + (4*c^2*x^2 - 1)*
(c*x + 1)*(c*x - 1) + 4*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - 1)/
(((b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2
*(b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^8*x^8 - 4*b^2*c^6*x^6 + 6*b^2*c^4*x^4 - 4*b^2*c^2*x^2 + b^2)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*(c*x + 1)*sqrt(c*x - 1
) + (a*b*c^8*x^8 - 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 - 4*a*b*c^2*x^2 + a*b)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2 +
 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
[Out] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.361 $\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 26.68, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c
^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^5*x^5 - 5*c^3*x^3 + (5*c^3*x
^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (10*c^4*x^4 - 7*c^2*x^2 + 1)*sqrt(c*x + 1
)*sqrt(c*x - 1))/((((b^2*c^7*x^8 - 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c*x + 1)^(3
/2)*(c*x - 1) + 2*(b^2*c^8*x^9 - 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 - b^2*c^2*x^
3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^10 - 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6
- 4*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^8 - 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^9 - 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^10 - 4*a*b*c^7*x^8 + 6*a*b*c
^5*x^6 - 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^7 - 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 -
 a^2*x + (b^2*c^6*x^7 - 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 - b^2*x)*arccosh(c*x)
^2 + 2*(a*b*c^6*x^7 - 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 - a*b*x)*arccosh(c*x)),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.362 $\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 10.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^
2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((6*c^5*x^5 - 7*c^3*x^3 + 3*(2
*c^3*x^3 - c*x)*(c*x + 1)*(c*x - 1) + 2*(6*c^4*x^4 - 5*c^2*x^2 + 1)*sqrt(c*
x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^7*x^9 - 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(
c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^10 - 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6
- b^2*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^11 - 4*b^2*c^7*x^9 + 6*
b^2*c^5*x^7 - 4*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^9 - 2*a*b*c^5*x^7 + a*b*c^
3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^10 - 3*a*b*c^6*x^8 + 3*a*b*
c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^11 - 4*a*b*c^7*
x^9 + 6*a*b*c^5*x^7 - 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x +
 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^8 - 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 -
 a^2*x^2 + (b^2*c^6*x^8 - 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 - b^2*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^6*x^8 - 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 - a*b*x^2)*arccosh(
c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.363 $\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2} \, dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
[Out] Defer[Int][((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2} \, dx = -\frac{\sqrt{1-c^2x^2} \int \frac{(fx)^m (-1+cx)^{3/2} (1+cx)^{3/2}}{(a+b\cosh^{-1}(cx))^2} \, dx}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (-c^2x^2+1)^{\frac{3}{2}}}{(a+b\,\mathrm{arccosh}\,(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] ((c^4*f^m*x^4 - 2*c^2*f^m*x^2 + f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*x^5 - 2*c^3*f^m*x^3 + c*f^m*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*
x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c
*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))) - integrate(((c^5*f^m*(m + 4)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m
- 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^6*f^m*(m + 4)*x^6 - c^4*f^m*(5
*m + 12)*x^4 + 4*c^2*f^m*(m + 1)*x^2 - f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m +
 (c^7*f^m*(m + 4)*x^7 - 3*c^5*f^m*(m + 3)*x^5 + 3*c^3*f^m*(m + 2)*x^3 - c*f
^m*(m + 1)*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^3 - 2*a*b*c^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^
2)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3
*x^3 - 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1
)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*
a*b*arccosh(c*x) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(-(cx-1)(cx+1)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

3.364 $\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
[Out] Defer[Int][((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(fx)^m\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\sqrt{-c^2x^2+1}}{\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -((c^2*f^m*x^2 - f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^m*x)
*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*f^m*(m
+ 2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^4*f^m*(m +
 2)*x^4 - c^2*f^m*(3*m + 2)*x^2 + f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5
*f^m*(m + 2)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m + 1)*x)*sqrt(c*x + 1)*x
^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^3 - 2*a*b*c
^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1
) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^3 - 2*b^2*c^3*x^3 + b^2*c*
x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x
) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

3.365 $\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{(fx)^m\sqrt{-1+cx}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{fm\sqrt{-1+cx}\,\mathrm{Int}\left(\frac{(fx)^{-1+m}}{a+b\cosh^{-1}(cx)},x\right)}{bc\sqrt{1-cx}}$$

```
[Out] -(f*x)^m*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+f*m*(c*x-1)^(1
/2)*Unintegrable((f*x)^(-1+m)/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(((f*x)^m*Sqrt[-1 + c*x])/(b*c*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x]))) + (f*m
*Sqrt[-1 + c*x]*Defer[Int][(f*x)^(-1 + m)/(a + b*ArcCosh[c*x]), x])/(b*c*Sq
rt[1 - c*x])
```

Rubi steps

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{(fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(fm\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int}{bc\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\sqrt{-c^2x^2+1}\,\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
[Out] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
[Out] -((c^2*f^m*x^2 - f^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^
m*x)*x^m)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1
)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)) + integrate(((c^3*f^m*m*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^
m + (2*c^4*f^m*m*x^4 - 3*c^2*f^m*m*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)
*x^m + (c^5*f^m*m*x^5 - c^3*f^m*(2*m + 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((c*
x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x
 - 1)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) +
 (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arc
cosh(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral((f*x)**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
[Out] integrate((f*x)^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m}{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

3.366 $\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] (c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/((((c*x + 1)*sqrt(c*x -
1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*
c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate(((c^3*f^m*(m -
2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 2)*x^4
- c^2*f^m*(3*m - 2)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*(m - 2)*x^5 - c^3*f^m*(2*m - 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^5*x^5
 - b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^6 - 2*b^2*c^4*x^4
+ b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b
^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) + ((a*b*c^5*x^5 - a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) +
2*(a*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c
*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4
*x^4 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c
^2*x^2 + a*b)*arccosh(c*x)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,(1-c^2\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

3.367 $\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2},x\right)$$

`[Out] Unintegrable((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)`

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]`

`[Out] Defer[Int][(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]`

Rubi steps

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]`

`[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]`

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}\,(a+b\,\text{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -(c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/(((b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4
*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 +
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(((c^3*f^m*(m - 4)*x^3 - c
*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 4)*x^4 - c^2*f^m*
(3*m - 4)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m*(m - 4)*x
^5 - c^3*f^m*(2*m - 3)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^7*x^7 - 2*b^2*c
^5*x^5 + b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^8 - 3*b^2*c^
6*x^6 + 3*b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^9
 - 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 - 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^7 - 2*a
*b*c^5*x^5 + a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^8 - 3*a*
b*c^6*x^6 + 3*a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9
*x^9 - 4*a*b*c^7*x^7 + 6*a*b*c^5*x^5 - 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x +
1))*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c
^2*x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2
 - a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,(1-c^2\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

3.368 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=32

$$-\frac{\sqrt{-1+ax}}{2a\sqrt{1-ax}\cosh^{-1}(ax)^2}$$

```
[Out] -1/2*(a*x-1)^(1/2)/a/arccosh(a*x)^2/(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5892}

$$-\frac{\sqrt{ax-1}}{2a\sqrt{1-ax}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]
```

```
[Out] -1/2*Sqrt[-1 + a*x]/(a*Sqrt[1 - a*x]*ArcCosh[a*x]^2)
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3} \, dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3} \, dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.41

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]
```

```
[Out] -1/2*(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)
```

Maple [A]
time = 1.42, size = 51, normalized size = 1.59

method	result	size
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}}{2a(a^2x^2-1)\operatorname{arccosh}(ax)^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)/arcc
osh(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - a^2*x^2)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) + (3*a^5*x^5 - 5*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (3
*a^6*x^6 - 7*a^4*x^4 + 5*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x - (
a^5*x^5 - 2*a^3*x^3 - (a^2*x^2 - 1)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - (a^3*
x^3 - a*x)*(a*x + 1)*(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(a*x + 1)*sq
rt(a*x - 1) + a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/(((a*x + 1)^2*(a
*x - 1)^(3/2)*a^4*x^3 + 3*(a^5*x^4 - a^3*x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 3
*(a^6*x^5 - 2*a^4*x^3 + a^2*x)*(a*x + 1)*sqrt(a*x - 1) + (a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*s
qrt(a*x - 1))^2) - integrate(-1/2*(2*a^6*x^6 - 3*a^4*x^4 - (2*a^2*x^2 - 3)*
(a*x + 1)^2*(a*x - 1)^2 - 4*(a^3*x^3 - a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
 - 4*(a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + 4*(a^5*x^5 - 2*a^3*x^3 + a*x)*sqrt
(a*x + 1)*sqrt(a*x - 1) + 1)/(((a*x + 1)^(5/2)*(a*x - 1)^2*a^4*x^4 + 4*(a^5
*x^5 - a^3*x^3)*(a*x + 1)^2*(a*x - 1)^(3/2) + 6*(a^6*x^6 - 2*a^4*x^4 + a^2*
x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 4*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 - a*x)*
(a*x + 1)*sqrt(a*x - 1) + (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.34, size = 56, normalized size = 1.75

$$\frac{\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}}{2\,(a^3x^2-a)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)/((a^3*x^2 - a)*log(a*x + sqrt(a^2*
x^2 - 1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3), x)
```

Mupad [B]
time = 0.41, size = 48, normalized size = 1.50

$$\frac{\sqrt{1-a^2\,x^2}\,\sqrt{a\,x-1}\,\sqrt{a\,x+1}}{a\,\operatorname{acosh}\left(a\,x\right)^2\,\left(2\,a^2\,x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
[Out] ((1 - a^2*x^2)^(1/2)*(a*x - 1)^(1/2)*(a*x + 1)^(1/2))/(a*acosh(a*x)^2*(2*a^
2*x^2 - 2))
```

3.369 $\int \frac{x^3(d-c^2dx^2)}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{2dx^3(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh}}{\sqrt{b}}\right.}{16b^{3/2}c^4}$$

```
[Out] 3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)/c^4+3/32*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)-1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b*arc
cosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4-1/32*d*erfi(6^(1/2)*
(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)+2
*d*x^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 1.11, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5942, 5953, 5556, 3388, 2211, 2236, 2235}

$$\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\,\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}\,de^{-\frac{6a}{b}}\,\text{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{2dx^3(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*x^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
 (3*d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(16*b^(3/2)*c^4) - (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4) + (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) - (d*Sqrt[
(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4
*E^((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\int \frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - -$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\text{Subst}\Big(\int \frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cos}{bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\text{Subst}\Big(\int \Big(-\frac{1}{8\sqrt{a+bx}} + \frac{\cosh(}{8\sqrt{a-}}}{bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(3d)\text{Subst}\Big(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}}{8bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\text{Subst}\Big(\int \frac{e^{-6x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}}{16bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\text{Subst}\Big(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\,\text{c}}}{8b^2c^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\Bigg(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}}{16b^{3/2}c^4}$$

Mathematica [A]
time = 1.86, size = 300, normalized size = 1.16

$$\frac{de^{-\frac{6a}{b}}\left(-\sqrt{6}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)+3\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+e^{\frac{6a}{b}}\left(-64c^3x^3\sqrt{\frac{-1+cx}{1+cx}}-64c^4x^4\sqrt{\frac{-1+cx}{1+cx}}-3\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+\sqrt{6}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\cosh^{-1}(cx))}{b}\right)+10\sinh\left(2\cosh^{-1}(cx)\right)+8\sinh\left(4\cosh^{-1}(cx)\right)+2\sinh\left(6\cosh^{-1}(cx)\right)\right)\right)}{32bc^4\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-(Sqrt[6]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh
[c*x]))/b]) + 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1
/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((6*a)/b)*(-64*c^3*x^3*Sqrt[(-1 + c*x)
```

```
/(1 + c*x)] - 64*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*Sqrt[2]*E^((2*a)/b)
*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*
E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b]
 + 10*Sinh[2*ArcCosh[c*x]] + 8*Sinh[4*ArcCosh[c*x]] + 2*Sinh[6*ArcCosh[c*x]
])))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-c^{2} d\,x^{2}+d\right)}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \left(-\frac{x^{3}}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\right) dx+\int \frac{c^{2}x^{5}}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(d-c^2\,d\,x^2\right)}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

3.370 $\int \frac{x^2(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=340

$$\frac{2dx^2(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx}}{\sqrt{b}}\right.}{16b^{3/2}c^3}$$

```
[Out] 1/8*d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+1
/8*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)+1
/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b*arcc
osh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3-1/16*d*erfi(5^(1/2)*(
a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*a/b)+2*
d*x^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 1.11, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5942, 5953, 5556, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{2dx^2(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
 (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3)
 + (d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(16*b^(3/2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Arc
Cosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) - (d*Sqrt[5*P
i]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*
a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
```

```
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\int \frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \cdots$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cos\cdots\right.}{bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\mathrm{Subst}\left(\int\left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cos\cdots}{4\sqrt{a\cdots}}\right.\right.}{bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\mathrm{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{\cdots}\right.}{8bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\mathrm{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{\cdots}\right.}{16bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\mathrm{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cdots}\right.}{8b^2c^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}\cdots$$

Mathematica [A]

time = 1.05, size = 384, normalized size = 1.13

$$\frac{de^{-\frac{5a}{b}}\left(-4e^{\frac{5a}{b}}\sqrt{\frac{-1+cx}{1+cx}} - 4ce^{\frac{5a}{b}}x\sqrt{\frac{-1+cx}{1+cx}} - 2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right) - \sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(cx))}{b}\right) + \sqrt{3}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right) + 2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right) - \sqrt{3}\,e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right) + \sqrt{5}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(cx))}{b}\right) - 2e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(cx)\right) + 2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(cx)\right)\right)}{16bc^3\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-4*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*c*E^((5*a)/b)*x*Sqrt[(-1
+ c*x)/(1 + c*x)] - 2*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b +
 ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcCosh[c*x]))/b] + Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]
*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*ArcC
osh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - Sqrt[3]*E^((8*a)/b)*S
qrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + Sqrt[5]*E^
((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b]
- 2*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5*ArcCosh[c*x]]))
/(16*b*c^3*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-c^{2} d \,x^{2}+d \right)}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{x^2}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^2x^4}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,\left(d-c^2\,d\,x^2\right)}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

3.371 $\int \frac{x(d-c^2dx^2)}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{2dx(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}$$

```
[Out] 1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)-1/4*d*exp(4*a/b)*erf(2*(a+b*arccosh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2-1/4*d*erfi(2*(a+b*arccosh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+2*d*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)
/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5942, 5907, 3393, 3388, 2211, 2236, 2235, 5953, 5556}

$$-\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{2dx(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (
d*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^2) + (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(2*b^(3/2)*c^2) - (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5907

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Sinh[-a/b + x/
b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2,
 e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
```

```
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(8c$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(\right.}{bc^2}$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+b}\right.\right.}{bc^2}$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right.}{bc^2}$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right.}{2bc^2}$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{\cdot}}\right.}{b^2c^2}$$

$$= -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}$$

Mathematica [A]
time = 2.85, size = 331, normalized size = 1.37

$$\frac{de^{-\frac{4a}{b}}\left(2e^{\frac{6a}{b}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+2e^{\frac{2a}{b}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\frac{\sqrt{b}\left(-\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+e^{\frac{4a}{b}}\left(8cx\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3+\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\cosh^{-1}(cx))}{b}\right)\right)\right)}{\sqrt{a+b\cosh^{-1}(cx)}}\right)}{4b^{3/2}c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(2*E^((6*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
] + 2*E^((2*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]] + (Sqrt[b]*(-(Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcC
osh[c*x]))/b]) - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*(8*c*x*((-1 + c*x)/(1 + c*x
))^(3/2)*(1 + c*x)^3 + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1
/2, (2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b])))/Sqrt[a + b*ArcCosh[c*x]]))/(4*b^(3/2)
*c^2*E^((4*a)/b))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(-c^2 d\,x^2+d)}{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{x}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^2x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh(
c*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*aco
sh(c*x))*acosh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x\,(d-c^2\,d\,x^2)}{(a+b\operatorname{acosh}(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

3.372 $\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{3de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

```
[Out] 3/4*d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3/4
*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*d
*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/
b^(3/2)/c-1/4*d*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c/exp(3*a/b)+2*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(
c*x))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5904, 5953, 5556, 3388, 2211, 2236, 2235}

$$\frac{3\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{2d(cx-1)^{3/2}(cx+1)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (3*
d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (
d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*b^(3/2)*c) + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(
3/2)*c*E^(a/b)) - (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(4*b^(3/2)*c*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6cd)\int \frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right)\,dx\right.}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(3d)\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2bc} -$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4bc} +$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right.}{2b^2c}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{}}{}$$

Mathematica [A]
time = 1.20, size = 246, normalized size = 1.06

$$\frac{e^{-\frac{3a}{b}}\left(-3de^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{3}\,d\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+de^{\frac{2a}{b}}\left(3\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+e^{a/b}\left(-6\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)+\sqrt{3}\,e^{\frac{3a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+2\sinh\left(3\cosh^{-1}(cx)\right)\right)\right)\right)}{4bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*d*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] -
 Sqrt[3]*d*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*
x]))/b] + d*E^((2*a)/b)*(3*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a
```

```
+ b*ArcCosh[c*x])/b)] + E^(a/b)*(-6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) +
Sqrt[3]*E^((3*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c
*x]))/b] + 2*Sinh[3*ArcCosh[c*x]])))/(4*b*c*E^((3*a)/b)*Sqrt[a + b*ArcCosh[
c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{-c^{2} d\,x^{2}+d}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
[Out] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] -integrate((c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \frac{c^{2}x^{2}}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx+\int\left(-\frac{1}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))
*acosh(c*x)), x) + Integral(-1/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*aco
sh(c*x))*acosh(c*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(-(c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d-c^2\,d\,x^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2),x)
[Out] int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2), x)
```

3.373 $\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bcx\sqrt{a+b\cosh^{-1}(cx)}}-\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}$$

```
[Out] -1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)-1/2*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/exp(2*a/b)+2*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/x/(a+b*arcc
osh(c*x))^(1/2)+2*d*Unintegrable(1/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(a+b*arc
cosh(c*x))^(1/2),x)/b/c
```

Rubi [A]
time = 0.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*x*Sqrt[a + b*ArcCosh[c*x]]) - (
d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/b
^(3/2) - (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b
^(3/2)*E^((2*a)/b)) + (2*d*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*S
qrt[a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \cdots$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d)\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-}\cdots\right.}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2\cdots}{2\sqrt{a+\cdots}}\right.\right.}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-}\cdots\right.}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\cdots\right.}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cdots}\right.}{b^2}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(c\cdots}}{\sqrt{b}}\right.}{b^{3/2}}$$

Mathematica [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{-c^{2} d\,x^{2}+d}{x \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
[Out] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] -integrate((c^2*d*x^2 - d)/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- d \left(\int \frac{c^{2} x^{2}}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx + \int \left(- \frac{1}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)/x/(a+b*acosh(c*x))**(3/2),x)
[Out] -d*(Integral(c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*acosh(c
*x))*acosh(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a
 + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d-c^2\,d\,x^2}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)),x)
```

```
[Out] int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)), x)
```

3.374 $\int \frac{x^3(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=479

$$-\frac{2d^2x^3(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh}}{\sqrt{b}}\right.}{32b^{3/2}c^4}$$

```
[Out] 3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arccosh(c*x))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4+3/64*d^2*erfi(2^(1/2)*(a+b*arccos
h(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^2*erf
i(2*2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/
exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(
1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)
/b^(3/2)/c^4/exp(4*a/b)-1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arccosh(c*x))^
(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4-1/64*d^2*erfi(6^(1/2)*(a+b*arcc
osh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-2*d^2*x^3*
(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 1.32, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5942, 5953, 5556, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\mathrm{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\pi}d^2e^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}d^2e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{-\frac{8a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}d^2e^{-\frac{6a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{2d^2x^3(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]
) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(3
2*b^(3/2)*c^4) + (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcC
osh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*
Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) - (d^2*E^((6*a
)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^
(3/2)*c^4) - (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*
b^(3/2)*c^4*E^((4*a)/b)) + (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*
Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) -
(d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b
^(3/2)*c^4*E^((6*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d-c^2dx^2)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d^2)\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \cdots$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d^2)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cdots\right.}{bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d^2)\,\text{Subst}\left(\int \left(\frac{1}{16\sqrt{a+bx}} - \frac{\cdots}{32\sqrt{\cdots}}\right.\right.}{\cdots}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\cdots\right.}{8bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\text{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\cdots\right.}{16bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}\cdots}\right.}{8b^2c^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\cdots\right.}{32b^{3/2}c^4}$$

Mathematica [A]
time = 2.53, size = 527, normalized size = 1.10

$$-\frac{d^2e^{-\frac{8a}{b}}\left(128c^3e^{\frac{8a}{b}}x^3\sqrt{\frac{-1+cx}{1+cx}}+128c^4e^{\frac{8a}{b}}x^4\sqrt{\frac{-1+cx}{1+cx}}-\sqrt{2}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{8(a+b\cosh^{-1}(cx))}{b}\right)+\sqrt{6}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)+2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-3\sqrt{2}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+3\sqrt{2}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(cx))}{b}\right)-2e^{\frac{12a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-\sqrt{6}\,e^{\frac{14a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\cosh^{-1}(cx))}{b}\right)+\sqrt{2}\,e^{\frac{16a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{8(a+b\cosh^{-1}(cx))}{b}\right)-26e^{\frac{8a}{b}}\sinh\left(2\cosh^{-1}(cx)\right)-18e^{\frac{8a}{b}}\sinh\left(4\cosh^{-1}(cx)\right)-2e^{\frac{8a}{b}}\sinh\left(6\cosh^{-1}(cx)\right)+e^{\frac{8a}{b}}\sinh\left(8\cosh^{-1}(cx)\right)\right)}{64bc^4\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(128*c^3*E^((8*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128*c^4*E^
((8*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - Sqrt[2]*Sqrt[-((a + b*ArcCosh[c*
x])/b)]*Gamma[1/2, (-8*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[
-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] + 2*E^((
4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x])
)/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2
*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[2]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]
*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] - 2*E^((12*a)/b)*Sqrt[a/b + ArcCosh
[c*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a
/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] + Sqrt[2]*E^((16*
a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (8*(a + b*ArcCosh[c*x]))/b] - 26*
E^((8*a)/b)*Sinh[2*ArcCosh[c*x]] - 18*E^((8*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*
E^((8*a)/b)*Sinh[6*ArcCosh[c*x]] + E^((8*a)/b)*Sinh[8*ArcCosh[c*x]]))/(b*c^
4*E^((8*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-c^{2} d\,x^{2}+d\right)^{2}}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x^3}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx+\int\left(-\frac{2c^2x^5}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\right)dx+\int \frac{c^4x^7}{a\sqrt{a+b\,\mathrm{acosh}\,(cx)}+b\sqrt{a+b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

3.375 $\int \frac{x^2(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=462

$$-\frac{2d^2x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{5d^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cos}}{\sqrt{b}}\right.}{64b^{3/2}c^3}$$

```
[Out] 5/64*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^
3+5/64*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(
a/b)+1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(5^(1/
2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3-3/64*d^2*
erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3
/exp(5*a/b)+1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2
))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arccosh(c*x))^(1
/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c*x-1)^(5/2
)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 1.43, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5942, 5953, 5556, 3388, 2211, 2236, 2235}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{7\pi}\,d^2e^{\frac{7a}{b}}\mathrm{Erf}\left(\frac{\sqrt{7}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{5\sqrt{\pi}\,d^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}\,d^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{7\pi}\,d^2e^{-\frac{7a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{7}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{2d^2x^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]
) + (5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^(3
/2)*c^3) + (d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]
])/Sqrt[b]])/(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) + (d^2*E^((7*a)/b)*Sqr
t[7*Pi]*Erf[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) +
 (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^
(a/b)) + (d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3*E^((3*a)/b)) - (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*A
rcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi
[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
```

```
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx = -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d^2)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} +$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d^2)\,\mathrm{Subst}\Big(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x,}{bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d^2)\,\mathrm{Subst}\Big(\int \Big(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3}{16}}{}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(7d^2)\,\mathrm{Subst}\Big(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \mathrm{co}}{32bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(7d^2)\,\mathrm{Subst}\Big(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \mathrm{co}}{64bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(7d^2)\,\mathrm{Subst}\Big(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a}}{32b^2c^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\Bigg(\frac{\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}}{64b^{3/2}c^3}$$

Mathematica [A]
time = 1.95, size = 498, normalized size = 1.08

$$-\frac{d^2e^{-\frac{7a}{b}}\left(10e^{\frac{7a}{b}}\sqrt{\frac{-1+cx}{1+cx}}+10ce^{\frac{7a}{b}}x\sqrt{\frac{-1+cx}{1+cx}}+5e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)-\sqrt{7}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{7(a+b\cosh^{-1}(cx))}{b}\right)+3\sqrt{5}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(cx))}{b}\right)-\sqrt{3}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)-5e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+\sqrt{3}e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right)-3\sqrt{5}e^{\frac{12a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(cx))}{b}\right)+\sqrt{7}e^{\frac{14a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{7(a+b\cosh^{-1}(cx))}{b}\right)+2e^{\frac{7a}{b}}\sinh\left(3\cosh^{-1}(cx)\right)-6e^{\frac{7a}{b}}\sinh\left(5\cosh^{-1}(cx)\right)+2e^{\frac{7a}{b}}\sinh\left(7\cosh^{-1}(cx)\right)\right)}{64bc^3\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(10*E^((7*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 10*c*E^((7*a)/b)*x*
Sqrt[(-1 + c*x)/(1 + c*x)] + 5*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1
/2, a/b + ArcCosh[c*x]] - Sqrt[7]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2
, (-7*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*E^((2*a)/b)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] - Sqrt[3]*E^((4*a)/b)*S
qrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 5*
E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x
])/b)] + Sqrt[3]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b
*ArcCosh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[
1/2, (5*(a + b*ArcCosh[c*x]))/b] + Sqrt[7]*E^((14*a)/b)*Sqrt[a/b + ArcCosh[
c*x]]*Gamma[1/2, (7*(a + b*ArcCosh[c*x]))/b] + 2*E^((7*a)/b)*Sinh[3*ArcCosh
[c*x]] - 6*E^((7*a)/b)*Sinh[5*ArcCosh[c*x]] + 2*E^((7*a)/b)*Sinh[7*ArcCosh[
c*x]]))/(b*c^3*E^((7*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-c^{2} d\,x^{2}+d\right)^{2}}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x^2}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int\left(-\frac{2c^2x^4}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^4x^6}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\operatorname{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

3.376 $\int \frac{x\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{2d^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}}}{\sqrt{b}}\right.}{16b^{3/2}c^2}$$

[Out] 5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)-1/4*d^2*exp(4*a/b)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2-1/4*d^2*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2+1/32*d^2*erfi(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(6*a/b)-2*d^2*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)

Rubi [A]
time = 1.12, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5942, 5907, 3393, 3388, 2211, 2236, 2235, 5953, 5556}

$$-\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{\frac{6a}{b}}\text{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{\sqrt{\pi}\,d^2e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{-\frac{6a}{b}}\text{Erfi}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{2d^2x(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]

[Out] (-2*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) - (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5907

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Sinh[-a/b + x/
b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2,
 e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
```

```
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d-c^2dx^2)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d^2)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \cdots$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cos\cdots\right.}{bc^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d^2)\,\mathrm{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} - \frac{\mathrm{co}\cdots}{2\sqrt{a}\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\mathrm{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\cdots\right.}{4bc^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\cdots\right.}{8bc^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{c}\cdots}\right.}{4b^2c^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\cdots\right.}{4b^{3/2}c^2}$$

Mathematica [A]
time = 4.80, size = 508, normalized size = 1.40

$$\frac{d^2e^{-\frac{8a}{b}}\left(16e^{\frac{8a}{b}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+16e^{\frac{4a}{b}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\frac{\sqrt{b}\left(128c^3e^{\frac{6a}{b}}x^3\sqrt{\frac{-1+cx}{1+cx}}+128c^4e^{\frac{6a}{b}}x^4\sqrt{\frac{-1+cx}{1+cx}}+\sqrt{6}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-11\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+11\sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+8e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-\sqrt{6}e^{\frac{12a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6(a+b\cosh^{-1}(cx))}{b}\right)-42e^{\frac{6a}{b}}\sinh(2\cosh^{-1}(cx))-8e^{\frac{6a}{b}}\sinh(4\cosh^{-1}(cx))-2e^{\frac{6a}{b}}\sinh(6\cosh^{-1}(cx))\right)}{\sqrt{a+b\cosh^{-1}(cx)}}\right)}{32b^{3/2}c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d^2*(16*E^((8*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]] + 16*E^((4*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]] + (Sqrt[b]*(128*c^3*E^((6*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128
```

```
*c^4*E^((6*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] + Sqrt[6]*Sqrt[-((a + b*Arc
Cosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] - 8*E^((2*a)/b)*Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x]))/b] - 11*Sq
rt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2*(a + b*Arc
Cosh[c*x]))/b] + 11*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2,
 (2*(a + b*ArcCosh[c*x]))/b] + 8*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcCos
h[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] - 42*E^((6*a)/b)*Sinh[2*ArcC
osh[c*x]] - 8*E^((6*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*E^((6*a)/b)*Sinh[6*ArcCo
sh[c*x]]))/Sqrt[a + b*ArcCosh[c*x]]))/(32*b^(3/2)*c^2*E^((6*a)/b))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(-c^2 d\,x^2+d)^2}{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{x}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx+\int\left(-\frac{2c^2x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^4x^5}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh
(c*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b
*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

3.377 $\int \frac{(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=351

$$-\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{5d^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}$$

```
[Out] 5/8*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+5
/8*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-5
/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/b^(3/2)/c-5/16*d^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)+1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(a+b*ar
ccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(5^(1/2)
*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5*a/b)-2*
d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5904, 5953, 5556, 3388, 2211, 2236, 2235}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}\,d^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}\,d^2e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{2d^2(cx-1)^{5/2}(cx+1)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
(5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c
) - (5*d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*b^(3/2)*c) + (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) - (5*d^2*Sqrt[3*Pi]*Erfi[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2
*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c
*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(10cd^2)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(10d^2)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx\right.}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(10d^2)\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3\cosh(3x)}{16\sqrt{a+bx}} + \right.\right.}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(5d^2)\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(5d^2)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(5d^2)\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c\right.}{8b^2c}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2}{}$$

Mathematica [**A**]
time = 1.27, size = 387, normalized size = 1.10

$$-\frac{d^2e^{-\frac{5a}{b}}\left(20e^{\frac{5a}{b}}\sqrt{\frac{-1+cx}{1+cx}} + 20ce^{\frac{5a}{b}}x\sqrt{\frac{-1+cx}{1+cx}} + 10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right) - \sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(cx))}{b}\right) + 5\sqrt{3}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right) - 10e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right) - 5\sqrt{3}e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right) + \sqrt{5}e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(cx))}{b}\right) - 10e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(cx)\right) + 2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(cx)\right)\right)}{16bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/16*(d^2*(20*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 20*c*E^((5*a)/b)*x*
Sqrt[(-1 + c*x)/(1 + c*x)] + 10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[
1/2, a/b + ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/
```

```
2, (-5*(a + b*ArcCosh[c*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCo
sh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 10*E^((4*a)/b)*Sqrt[
-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - 5*Sqrt[3
]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/
b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*Arc
Cosh[c*x]))/b] - 10*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5
*ArcCosh[c*x]]))/(b*c*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d \,x^{2}+d \right)^{2}}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
[Out] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \left(- \frac{2 c^{2} x^{2}}{a \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\right)\, dx + \int \frac{c^{4} x^{4}}{a \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx + \int \frac{1}{a \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(
c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sq
rt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2), x)
```

3.378 $\int \frac{(d-c^2dx^2)^2}{x(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$-\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bcx\sqrt{a+b\cosh^{-1}(cx)}}+\frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{2b^{3/2}}$$

```
[Out] -3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)-3/4*d^2*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arccosh(c*x))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/x/
(a+b*arccosh(c*x))^(1/2)+2*d^2*Unintegrable(1/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/(a+b*arccosh(c*x))^(1/2),x)/b/c
```

Rubi [A]
time = 1.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d-c^2dx^2)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*x*Sqrt[a + b*ArcCosh[c*x]])
+ (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b
^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]]
)/Sqrt[b]])/(2*b^(3/2)) - (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c*x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcC
osh[c*x]])/Sqrt[b]])/(4*b^(3/2)*E^((4*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) - (d^2*Sqrt[2
*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)
) + (2*d^2*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCos
h[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d^2)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} +$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(8d^2)\,\mathrm{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh\right.}{b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(8d^2)\,\mathrm{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh}{2\sqrt{a}}\right.\right.}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh}\right.}{b^2}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{4b^{3/2}}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{4b^{3/2}}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(1-c^2x^2)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{4b^{3/2}}$$

<u>Mathematica</u> [A]

time = 2.15, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2 d x^2\right)^2}{x \left(a+b \cosh^{-1}(c x)\right)^{3/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d \,x^{2}+d \right)^{2}}{x \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \left(- \frac{2 c^{2} x^{2}}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\right)\, dx + \int \frac{c^{4} x^{4}}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx + \int \frac{1}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*ac
osh(c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*acosh(c*x))
+ b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*a
cosh(c*x)) + b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d-c^2\,d\,x^2\right)}^2}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)),x)
```

```
[Out] int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)), x)
```

3.379 $\int (c-a^2cx^2)^{3/2} \sqrt{\cosh^{-1}(ax)}\, dx$

Optimal. Leaf size=351

$$\frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}-\frac{c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{4a\sqrt{-1+ax}\sqrt{1+ax}}-\frac{c\sqrt{\pi}\sqrt{c-}}{256}$$

```
[Out] -1/4*c*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2
)+1/32*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1
/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2
^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/256*c*
erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*
x+1)^(1/2)+1/256*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)
/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2
)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5897, 5895, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236, 5912, 5952}

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}-\frac{c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/8 + (x*(c - a^2*c*x^2)^(3/2)
*Sqrt[ArcCosh[a*x]])/4 - (c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(4*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[Ar
cCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c -
 a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(256*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
```

```
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int}{4}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{c}$$

<u>**Mathematica**</u> **[A]**
time = 0.19, size = 154, normalized size = 0.44

$$-\frac{c\sqrt{c-a^2cx^2}\left(-\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-4\cosh^{-1}(ax)\right)+8\sqrt{2}\,\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)+\sqrt{\cosh^{-1}(ax)}\left(32\cosh^{-1}(ax)^{3/2}+8\sqrt{2}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}(ax)\right)-\Gamma\left(\frac{3}{2},4\cosh^{-1}(ax)\right)\right)\right)}{128a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/128*(c*Sqrt[c - a^2*c*x^2]*(-(Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -4*ArcCosh[
a*x]]) + 8*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + Sqrt[A
```

```
rcCosh[a*x]]*(32*ArcCosh[a*x]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[a*x]]
- Gamma[3/2, 4*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqr
t[ArcCosh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{acosh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(1/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acosh(a*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2), x)
```

3.380 $\int \sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=205

$$\frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{-1+ax}\sqrt{1+ax}}-\cdots$$

```
[Out] -1/3*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+
1/32*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)
*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*x*(-a^2*c*
x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5895, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*
c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}\,dx = \frac{\sqrt{c-a^2cx^2}\,\int \sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,d}{2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\sqrt{c-a^2}}{}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\sqrt{c-a^2}}{}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\sqrt{c-a^2}}{}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\sqrt{c-a^2}}{}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\sqrt{c-a^2}}{}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c}}{}$$

Mathematica [A]
time = 0.10, size = 117, normalized size = 0.57

$$-\frac{\sqrt{-c(-1+ax)(1+ax)}\left(16\cosh^{-1}(ax)^2+3\sqrt{2}\,\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)+3\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}(ax)\right)\right)}{48a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/48*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(16*ArcCosh[a*x]^2 + 3*Sqrt[2]*Sqrt[
-ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + 3*Sqrt[2]*Sqrt[ArcCosh[a*x]]*G
amma[3/2, 2*ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[Ar
cCosh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{\operatorname{arccosh}\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \sqrt{\operatorname{acosh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2), x)
```

3.381 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*arccosh(a*x)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.00

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Maple [A]
time = 1.25, size = 41, normalized size = 0.85

method	result	size
default	$\frac{2\operatorname{arccosh}(ax)^{\frac{3}{2}}\sqrt{ax-1}\,\sqrt{ax+1}}{3a\sqrt{-c\,(ax-1)\,(ax+1)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arccosh(a*x)^(3/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(a*x))/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\mathrm{acosh}\left(a\,x\right)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.382 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}+\frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{2c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)+1/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x/(-a^2*x^2+1)/arccosh(a*x)^(1/2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(c*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Defer[Int][x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(2*c*Sqrt[c - a^
2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}}\,dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 4.92, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}\left(a x\right)}}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(a x\right)}}{\left(-c\left(a x-1\right)\left(a x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{acosh}\left(a\,x\right)}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.383 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{\cosh^{-1}(ax)}}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x\sqrt{\cosh^{-1}(ax)}}{3c^2\sqrt{c-a^2cx^2}}+\frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{a\sqrt{-1+ax}\,\sqrt{}}{}$$

```
[Out] 1/3*x*arccosh(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(3/2)+2/3*x*arccosh(a*x)^(1/2)/c^
2/(-a^2*c*x^2+c)^(1/2)+1/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(-a
^2*x^2+1)/arccosh(a*x)^(1/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)+1/6*a*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)^(1/2),x)/c^2/(-a^
2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcCosh[a*x]
])/(3*c^2*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int]
[x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(3*c^2*Sqrt[c - a^2*c*x^2]) + (a
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x^2)^2*Sqrt[ArcCosh[a
*x]]), x])/(6*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{2x\sqrt{\cosh^{-1}(ax)}}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int}{6c^2\sqrt{c}}$$

Mathematica [A]
time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 4.57, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}(ax)}}{(-a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{acosh}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.384 $\int (c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=511

$$\frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{3c(1-a^2x^2)^2\,\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2)+3/8*c*x*arccosh(a*x)^(3/2)*(-
a^2*c*x^2+c)^(1/2)-3/20*c*arccosh(a*x)^(5/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)
^(1/2)/(a*x+1)^(1/2)+3/128*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*c*erfi(2^(1/2)*
arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(
a*x+1)^(1/2)-3/2048*c*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/2048*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/
2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+27/256*c*(-a^2*c*x^2+
c)^(1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-9/32*a*c*x^2*(-a^
2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/32*c*(-a^
2*x^2+1)^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5897, 5895, 5893, 5884, 5953, 3393, 3388, 2211, 2235, 2236, 5912, 5914, 5907}

$$-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3c\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2}}{20a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{4}x(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}+\frac{3c(1-a^2x^2)^2\,\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (27*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(32*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh
[a*x]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*Ar
cCosh[a*x]^(3/2))/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2))/4 - (3*c
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/(20*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(2048*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*c*Sqrt[Pi]
*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh
[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5907

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Sinh[-a/b + x/
b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2,
 e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
```

```
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int}{4\sqrt{}}$$

$$= \frac{3c(1-a^2x^2)^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3c(1-a^2x^2)^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3c(1-a^2x^2)^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{9c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + $$

$$= \frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{}$$

Mathematica [A]

time = 0.35, size = 198, normalized size = 0.39

$$\frac{c\sqrt{c-a^2cx^2}\left(-384\cosh^{-1}(ax)^3-480\cosh^{-1}(ax)\cosh\left(2\cosh^{-1}(ax)\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-5\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-4\cosh^{-1}(ax)\right)+5\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},4\cosh^{-1}(ax)\right)+640\cosh^{-1}(ax)^2\sinh\left(2\cosh^{-1}(ax)\right)\right)}{2560a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-384*ArcCosh[a*x]^3 - 480*ArcCosh[a*x]*Cosh[2*ArcCo
sh[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
 + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 5*Sq
rt[-ArcCosh[a*x]]*Gamma[5/2, -4*ArcCosh[a*x]] + 5*Sqrt[ArcCosh[a*x]]*Gamma[
5/2, 4*ArcCosh[a*x]] + 640*ArcCosh[a*x]^2*Sinh[2*ArcCosh[a*x]]))/(2560*a*Sq
rt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^2 c\,x^2+c\right)^{\frac{3}{2}} \operatorname{arccosh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(a\,x)^{3/2}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2), x)
```

3.385 $\int \sqrt{c-a^2cx^2} \cosh^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=302

$$\frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}}-\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}-\frac{\sqrt{c-a^2cx^2}}{5a\sqrt{-1+}}$$

```
[Out] 1/2*x*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)-1/5*arccosh(a*x)^(5/2)*(-a^2*
c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*erf(2^(1/2)*arccosh(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
+3/128*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/16*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x
)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5895, 5893, 5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}-\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/2 - (Sqrt[c - a^2
*c*x^2]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi
/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} -$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} -$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} -$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x$$

Mathematica [A]

time = 0.25, size = 136, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(15\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+15\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-8\sqrt{\cosh^{-1}(ax)}\left(16\cosh^{-1}(ax)^2+15\cosh\left(2\cosh^{-1}(ax)\right)-20\cosh^{-1}(ax)\sinh\left(2\cosh^{-1}(ax)\right)\right)\right)}{640a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]

[Out] (Sqrt[c - a^2*c*x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 15*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 8*Sqrt[ArcCosh[a*x]]*(16*ArcCosh[a*x]^2 + 15*Cosh[2*ArcCosh[a*x]] - 20*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]]))
)/(640*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2c\,x^2+c}\;\operatorname{arccosh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)

[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}\ \operatorname{acosh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^{3/2}\,\sqrt{c-a^2\,c\,x^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2), x)
```

3.386 $\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/5*arccosh(a*x)^(5/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5892\}$

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.00

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Maple [A]
time = 1.44, size = 41, normalized size = 0.85

method	result	size
default	$\frac{2\operatorname{arccosh}(ax)^{\frac{5}{2}}\sqrt{ax-1}\,\sqrt{ax+1}}{5a\sqrt{-c\,(ax-1)\,(ax+1)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*arccosh(a*x)^(5/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}(a\,x)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.387 $\int \frac{\cosh^{-1}(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}+\frac{3a\sqrt{-1+ax}\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(3/2)/c/(-a^2*c*x^2+c)^(1/2)+3/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x*arccosh(a*x)^(1/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(3/2))/(c*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*Sqrt[ArcCosh[a*x]])/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$=\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(3a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 4.34, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^{\frac{3}{2}}}{(-a^2 c\, x^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)^{3/2}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.388 $\int \left(c-a^2cx^2\right)^{3/2} \cosh^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=580

$$\frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}+\frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}+\frac{45c\sqrt{c-a^2cx^2}\cosh^{-1}}{256a\sqrt{-1+ax}\sqrt{1+}}$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2)+3/8*c*x*arccosh(a*x)^(5/2)*(-
a^2*c*x^2+c)^(1/2)+45/256*c*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)-15/32*a*c*x^2*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2
)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+5/32*c*(-a^2*x^2+1)^2*arccosh(a*x)^(3/2)*(-a^
2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/28*c*arccosh(a*x)^(7/2)*(-
a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/512*c*erf(2^(1/2)*arcco
sh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1
)^(1/2)-15/512*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*
x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-15/16384*c*erf(2*arccosh(a*x)^(1
/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/16384*c
*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(
a*x+1)^(1/2)+225/512*c*x*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)+15/256*c*x
*(-a*x+1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5897, 5895, 5893, 5884, 5939, 5887, 5556, 12, 3389, 2211, 2235, 2236, 5912, 5914, 5898, 5896, 5952}

$$-\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15\sqrt{\pi}c\sqrt{c-a^2cx^2}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{3c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{7/2}}{28a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}+\frac{5c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{32a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{32\sqrt{ax-1}\sqrt{ax+1}}+\frac{45c\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{256a\sqrt{ax-1}\sqrt{ax+1}}+\frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}+\frac{15}{256}cx(1-ax)(ax+1)\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/512 + (15*c*x*(1 - a*x)*(1
 + a*x)*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/256 + (45*c*Sqrt[c - a^2*c*
x^2]*ArcCosh[a*x]^(3/2))/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c*x^2
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
+ (5*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*a*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/8
+ (x*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2))/4 - (3*c*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^(7/2))/(28*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c*Sqrt[Pi]*S
qrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*x]*Sqrt[
1 + a*x]) + (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqrt[Pi]*Sqrt[c - a^2*c
*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) -
```

```
(15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
```

```
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int}{4\sqrt{}}$$

$$= \frac{5c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax$$

$$= \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{15acx^2\sqrt{c-a^2cx}}{32\sqrt{-1+ax}}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}$$

Mathematica [A]
time = 0.36, size = 213, normalized size = 0.37

$$\frac{c\sqrt{c-a^2cx^2}\left(-1536\cosh^{-1}(ax)^4-4480\cosh^{-1}(ax)^2\cosh\left(2\cosh^{-1}(ax)\right)+420\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)-420\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)+7\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-4\cosh^{-1}(ax)\right)+7\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},4\cosh^{-1}(ax)\right)+3360\cosh^{-1}(ax)\sinh\left(2\cosh^{-1}(ax)\right)+3584\cosh^{-1}(ax)^3\sinh\left(2\cosh^{-1}(ax)\right)\right)}{14336a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-1536*ArcCosh[a*x]^4 - 4480*ArcCosh[a*x]^2*Cosh[2*A
rcCosh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a
*x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
+ 7*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, -4*ArcCosh[a*x]] + 7*Sqrt[ArcCosh[a*x]]*
Gamma[7/2, 4*ArcCosh[a*x]] + 3360*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]] + 3584*
ArcCosh[a*x]^3*Sinh[2*ArcCosh[a*x]]))/(14336*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(
1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arccosh}\left(a x \right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(a\,x)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2), x)
```

3.389 $\int \sqrt{c-a^2cx^2} \cosh^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=330

$$\frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}+\frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}}-\frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\sqrt{1+ax}}+\frac{1}{2}x\sqrt{c-a}$$

```
[Out] 1/2*x*arccosh(a*x)^(5/2)*(-a^2*c*x^2+c)^(1/2)+5/16*arccosh(a*x)^(3/2)*(-a^2
*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/8*a*x^2*arccosh(a*x)^(3/2)*
(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/7*arccosh(a*x)^(7/2)*(-a
^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/512*erf(2^(1/2)*arccosh(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(
1/2)-15/512*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/32*x*(-a^2*c*x^2+c)^(1/2)*arccosh(
a*x)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5895, 5893, 5884, 5939, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{7/2}}{7a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}-\frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{ax-1}\sqrt{ax+1}}+\frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{ax-1}\sqrt{ax+1}}+\frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/32 + (5*Sqrt[c - a^2*c*x^2]*A
rcCosh[a*x]^(3/2))/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (5*a*x^2*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c
 - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/
2))/(7*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (1
5*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2} - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5ax^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{5ax^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{1}{2}x\cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\,\sqrt{1+ax}} - \cdots$$

Mathematica [A]
time = 0.34, size = 148, normalized size = 0.45

$$-\frac{\sqrt{-c(-1+ax)(1+ax)}\left(-105\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+105\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+8\sqrt{\cosh^{-1}(ax)}\left(64\cosh^{-1}(ax)^3+140\cosh^{-1}(ax)\cosh\left(2\cosh^{-1}(ax)\right)-7\left(15+16\cosh^{-1}(ax)^2\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)\right)}{3584a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```

```
[Out] -1/3584*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(-105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[
ArcCosh[a*x]]] + 105*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 8*Sqrt[A
rcCosh[a*x]]*(64*ArcCosh[a*x]^3 + 140*ArcCosh[a*x]*Cosh[2*ArcCosh[a*x]] - 7
*(15 + 16*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a
*x)]*(1 + a*x))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c\,x^{2}+c}\,\operatorname{arccosh}(a x)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^{5/2}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2), x)
```

3.390 $\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/7*arccosh(a*x)^(7/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5892\}$

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.00

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]

[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])

Maple [A]
time = 1.42, size = 41, normalized size = 0.85

method	result	size
default	$\frac{2 \operatorname{arccosh}(a x)^{\frac{7}{2}} \sqrt{a x-1}\, \sqrt{a x+1}}{7 a \sqrt{-c \left(a x-1\right) \left(a x+1\right)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/7*arccosh(a*x)^(7/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(arccosh(a*x)^(5/2)/sqrt(-a^2*c*x^2 + c), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^(5/2)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a\,x)^{5/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2),x)
[Out] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.391 $\int \frac{\cosh^{-1}(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}+\frac{5a\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(5/2)/c/(-a^2*c*x^2+c)^(1/2)+5/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x*arccosh(a*x)^(3/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(5/2))/(c*Sqrt[c - a^2*c*x^2]) + (5*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*ArcCosh[a*x]^(3/2))/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(5a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^{\frac{5}{2}}}{(-a^2 c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(a\,x)^{5/2}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.392 $\int \left(a^2-x^2\right)^{3/2} \sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=368

$$\frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a^3\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}-\frac{a^3\sqrt{\pi}\,\sqrt{a^2-}}{256\sqrt{}}$$

```
[Out] -1/4*a^3*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/
32*a^3*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1
+x/a)^(1/2)/(1+x/a)^(1/2)-1/32*a^3*erfi(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)
*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)-1/256*a^3*erf(2*arcc
osh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/256
*a^3*erfi(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+
x/a)^(1/2)+1/4*x*(a^2-x^2)^(3/2)*arccosh(x/a)^(1/2)+3/8*a^2*x*(a^2-x^2)^(1/
2)*arccosh(x/a)^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5897, 5895, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236, 5912, 5952}

$$\frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{\sqrt{\pi}\,a^3\sqrt{a^2-x^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\frac{\pi}{2}}\,a^3\sqrt{a^2-x^2}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\pi}\,a^3\sqrt{a^2-x^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}\,a^3\sqrt{a^2-x^2}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}-\frac{a^3\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/8 + (x*(a^2 - x^2)^(3/2)*Sqrt[
ArcCosh[x/a]])/4 - (a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/(4*Sqrt[-1 + x/
a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[2*Sqrt[ArcCosh[x/a]]]
)/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf[
Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[
Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(256*Sqrt[-1 + x/a]*Sqrt[1
+ x/a]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])
/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
```

```
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2} \sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\, dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int \left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\, dx}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{\left(a\sqrt{a^2-x^2}\right)\int \frac{x\left(-1+\right.}{\sqrt{\cosh^{-}}}}{8\sqrt{-1+\frac{x}{a}}\,\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}}$$

Mathematica [A]
time = 0.20, size = 165, normalized size = 0.45

$$-\frac{a^4\sqrt{a^2-x^2}\left(-\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-4\cosh^{-1}\left(\frac{x}{a}\right)\right)+8\sqrt{2}\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}\left(\frac{x}{a}\right)\right)+\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\left(32\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}+8\sqrt{2}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}\left(\frac{x}{a}\right)\right)-\Gamma\left(\frac{3}{2},4\cosh^{-1}\left(\frac{x}{a}\right)\right)\right)\right)}{128\sqrt{\frac{-a+x}{a+x}}\,(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -1/128*(a^4*Sqrt[a^2 - x^2]*(-(Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -4*ArcCosh[x/
a]]) + 8*Sqrt[2]*Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -2*ArcCosh[x/a]] + Sqrt[Arc
Cosh[x/a]]*(32*ArcCosh[x/a]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[x/a]] -
Gamma[3/2, 4*ArcCosh[x/a]])))/(Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[
x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^{2}-x^{2}\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

```
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*sqrt(arccosh(x/a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(1/2),x)
```

```
[Out] Integral((-(-a + x)*(a + x))**(3/2)*sqrt(acosh(x/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*sqrt(arccosh(x/a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,{\left(a^2-x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(3/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(3/2), x)
```

3.393 $\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=211

$$\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{a\sqrt{}}{}$$

```
[Out] -1/3*a*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/32
*a*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)-1/32*a*erfi(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1
/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/2*x*(a^2-x^2)^(1/2)*arcc
osh(x/a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5895, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] (x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCosh[x/a]^
(3/2))/(3*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a*Sqrt[P
i/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*S
qrt[1 + x/a])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rubi steps

$$\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx=\frac{\sqrt{a^2-x^2}\int\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{\sqrt{a^2-x^2}\int\frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{\left(a\sqrt{a^2-x^2}\right.}{}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{\left(a\sqrt{a^2-x^2}\right.}{}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{\left(a\sqrt{a^2-x^2}\right.}{}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{\left(a\sqrt{a^2-x^2}\right.}{}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{\left(a\sqrt{a^2-x^2}\right.}{}$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2-}}{16}$$

Mathematica [A]

time = 0.11, size = 121, normalized size = 0.57

$$-\frac{a^2\sqrt{a^2-x^2}\left(16\cosh^{-1}\left(\frac{x}{a}\right)^2+3\sqrt{2}\,\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}\left(\frac{x}{a}\right)\right)+3\sqrt{2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}\left(\frac{x}{a}\right)\right)\right)}{48\sqrt{\frac{-a+x}{a+x}}\,(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -1/48*(a^2*Sqrt[a^2 - x^2]*(16*ArcCosh[x/a]^2 + 3*Sqrt[2]*Sqrt[-ArcCosh[x/a
]]*Gamma[3/2, -2*ArcCosh[x/a]] + 3*Sqrt[2]*Sqrt[ArcCosh[x/a]]*Gamma[3/2, 2*
ArcCosh[x/a]]))/(Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2-x^2}\,\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
```

```
[Out] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 - x^2)*sqrt(arccosh(x/a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(-a+x\right)\left(a+x\right)}\,\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(1/2)*acosh(x/a)**(1/2),x)
[Out] Integral(sqrt(-(-a + x)*(a + x))*sqrt(acosh(x/a)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 - x^2)*sqrt(arccosh(x/a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,\sqrt{a^2-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(1/2),x)
[Out] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(1/2), x)
```

3.394 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

```
[Out] 2/3*a*arccosh(x/a)^(3/2)*(-1+x/a)^(1/2)*(1+x/a)^(1/2)/(a^2-x^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx &= \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}\\
&= \frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 1.00

$$\frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Maple [A]
time = 1.69, size = 44, normalized size = 0.88

method	result	size
default	$\frac{2\operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} a\sqrt{-\frac{a-x}{a}}\sqrt{\frac{a+x}{a}}}{3\sqrt{(a-x)(a+x)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arccosh(x/a)^(3/2)*a/((a-x)*(a+x))^(1/2)*(-(a-x)/a)^(1/2)*((a+x)/a)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/sqrt(a^2 - x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/sqrt(-(-a + x)*(a + x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(x/a))/sqrt(a^2 - x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(1/2), x)
```

3.395 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\mathrm{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2-x^2}}$$

```
[Out] x*arccosh(x/a)^(1/2)/a^2/(a^2-x^2)^(1/2)+1/2*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*U
nintegrable(x/(1-x^2/a^2)/arccosh(x/a)^(1/2),x)/a^3/(a^2-x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(a^2*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a
]*Defer[Int][x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(2*a^3*Sqrt[a^2 - x^
2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}\\ &= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}+\frac{\left(\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2), x]
```

Maple [A]
time = 4.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/(-(-a + x)*(a + x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{{\left(a^2-x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(3/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(3/2), x)
```

3.396 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}+\frac{2x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\,\mathrm{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}}\sqrt{1$$

```
[Out] 1/3*x*arccosh(x/a)^(1/2)/a^2/(a^2-x^2)^(3/2)+2/3*x*arccosh(x/a)^(1/2)/a^4/(
a^2-x^2)^(1/2)+1/3*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*Unintegrable(x/(1-x^2/a^2)/
arccosh(x/a)^(1/2),x)/a^5/(a^2-x^2)^(1/2)+1/6*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*
Unintegrable(x/(-1+x^2/a^2)^2/arccosh(x/a)^(1/2),x)/a^5/(a^2-x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(3*a^2*(a^2 - x^2)^(3/2)) + (2*x*Sqrt[ArcCosh[x/a]])
/(3*a^4*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Defer[Int][x/((1 -
 x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(3*a^5*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a
]*Sqrt[1 + x/a]*Defer[Int][x/((-1 + x^2/a^2)^2*Sqrt[ArcCosh[x/a]]), x])/(6*
a^5*Sqrt[a^2 - x^2])
```

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx = \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}dx}{a^4\sqrt{a^2-x^2}}$$

$$= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}} + \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(-1+\frac{x^2}{a^2}\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}}{6a^5\sqrt{a^2-x^2}}$$

$$= \frac{2x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}} + \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}} + \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{}{(-}}{6a^5\sqrt{a^2-}}$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2), x]
```

Maple [A]
time = 4.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}}{(a^2-x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(5/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/(-(-a + x)*(a + x))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{{\left(a^2-x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(5/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(5/2), x)
```

3.397 $\int (a^2-x^2)^{3/2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=525

$$\frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3(a^2-x^2)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3}{8}a^2x\sqrt{a^2-}$$

```
[Out] 1/4*x*(a^2-x^2)^(3/2)*arccosh(x/a)^(3/2)+3/8*a^2*x*arccosh(x/a)^(3/2)*(a^2-
x^2)^(1/2)-3/20*a^3*arccosh(x/a)^(5/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/
a)^(1/2)+3/128*a^3*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^
2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a^3*erfi(2^(1/2)*arccosh(x/a)^(
1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)-3/2048*
a^3*erf(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/
a)^(1/2)-3/2048*a^3*erfi(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1
+x/a)^(1/2)/(1+x/a)^(1/2)+3/32*(a^2-x^2)^(5/2)*arccosh(x/a)^(1/2)/a/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)+27/256*a^3*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)-9/32*a*x^2*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5897, 5895, 5893, 5884, 5953, 3393, 3388, 2211, 2235, 2236, 5912, 5914, 5907}

$$\frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{4}x(a^2-x^2)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{3(a^2-x^2)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3a^3\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (27*a^3*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/
a]) - (9*a*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(32*Sqrt[-1 + x/a]*Sqrt[
1 + x/a]) + (3*(a^2 - x^2)^(5/2)*Sqrt[ArcCosh[x/a]])/(32*a*Sqrt[-1 + x/a]*S
qrt[1 + x/a]) + (3*a^2*x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/8 + (x*(a^2 -
x^2)^(3/2)*ArcCosh[x/a]^(3/2))/4 - (3*a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(5/2
))/(20*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[
2*Sqrt[ArcCosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi
/2]*Sqrt[a^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqr
t[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(
2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5897

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)
], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
 && GtQ[p, 0]
```

Rule 5907

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Sinh[-a/b + x/
b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2,
 e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5914

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 +
 c*x)^p)], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x]
)^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && G
tQ[n, 0] && NeQ[p, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
```

```
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{\left(3a\sqrt{a^2-x^2}\right)\int x\Big(-1}{8\sqrt{-1+\frac{x}{a}}}$$

$$= \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}(a-$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a$$

$$= -\frac{9a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3(}{}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3(a}{3}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3(a}{3}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3(a}{3}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{3(a}{3}$$

Mathematica [A]
time = 0.33, size = 219, normalized size = 0.42

$$\frac{a^4\sqrt{a^2-x^2}\left(-384\cosh^{-1}\left(\frac{x}{a}\right)^3-480\cosh^{-1}\left(\frac{x}{a}\right)\cosh\left(2\cosh^{-1}\left(\frac{x}{a}\right)\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)-5\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\Gamma\left(\frac{5}{2},-4\cosh^{-1}\left(\frac{x}{a}\right)\right)+5\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\Gamma\left(\frac{5}{2},4\cosh^{-1}\left(\frac{x}{a}\right)\right)+640\cosh^{-1}\left(\frac{x}{a}\right)^2\sinh\left(2\cosh^{-1}\left(\frac{x}{a}\right)\right)\right)}{2560\sqrt{\frac{-a+x}{a+x}}\,(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^4*Sqrt[a^2 - x^2]*(-384*ArcCosh[x/a]^3 - 480*ArcCosh[x/a]*Cosh[2*ArcCosh
[x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] +
 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 5*Sqrt
[-ArcCosh[x/a]]*Gamma[5/2, -4*ArcCosh[x/a]] + 5*Sqrt[ArcCosh[x/a]]*Gamma[5/
2, 4*ArcCosh[x/a]] + 640*ArcCosh[x/a]^2*Sinh[2*ArcCosh[x/a]]))/(2560*Sqrt[(
-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^2-x^2\right)^{\frac{3}{2}} \operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
```

```
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*arccosh(x/a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="giac")
[Out] integrate((a^2 - x^2)^(3/2)*arccosh(x/a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(\frac{x}{a}\right)^{3/2}\left(a^2-x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(3/2),x)
[Out] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(3/2), x)
```

3.398 $\int \sqrt{a^2-x^2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=316

$$\frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{a\sqrt{a^2-x^2}\cosh}{5\sqrt{-1+\frac{x}{a}}\sqrt{}}$$

```
[Out] 1/2*x*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)-1/5*a*arccosh(x/a)^(5/2)*(a^2-x^2)
^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a*erf(2^(1/2)*arccosh(x/a)^(1/2))
*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a*erfi
(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2
)/(1+x/a)^(1/2)+3/16*a*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a)^(1/2)/(1
+x/a)^(1/2)-3/8*x^2*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/a/(-1+x/a)^(1/2)/(1+
x/a)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5895, 5893, 5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (3*a*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
- (3*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(8*a*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]) + (x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos
h[x/a]^(5/2))/(5*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 -
 x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[
-1 + x/a]*Sqrt[1 + x/a])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5895

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(
1/2)*Simp[Sqrt[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[(a + b*ArcCo
sh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqr
t[d + e*x^2]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], Int[x*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx = \frac{\sqrt{a^2-x^2}\,\int \sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{\sqrt{a^2-x^2}\,\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}\,dx}{2\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \cdots$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2 \cdots}}{5\sqrt{\cdots}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2 \cdots}}{5\sqrt{\cdots}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2 \cdots}}{5\sqrt{\cdots}}$$

$$= \frac{3a\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2 \cdots}$$

$$= \frac{3a\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2 \cdots}$$

$$= \frac{3a\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2 \cdots}$$

$$= \frac{3a\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2 \cdots}$$

Mathematica [A]
time = 0.27, size = 144, normalized size = 0.46

$$\frac{a^2\sqrt{a^2-x^2}\left(15\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)-8\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\left(16\cosh^{-1}\left(\frac{x}{a}\right)^2+15\cosh\left(2\cosh^{-1}\left(\frac{x}{a}\right)\right)-20\cosh^{-1}\left(\frac{x}{a}\right)\sinh\left(2\cosh^{-1}\left(\frac{x}{a}\right)\right)\right)\right)}{640\sqrt{\frac{-a+x}{a+x}}\,(a+x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a^2 - x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] + 15*Sq
rt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 8*Sqrt[ArcCosh[x/a]]*(16*ArcCos
h[x/a]^2 + 15*Cosh[2*ArcCosh[x/a]] - 20*ArcCosh[x/a]*Sinh[2*ArcCosh[x/a]]))
)/(640*Sqrt[(-a + x)/(a + x)]*(a + x))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\sqrt{a^2-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 - x^2)*arccosh(x/a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(-a+x\right)\left(a+x\right)}\, \operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)*(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(-a + x)*(a + x))*acosh(x/a)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2 - x^2)*arccosh(x/a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}\left(\frac{x}{a}\right)^{3/2} \sqrt{a^2-x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(1/2), x)
```

3.399 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] 2/5*a*arccosh(x/a)^(5/2)*(-1+x/a)^(1/2)*(1+x/a)^(1/2)/(a^2-x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$\frac{2a\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\,\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx &= \frac{\left(\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}\\
&= \frac{2a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 1.00

$$\frac{2a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Maple [A]
time = 1.69, size = 44, normalized size = 0.88

method	result	size
default	$\frac{2\mathrm{arccosh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{-\frac{a-x}{a}}\,\sqrt{\frac{a+x}{a}}}{5\sqrt{(a-x)(a+x)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arccosh(x/a)^(5/2)*a/((a-x)*(a+x))^(1/2)*(-(a-x)/a)^(1/2)*((a+x)/a)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(acosh(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{acosh}\left(\frac{x}{a}\right)}^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

3.400 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}+\frac{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\,\mathrm{Int}\left(\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2-x^2}}$$

```
[Out] x*arccosh(x/a)^(3/2)/a^2/(a^2-x^2)^(1/2)+3/2*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*U
nintegrable(x*arccosh(x/a)^(1/2)/(1-x^2/a^2),x)/a^3/(a^2-x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[x/a]^(3/2))/(a^2*Sqrt[a^2 - x^2]) + (3*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]*Defer[Int][(x*Sqrt[ArcCosh[x/a]])/(1 - x^2/a^2), x])/(2*a^3*Sqrt[a^2 -
x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}\\ &= \frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}+\frac{\left(3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2), x]
```

Maple [A]
time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^{2}-x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Integral(acosh(x/a)**(3/2)/(-(-a + x)*(a + x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
[Out] integrate(arccosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(3/2),x)
[Out] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

3.401 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\sqrt{-1+x}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}+\frac{\sqrt{\pi}\sqrt{-1+x}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}$$

```
[Out] 1/2*erf(arccosh(x)^(1/2))*Pi^(1/2)*(-1+x)^(1/2)/(1-x)^(1/2)+1/2*erfi(arccos
h(x)^(1/2))*Pi^(1/2)*(-1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5952, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\sqrt{x-1}\,\mathrm{Erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}+\frac{\sqrt{\pi}\sqrt{x-1}\,\mathrm{Erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] (Sqrt[Pi]*Sqrt[-1 + x]*Erf[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x]) + (Sqrt[Pi]*S
qrt[-1 + x]*Erfi[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx = \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{\cosh^{-1}(x)}}\,dx}{\sqrt{1-x^2}}$$

$$= \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(x)\right)}{\sqrt{1-x^2}}$$

$$= \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(x)\right)}{2\sqrt{1-x^2}} + \frac{\left(\sqrt{-1+x}\,\sqrt{1+\right.}{}$$

$$= \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\cosh^{-1}(x)}\right)}{\sqrt{1-x^2}} + \frac{\left(\sqrt{-1+x}\,\sqrt{\right.}{}$$

$$= \frac{\sqrt{\pi}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}} + \frac{\sqrt{\pi}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{erfi}\left(\right.}{2\sqrt{1-x^2}}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 1.11

$$-\frac{\sqrt{-((-1+x)(1+x))}\left(\sqrt{-\cosh^{-1}(x)}\,\Gamma\left(\frac{1}{2},-\cosh^{-1}(x)\right)-\sqrt{\cosh^{-1}(x)}\,\Gamma\left(\frac{1}{2},\cosh^{-1}(x)\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{\cosh^{-1}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] -1/2*(Sqrt[-((-1 + x)*(1 + x))]*(Sqrt[-ArcCosh[x]]*Gamma[1/2, -ArcCosh[x]]
- Sqrt[ArcCosh[x]]*Gamma[1/2, ArcCosh[x]]))/(Sqrt[(-1 + x)/(1 + x)]*(1 + x)
*Sqrt[ArcCosh[x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^2+1}\,\sqrt{\operatorname{arccosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

```
[Out] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\,\sqrt{\operatorname{acosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2)/acosh(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acosh(x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\mathrm{acosh}\left(x\right)}\,\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)),x)
[Out] int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

3.402 $\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=438

$$-\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{15c^2\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{\right.}}{64a\sqrt{-1+ax}\,\sqrt{1}}$$

```
[Out] 1/384*c^2*erf(6^(1/2)*arccosh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arccosh(a*x)^(1/2
))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/1
28*c^2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/128*c^2*erfi(2^(1/2)*arccosh(a*x)^(1/2))
*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/64*c
^2*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/
(a*x+1)^(1/2)-3/64*c^2*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/8*c^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5906, 3393, 3388, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{6}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{6}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (-5*c^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(6
4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*
Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c^2
*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt
[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*S
qrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (c^2*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[ArcCo
sh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^6(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}} - \frac{15\cosh(2x)}{32\sqrt{x}} + \frac{3\cosh(4x)}{16\sqrt{x}} - \frac{\cosh(6x)}{32\sqrt{x}}\right)dx, x, \cosh^{-1}\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax\right.}{64a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cosh^{-1}(}\right.}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{15c}{}$$

Mathematica [A]
time = 0.31, size = 209, normalized size = 0.48

$$-\frac{c^2\sqrt{c-a^2cx^2}\left(240\cosh^{-1}(ax) - \sqrt{6}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -6\cosh^{-1}(ax)\right) + 18\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -4\cosh^{-1}(ax)\right) - 45\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -2\cosh^{-1}(ax)\right) + 45\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, 2\cosh^{-1}(ax)\right) - 18\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, 4\cosh^{-1}(ax)\right) + \sqrt{6}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, 6\cosh^{-1}(ax)\right)\right)}{384a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/384*(c^2*Sqrt[c - a^2*c*x^2]*(240*ArcCosh[a*x] - Sqrt[6]*Sqrt[-ArcCosh[a
*x]]*Gamma[1/2, -6*ArcCosh[a*x]] + 18*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*Arc
Cosh[a*x]] - 45*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + 4
5*Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 18*Sqrt[ArcCosh[a
*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*A
rcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-a^2 c\, x^2 + c)^{\frac{5}{2}}}{\sqrt{\operatorname{arccosh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt(arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}}{\sqrt{\mathrm{acosh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(1/2), x)
```

3.403 $\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=294

$$-\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\sqrt{1+ax}}-\frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{c\ldots}\right.}{4a\sqrt{-1+ax}\sqrt{1+\ldots}}$$

```
[Out] 1/8*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)
/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/8*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/
2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c*erf(2
*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)-1/32*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*
x-1)^(1/2)/(a*x+1)^(1/2)-3/4*c*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/a/(a
*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5906, 3393, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\pi}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (-3*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt
[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}-\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right.}{16a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{3c\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{c\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.19, size = 153, normalized size = 0.52

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)-4\sqrt{2}\,\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{\cosh^{-1}(ax)}\left(24\sqrt{\cosh^{-1}(ax)}+4\sqrt{2}\,\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)-\Gamma\left(\frac{1}{2},4\cosh^{-1}(ax)\right)\right)\right)}{32a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/32*(c*Sqrt[c - a^2*c*x^2]*(Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x
]] - 4*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[ArcCo
sh[a*x]]*(24*Sqrt[ArcCosh[a*x]] + 4*Sqrt[2]*Gamma[1/2, 2*ArcCosh[a*x]] - Ga
mma[1/2, 4*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[Ar
cCosh[a*x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt(acosh(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt(arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\mathrm{acosh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(1/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(1/2), x)
```

3.404 $\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}}$$

```
[Out] 1/8*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/8*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-(-a^2*c*x^2+c)^(
1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5906, 3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -((Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c}}{}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}} +$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.12, size = 114, normalized size = 0.65

$$-\frac{\sqrt{-c(-1+ax)(1+ax)}\left(8\cosh^{-1}(ax)-\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)\right)}{8a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/8*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(8*ArcCosh[a*x] - Sqrt[2]*Sqrt[-ArcCo
sh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2
, 2*ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*
x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2 c\, x^2 + c}}{\sqrt{\operatorname{arccosh}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt(arccosh(a*x)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax - 1\right)\left(ax + 1\right)}}{\sqrt{\operatorname{acosh}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt(acosh(a*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt(arccosh(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\mathrm{acosh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(1/2), x)
```

3.405 $\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 46, normalized size = 1.00

$$\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 1.57, size = 41, normalized size = 0.89

method	result	size
default	$\frac{2\sqrt{\operatorname{arccosh}\left(a x\right)}\,\sqrt{a x -1}\,\sqrt{a x +1}}{a\sqrt{-c \left(a x -1\right) \left(a x +1\right)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arccosh(a*x)^(1/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\sqrt{\operatorname{acosh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\mathrm{acosh}\left(a\,x\right)}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)), x)
```

3.406 $\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
[Out] Defer[Int][1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A]
time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(a x \right)}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{acosh}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acosh(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{acosh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)), x)
```

3.407 $\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
[Out] Defer[Int][1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]), x]
```

Maple [A]
time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\operatorname{arccosh}\left(a x \right)}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{acosh}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(5/2)*sqrt(acosh(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccosh(a*x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{acosh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)), x)
```

3.408 $\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=433

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,(c-a^2cx^2)^{5/2}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{15c^2\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{E}}{16a\sqrt{-1+}}$$

```
[Out] -15/32*c^2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^
(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arccosh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/8
*c^2*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2
)/(a*x+1)^(1/2)-3/8*c^2*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^
(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c^2*erf(6^(1/2)*arccosh(a*x)^(1/2)
)*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/32*
c^2*erfi(6^(1/2)*arccosh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2*(-a^2*c*x^2+c)^(5/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)/a/arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5904, 5912, 5952, 5556, 3389, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{6}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{6}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,(c-a^2cx^2)^{5/2}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(5/2))/(a*Sqrt[ArcCosh[a*x
]]) + (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(8*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[S
qrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c^2*Sqrt
[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[
ArcCosh[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqr
t[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[Arc
Cosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
```

```
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(c-a^2cx^2)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)^2}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{\sqrt{x}}\,dx, \ldots\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}\ldots}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} - \frac{\sinh\ldots}{8\sqrt{\ldots}}\right.\right.}{a\sqrt{-1+ax}\,\sqrt{1\ldots}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \cosh\ldots\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-}\ldots\right.}{16a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cos\ldots}\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\operatorname{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} - \ldots$$

Mathematica [A]
time = 0.85, size = 411, normalized size = 0.95

$$\frac{c^2e^{-6\cosh^{-1}(ax)}\sqrt{c-a^2cx^2}\left(-1+6e^{2\cosh^{-1}(ax)}+e^{4\cosh^{-1}(ax)}+52e^{6\cosh^{-1}(ax)}+e^{8\cosh^{-1}(ax)}+6e^{10\cosh^{-1}(ax)}-e^{12\cosh^{-1}(ax)}-64a^2e^{6\cosh^{-1}(ax)}x^2-16e^{6\cosh^{-1}(ax)}\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\,\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+16e^{6\cosh^{-1}(ax)}\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\,\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\sqrt{6}e^{6\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-6\cosh^{-1}(ax)\right)-12e^{6\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)-\sqrt{2}e^{6\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)-\sqrt{2}e^{6\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)-12e^{6\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\cosh^{-1}(ax)\right)+\sqrt{6}e^{6\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},6\cosh^{-1}(ax)\right)\right)}{32a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]

[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-1 + 6*E^(2*ArcCosh[a*x]) + E^(4*ArcCosh[a*x]) + 52*E^(6*ArcCosh[a*x]) + E^(8*ArcCosh[a*x]) + 6*E^(10*ArcCosh[a*x]) - E^(12*ArcCosh[a*x]) - 64*a^2*E^(6*ArcCosh[a*x])*x^2 - 16*E^(6*ArcCosh[a*x])*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 16*E^(6*ArcCosh[a*x])*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Sqrt[6]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -6*ArcCosh[a*x]] - 12*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] - Sqrt[2]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] - Sqrt[2]*E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 12*E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*ArcCosh[a*x]]))/(32*a*E^(6*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{\operatorname{arccosh}\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)

[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^(5/2)/arccosh(a*x)^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/arccosh(a*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{{\mathrm{acosh}\left(a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(3/2), x)
```

3.409 $\int \frac{(c-a^2cx^2)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,(c-a^2cx^2)^{3/2}}{a\sqrt{\cosh^{-1}(ax)}}+\frac{c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{c\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\right.}{a\sqrt{-1+ax}}$$

```
[Out] -1/2*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/4*c*erf(2
*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)-1/4*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x
-1)^(1/2)/(a*x+1)^(1/2)-2*(-a^2*c*x^2+c)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
a/arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5904, 5912, 5952, 5556, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,(c-a^2cx^2)^{3/2}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(3/2))/(a*Sqrt[ArcCosh[a*x
]]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sq
rt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c -
a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
 (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x(-1+a^2x^2)}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4}{8\sqrt{x}}\right.\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-}\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(\right.}{2a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh}\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{c}{}$$

Mathematica [A]
time = 0.33, size = 239, normalized size = 0.84

$$-\frac{ce^{-4\cosh^{-1}(ax)}\sqrt{c-a^2cx^2}\left(-1-14e^{4\cosh^{-1}(ax)}-e^{8\cosh^{-1}(ax)}+16a^2e^{4\cosh^{-1}(ax)}x^2+4e^{4\cosh^{-1}(ax)}\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\text{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)-4e^{4\cosh^{-1}(ax)}\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\text{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)+2e^{4\cosh^{-1}(ax)}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)+2e^{4\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\cosh^{-1}(ax)\right)\right)}{8a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -1/8*(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x]
) + 16*a^2*E^(4*ArcCosh[a*x])*x^2 + 4*E^(4*ArcCosh[a*x])*Sqrt[2*Pi]*Sqrt[Ar
```

```
cCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 4*E^(4*ArcCosh[a*x])*Sqrt[2*P
i]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 2*E^(4*ArcCosh[a*x
])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] + 2*E^(4*ArcCosh[a*x])*S
qrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]]))/(a*E^(4*ArcCosh[a*x])*Sqrt[(
-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{\operatorname{arccosh}\left(a x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\operatorname{acosh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/acosh(a*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\mathrm{acosh}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(3/2), x)
```

3.410 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{\cosh^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\right.}{a\sqrt{-1+ax}}$$

```
[Out] -1/2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*
Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2*(a*x-1)^(1/2)
*(a*x+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5904, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\sqrt{c-a^2cx^2}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(a*Sqrt[ArcCosh[a*x]]
) - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqr
t[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*S
qrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\Big(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-}}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\Big(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(ax}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(2\sqrt{c-a^2cx^2}\right)\text{Subst}\Big(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\Big(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\Big)}{a\sqrt{-1+ax}\,\sqrt{1+ax}} + \cdots$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(2\sqrt{c-a^2cx^2}\right)\text{Subst}\Big(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(a}}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2cx^2}\,\text{erf}\Big(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\Big)}{a\sqrt{-1+ax}\,\sqrt{1+ax}} + \sqrt{\frac{\pi}{2}}\cdots$$

Mathematica [A]
time = 0.19, size = 127, normalized size = 0.75

$$\frac{\sqrt{c-a^2cx^2}\left(4-4a^2x^2-\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\text{Erf}\Big(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\Big)+\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\text{Erfi}\Big(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\Big)\right)}{2a\sqrt{\frac{-1+ax}{1+ax}}\,(1+ax)\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(4 - 4*a^2*x^2 - Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqr
t[2]*Sqrt[ArcCosh[a*x]]] + Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[
ArcCosh[a*x]]]))/(2*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x
]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} c \,x^{2}+c}}{\operatorname{arccosh}\left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}{\operatorname{acosh}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acosh(a*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{{\mathrm{acosh}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(3/2), x)
```

3.411 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.00

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Maple [A]
time = 1.44, size = 41, normalized size = 0.89

method	result	size
default	$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{\sqrt{\operatorname{arccosh}\left(ax\right)}\,a\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/arccosh(a*x)^(1/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 59, normalized size = 1.28

$$\frac{2\sqrt{-a^2cx^2+c}\,\sqrt{a^2x^2-1}}{\left(a^3cx^2-ac\right)\sqrt{\log\left(ax+\sqrt{a^2x^2-1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 - 1))))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{3/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)), x)
```

3.412 $\int \frac{1}{(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{a\,(c-a^2cx^2)^{3/2}\sqrt{\cosh^{-1}(ax)}}+\frac{4a\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x}{(-1+a^2x^2)^2\sqrt{\cosh^{-1}(ax)}},x\right)}{c\sqrt{c-a^2cx^2}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2)+4*
a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)^(1/
2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x
]]) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x^2)^2*Sqrt
[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{2\sqrt{-1+ax}}{ac(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}+\frac{\left(4a\sqrt{-1+ax}\,\sqrt{1\right.}{}$$

Mathematica [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)), x]
```

Maple [A]
time = 4.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arccosh}\left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \operatorname{acosh}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*acosh(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)), x)
```

3.413 $\int \frac{1}{(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{a\,(c-a^2cx^2)^{5/2}\,\sqrt{\cosh^{-1}(ax)}}-\frac{8a\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x}{(-1+a^2x^2)^3\sqrt{\cosh^{-1}(ax)}},x\right)}{c^2\sqrt{c-a^2cx^2}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2)-8*
a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^3/arccosh(a*x)^(1/
2),x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*(c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x
]]) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x^2)^3*Sqrt
[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx=\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}-\frac{\left(8a\sqrt{-1+a\right.}{}$$

Mathematica [A]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)), x]
```

Maple [A]
time = 4.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \operatorname{arccosh}\left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(3/2)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)), x)
```

3.414 $\int \frac{(c-a^2cx^2)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,(c-a^2cx^2)^{3/2}}{3a\cosh^{-1}(ax)^{3/2}}-\frac{16cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\Big(2\sqrt{\cosh}}{3a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

```
[Out] -2/3*(-a^2*c*x^2+c)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(3/2)-
2/3*c*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/
2)/(a*x+1)^(1/2)-2/3*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*c*erf
i(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)-16/3*c*x*(-a*x+1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/arcco
sh(a*x)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5904, 5912, 5942, 5907, 3393, 3388, 2211, 2235, 2236, 5953, 5556}

$$-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,(c-a^2cx^2)^{3/2}}{3a\cosh^{-1}(ax)^{3/2}}-\frac{16cx(1-ax)(ax+1)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c - a^2*c*x^2)^(3/2))/(3*a*ArcCosh[a*x]^(
3/2)) - (16*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*
x]]) - (2*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[
2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (2*c*Sqrt[Pi]*
Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x
]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 5907

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c))*Simp[(d1 + e1*x)^p/
(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Sinh[-a/b + x/
b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2,
 e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[2*p, 0]
```

Rule 5912

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (
e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d1
*d2 + e1*e2*x^2)^p*(a + b*ArcCosh[c*x])^n, x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m, n}, x] && EqQ[d2*e1 + d1*e2, 0] && IntegerQ[p]
```

Rule 5942

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*
x]*(d + e*x^2)^p]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Dist[
f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[(f*
x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/((1
 + c*x)^p*(-1 + c*x)^p)], Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^
(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f,
 m, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*
p + 1, 0] && IGtQ[m, -3]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x(-1+a^2x^2)}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2c\cdots\right.}{3\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2c\cdots\right.}{\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(16c\sqrt{c-a^2c\cdots\right.}{\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8c\sqrt{c-a^2cx\cdots\right.}{\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(4c\sqrt{c-a^2cx\cdots\right.}{\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx\cdots\right.}{\cdots}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2c\sqrt{\pi}\,\sqrt{c-a\cdots}}{3a\sqrt{\cdots}}$$

__**Mathematica**__ [A]
time = 0.42, size = 317, normalized size = 0.96

$$-\frac{ce^{-4\cosh^{-1}(ax)}\sqrt{c-a^2cx^2}\left(-1-14e^{4\cosh^{-1}(ax)}-e^{8\cosh^{-1}(ax)}+16a^2e^{4\cosh^{-1}(ax)}x^2+8\cosh^{-1}(ax)-8e^{8\cosh^{-1}(ax)}\cosh^{-1}(ax)+64ae^{4\cosh^{-1}(ax)}x\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)+64a^2e^{4\cosh^{-1}(ax)}x^2\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)-16e^{4\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)+16\sqrt{2}e^{4\cosh^{-1}(ax)}\left(-\cosh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+16\sqrt{2}e^{4\cosh^{-1}(ax)}\cosh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)-16e^{4\cosh^{-1}(ax)}\cosh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},4\cosh^{-1}(ax)\right)\right)}{24a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x
]) + 16*a^2*E^(4*ArcCosh[a*x])*x^2 + 8*ArcCosh[a*x] - 8*E^(8*ArcCosh[a*x])*
ArcCosh[a*x] + 64*a*E^(4*ArcCosh[a*x])*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh
[a*x] + 64*a^2*E^(4*ArcCosh[a*x])*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*
x] - 16*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -4*ArcCosh[a*x]
] + 16*Sqrt[2]*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCo
sh[a*x]] + 16*Sqrt[2]*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*Ar
cCosh[a*x]] - 16*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 4*ArcCosh
[a*x]]))/(a*E^(4*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh
[a*x]^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c\,x^{2}+c \right)^{\frac{3}{2}}}{\operatorname{arccosh}\left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}{\operatorname{acosh}^{\frac{5}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/acosh(a*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\mathrm{acosh}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(5/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(5/2), x)
```

3.415 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c-a^2cx^2}}{3a\cosh^{-1}(ax)^{3/2}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{2\sqrt{2}}{}$$

```
[Out] 2/3*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2/3*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccosh(a*x)^(3/2)-8/3*x*(-a^2*c*x^2
+c)^(1/2)/arccosh(a*x)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5904, 5885, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{2\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\sqrt{c-a^2cx^2}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(3*a*ArcCosh[a*x]^(3/
2)) - (8*x*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt
[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (2*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]
]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5904

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(d + e*x^2)^p]*((a + b*A
rcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Si
mp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)], Int[x*(1 + c*x)^(p - 1/2)*(-1
 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2\cdots}{\sqrt{x\cdots}}\cdots\right.}{3a\sqrt{-1+ax}\sqrt{\cdots}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\cdots\right.}{3a\sqrt{-1+ax}\sqrt{1\cdots}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-2x\cdots}\cdots\right.}{3a\sqrt{-1+ax}\sqrt{\cdots}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cdots}\right.}{3a\sqrt{-1+ax}\sqrt{1+\cdots}}$$

Mathematica [A]
time = 0.23, size = 141, normalized size = 0.70

$$-\frac{2\sqrt{c-a^2cx^2}\left((1+ax)\left(-1+ax+4ax\sqrt{\frac{-1+ax}{1+ax}}\cosh^{-1}(ax)\right)+\sqrt{2}\left(-\cosh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{2}\cosh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)\right)}{3a\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c - a^2*c*x^2]*((1 + a*x)*(-1 + a*x + 4*a*x*Sqrt[(-1 + a*x)/(1 + a
*x)]*ArcCosh[a*x]) + Sqrt[2]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCosh[a*
x]] + Sqrt[2]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*ArcCosh[a*x]]))/(3*a*Sqrt[(-1
 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2c\,x^2+c}}{\operatorname{arccosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acosh(a*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\operatorname{acosh}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(5/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(5/2), x)
```

3.416 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{3a\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5892}

$$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{3a\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.00

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{3a\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Maple [A]
time = 1.40, size = 41, normalized size = 0.85

method	result	size
default	$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{3\operatorname{arccosh}(ax)^{\frac{3}{2}} a\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/arccosh(a*x)^(3/2)/a/(-c*(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 1.23

$$\frac{2\sqrt{-a^2cx^2+c}\,\sqrt{a^2x^2-1}}{3\left(a^3cx^2-ac\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*log(a*x + sqr
t(a^2*x^2 - 1))^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{5/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)), x)
```

3.417 $\int \frac{1}{(c-a^2cx^2)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{3a\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}+\frac{4a\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Int}\left(\frac{x}{\left(-1+a^2x^2\right)^2\cosh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2)+
4/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)
^(3/2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(
3/2)) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x^2)^2*Ar
cCosh[a*x]^(3/2)), x])/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}}{3ac(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}+\frac{\left(4a\sqrt{-1+ax}\,\sqrt{\right.}{}}{}\end{aligned}$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)), x]
```

Maple [A]
time = 4.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \operatorname{arccosh}\left(a x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)), x)
```

3.418 $\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{3a\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}-\frac{8a\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{Int}\left(\frac{x}{\left(-1+a^2x^2\right)^3\cosh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{c-a^2cx^2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2)-
8/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^3/arccosh(a*x)
^(3/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(
3/2)) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x^2)^3*Ar
cCosh[a*x]^(3/2)), x])/(3*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}}{3ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{-1+a\right.}{}\end{aligned}$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)), x]
```

Maple [A]
time = 4.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \operatorname{arccosh}\left(a x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(5/2)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)), x)
```

3.419 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-2(3+n)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] -1/8*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c
^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*
arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arccosh(c*x))/
b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5952, 5556, 3388, 2212}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{8bc^3(n+1)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/8*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c^3*(1 + n)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x])
)/b])/(2^(2*(3 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])
/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d-c^2dx^2}\,\int x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\sinh^2(x)\,dx, x, \cosh^{-1}(c\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)\,dx\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a\right.}{8c^3\sqrt{-}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e\right.}{16c^3\sqrt{-}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}}{}$$

Mathematica [A]
time = 0.74, size = 181, normalized size = 0.72

$$-\frac{d\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b+bn}+4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-e^{\frac{8a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)\right)}{64c^3\sqrt{-d(-1+cx)(1+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/64*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((-8*(
a + b*ArcCosh[c*x]))/(b + b*n) + ((a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(
a + b*ArcCosh[c*x]))/b] - E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcCosh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])
^2/b^2))^n)))/(c^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{n} \sqrt{-c^{2} d\,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{- d \left(c x - 1\right) \left(c x + 1\right)} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARc near
 0Simplification assuming sageVARc near 0Simplification assuming sageVARc n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

3.420 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=379

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/8*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+1/8*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*
(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(
c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5952, 5556, 3388, 2212}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{e^{a/b}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3^{-n-1}e^{\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcCosh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x])
)/b])/(8*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^2(x)\,dx,x,\cosh^{-1}(cx\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3\right.\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{}}{}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c}}{}$$

$$=\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma}{8c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]

time = 0.87, size = 241, normalized size = 0.64

$$-\frac{de^{-\frac{3a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(3e^{\frac{4a}{b}}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)+\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\left(3^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-3e^{\frac{2a}{b}}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\Gamma\left(1+n,\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)\right)}{24c^2\sqrt{-d(-1+cx)(1+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/24*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((3*E^
((4*a)/b)*Gamma[1 + n, a/b + ArcCosh[c*x]])/(a/b + ArcCosh[c*x])^n + (Gamma
[1 + n, (-3*(a + b*ArcCosh[c*x]))/b]/3^n - 3*E^((2*a)/b)*Gamma[1 + n, -((a
+ b*ArcCosh[c*x])/b)] - (E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamm
a[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(3^n*(-((a + b*ArcCosh[c*x])^2/b^2))^
n))/(-((a + b*ArcCosh[c*x])/b))^n))/(c^2*E^((3*a)/b)*Sqrt[-(d*(-1 + c*x)*(1
 + c*x))])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{arccosh}(cx))^{n}\sqrt{-c^{2}d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}\,\mathrm{d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

3.421 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1+}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+2^(-3-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))
/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2^(-3-n)*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+
b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5906, 3393, 3388, 2212}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{2bc(n+1)\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (2^(-3 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c
*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\sinh^2(x)\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)\right)\,dx, x$$
$$c\sqrt{-1+cx}\,\sqrt{1+cx}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a+\right.}{2c\sqrt{-1}}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-2x}\right.}{4c\sqrt{-1+}}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\,(a}{}$$

Mathematica [A]
time = 0.50, size = 214, normalized size = 0.85

$$\frac{2^{-3-n}de^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n-b(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+be^{\frac{4a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)}{bc(1+n)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]

[Out] (2^(-3 - n)*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(
2^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^
n - b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))
```

```
/b] + b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(
a + b*ArcCosh[c*x]))/b]))/(b*c*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arccosh}(cx))^{n} \sqrt{-c^{2} d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))^{n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

3.422 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=212

$$-\frac{de^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2\sqrt{d-c^2dx^2}}+\frac{de^{a/b}\sqrt{-1+c}}{}$$

[Out] -1/2*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+1/2*d*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)

Rubi [A]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]

[Out] -1/2*(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + d*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^n}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(-\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2x(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\sqrt{1+cx}}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int}{\sqrt{-1+}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2dx^2}\,\text{Subst}(}{\sqrt{}}$$

$$= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{d-c^2}}{}$$

$$= \frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\right.}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n\sqrt{-c^2d\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x, x)
```

3.423 $\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{cd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{d-c^2dx^2}}+d\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] -c*d*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-c^2*d*x
^2+d)^(1/2)+d*Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] -((c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*(1 + n
)*Sqrt[d - c^2*d*x^2])) + d*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d -
 c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx=\frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\int \left(\frac{c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int}{\sqrt{-1+}}$$

$$=\frac{c\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\cosh}{x^2\sqrt{-1+c}}}{\sqrt{-1+cx}\,\sqrt{1-}}$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(cx))^{n}\sqrt{-c^{2}d\,x^{2}+d}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

3.424 $\int x^2(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=658

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-}}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -1/16*d*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2^(-7-n)*3^(-1-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-6*(a
+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arccosh(c*x
))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-2*n)*d*(a+b*arccosh(c*x))^n*GAMM
A(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*
arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-n)*d*(a+b*arccosh(c*x
))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)
/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-n)*d*exp(2*a
/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(
1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-2*n)*
d*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)*(-c^2*d
*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^
(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(a+b*arccosh(
c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.138$, Rules used = {5952, 5556, 3388, 2212}

$$-\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{d2^{-n-7}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d2^{-n-7}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d2^{-2n-7}e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{d2^{-n-7}3^{-n-1}e^{\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{16bc^3(n+1)\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/16*(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c^3*(1 + n)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^(
(6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(
-7 - 2*n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a
+ b*ArcCosh[c*x]))/b])/(c^3*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a
+ b*ArcCosh[c*x])/b))^n) + (2^(-7 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d*E^((2
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*Ar
cCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)
```

```
^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((a + b*ArcCosh[c*x])/b)^n) + (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b)*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))
/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^4(x)\,dx,x,\mathrm{c}\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{16}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2\right.\right.}{c^3\sqrt{-1}}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Sub}}{32}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Sub}}{64c^3}$$

$$= -\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d}}{}$$

Mathematica [A]
time = 2.43, size = 438, normalized size = 0.67

$$\frac{2^{-7-2n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{-n}\left(2^nb(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)-3^{1+n}be^{\frac{2a}{b}}(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-2^n3^{1+n}be^{\frac{4a}{b}}(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+2^n3^{1+n}be^{\frac{8a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+3^{1+n}be^{\frac{10a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)+2^ne^{\frac{6a}{b}}\left(2^{3+n}3^{1+n}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n-be^{\frac{6a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{6(a+b\cosh^{-1}(cx))}{b}\right)\right)\right)}{bc^3(1+n)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b
*ArcCosh[c*x]))/b] - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n
*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^n*3^(1 + n)*b*E^((4*a)/b)*(1
 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + 2^
n*3^(1 + n)*b*E^((8*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, (2*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] + 2^n*E^((6*a)
/b)*(2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2
))^n - b*E^((6*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*
(a + b*ArcCosh[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*
(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARc near
 0Simplification assuming sageVARc near 0Simplification assuming sageVARc n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a+b\operatorname{acosh}(cx))^n (d-c^2 dx^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

3.425 $\int x(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=578

$$-\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{32c^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3^{-n}de^{-\frac{3a}{b}}\sqrt{}}{}$$

[Out] -1/32*5^(-1-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/32*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*d*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(3^n)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/32*5^(-1-n)*d*exp(5*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)

Rubi [A]

time = 0.37, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5952, 5556, 3388, 2212}

$$-\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{16c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{de^{a/b}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{16c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d3^{-n}e^{\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d5^{-n-1}e^{\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]

[Out] -1/32*(5^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(c^2*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(16*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^4(x)\,dx,x,\text{c}\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\cosh(x)-\frac{3}{16}(a+bx)\right.\right.}{c^2\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(5x)\,dx,x,\cosh^{-1}(c\right.}{16c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{32c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right.}{32c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 1.48, size = 500, normalized size = 0.87

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/32*(15^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[
c*x])^n*(2*15^(1 + n)*E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*A
rcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]] + (a/b + ArcCos
h[c*x])^n*(-(3^(1 + n)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (
-5*(a + b*ArcCosh[c*x]))/b]) + 3*5^(1 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*
x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] - 2*15^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
Cosh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*Arc
Cosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] - 4*5^(1 + n)
*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^
2))^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*E^((10*a)/b)*(a/
b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a +
b*ArcCosh[c*x]))/b])))/(c^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCo
sh[c*x])^2/b^2))^(3*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

3.426 $\int (d-c^2dx^2)^{3/2} (a+b\cosh^{-1}(cx))^n \, dx$

Optimal. Leaf size=450

$$-\frac{3d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2^{-2(3+n)}de^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -3/8*d*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(
-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-3-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b
*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arccosh(c*x))/b
)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-3-n)*d*exp(2*a/b)*(a+b*arccosh(c*x))^n
*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*
x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAM
MA(1+n,4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c/(((a+b*ar
ccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5906, 3393, 3388, 2212}

$$-\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d2^{-n-3}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{d2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3d\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^{n+1}}{8bc(n+1)\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x
]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])
/b))^n) - (2^(-3 - n)*d*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((a + b*ArcCosh[c*x])/b)^n) + (d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
```

```
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx &= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)^n\sinh^4(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)\right.\right.}{c\sqrt{-1+cx}\,\sqrt{1+c}} \\
&= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right)\operatorname{Sub}}{8} \\
&= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right)\operatorname{Sub}}{16c} \\
&= -\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d-c^2}}{}
\end{aligned}$$

Mathematica [A]
time = 1.55, size = 384, normalized size = 0.85

$$\frac{4^{-3-n}d^2e^{-\frac{4a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\,(a+b\cosh^{-1}(cx))^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{-2n}\left(3\,2^{3+2n}e^{\frac{4a}{b}}(a+b\cosh^{-1}(cx))\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{2n}+b(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^{2n}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)-2^{3+n}be^{\frac{2a}{b}}(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)+2^{3+n}be^{\frac{6a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n\Gamma\left(1+n,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)-be^{\frac{8a}{b}}(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\Gamma\left(1+n,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)\right)}{bc(1+n)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (4^(-3 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n
*(3*2^(3 + 2*n)*E^((4*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^(2*n) + b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/
b))^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*b*E^((2*a)/b)*(
1 + n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n
, (-2*(a + b*ArcCosh[c*x]))/b] + 2^(3 + n)*b*E^((6*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a +
b*ArcCosh[c*x]))/b] - b*E^((8*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*(-((a +
b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b]))/(b*c*E
^((4*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n
))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-c^2 d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}(cx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

3.427 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x} dx$

Optimal. Leaf size=415

$$\frac{3^{-1-n}d^2e^{-\frac{3a}{b}}\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}-\frac{5d^2e^{-}}{}$$

```
[Out] 1/8*3^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^
2+d)^(1/2)-5/8*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+
d)^(1/2)+5/8*d^2*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))
/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(
1/2)-1/8*3^(-1-n)*d^2*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arcc
osh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d
*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),
x)
```

Rubi [A]
time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] (3^(-1 - n)*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1
 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])/b))^n) - (5*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Arc
Cosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^(a/b)*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8
*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*d^2*E^((3*a)
/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a
+ b*ArcCosh[c*x]))/b])/(8*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) +
 d^2*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2c^2x(a+b\cosh^{-1}(c}{\sqrt{-1+cx}\,\sqrt{1}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c^2\right.}{\sqrt{}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right.}{}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d\sqrt{d-c^2dx^2}\right.}{}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1+}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1+}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{3^{-1-n}de^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8\sqrt{-1+cx}\,\sqrt{1+cx}}$$

<u>**Mathematica**</u> [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

<u>**Maple**</u> [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**n/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n\,(d - c^2\,d\,x^2)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x,x)
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x, x)
```

3.428 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x^2}\,dx$

Optimal. Leaf size=292

$$-\frac{3cd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{2b(1+n)\sqrt{d-c^2dx^2}}+\frac{2^{-3-n}cd^2e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^n\,(-}{\sqrt{d-c^2dx^2}}$$

```
[Out] -3/2*c*d^2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-c
^2*d*x^2+d)^(1/2)+2^(-3-n)*c*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arc
cosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(2*a/b)/(((-a-b*arccosh(c*x))/
b)^n)/(-c^2*d*x^2+d)^(1/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*G
AMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh
(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arccosh(c*x))^n/x^2
/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]

time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cosh^{-1}(cx))^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (-3*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*(
1 + n)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*
a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*c*d^
2*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 +
 n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])
/b)^n) + d^2*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x
]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))^n}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(-\frac{2c^2(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(a+b\cosh^{-1}(c}{x^2\sqrt{-1+cx}\,\sqrt{}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c\right.}{}$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2^{-3-n}cde^{-\frac{2a}{b}}\sqrt{d-c^2d}}{}$$

Mathematica [**A**]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [**A**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-c^2d\,x^2+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2,x)
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

3.429 $\int x^2(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=870

$$-\frac{5d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -5/128*d^2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-8*(a+
b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arccosh(c*x)
)/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*(a+b*arccosh(c*x)
)^n*GAMMA(1+n,-6*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/
(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d^2*(a+b*arccosh(c*
x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c
^3/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7
-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2
+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2^(-7-n)*d^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh
(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh
(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*arccosh(c*x))/b)^n)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,6*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arc
cosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*(a+
b*arccosh(c*x))^n*GAMMA(1+n,8*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^
3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 870, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5952, 5556, 3388, 2212}

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-11 - 3*n)*d^2*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((8*
a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7
- n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n,
 (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
```

```
c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(4 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) + (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcCosh[
c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^6(x)\,dx,x,\mathrm{co}\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n+\frac{1}{32}(a+bx)^n\cosh\right.\right.}{}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{S}}{}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{S}}{25}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d-}}{}$$

Mathematica [A]
time = 4.94, size = 677, normalized size = 0.78

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcCosh[c*x])^n*(-(3^(1 + n)*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (
-8*(a + b*ArcCosh[c*x]))/b]) + 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCo
sh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*
b*E^((4*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCos
h[c*x]))/b] - 3^(1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*
3^(1 + n)*a*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*A
rcCosh[c*x]*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 3^(1 + n)*4^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcCosh
[c*x]))/b] + 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*
x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] - 4^(2 + n)*b*E^((6*a)/b
)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b] -
4^(2 + n)*b*E^((6*a)/b)*n*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a
+ b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))
```

```
^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-(
(a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b])))/(b*c
^3*E^((8*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^
n)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*ar
ccosh(c*x) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARc near
 0Simplification assuming sageVARc near 0Simplification assuming sageVARc n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

3.430 $\int x(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=793

$$\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{}}{}$$

```
[Out] 1/128*7^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-7*(a+b*arccosh(c*x))/b)*
(-c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-1/128*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c
*x))/b)*(-c^2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^
n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA
(1+n,-3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*a
rccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/128*d^2*(a+b*arccosh(c*x))
^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a
-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/128*d^2*exp(a/b)*(a+b*
arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/((
(a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/128*3^(1-n)*d^2*exp(
3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d
)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/128*d^
2*exp(5*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(-c^2*d
*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,7*(a+b*arc
cosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 793, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5952, 5556, 3388, 2212}

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcCosh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (3^(1 - n)*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCo
sh[c*x]))/b])/(128*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*A
rcCosh[c*x])/b))^n) - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gam
```

```
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(128*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(128*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(1 - n)*d^2*E^((
3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcCosh[c*x]))/b])/(128*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x
])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b])/(1
28*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^6(x)\,dx,x,\cos\right.}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int\left(-\frac{5}{64}(a+bx)^n\cosh(x)+\frac{9}{64}(a+b\right.\right.}{c^2}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh(7x)\,dx,x,\cosh^{-1}(cx\right.}{64c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}+$$

$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A]
time = 2.67, size = 633, normalized size = 0.80

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (21^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(-(105^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCos
h[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + ArcCosh[c*
x])^n*(-(3^(1 + n)*5^n*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (
-7*(a + b*ArcCosh[c*x]))/b]) + E^((2*a)/b)*(21^(1 + n)*(-((a + b*ArcCosh[c*
x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b] - 9*5^n*7^(1 +
n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a +
b*ArcCosh[c*x]))/b] + 105^(1 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2
))^(2*n)*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] - 5^n*7^(2 + n)*E^((8*a)/b
)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*
(a + b*ArcCosh[c*x]))/b] + 16*5^n*7^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c
*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (3*(a + b*Arc
Cosh[c*x]))/b] - 21^(1 + n)*E^((10*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*A
rcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)
*5^n*E^((12*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*
```

```
Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b]))))/(128*5^n*c^2*E^((7*a)/b)*Sqrt[
d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(3*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arcc
osh(c*x) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

3.431 $\int \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=674

$$-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right.}{c\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] -5/16*d^2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-6*
(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arccosh(c*x
))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*2^(-7-2*n)*d^2*(a+b*arccosh(c*x))^n*
GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-
b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2
*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*2^(-7-n)*d
^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*2
^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x)
)/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(
a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5906, 3393, 3388, 2212}

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3d^2 2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{15d^2 2^{-n-7}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{15d^2 2^{-n-7}e^{\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{3d^2 2^{-2n-7}e^{\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2 2^{-n-7}3^{-n-1}e^{\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{16bc(n+1)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c*(1 + n)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c*E^(
(6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*2
^(-7 - 2*n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcCosh[c*x]))/b])/(c*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) + (15*2^(-7 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (15*2^(-7 - n)
*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2
*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[
```

```
c*x])/b)^n) + (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*Ar
cCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5906

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/((1 + c*x)^p*(-1 + c*x)^p)],
 Subst[Int[x^n*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\operatorname{Subst}\left(\int(a+bx)^n\sinh^6(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\operatorname{Subst}\left(\int\left(\frac{5}{16}(a+bx)^n-\frac{15}{32}(a+bx)^n\cosh(2x\right.\right.}{c\sqrt{-1+}}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\operatorname{Sul}}{3}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\operatorname{Sul}}{64c}$$

$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{c}}{}$$

Mathematica [A]
time = 3.76, size = 538, normalized size = 0.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(-(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh
[c*x])/b))^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b]) + 3^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/b))^n*Gam
ma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 +
 n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (
-2*(a + b*ArcCosh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((8*a)/b)*(1 + n)*(-((a +
 b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a
 + b*ArcCosh[c*x]))/b] - 3^(2 + n)*b*E^((10*a)/b)*(1 + n)*(a/b + ArcCosh[c*
x])^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]
))/b] + 2^n*E^((6*a)/b)*(5*2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a +
 b*ArcCosh[c*x])^2/b^2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])
^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/
b])))/(b*c*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^
2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}(cx)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

3.432 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=805

$$-\frac{5^{-1-n}d^3e^{-\frac{5a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}-\frac{5\ 3}{}$$

```
[Out] -1/32*5^(-1-n)*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c*x))/b)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*
x^2+d)^(1/2)-5/32*3^(-1-n)*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arcco
sh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)
^n)/(-c^2*d*x^2+d)^(1/2)+1/8*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arc
cosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arccosh(
c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-11/16*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,(
-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(
c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+11/16*d^3*exp(a/b)*(a+b*arccosh(c*x))^n*GA
MMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*
x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+5/32*3^(-1-n)*d^3*exp(3*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*
arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-1/8*d^3*exp(3*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(3^n)/(
((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3*exp(5*a/b)
*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((
a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 1.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] -1/32*(5^(-1 - n)*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*
(-((a + b*ArcCosh[c*x])/b))^n) - (5*3^(-1 - n)*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*
E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^3*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcC
```

```
osh[c*x]))/b])/(8*3^n*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x
])/b))^n) - (11*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a
 + b*ArcCosh[c*x])/b))^n) + (11*d^3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(16*Sqrt[d - c^2
*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + (5*3^(-1 - n)*d^3*E^((3*a)/b)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh
[c*x]))/b])/(32*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (d^3*E^((
3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3
*(a + b*ArcCosh[c*x]))/b])/(8*3^n*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])
/b)^n) + (5^(-1 - n)*d^3*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*Sqrt[d - c^2*d*
x^2]*((a + b*ArcCosh[c*x])/b)^n) + d^3*Defer[Int][(a + b*ArcCosh[c*x])^n/(x
*Sqrt[d - c^2*d*x^2]), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx &= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3c^2x\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1-}}\right.}{\sqrt{-1+cx}\,\sqrt{}}\\
&= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(3c^2d^2\sqrt{d-c\right.}{\sqrt{}}\\
&= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx\right.}{}\\
&= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx\right.}{}\\
&= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1\right.}{2\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1\right.}{2\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{5^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}
$$

Mathematica [A]

time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x, x)
```

3.433 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))^n}{x^2}\,dx$

Optimal. Leaf size=486

$$-\frac{15cd^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{8b(1+n)\sqrt{d-c^2dx^2}}-\frac{2^{-2(3+n)}cd^3e^{-\frac{4a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2d}}$$

```
[Out] -15/8*c*d^3*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-
c^2*d*x^2+d)^(1/2)-c*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x
))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arccosh(c*
x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+2^(-2-n)*c*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+
n,-2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(2*a/b)/(((-a-b*a
rccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(
((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+c*d^3*exp(4*a/b)*(a+b*arccos
h(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(2^
(6+2*n))/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((
a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\cosh^{-1}(cx))^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (-15*c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*
(1 + n)*Sqrt[d - c^2*d*x^2]) - (c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*A
rcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*E^
((4*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-2 - n)*
c*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*
(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCos
h[c*x])/b))^n) - (2^(-2 - n)*c*d^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[d -
 c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + (c*d^3*E^((4*a)/b)*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))
/b])/(2^(2*(3 + n))*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) + d^3*D
efer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+}}\right.}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2d^2\sqrt{d-c}\right.}{}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{}{x^2}}{\sqrt{-1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int}{\sqrt{-1+}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int}{\sqrt{-1+}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4^{-3-n}cd^2e^{-\frac{4a}{b}}\sqrt{d-c}}{}$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2,x)
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

3.434 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=323

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-cx}}+\frac{3e^{-\frac{a}{b}}\sqrt{-1+cx}}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c*x-1
)^(1/2)/c^4/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)+3/8*(a+b*
arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/exp(a/b)
/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-3/8*exp(a/b)*(a+b*arccosh(c*x))
^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b
)^n)/(-c*x+1)^(1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,
3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c*x+
1)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5952, 3393, 3388, 2212}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-cx}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^4\sqrt{1-cx}}-\frac{3e^{a/b}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^4\sqrt{1-cx}}-\frac{3^{-n-1}e^{\frac{3a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*A
rcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/
b))^n) + (3*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*Arc
Cosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n)
 - (3*E^(a/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*Arc
Cosh[c*x])/b])/(8*c^4*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 -
n)*E^((3*a)/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b
*ArcCosh[c*x]))/b])/(8*c^4*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^3(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{3}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)\right.}{c^4\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh(3x)\,dx, x, \cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} + \frac{\left(3\sqrt{\right.}}{}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{8c^4\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+\right.}}{}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+\right.}{8c^4\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.92, size = 292, normalized size = 0.90

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{2+n}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)-\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+3^{2+n}e^{\frac{2a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-e^{\frac{6a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\Gamma\left(1+n,\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)\right)}{8c^4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*(3^(2 + n)*E^((4*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 +
n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-((a + b*ArcCosh[c*x])^2/
b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] + 3^(2 + n)*E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] -
E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])))/(8*c^4*E^((3*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((
a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}(a+b\,\mathrm{arccosh}(cx))^{n}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^3/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}(a+b\operatorname{acosh}(cx))^{n}}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}\,(a+b\operatorname{acosh}(c\,x))^{n}}{\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
[Out] int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.435 $\int \frac{x^2(a+b\cosh^{-1}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{2bc^3(1+n)\sqrt{1-cx}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(1+n,-}{c^3\sqrt{1-cx}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c^3/(1+n)/(-c*x+1)^(1/2)+2^(-3
-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c
^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-2^(-3-n)*exp(2*a/b
)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^3/
(((a+b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5952, 3393, 3388, 2212}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{1-cx}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\,(a+b\cosh^{-1}(cx))^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,(a+b\cosh^{-1}(cx))^{n+1}}{2bc^3(n+1)\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n)*Sqrt[1 - c*x
]) + (2^(-3 - n)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a
+ b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x
])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[1 - c*x]*((a + b*ArcCosh
[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)\,dx,x,\text{c}\right.}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{2c^3}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{4c^3\sqrt{}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}}{}$$

Mathematica [A]
time = 0.56, size = 212, normalized size = 1.00

$$\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n+b(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-be^{\frac{4a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)}{bc^3(1+n)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a
+ b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^2/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b \operatorname{acosh}(cx))^n}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.436 $\int \frac{x(a+b\cosh^{-1}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=154

$$\frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{e^{a/b}\sqrt{-1+cx}\,(a+b\cosh$$

```
[Out] 1/2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2
/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-1/2*exp(a/b)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/(((a+b*arccos
h(c*x))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5952, 3388, 2212}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{e^{a/b}\sqrt{cx-1}\,(a+b\cosh^{-1}(cx))^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/
b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)
*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b]
)/(2*c^2*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\operatorname{Subst}\left(\int (a+bx)^n\cosh(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\operatorname{Subst}\left(\int e^{-x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{2c^2\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+c\right.}{}\\
&= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+}{}\right.}{2c^2\sqrt{1-c^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 154, normalized size = 1.00

$$-\frac{e^{-\frac{a}{b}}\sqrt{-((-1+cx)(1+cx))}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(-e^{\frac{2a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)+\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] -1/2*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + Arc
Cosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(c^2*E^(a/b)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{arccosh}(cx))^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acosh}(cx))^n}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\operatorname{acosh}(c\,x))^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.437 $\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}}$$

```
[Out] (a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c/(1+n)/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5892}

$$\frac{\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*x])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.30

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[1 - c^2*x^2])
```

Maple [A]
time = 1.09, size = 53, normalized size = 1.23

method	result	size
default	$\frac{(a+b\,\mathrm{arccosh}(cx))^{1+n}\sqrt{cx-1}\,\sqrt{cx+1}}{b(1+n)c\sqrt{-(cx-1)(cx+1)}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(39) = 78$.
time = 0.36, size = 213, normalized size = 4.95

$$\frac{\left(\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}\,a\right)\cosh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right)+a\right)\right)+\left(\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}\,a\right)\sinh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right)+a\right)\right)}{bcn-(bc^3n+bc^3)x^2+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + sqr
t(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^2 -
1)) + a)) + (sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2
- 1)) + sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*sinh(n*log(b*log(c*x + sqrt
(c^2*x^2 - 1)) + a)))/(b*c*n - (b*c^3*n + b*c^3)*x^2 + b*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2), x)
```

3.438 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n}{x\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^3 - x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*x**2+1)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n}{x\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)), x)
```

3.439 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccosh}(cx)\right)^n}{x^2\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^4 - x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x^2\sqrt{-(cx - 1)(cx + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n}{x^2\,\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)), x)
```

3.440 $\int \frac{x^3(a+b\cosh^{-1}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=379

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{}}{}$$

[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+3/8*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-3/8*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)

Rubi [A]

time = 0.28, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5952, 3393, 3388, 2212}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}-\frac{3e^{a/b}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}-\frac{3^{-n-1}e^{\frac{3a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]

[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh^3(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\left(\frac{3}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)\cdots\right.}{c^4\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh(3x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{d-c^2dx^2}}+\frac{\left(3\sqrt{\cdots}\right.}{\cdots}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+\cdots}\right.}{\cdots}\\
&= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+\cdots\right.}{8c^4\sqrt{d-c^2dx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.78, size = 291, normalized size = 0.77

$$
-\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{2+n}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)-\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+3^{2+n}e^{\frac{2a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-e^{\frac{6a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\Gamma\left(1+n,\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)\right)}{8c^4\sqrt{d-c^2dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -1/8*(3^(-1 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(3^(2 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*
x])^2/b^2))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-
((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] +
 3^(2 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a
+ b*ArcCosh[c*x])/b)] - E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma
[1 + n, (3*(a + b*ArcCosh[c*x]))/b])))/(c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]
*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} (a +b \,\mathrm{arccosh}\left(c x \right))^{n}}{\sqrt{-c^{2} d \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^3/(c^2*d*x^2 - d),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b \operatorname{acosh}(cx))^n}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\operatorname{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

3.441 $\int \frac{x^2(a+b\cosh^{-1}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=253

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-}}{b}\right.}{c^3\sqrt{d-c^2dx^2}}$$

[Out] 1/2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(1+n)/(-c^2*d*x^2+d)^(1/2)+2^(-3-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-2^(-3-n)*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5952, 3393, 3388, 2212}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{d-c^2dx^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c^3\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^{n+1}}{2bc^3(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)\,dx, x, \text{c}\right.}{c^3\sqrt{d-c^2dx^2}}\\
&= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{2c^3}\\
&= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{4c^3\sqrt{d}}\\
&= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.55, size = 213, normalized size = 0.84

$$
\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{2+n}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n+b(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-be^{\frac{4a}{b}}(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)}{bc^3(1+n)\sqrt{d-c^2dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}}{\sqrt{-c^{2} d \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^2*d*x^2 - d),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}}{\sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

3.442 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{-1+cx}\,\cdots}{\cdots}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-1/
2*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5952, 3388, 2212}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x
])/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d - c^2*d*x^2]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5952

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/((1 + c*x
)^p*(-1 + c*x)^p)], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p
+ 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ
[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+c\right.}{}}{}$$

$$= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+}{}\right.}{2c^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.17, size = 153, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(-e^{\frac{2a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)+\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c^2\sqrt{-d(-1+cx)(1+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + A
rcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(2*c^2*E^(a/b)*Sqr
t[-(d*(-1 + c*x)*(1 + c*x))]*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\text{arccosh}(cx))^n}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)

[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)

[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^2*d*x^2 - d), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{acosh}\,(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)

[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

3.443 $\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(-c^2*d*x^2+
d)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {5892}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.00

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Maple [A]
time = 1.05, size = 54, normalized size = 0.95

method	result	size
default	$\frac{(a+b\,\mathrm{arccosh}(cx))^{1+n}\sqrt{cx-1}\,\sqrt{cx+1}}{b(1+n)c\sqrt{-d\,(cx-1)\,(cx+1)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-d*(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(51) = 102.
time = 0.37, size = 221, normalized size = 3.88

$$\frac{\left(\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,a\right)\cosh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right)+a\right)\right)+\left(\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,a\right)\sinh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right)+a\right)\right)}{bcdn+bcd-(bc^3dn+bc^3d)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^
2 - 1)) + a)) + (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^
2*x^2 - 1)) + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*sinh(n*log(b*log(c*
x + sqrt(c^2*x^2 - 1)) + a)))/(b*c*d*n + b*c*d - (b*c^3*d*n + b*c^3*d)*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2), x)
```

3.444 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n}{x\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^3 - d*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)), x)
```

3.445 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n}{x^2\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^4 - d*x^2),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n}{x^2\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.446 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^4*d^2*x^4 - 2*c
^2*d^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{acosh}\,(cx))^n}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

3.447 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{arccosh}(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^4*d^2*x^4 - 2*c^2
*d^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acosh}(cx))^n}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,(a+b\,\mathrm{acosh}(c\,x))^n}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

3.448 $\int \frac{(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n}{(-c^2d\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
[Out] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d
^2*x^2 + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^n}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2),x)
[Out] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2), x)
```

3.449 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^5 - 2*c^2*d
^2*x^3 + d^2*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{x\left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n}{x\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)), x)
```

3.450 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}},x\right)$$

[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]

[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)

[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")

[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")

[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^6 - 2*c^2*d
^2*x^4 + d^2*x^2), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^n}{x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(3/2),x)

[Out] Integral((a + b*acosh(c*x))**n/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n}{x^2\,\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

3.451 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\text{arccosh}(cx)\right)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b \operatorname{acosh}(cx)\right)^n}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n\,(f\,x)^m}{\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.452 $\int (fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n dx$

Optimal. Leaf size=32

$$\text{Int}\left((fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n, x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n dx = \int (fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n dx$$

Mathematica [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int (fx)^m (d-c^2dx^2)^2 (a+b\cosh^{-1}(cx))^n dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m (-c^2d\,x^2+d)^2 (a+b\,\mathrm{arccosh}(cx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*(f*x)^m*(b*arccosh(c*x) + a)^n
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^2\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

3.453 $\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=30

$$\text{Int}\left((fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right) \left(a+b\,\mathrm{arccosh}(cx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \left(-(fx)^{m}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}\right)\,dx+\int c^{2}x^{2}(fx)^{m}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x))**n,x)
```

```
[Out] -d*(Integral(-(f*x)**m*(a + b*acosh(c*x))**n, x) + Integral(c**2*x**2*(f*x)
**m*(a + b*acosh(c*x))**n, x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m, x)
```

3.454 $\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((fx)^m \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n\,dx = \int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(a+b\,\mathrm{arccosh}(cx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\operatorname{acosh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n,x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\operatorname{acosh}(c\,x))^n\,(f\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(f*x)^m,x)
[Out] int((a + b*acosh(c*x))^n*(f*x)^m, x)
```

3.455 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{-c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="maxima"
)
```

```
[Out] -integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="fricas"
)
```

```
[Out] integral(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(fx)^m(a+b\operatorname{acosh}(cx))^n}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d),x)
```

```
[Out] -Integral((f*x)**m*(a + b*acosh(c*x))**n/(c**2*x**2 - 1), x)/d
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,(f\,x)^m}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2),x)
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2), x)
```

3.456 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Mathematica [A]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{\left(-c^2d\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="frica
s")
```

```
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="giac"
)
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))^n\,(f\,x)^m}{(d - c^2\,d\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2,x)
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

3.457 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=34

$$\text{Int}\left((fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n ,x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}(cx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
```

```
[Out] integral(-(c^2*d*x^2 - d)*sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)
^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a + b\,\mathrm{acosh}(c\,x))^n\,\left(d - c^2\,d\,x^2\right)^{3/2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

3.458 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=34

$$\text{Int}\left((fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Defer[Int][(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \frac{\sqrt{d-c^2dx^2} \int (fx)^m \sqrt{-1+cx} \sqrt{1+cx} \left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-c^2d\,x^2+d}\, (a+b\,\mathrm{arccosh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int (a+b\,\mathrm{acosh}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

3.459 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)

[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b\operatorname{acosh}(cx))^n}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)

[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^n\,(f\,x)^m}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

3.460 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m(a+b\cosh^{-1}(cx))^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 -
 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{acosh}(c\,x)\right)^n\,\left(f\,x\right)^m}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

3.461 $\int x^4(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=177

$$-\frac{8b(49c^2d+30e)\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^7}-\frac{4b(49c^2d+30e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{b(49c^2d+30e)\,x^4\sqrt{-1+}}{1225c^3}$$

```
[Out] 1/5*d*x^5*(a+b*arccosh(c*x))+1/7*e*x^7*(a+b*arccosh(c*x))-8/3675*b*(49*c^2*
d+30*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-4/3675*b*(49*c^2*d+30*e)*x^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^5-1/1225*b*(49*c^2*d+30*e)*x^4*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^3-1/49*b*e*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5956, 471, 102, 12, 75}

$$\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{8b\sqrt{cx-1}\,\sqrt{cx+1}\,(49c^2d+30e)}{3675c^7}-\frac{4bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,(49c^2d+30e)}{3675c^5}-\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}\,(49c^2d+30e)}{1225c^3}-\frac{bex^6\sqrt{cx-1}\,\sqrt{cx+1}}{49c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-8*b*(49*c^2*d + 30*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^7) - (4*b*(49
*c^2*d + 30*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (b*(49*c^2*d
+ 30*e)*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1225*c^3) - (b*e*x^6*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(49*c) + (d*x^5*(a + b*ArcCosh[c*x]))/5 + (e*x^7*(a + b*
ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x^4(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{35}(bc)\int \frac{}{\sqrt{-}}$$

$$= -\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+cx}}{49c}+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+\right.$$

$$= -\frac{b(49c^2d+30e)\,x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+c}}{49c}$$

$$= -\frac{b(49c^2d+30e)\,x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+c}}{49c}$$

$$= -\frac{4b(49c^2d+30e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{b(49c^2d+30e)\,x^4\sqrt{-}}{1225c^3}$$

$$= -\frac{4b(49c^2d+30e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{b(49c^2d+30e)\,x^4\sqrt{-}}{1225c^3}$$

$$= -\frac{8b(49c^2d+30e)\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^7}-\frac{4b(49c^2d+30e)\,x^2\sqrt{-1}}{3675c^5}$$

Mathematica [A]

time = 0.08, size = 122, normalized size = 0.69

$$\frac{1}{35}ax^5(7d+5ex^2)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(240e+8c^2(49d+15ex^2)+2c^4(98dx^2+45ex^4)+3c^6(49dx^4+25ex^6))}{3675c^7}+\frac{1}{35}bx^5(7d+5ex^2)\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^5*(7*d + 5*e*x^2))/35 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e + 8*c^2
*(49*d + 15*e*x^2) + 2*c^4*(98*d*x^2 + 45*e*x^4) + 3*c^6*(49*d*x^4 + 25*e*x
^6)))/(3675*c^7) + (b*x^5*(7*d + 5*e*x^2)*ArcCosh[c*x])/35
```

Maple [A]
time = 2.72, size = 133, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}dc^7x^5+\frac{1}{7}ec^7x^7\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)dc^7x^5}{5}+\frac{\operatorname{arccosh}(cx)ec^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75c^6ex^6+147c^6dx^4+90c^4ex^4+19}{3675}\right)}{c^2}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}dc^7x^5+\frac{1}{7}ec^7x^7\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)dc^7x^5}{5}+\frac{\operatorname{arccosh}(cx)ec^7x^7}{7}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75c^6ex^6+147c^6dx^4+90c^4ex^4+19}{3675}\right)}{c^2}}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/c^2*(1/5*d*c^7*x^5+1/7*e*c^7*x^7)+b/c^2*(1/5*arccosh(c*x)*d*c^7*x^
5+1/7*arccosh(c*x)*e*c^7*x^7-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e*x
^6+147*c^6*d*x^4+90*c^4*e*x^4+196*c^4*d*x^2+120*c^2*e*x^2+392*c^2*d+240*e))
)
```

Maxima [A]
time = 0.27, size = 180, normalized size = 1.02

$$\frac{1}{7}ax^7e+\frac{1}{5}adx^5+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd+\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e + 1/5*a*d*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1
)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d +
 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2
 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*
b*e
```

Fricas [A]
time = 0.35, size = 200, normalized size = 1.13

$$\frac{525\,ac^7x^7\cosh(1)+525\,ac^7x^7\sinh(1)+735\,ac^7dx^5+105\,(5\,bc^7x^7\cosh(1)+5\,bc^7x^7\sinh(1)+7\,bc^7dx^5)\log\left(cx+\sqrt{c^2x^2-1}\right)-(147\,bc^6dx^4+196\,bc^4dx^2+392\,bc^2d+15\,(5\,bc^6x^6+6\,bc^4x^4+8\,bc^2x^2+16\,b)\cosh(1)+15\,(5\,bc^6x^6+6\,bc^4x^4+8\,bc^2x^2+16\,b)\sinh(1))\sqrt{c^2x^2-1}}{3675\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*x^7*cosh(1) + 525*a*c^7*x^7*sinh(1) + 735*a*c^7*d*x^5 + 1
05*(5*b*c^7*x^7*cosh(1) + 5*b*c^7*x^7*sinh(1) + 7*b*c^7*d*x^5)*log(c*x + sq
rt(c^2*x^2 - 1)) - (147*b*c^6*d*x^4 + 196*b*c^4*d*x^2 + 392*b*c^2*d + 15*(5
*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*cosh(1) + 15*(5*b*c^6*x^6 +
6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*sinh(1))*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 230, normalized size = 1.30

$$\begin{cases} \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\operatorname{acosh}(cx)}{5}+\frac{bex^7\operatorname{acosh}(cx)}{7}-\frac{bdx^4\sqrt{c^2x^2-1}}{25c}-\frac{bex^6\sqrt{c^2x^2-1}}{49c}-\frac{4bdx^2\sqrt{c^2x^2-1}}{75c^3}-\frac{6bex^4\sqrt{c^2x^2-1}}{245c^3}-\frac{8bd\sqrt{c^2x^2-1}}{75c^5}-\frac{8bex^2\sqrt{c^2x^2-1}}{245c^5}-\frac{16be\sqrt{c^2x^2-1}}{245c^7} & \text{for } c\neq 0 \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^5}{5}+\frac{ex^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acosh(c*x)/5 + b*e*x**7*acosh
(c*x)/7 - b*d*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b*e*x**6*sqrt(c**2*x**2 - 1
)/(49*c) - 4*b*d*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e*x**4*sqrt(c**2*
x**2 - 1)/(245*c**3) - 8*b*d*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e*x**2*sqr
t(c**2*x**2 - 1)/(245*c**5) - 16*b*e*sqrt(c**2*x**2 - 1)/(245*c**7), Ne(c,
0)), ((a + I*pi*b/2)*(d*x**5/5 + e*x**7/7), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2), x)
```

3.462 $\int x^3(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=161

$$-\frac{b(9c^2d+5e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c^5}-\frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}-b(9$$

```
[Out] -1/96*b*(9*c^2*d+5*e)*arccosh(c*x)/c^6+1/4*d*x^4*(a+b*arccosh(c*x))+1/6*e*x
^6*(a+b*arccosh(c*x))-1/96*b*(9*c^2*d+5*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
5-1/144*b*(9*c^2*d+5*e)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*e*x^5*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {5956, 471, 102, 12, 92, 54}

$$\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cosh^{-1}(cx)\right)-\frac{b(9c^2d+5e)\cosh^{-1}(cx)}{96c^6}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^2d+5e)}{96c^5}-\frac{bx^3\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^2d+5e)}{144c^3}-\frac{bex^5\sqrt{cx-1}\,\sqrt{cx+1}}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/96*(b*(9*c^2*d + 5*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^5 - (b*(9*c^2*d
+ 5*e)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e*x^5*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(36*c) - (b*(9*c^2*d + 5*e)*ArcCosh[c*x])/(96*c^6) + (d*x^
4*(a + b*ArcCosh[c*x]))/4 + (e*x^6*(a + b*ArcCosh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x^3(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{24}(bc)\int \frac{}{\sqrt{-}}$$

$$= -\frac{bex^5\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6(a+$$

$$= -\frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}$$

$$= -\frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\,\sqrt{1+cx}}{36c}$$

$$= -\frac{b(9c^2d+5e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c^5}-\frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}}{144c^3}$$

$$= -\frac{b(9c^2d+5e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c^5}-\frac{b(9c^2d+5e)\,x^3\sqrt{-1+cx}}{144c^3}$$

Mathematica [A]
time = 0.12, size = 140, normalized size = 0.87

$$\frac{24ac^6x^4(3d+2ex^2)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(15e+c^2(27d+10ex^2)+2c^4(9dx^2+4ex^4))+24bc^6x^4(3d+2ex^2)\cosh^{-1}(cx)-6b(9c^2d+5e)\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{288c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^6*x^4*(3*d + 2*e*x^2) - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15*e +
c^2*(27*d + 10*e*x^2) + 2*c^4*(9*d*x^2 + 4*e*x^4)) + 24*b*c^6*x^4*(3*d + 2*
e*x^2)*ArcCosh[c*x] - 6*b*(9*c^2*d + 5*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)
]])/(288*c^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(137) = 274$.
time = 3.07, size = 332, normalized size = 2.06

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}c^6dx^4+\frac{1}{6}c^6ex^6\right)}{c^2}-\frac{bc^4\operatorname{arccosh}(cx)d^3}{12e^2}+\frac{b\operatorname{arccosh}(cx)dc^4x^4}{4}+\frac{bc^4e\operatorname{arccosh}(cx)x^6}{6}+\frac{bc^4\sqrt{cx-1}\,\sqrt{cx+1}\,d^3\ln\left(cx\right.}{12e^2\sqrt{c^2x^2-1}}}{}$
default	$\frac{\frac{a\left(\frac{1}{4}c^6dx^4+\frac{1}{6}c^6ex^6\right)}{c^2}-\frac{bc^4\operatorname{arccosh}(cx)d^3}{12e^2}+\frac{b\operatorname{arccosh}(cx)dc^4x^4}{4}+\frac{bc^4e\operatorname{arccosh}(cx)x^6}{6}+\frac{bc^4\sqrt{cx-1}\,\sqrt{cx+1}\,d^3\ln\left(cx\right.}{12e^2\sqrt{c^2x^2-1}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^2*(1/4*c^6*d*x^4+1/6*c^6*e*x^6)-1/12*b*c^4/e^2*arccosh(c*x)*d^3+
1/4*b*arccosh(c*x)*d*c^4*x^4+1/6*b*c^4*e*arccosh(c*x)*x^6+1/12*b*c^4/e^2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1
/16*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*c^3*x^3-1/36*b*c^3*e*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*x^5-3/32*b*c*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-5/144*b*c*e*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*x^3-3/32*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2
)*ln(c*x+(c^2*x^2-1)^(1/2))*d-5/96*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/96
*b/c^2*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(
1/2)))
```

Maxima [A]
time = 0.26, size = 198, normalized size = 1.23

$$\frac{1}{6}\,ax^6e+\frac{1}{4}\,adx^4+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}\left(cx\right)-\left(\frac{2\,\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd+\frac{1}{288}\left(48\,x^6\operatorname{arcosh}\left(cx\right)-\left(\frac{8\,\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\,\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\,\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e + 1/4*a*d*x^4 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)
*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*
c)/c^5)*c)*b*d + 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2
+ 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*
x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*e
```

Fricas [A]
time = 0.37, size = 192, normalized size = 1.19

$$\frac{48\,ac^6x^6\cosh\left(1\right)+48\,ac^6x^6\sinh\left(1\right)+72\,ac^6dx^4+3\,(24\,bc^6dx^4-9\,bc^2d+(16\,bc^6x^6-5\,b)\cosh\left(1\right)+(16\,bc^6x^6-5\,b)\sinh\left(1\right))\log\left(cx+\sqrt{c^2x^2-1}\right)-(18\,bc^5dx^3+27\,bc^3dx+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\cosh\left(1\right)+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\sinh\left(1\right))\sqrt{c^2x^2-1}}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*x^6*cosh(1) + 48*a*c^6*x^6*sinh(1) + 72*a*c^6*d*x^4 + 3*(24
*b*c^6*d*x^4 - 9*b*c^2*d + (16*b*c^6*x^6 - 5*b)*cosh(1) + (16*b*c^6*x^6 - 5
*b)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - (18*b*c^5*d*x^3 + 27*b*c^3*d*x
+ (8*b*c^5*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*cosh(1) + (8*b*c^5*x^5 + 10*b*c^3
*x^3 + 15*b*c*x)*sinh(1))*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 212, normalized size = 1.32

$$\begin{cases} \frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{acosh}\left(cx\right)}{4}+\frac{bex^6\operatorname{acosh}\left(cx\right)}{6}-\frac{bdx^3\sqrt{c^2x^2-1}}{16c}-\frac{bex^5\sqrt{c^2x^2-1}}{36c}-\frac{3bdx\sqrt{c^2x^2-1}}{32c^3}-\frac{5bex^3\sqrt{c^2x^2-1}}{144c^3}-\frac{3bd\operatorname{acosh}\left(cx\right)}{32c^4}-\frac{5bex\sqrt{c^2x^2-1}}{96c^5}-\frac{5be\operatorname{acosh}\left(cx\right)}{96c^6} & \text{for}\ c\neq 0\\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acosh(c*x)/4 + b*e*x**6*acosh
(c*x)/6 - b*d*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*e*x**5*sqrt(c**2*x**2 - 1
)/(36*c) - 3*b*d*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*e*x**3*sqrt(c**2*x**
2 - 1)/(144*c**3) - 3*b*d*acosh(c*x)/(32*c**4) - 5*b*e*x*sqrt(c**2*x**2 - 1
)/(96*c**5) - 5*b*e*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**
4/4 + e*x**6/6), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2), x)
```

3.463 $\int x^2(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=138

$$-\frac{2b(25c^2d+12e)\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^5}-\frac{b(25c^2d+12e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\,\sqrt{1+cx}}{25c}+$$

```
[Out] 1/3*d*x^3*(a+b*arccosh(c*x))+1/5*e*x^5*(a+b*arccosh(c*x))-2/225*b*(25*c^2*d
+12*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/225*b*(25*c^2*d+12*e)*x^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^3-1/25*b*e*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5956, 471, 102, 12, 75}

$$\frac{1}{3}dx^3(a+b\cosh^{-1}(cx))+\frac{1}{5}ex^5(a+b\cosh^{-1}(cx))-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,(25c^2d+12e)}{225c^5}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,(25c^2d+12e)}{225c^3}-\frac{bex^4\sqrt{cx-1}\,\sqrt{cx+1}}{25c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*b*(25*c^2*d + 12*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^5) - (b*(25*c^
2*d + 12*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (b*e*x^4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(25*c) + (d*x^3*(a + b*ArcCosh[c*x]))/3 + (e*x^5*(a +
 b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx &= \frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}(bc)\int \frac{}{\sqrt{-}}\\ &= -\frac{bex^4\sqrt{-1+cx}\,\sqrt{1+cx}}{25c}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+\right.\\ &= -\frac{b(25c^2d+12e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\,\sqrt{1+c}}{25c}\\ &= -\frac{b(25c^2d+12e)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\,\sqrt{1+c}}{25c}\\ &= -\frac{2b(25c^2d+12e)\,\sqrt{-1+cx}\,\sqrt{1+cx}}{225c^5}-\frac{b(25c^2d+12e)\,x^2\sqrt{-1+}}{225c^3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 0.73

$$\frac{1}{225}\left(15ax^3\left(5d+3ex^2\right)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(24e+2c^2\left(25d+6ex^2\right)+c^4\left(25dx^2+9ex^4\right)\right)}{c^5}+15bx^3\left(5d+3ex^2\right)\cosh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x^3*(5*d + 3*e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e + 2*c^2*(
25*d + 6*e*x^2) + c^4*(25*d*x^2 + 9*e*x^4)))/c^5 + 15*b*x^3*(5*d + 3*e*x^2)
*ArcCosh[c*x])/225
```

Maple [A]
time = 2.58, size = 115, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d\,c^5x^3}{3}+\frac{\operatorname{arccosh}(cx)e\,c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4e\,x^4+25c^4d\,x^2+12c^2e\,x^2+50c^2}{225}\right.}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d\,c^5x^3}{3}+\frac{\operatorname{arccosh}(cx)e\,c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4e\,x^4+25c^4d\,x^2+12c^2e\,x^2+50c^2}{225}\right.}{c^2}}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^2*(1/3*d*c^5*x^3+1/5*e*c^5*x^5)+b/c^2*(1/3*arccosh(c*x)*d*c^5*x^
3+1/5*arccosh(c*x)*e*c^5*x^5-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e*x^4
+25*c^4*d*x^2+12*c^2*e*x^2+50*c^2*d+24*e)))
```

Maxima [A]
time = 0.26, size = 141, normalized size = 1.02

$$\frac{1}{5}\,ax^5e+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e + 1/3*a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
/c^6)*c)*b*e
```

Fricas [A]
time = 0.38, size = 172, normalized size = 1.25

$$\frac{45\,ac^5x^5\cosh\left(1\right)+45\,ac^5x^5\sinh\left(1\right)+75\,ac^5dx^3+15\left(3\,bc^5x^5\cosh\left(1\right)+3\,bc^5x^5\sinh\left(1\right)+5\,bc^5dx^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(25\,bc^4dx^2+50\,bc^2d+3\left(3\,bc^4x^4+4\,bc^2x^2+8\,b\right)\cosh\left(1\right)+3\left(3\,bc^4x^4+4\,bc^2x^2+8\,b\right)\sinh\left(1\right)\right)\sqrt{c^2x^2-1}}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*x^5*cosh(1) + 45*a*c^5*x^5*sinh(1) + 75*a*c^5*d*x^3 + 15*(3
*b*c^5*x^5*cosh(1) + 3*b*c^5*x^5*sinh(1) + 5*b*c^5*d*x^3)*log(c*x + sqrt(c^
2*x^2 - 1)) - (25*b*c^4*d*x^2 + 50*b*c^2*d + 3*(3*b*c^4*x^4 + 4*b*c^2*x^2 +
 8*b)*cosh(1) + 3*(3*b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sinh(1))*sqrt(c^2*x^2 -
 1))/c^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 178, normalized size = 1.29

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3 \operatorname{acosh}(cx)}{3} + \frac{bex^5 \operatorname{acosh}(cx)}{5} - \frac{bdx^2\sqrt{c^2x^2-1}}{9c} - \frac{bex^4\sqrt{c^2x^2-1}}{25c} - \frac{2bd\sqrt{c^2x^2-1}}{9c^3} - \frac{4bex^2\sqrt{c^2x^2-1}}{75c^3} - \frac{8be\sqrt{c^2x^2-1}}{75c^5} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^3}{3} + \frac{ex^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acosh(c*x)/3 + b*e*x**5*acosh
(c*x)/5 - b*d*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e*x**4*sqrt(c**2*x**2 - 1)
/(25*c) - 2*b*d*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e*x**2*sqrt(c**2*x**2 -
1)/(75*c**3) - 8*b*e*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c, 0)), ((a + I*pi*b
/2)*(d*x**3/3 + e*x**5/5), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2), x)
```

3.464 $\int x(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=122

$$-\frac{b(8c^2d+3e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{b(8c^2d+3e)\cosh^{-1}(cx)}{32c^4}+\frac{1}{2}dx^2\left(a+b\cosh\right.$$

```
[Out] -1/32*b*(8*c^2*d+3*e)*arccosh(c*x)/c^4+1/2*d*x^2*(a+b*arccosh(c*x))+1/4*e*x
^4*(a+b*arccosh(c*x))-1/32*b*(8*c^2*d+3*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
3-1/16*b*e*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5956, 471, 92, 54}

$$\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{b(8c^2d+3e)\cosh^{-1}(cx)}{32c^4}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,(8c^2d+3e)}{32c^3}-\frac{bex^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/32*(b*(8*c^2*d + 3*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (b*e*x^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (b*(8*c^2*d + 3*e)*ArcCosh[c*x])/(32*c^
4) + (d*x^2*(a + b*ArcCosh[c*x]))/2 + (e*x^4*(a + b*ArcCosh[c*x]))/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
```

```
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{8}(bc)\int\frac{x^2}{\sqrt{-1}}$$

$$= -\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4(a+$$

$$= -\frac{b(8c^2d+3e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+$$

$$= -\frac{b(8c^2d+3e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-$$

Mathematica [A]
time = 0.10, size = 120, normalized size = 0.98

$$\frac{cx\left(8ac^3x(2d+ex^2)-b\sqrt{-1+cx}\,\sqrt{1+cx}\,(3e+2c^2(4d+ex^2))\right)+8bc^4x^2(2d+ex^2)\cosh^{-1}(cx)-2b(8c^2d+3e)\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{32c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(8*a*c^3*x*(2*d + e*x^2) - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*e + 2*c^2
*(4*d + e*x^2))) + 8*b*c^4*x^2*(2*d + e*x^2)*ArcCosh[c*x] - 2*b*(8*c^2*d +
3*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(32*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(104) = 208.
time = 2.27, size = 285, normalized size = 2.34

method	result
derivativedivides	$\frac{\frac{\left(c^2 e\, x^2+c^2 d\right)^2 a}{4c^2 e}+\frac{b\, c^2 \operatorname{arccosh}(cx) d^2}{4e}+\frac{b \operatorname{arccosh}(cx) d\, c^2 x^2}{2}+\frac{b\, c^2 e \operatorname{arccosh}(cx) x^4}{4}-\frac{b\, c^2 \sqrt{cx-1}\, \sqrt{cx+1}\, d^2 \ln\left(cx+\sqrt{c^2\right.}}{4e\sqrt{c^2x^2-1}}}{}$
default	$\frac{\frac{\left(c^2 e\, x^2+c^2 d\right)^2 a}{4c^2 e}+\frac{b\, c^2 \operatorname{arccosh}(cx) d^2}{4e}+\frac{b \operatorname{arccosh}(cx) d\, c^2 x^2}{2}+\frac{b\, c^2 e \operatorname{arccosh}(cx) x^4}{4}-\frac{b\, c^2 \sqrt{cx-1}\, \sqrt{cx+1}\, d^2 \ln\left(cx+\sqrt{c^2\right.}}{4e\sqrt{c^2x^2-1}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*(c^2*e*x^2+c^2*d)^2*a/c^2/e+1/4*b*c^2/e*arccosh(c*x)*d^2+1/2*b*a
rccosh(c*x)*d*c^2*x^2+1/4*b*c^2*e*arccosh(c*x)*x^4-1/4*b*c^2/e*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-1/4*b*c*d*x
*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/16*b*c*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-1/4
*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*
d-3/32*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32*b/c^2*e*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.28, size = 158, normalized size = 1.30

$$\frac{1}{4}\,ax^4e+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}\left(cx\right)-\left(\frac{2\,\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\,\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d + 1/32*(8*x^4*arccos
h(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2
*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*e
```

Fricas [A]
time = 0.36, size = 163, normalized size = 1.34

$$\frac{8\,ac^4x^4\cosh\left(1\right)+8\,ac^4x^4\sinh\left(1\right)+16\,ac^4dx^2+\left(16\,bc^4dx^2-8\,bc^2d+\left(8\,bc^4x^4-3\,b\right)\cosh\left(1\right)+\left(8\,bc^4x^4-3\,b\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(8\,bc^3dx+\left(2\,bc^3x^3+3\,bcx\right)\cosh\left(1\right)+\left(2\,bc^3x^3+3\,bcx\right)\sinh\left(1\right)\right)\sqrt{c^2x^2-1}}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*x^4*cosh(1) + 8*a*c^4*x^4*sinh(1) + 16*a*c^4*d*x^2 + (16*b*c^
4*d*x^2 - 8*b*c^2*d + (8*b*c^4*x^4 - 3*b)*cosh(1) + (8*b*c^4*x^4 - 3*b)*sin
h(1))*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^3*d*x + (2*b*c^3*x^3 + 3*b*c*x)
*cosh(1) + (2*b*c^3*x^3 + 3*b*c*x)*sinh(1))*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 0.24, size = 160, normalized size = 1.31

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2 \operatorname{acosh}(cx)}{2} + \frac{bex^4 \operatorname{acosh}(cx)}{4} - \frac{bdx\sqrt{c^2x^2-1}}{4c} - \frac{bex^3\sqrt{c^2x^2-1}}{16c} - \frac{bd\operatorname{acosh}(cx)}{4c^2} - \frac{3bex\sqrt{c^2x^2-1}}{32c^3} - \frac{3be\operatorname{acosh}(cx)}{32c^4} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acosh(c*x)/2 + b*e*x**4*acosh
(c*x)/4 - b*d*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*x**3*sqrt(c**2*x**2 - 1)/(1
6*c) - b*d*acosh(c*x)/(4*c**2) - 3*b*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*
b*e*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**2/2 + e*x**4/4),
 True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2), x)
```

3.465 $\int (d+ex^2) \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=94

$$-\frac{b(9c^2d+2e)\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3}-\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(c\right.$$

```
[Out] d*x*(a+b*arccosh(c*x))+1/3*e*x^3*(a+b*arccosh(c*x))-1/9*b*(9*c^2*d+2*e)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/9*b*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5908, 471, 75}

$$dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^2d+2e)}{9c^3}-\frac{bex^2\sqrt{cx-1}\,\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/9*(b*(9*c^2*d + 2*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^3 - (b*e*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(9*c) + d*x*(a + b*ArcCosh[c*x]) + (e*x^3*(a + b*Ar
cCosh[c*x]))/3
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
```

```
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{x\left(d+\right.}{\sqrt{-1+cx}}$$
$$= -\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cos\right.$$
$$= -\frac{b(9c^2d+2e)\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3}-\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+dx$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.81

$$\frac{1}{9}\left(3ax\left(3d+ex^2\right)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(2e+c^2\left(9d+ex^2\right)\right)}{c^3}+3bx\left(3d+ex^2\right)\cosh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*
x^2)))/c^3 + 3*b*x*(3*d + e*x^2)*ArcCosh[c*x])/9
```

Maple [A]
time = 1.91, size = 90, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccosh}(cx)d\,c^3x+\frac{\operatorname{arccosh}(cx)e\,c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2e\,x^2+9c^2d+2e\right)}{9}\right)}{c^2}}{c}$	90
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccosh}(cx)d\,c^3x+\frac{\operatorname{arccosh}(cx)e\,c^3x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2e\,x^2+9c^2d+2e\right)}{9}\right)}{c^2}}{c}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^2*(d*c^3*x+1/3*e*c^3*x^3)+b/c^2*(arccosh(c*x)*d*c^3*x+1/3*arccosh(
c*x)*e*c^3*x^3-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^2*d+2*e)))
```

Maxima [A]
time = 0.26, size = 93, normalized size = 0.99

$$\frac{1}{3}\,ax^3e + adx + \frac{1}{9}\left(3\,x^3\operatorname{arcosh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2} + \frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)be + \frac{\left(cx\operatorname{arcosh}\left(cx\right) - \sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e + a*d*x + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^
2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*e + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*
b*d/c
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.43

$$\frac{3\,ac^3x^3\cosh\left(1\right) + 3\,ac^3x^3\sinh\left(1\right) + 9\,ac^3dx + 3\,(bc^3x^3\cosh\left(1\right) + bc^3x^3\sinh\left(1\right) + 3\,bc^3dx)\log\left(cx + \sqrt{c^2x^2-1}\right) - (9\,bc^2d + (bc^2x^2 + 2\,b)\cosh\left(1\right) + (bc^2x^2 + 2\,b)\sinh\left(1\right))\sqrt{c^2x^2-1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*x^3*cosh(1) + 3*a*c^3*x^3*sinh(1) + 9*a*c^3*d*x + 3*(b*c^3*x^3
*cosh(1) + b*c^3*x^3*sinh(1) + 3*b*c^3*d*x)*log(c*x + sqrt(c^2*x^2 - 1)) -
(9*b*c^2*d + (b*c^2*x^2 + 2*b)*cosh(1) + (b*c^2*x^2 + 2*b)*sinh(1))*sqrt(c^
2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.15, size = 116, normalized size = 1.23

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\operatorname{acosh}\left(cx\right) + \frac{bex^3\operatorname{acosh}\left(cx\right)}{3} - \frac{bd\sqrt{c^2x^2-1}}{c} - \frac{bex^2\sqrt{c^2x^2-1}}{9c} - \frac{2be\sqrt{c^2x^2-1}}{9c^3} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*acosh(c*x) + b*e*x**3*acosh(c*x)/3 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*e*sqrt
(c**2*x**2 - 1)/(9*c**3), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**3/3), True
))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2), x)
```

3.466 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=264

$$-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2(a+b\cosh^{-1}(cx))-\frac{ibd\sqrt{1-c^2x^2}\operatorname{ArcSin}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd\sqrt{1-c^2}}{}$$

```
[Out] -1/4*b*e*arccosh(c*x)/c^2+1/2*e*x^2*(a+b*arccosh(c*x))+d*(a+b*arccosh(c*x))
*ln(x)-1/4*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*I*b*d*arcsin(c*x)^2*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*arcsin(c*x)*ln(1-(I*c*x+(-c^
2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d*arcsi
n(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d*polyl
og(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {14, 5958, 12, 6874, 92, 54, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$d\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}\operatorname{Li}_2\left(e^{2i\operatorname{ArcSin}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd\sqrt{1-c^2x^2}\operatorname{ArcSin}(cx)^2}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bd\sqrt{1-c^2x^2}\operatorname{ArcSin}(cx)\log\left(1-e^{2i\operatorname{ArcSin}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd\sqrt{1-c^2x^2}\log(x)\operatorname{ArcSin}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{be\cosh^{-1}(cx)}{4c^2}-\frac{bex\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/4*(b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*e*ArcCosh[c*x])/(4*c^2) +
(e*x^2*(a + b*ArcCosh[c*x]))/2 - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1
 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(a + b*ArcCos
h[c*x])*Log[x] - (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int \frac{ex^2}{2\sqrt{-1}}$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int \frac{ex}{\sqrt{-}}$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int \Bigg(\frac{}{\sqrt{}}$$

$$= \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bcd)\int \frac{}{\sqrt{-1}}$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-}\right.$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)$$

Mathematica [A]
time = 0.16, size = 119, normalized size = 0.45

$$\frac{1}{2}\left(aex^2+bex^2\cosh^{-1}(cx)-\frac{be\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}+2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{2c^2}+bd\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+2ad\log(x)-bd\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*e*x^2 + b*e*x^2*ArcCosh[c*x] - (b*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(2*c^2) + b*d*ArcCosh[c*x]*(ArcCosh
```

```
[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*d*Log[x] - b*d*PolyLog[2, -E^
(-2*ArcCosh[c*x])])/2
```

Maple [A]
time = 4.98, size = 130, normalized size = 0.49

method	result
derivativedivides	$\frac{ae\,x^{2}}{2}+ad\ln\left(cx\right)-\frac{b\mathrm{arccosh}(cx)^{2}d}{2}+\frac{b\,\mathrm{arccosh}(cx)e\,x^{2}}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{be\,\mathrm{arccosh}(cx)}{4c^{2}}+$
default	$\frac{ae\,x^{2}}{2}+ad\ln\left(cx\right)-\frac{b\mathrm{arccosh}(cx)^{2}d}{2}+\frac{b\,\mathrm{arccosh}(cx)e\,x^{2}}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{be\,\mathrm{arccosh}(cx)}{4c^{2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*e*x^2+a*d*ln(c*x)-1/2*b*arccosh(c*x)^2*d+1/2*b*arccosh(c*x)*e*x^2-1/4
*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*e*arccosh(c*x)/c^2+b*d*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*log(x) + integrate(b*x*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{acosh}(cx))\,(d+ex^{2})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x, x)
```

3.467 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{d(a+b\cosh^{-1}(cx))}{x}+ex(a+b\cosh^{-1}(cx))+bcd\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)$$

```
[Out] -d*(a+b*arccosh(c*x))/x+e*x*(a+b*arccosh(c*x))+b*c*d*arctan((c*x-1)^(1/2)*(
c*x+1)^(1/2))-b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5956, 471, 94, 211}

$$-\frac{d(a+b\cosh^{-1}(cx))}{x}+ex(a+b\cosh^{-1}(cx))+bcd\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)-\frac{be\sqrt{cx-1}\,\sqrt{cx+1}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c) - (d*(a + b*ArcCosh[c*x]))/x + e*x*
(a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+(bc)\int \frac{d-e\ldots}{x\sqrt{-1+cx}\ldots}$$
$$= -\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(c\ldots\right.$$
$$= -\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(c\ldots\right.$$
$$= -\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(c\ldots\right.$$

Mathematica [A]
time = 0.09, size = 105, normalized size = 1.40

$$-\frac{ad}{x}+aex-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{bd\cosh^{-1}(cx)}{x}+bex\cosh^{-1}(cx)+\frac{bcd\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
[Out] -((a*d)/x) + a*e*x - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*d*ArcCosh[c*
x])/x + b*e*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2
*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A]
time = 1.95, size = 108, normalized size = 1.44

method	result
derivativedivides	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\,\mathrm{arccosh}(cx)ex}{c}-\frac{b\,\mathrm{arccosh}(cx)d}{cx}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,d\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}-\ldots\right.$

default	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\,\mathrm{arccosh}(cx)ex}{c}-\frac{b\,\mathrm{arccosh}(cx)d}{cx}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,d\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^2*(e*c*x-d*c/x)+b/c*arccosh(c*x)*e*x-b*arccosh(c*x)*d/c/x-b*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*arctan(1/(c^2*x^2-1)^(1/2))-b/c^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*e)
```

Maxima [A]
time = 0.48, size = 65, normalized size = 0.87

$$-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bd+axe+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)be}{c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d + a*x*e + (c*x*arccosh(c*x)
- sqrt(c^2*x^2 - 1))*b*e/c - a*d/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(69) = 138.
time = 0.41, size = 174, normalized size = 2.32

$$\frac{2\,bc^2dx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+acx^2\cosh(1)+acx^2\sinh(1)-acd+\left(bcdx-bcd+(bcx^2-bcx)\cosh(1)+(bcx^2-bcx)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bcdx-bcx\cosh(1)-bcx\sinh(1)\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,(bx\cosh(1)+bx\sinh(1))}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (2*b*c^2*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + a*c*x^2*cosh(1) + a*c*x^2*s
inh(1) - a*c*d + (b*c*d*x - b*c*d + (b*c*x^2 - b*c*x)*cosh(1) + (b*c*x^2 -
b*c*x)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c*d*x - b*c*x*cosh(1) - b
*c*x*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*(b*x*cosh(1
) + b*x*sinh(1)))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{acosh}(cx))\,(d+ex^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^2, x)
```

3.468 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=251

$$\frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)^2}{2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)\log(1}{\sqrt{-1+cx}\sqrt{1+c}}$$

```
[Out] -1/2*d*(a+b*arccosh(c*x))/x^2+e*(a+b*arccosh(c*x))*ln(x)+1/2*b*c*d*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/x-1/2*I*b*e*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+b*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*e*arcsin(c*x)*ln(x)*(-c^2*x^2+1)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^
(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {14, 5958, 6874, 97, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{ibe\sqrt{1-c^2x^2}\,\text{Li}_2\left(e^{2i\text{ArcSin}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibe\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)^2}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{be\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)\log\left(1-e^{2i\text{ArcSin}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{be\sqrt{1-c^2x^2}\,\log(x)\text{ArcSin}(cx)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (d*(a + b*ArcCosh[c*x]))/(2*x^
2) - ((I/2)*b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + e*(a + b*ArcCosh[c*x])*Log[x] - (b*e*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*e
*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int \frac{-\frac{d}{2x^2}+}{\sqrt{-1+c}}$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int\left(-\frac{}{2x^2\sqrt{}}\right.$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)+\frac{1}{2}(bcd)\int\frac{}{x^2\sqrt{-}}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}}{2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}}{2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}}{2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}}{2\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [A]
time = 0.09, size = 101, normalized size = 0.40

$$\frac{-ad+bcdx\sqrt{-1+cx}\,\sqrt{1+cx}+bex^2\cosh^{-1}(cx)^2-b\cosh^{-1}(cx)\left(d-2ex^2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+2aex^2\log(x)-bex^2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
[Out] (-(a*d) + b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*e*x^2*ArcCosh[c*x]^2 - b
*ArcCosh[c*x]*(d - 2*e*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*e*x^2*Log[x]
 - b*e*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*x^2)
```

Maple [A]
time = 6.22, size = 147, normalized size = 0.59

method	result
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}-\frac{b e \operatorname{arccosh}(cx)^2}{2c^2}+\frac{bd\sqrt{cx+1}\,\sqrt{cx-1}}{2cx}-\frac{bd}{2}-\frac{b\operatorname{arccosh}(cx)d}{2c^2x^2}+\frac{be\operatorname{arcco}}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}-\frac{b e \operatorname{arccosh}(cx)^2}{2c^2}+\frac{bd\sqrt{cx+1}\,\sqrt{cx-1}}{2cx}-\frac{bd}{2}-\frac{b\operatorname{arccosh}(cx)d}{2c^2x^2}+\frac{be\operatorname{arcco}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] c^2*(-1/2*a*d/c^2/x^2+a/c^2*e*ln(c*x)-1/2*b/c^2*e*arccosh(c*x)^2+1/2*b*d/c/
x*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2*b*d-1/2*b*arccosh(c*x)*d/c^2/x^2+b/c^2*e*
arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b/c^2*e*polylog(
2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
[Out] 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + b*e*integrate(log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccosh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))(d + ex^2)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**3,x)
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\, x))\, (e\, x^2 + d)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^3,x)
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^3, x)
```

3.469 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=94

$$\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d(a+b\cosh^{-1}(cx))}{3x^3}-\frac{e(a+b\cosh^{-1}(cx))}{x}+\frac{1}{6}bc(c^2d+6e)\,\mathrm{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{\right.$$

```
[Out] -1/3*d*(a+b*arccosh(c*x))/x^3-e*(a+b*arccosh(c*x))/x+1/6*b*c*(c^2*d+6*e)*ar
ctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A]

time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5956, 465, 94, 211}

$$-\frac{d(a+b\cosh^{-1}(cx))}{3x^3}-\frac{e(a+b\cosh^{-1}(cx))}{x}+\frac{1}{6}bc\mathrm{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)(c^2d+6e)+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) - (e*(a + b*ArcCosh[c*x]))/x + (b*c*(c^2*d + 6*e)*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/6
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{3}(bc)\int\frac{-d-3}{x^3\sqrt{-1+cx}} \\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} \\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x} \\ &= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 128, normalized size = 1.36

$$\frac{-2b(d+3ex^2)\cosh^{-1}(cx)+\frac{bcdx(-1+c^2x^2)-2a\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+3ex^2)+bc(c^2d+6e)x^3\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*(d + 3*e*x^2)*ArcCosh[c*x] + (b*c*d*x*(-1 + c^2*x^2) - 2*a*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(d + 3*e*x^2) + b*c*(c^2*d + 6*e)*x^3*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(6*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(80) = 160$.
time = 1.86, size = 167, normalized size = 1.78

method	result

derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}-\frac{b\,\mathrm{arccosh}(cx)d}{3c^3x^3}-\frac{b\,\mathrm{arccosh}(cx)e}{c^3x}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,d\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}}\right.$
default	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}-\frac{b\,\mathrm{arccosh}(cx)d}{3c^3x^3}-\frac{b\,\mathrm{arccosh}(cx)e}{c^3x}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,d\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^2*(-1/3*d/c/x^3-e/c/x)-1/3*b*arccosh(c*x)*d/c^3/x^3-b/c^3*arccosh(
c*x)*e/x-1/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*arctan(1/(c^
2*x^2-1)^(1/2))-b/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(
1/(c^2*x^2-1)^(1/2))*e+1/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/x^2*d)
```

Maxima [A]
time = 0.46, size = 87, normalized size = 0.93

$$-\frac{1}{6}\left(\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\left(cx\right)}{x^3}\right)bd-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\left(cx\right)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)
/x^3)*b*d - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*e - a*e/x - 1/3*a*d
/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(82) = 164$.
time = 0.42, size = 192, normalized size = 2.04

$$\frac{\sqrt{c^2x^2-1}\,bcdx-6\,ax^2\cosh\left(1\right)-6\,ax^2\sinh\left(1\right)-2\,ad+2\left(bc^3dx^3+6\,bcx^3\cosh\left(1\right)+6\,bcx^3\sinh\left(1\right)\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\left(bdx^3-bd+3\left(bx^3-bx^2\right)\cosh\left(1\right)+3\left(bx^3-bx^2\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+2\left(bdx^3+3\,bx^3\cosh\left(1\right)+3\,bx^3\sinh\left(1\right)\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(c^2*x^2 - 1)*b*c*d*x - 6*a*x^2*cosh(1) - 6*a*x^2*sinh(1) - 2*a*d
 + 2*(b*c^3*d*x^3 + 6*b*c*x^3*cosh(1) + 6*b*c*x^3*sinh(1))*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + 2*(b*d*x^3 - b*d + 3*(b*x^3 - b*x^2)*cosh(1) + 3*(b*x^3 -
 b*x^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(b*d*x^3 + 3*b*x^3*cosh(1
) + 3*b*x^3*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))(d + ex^2)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,(e\,x^2 + d)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^4, x)
```

3.470 $\int x^4(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=319

$$\frac{b(63c^4d^2+90c^2de+35e^2)(1-c^2x^2)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2b(63c^4d^2+135c^2de+70e^2)(1-c^2x^2)^2}{945c^9\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b(21c^4d^2+90c^2de+70e^2}{525c^9\sqrt{-1+cx}\sqrt{1}}$$

```
[Out] 1/5*d^2*x^5*(a+b*arccosh(c*x))+2/7*d*e*x^7*(a+b*arccosh(c*x))+1/9*e^2*x^9*(
a+b*arccosh(c*x))+1/315*b*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)/c^9/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-2/945*b*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^
2+1)^2/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*(21*c^4*d^2+90*c^2*d*e+70*e^
2)*(-c^2*x^2+1)^3/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/441*b*e*(9*c^2*d+14*e)*
(-c^2*x^2+1)^4/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*e^2*(-c^2*x^2+1)^5/c^
9/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5958, 12, 534, 1265, 911, 1167}

$$\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{2be(1-c^2x^2)^4(9c^2d+14e)}{441c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{be^2(1-c^2x^2)^5}{81c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)^3(21c^4d^2+90c^2de+70e^2)}{525c^9\sqrt{cx-1}\sqrt{cx+1}}-\frac{2b(1-c^2x^2)^2(63c^4d^2+135c^2de+70e^2)}{945c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)(63c^4d^2+90c^2de+35e^2)}{315c^9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*b*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^2
)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(21*c^4*d^2 + 90*c^2*d*e + 70
*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*e*(9*c
^2*d + 14*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b
*ArcCosh[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCosh[c*x]))/7 + (e^2*x^9*(a + b*Ar
cCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\, dx = \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+\right.$$

$$= \frac{b(63c^4d^2+90c^2de+35e^2)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b(63c^4d^2+135c^2de+70}{945c^9\sqrt{-1+cx}\sqrt{}}$$

Mathematica [A]
time = 0.17, size = 192, normalized size = 0.60

$$\frac{315ax^5(63d^2+90dex^2+35e^2x^4)-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(4480e^2+160c^2e(81d+14ex^2)+24c^4(441d^2+270dex^2+70e^2x^4)+4c^6(1323d^2x^2+1215dex^4+350e^2x^6)+c^8(3969d^2x^4+4050dex^6+1225e^2x^8)\right)}{c^9}+315bx^5(63d^2+90dex^2+35e^2x^4)\cosh^{-1}(cx)}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(4480*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*x^
2 + 70*e^2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^8*(
3969*d^2*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 + 90*
d*e*x^2 + 35*e^2*x^4)*ArcCosh[c*x])/99225
```

Maple [A]
time = 3.25, size = 227, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d^2c^9x^5+\frac{2}{7}d\,c^9e\,x^7+\frac{1}{9}e^2c^9x^9\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^9x^5}{5}+\frac{2\operatorname{arccosh}(cx)d\,c^9e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)e^2c^9x^9}{9}-\frac{\sqrt{cx-1}\sqrt{cx+}}{}\right.}{}}{}$

default	$\frac{a\left(\frac{1}{5}d^2c^9x^5+\frac{2}{7}d\,c^9e\,x^7+\frac{1}{9}e^2c^9x^9\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^9x^5}{5}+\frac{2\operatorname{arccosh}(cx)d\,c^9e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)e^2c^9x^9}{9}-\frac{\sqrt{cx-1}\,\sqrt{cx+}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/c^4*(1/5*d^2*c^9*x^5+2/7*d*c^9*e*x^7+1/9*e^2*c^9*x^9)+b/c^4*(1/5*a
rccosh(c*x)*d^2*c^9*x^5+2/7*arccosh(c*x)*d*c^9*e*x^7+1/9*arccosh(c*x)*e^2*c
^9*x^9-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^2*x^8+4050*c^8*d*e*x
^6+3969*c^8*d^2*x^4+1400*c^6*e^2*x^6+4860*c^6*d*e*x^4+5292*c^6*d^2*x^2+1680
*c^4*e^2*x^4+6480*c^4*d*e*x^2+10584*c^4*d^2+2240*c^2*e^2*x^2+12960*c^2*d*e+
4480*e^2)))
```

Maxima [A]
time = 0.26, size = 305, normalized size = 0.96

$$\frac{1}{9}ax^9e^2+\frac{2}{7}adx^7e+\frac{1}{5}ad^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2+\frac{2}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bde+\frac{1}{2835}\left(315\,x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9*e^2 + 2/7*a*d*x^7*e + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) -
 (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^
6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt
(c^2*x^2 - 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x
^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c
^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^2
```

Fricas [A]
time = 0.38, size = 444, normalized size = 1.39

$$\frac{11025\,ac^9x^9\cosh(1)^2+11025\,ac^9x^9\sinh(1)^2+28350\,ac^9dx^7\cosh(1)+19845\,ac^9d^2x^5+315\left(35\,bc^9x^9\cosh(1)^2+35\,bc^9x^9\sinh(1)^2+90\,bc^9dx^7\cosh(1)+63\,bc^9d^2x^5+10\left(7\,bc^9x^9\cosh(1)+9\,bc^9dx^7\right)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+3150\left(7\,ac^9x^9\cosh(1)+9\,ac^9dx^7\right)\sinh(1)-\left(3969\,bc^8d^2x^4+5292\,bc^6d^2x^2+10584\,bc^4d^2+35\left(35\,bc^8x^8+40\,bc^6x^6+48\,bc^4x^4+64\,bc^2x^2+128\,b\right)\cosh(1)^2+35\left(35\,bc^8x^8+40\,bc^6x^6+48\,bc^4x^4+64\,bc^2x^2+128\,b\right)\sinh(1)^2+810\left(5\,bc^8dx^6+6\,bc^6dx^4+8\,bc^4dx^2+16\,bc^2d\right)\cosh(1)+10\left(405\,bc^8dx^6+486\,bc^6dx^4+648\,bc^4dx^2+1296\,bc^2d+7\left(35\,bc^8x^8+40\,bc^6x^6+48\,bc^4x^4+64\,bc^2x^2+128\,b\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2-1}}{99225\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*x^9*cosh(1)^2 + 11025*a*c^9*x^9*sinh(1)^2 + 28350*a*c^
9*d*x^7*cosh(1) + 19845*a*c^9*d^2*x^5 + 315*(35*b*c^9*x^9*cosh(1)^2 + 35*b*
c^9*x^9*sinh(1)^2 + 90*b*c^9*d*x^7*cosh(1) + 63*b*c^9*d^2*x^5 + 10*(7*b*c^9
*x^9*cosh(1) + 9*b*c^9*d*x^7)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 3150*
(7*a*c^9*x^9*cosh(1) + 9*a*c^9*d*x^7)*sinh(1) - (3969*b*c^8*d^2*x^4 + 5292*
b*c^6*d^2*x^2 + 10584*b*c^4*d^2 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^
4*x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1)^2 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 +
```

```
 48*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*sinh(1)^2 + 810*(5*b*c^8*d*x^6 + 6*b*
c^6*d*x^4 + 8*b*c^4*d*x^2 + 16*b*c^2*d)*cosh(1) + 10*(405*b*c^8*d*x^6 + 486
*b*c^6*d*x^4 + 648*b*c^4*d*x^2 + 1296*b*c^2*d + 7*(35*b*c^8*x^8 + 40*b*c^6*
x^6 + 48*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 -
 1))/c^9
```

Sympy [C] Result contains complex when optimal does not.
time = 1.45, size = 422, normalized size = 1.32

$$\begin{cases} \frac{ad^2x^5}{5} + \frac{2adex^7}{7} + \frac{ae^2x^9}{9} + \frac{bd^2x^5\operatorname{acosh}(cx)}{5} + \frac{2bdex^7\operatorname{acosh}(cx)}{7} + \frac{be^2x^9\operatorname{acosh}(cx)}{9} - \frac{bd^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{2bdex^6\sqrt{c^2x^2-1}}{49c} - \frac{be^2x^8\sqrt{c^2x^2-1}}{81c} - \frac{4bd^2x^2\sqrt{c^2x^2-1}}{75c^3} - \frac{12bdex^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8be^2x^6\sqrt{c^2x^2-1}}{567c^3} - \frac{8bd^2\sqrt{c^2x^2-1}}{75c^5} - \frac{16bdex^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16be^2x^4\sqrt{c^2x^2-1}}{945c^5} - \frac{32bde\sqrt{c^2x^2-1}}{245c^7} - \frac{64be^2x^2\sqrt{c^2x^2-1}}{2835c^7} - \frac{128be^2\sqrt{c^2x^2-1}}{2835c^9} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^5}{5} + \frac{2dex^7}{7} + \frac{e^2x^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 + b*d**2*x**5*aco
sh(c*x)/5 + 2*b*d*e*x**7*acosh(c*x)/7 + b*e**2*x**9*acosh(c*x)/9 - b*d**2*x
**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*b*d*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) -
b*e**2*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 4*b*d**2*x**2*sqrt(c**2*x**2 - 1)/
(75*c**3) - 12*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**2*x**6*sq
rt(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*sqrt(c**2*x**2 - 1)/(75*c**5) - 16*
b*d*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*x**4*sqrt(c**2*x**2 -
 1)/(945*c**5) - 32*b*d*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**2*x**2*s
qrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**2*sqrt(c**2*x**2 - 1)/(2835*c**9)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**5/5 + 2*d*e*x**7/7 + e**2*x**9/9), Tr
ue))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

3.471 $\int x^3(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=341

$$\frac{b(288c^4d^2+320c^2de+105e^2)\,x(1-c^2x^2)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b(288c^4d^2+320c^2de+105e^2)\,x^3(1-c^2x^2)}{4608c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be(64c^2d+21e)\,x^5(}{1152c^3\sqrt{-1+cx}\,\sqrt{}}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccosh(c*x))+1/3*d*e*x^6*(a+b*arccosh(c*x))+1/8*e^2*x^8*(
a+b*arccosh(c*x))+1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x*(-c^2*x^2+1)
/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4608*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)
*x^3*(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1152*b*e*(64*c^2*d+21*e
)*x^5*(-c^2*x^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/64*b*e^2*x^7*(-c^2*x^2
+1)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2
)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^8/(c*x-1)^(1/2)/(c*x+1
)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$,
Rules used = {272, 45, 5958, 12, 534, 1281, 470, 327, 223, 212}

$$\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(cx)\right)+\frac{be^2x^7(1-c^2x^2)}{64c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bex^5(1-c^2x^2)(64c^2d+21e)}{1152c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\,(288c^4d^2+320c^2de+105e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{3072c^8\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx(1-c^2x^2)(288c^4d^2+320c^2de+105e^2)}{3072c^7\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx^3(1-c^2x^2)(288c^4d^2+320c^2de+105e^2)}{4608c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x^3*(1 -
c^2*x^2))/(4608*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(64*c^2*d + 21*e)*
x^5*(1 - c^2*x^2))/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*x^7*(1
- c^2*x^2))/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c
*x]))/4 + (d*e*x^6*(a + b*ArcCosh[c*x]))/3 + (e^2*x^8*(a + b*ArcCosh[c*x]))
/8 - (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c
*x)/Sqrt[-1 + c^2*x^2]])/(3072*c^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+\right.$$

$$=\frac{be^2x^7(1-c^2x^2)}{64c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+\right.$$

$$=\frac{be(64c^2d+21e)\,x^5(1-c^2x^2)}{1152c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^2x^7(1-c^2x^2)}{64c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{1}{4}d^2x^4$$

$$=\frac{b(288c^4d^2+5e(64c^2d+21e))\,x^3(1-c^2x^2)}{4608c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be(64c^2d+21e)\,x^5(\cdots}{1152c^3\sqrt{-1+cx}\,\cdots}$$

$$=\frac{b(288c^4d^2+5e(64c^2d+21e))\,x(1-c^2x^2)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b(288c^4d^2+5e(64c^2\cdots}{4608c^5\sqrt{-1+\cdots}}$$

$$=\frac{b(288c^4d^2+5e(64c^2d+21e))\,x(1-c^2x^2)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b(288c^4d^2+5e(64c^2\cdots}{4608c^5\sqrt{-1+\cdots}}$$

$$=\frac{b(288c^4d^2+5e(64c^2d+21e))\,x(1-c^2x^2)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b(288c^4d^2+5e(64c^2\cdots}{4608c^5\sqrt{-1+\cdots}}$$

Mathematica [A]
time = 0.20, size = 220, normalized size = 0.65

$$\frac{384ac^8x^4(6d^2+8dex^2+3e^2x^4)-bcx\sqrt{-1+cx}\,\sqrt{1+cx}\,(315e^2+30c^2e(32d+7ex^2)+8c^4(108d^2+80dex^2+21e^2x^4)+16c^6(36d^2x^2+32dex^4+9e^2x^6))+384bc^8x^4(6d^2+8dex^2+3e^2x^4)\cosh^{-1}(cx)-3b(288c^4d^2+320c^2de+105e^2)\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{9216c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*c*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x^2
 + 21*e^2*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 384*b*c^8*
x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x] - 3*b*(288*c^4*d^2 + 320*c
^2*d*e + 105*e^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(9216*c^8)
```

Maple [A]
time = 2.87, size = 525, normalized size = 1.54

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}c^8e^2x^8\right)}{c^4}-\frac{b\,c^4\text{arccosh}(cx)d^4}{24e^2}+\frac{b\,\text{arccosh}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\text{arccosh}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\text{arccosh}(cx)x^8}{8}+\frac{b\,c^4\sqrt{}}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}c^8e^2x^8\right)}{c^4}-\frac{b\,c^4\text{arccosh}(cx)d^4}{24e^2}+\frac{b\,\text{arccosh}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\text{arccosh}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\text{arccosh}(cx)x^8}{8}+\frac{b\,c^4\sqrt{}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^4*(1/4*c^8*d^2*x^4+1/3*c^8*d*e*x^6+1/8*c^8*e^2*x^8)-1/24*b*c^4/e
^2*arccosh(c*x)*d^4+1/4*b*arccosh(c*x)*d^2*c^4*x^4+1/3*b*c^4*e*arccosh(c*x)
*d*x^6+1/8*b*c^4*e^2*arccosh(c*x)*x^8+1/24*b*c^4/e^2*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-1/16*b*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*d^2*c^3*x^3-1/18*b*c^3*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^5-1/
64*b*c^3*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^7-3/32*b*c*d^2*x*(c*x-1)^(1/2)*(
c*x+1)^(1/2)-5/72*b*c*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^3-7/384*b*c*e^2*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*x^5-3/32*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^2-5/48*b*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c-35/1536*b*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/48*b/c^2*e*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d-35/1024*b*
e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-35/1024*b/c^4*e^2*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.28, size = 332, normalized size = 0.97

$$\frac{1}{8}ax^8e^2+\frac{1}{3}adx^6e+\frac{1}{4}ad^2x^4+\frac{1}{32}\left(8x^4\text{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd^2+\frac{1}{144}\left(48x^6\text{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bde+\frac{1}{3072}\left(384x^8\text{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^2 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arccosh(c*x) -
(2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2 + 1/144*(48*x^6*arccosh(c*x) - (8*sqrt
(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)
*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e + 1/3072*(38
4*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x
^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*e^2
```

Fricas [A]
time = 0.39, size = 438, normalized size = 1.28

$$\frac{1152ac^8x^8\cosh(1)^2+1152ac^8x^8\sinh(1)^2+3072ac^8dx^6\cosh(1)+2304ac^8d^2x^4+3\left(768bc^8d^2x^4-288bc^4d^2+3\left(128bc^8x^8-35b\right)\cosh(1)^2+3\left(128bc^8x^8-35b\right)\sinh(1)^2+64\left(16bc^8dx^6-5bc^2d\right)\cosh(1)+2\left(512bc^8dx^6-160bc^2d+3\left(128bc^8x^8-35b\right)\cosh(1)\right)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+768\left(3ac^8x^8\cosh(1)+4ac^8dx^6\right)\sinh(1)-\left(576bc^7d^2x^3+864bc^5d^2x+3\left(48bc^7x^7+56bc^5x^5+70bc^3x^3+105bcx\right)\cosh(1)^2+3\left(48bc^7x^7+56bc^5x^5+70bc^3x^3+105bcx\right)\sinh(1)^2+64\left(8bc^7dx^5+10bc^5dx^3+15bc^3dx\right)\cosh(1)+2\left(256bc^7dx^5+320bc^5dx^3+480bc^3dx+3\left(48bc^7x^7+56bc^5x^5+70bc^3x^3+105bcx\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2-1}}{9216c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] 1/9216*(1152*a*c^8*x^8*cosh(1)^2 + 1152*a*c^8*x^8*sinh(1)^2 + 3072*a*c^8*d*x^6*cosh(1) + 2304*a*c^8*d^2*x^4 + 3*(768*b*c^8*d^2*x^4 - 288*b*c^4*d^2 + 3*(128*b*c^8*x^8 - 35*b)*cosh(1)^2 + 3*(128*b*c^8*x^8 - 35*b)*sinh(1)^2 + 64*(16*b*c^8*d*x^6 - 5*b*c^2*d)*cosh(1) + 2*(512*b*c^8*d*x^6 - 160*b*c^2*d + 3*(128*b*c^8*x^8 - 35*b)*cosh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 768*(3*a*c^8*x^8*cosh(1) + 4*a*c^8*d*x^6)*sinh(1) - (576*b*c^7*d^2*x^3 + 864*b*c^5*d^2*x + 3*(48*b*c^7*x^7 + 56*b*c^5*x^5 + 70*b*c^3*x^3 + 105*b*c*x)*cosh(1)^2 + 3*(48*b*c^7*x^7 + 56*b*c^5*x^5 + 70*b*c^3*x^3 + 105*b*c*x)*sinh(1)^2 + 64*(8*b*c^7*d*x^5 + 10*b*c^5*d*x^3 + 15*b*c^3*d*x)*cosh(1) + 2*(256*b*c^7*d*x^5 + 320*b*c^5*d*x^3 + 480*b*c^3*d*x + 3*(48*b*c^7*x^7 + 56*b*c^5*x^5 + 70*b*c^3*x^3 + 105*b*c*x)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^8

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 389, normalized size = 1.14

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{acosh}(cx)}{4} + \frac{bdex^6\operatorname{acosh}(cx)}{3} + \frac{be^2x^8\operatorname{acosh}(cx)}{8} - \frac{bd^2x^3\sqrt{c^2x^2-1}}{16c} - \frac{bdex^5\sqrt{c^2x^2-1}}{18c} - \frac{be^2x^7\sqrt{c^2x^2-1}}{64c} - \frac{3bd^2x\sqrt{c^2x^2-1}}{32c^3} - \frac{5bdex^3\sqrt{c^2x^2-1}}{72c^3} - \frac{7be^2x^5\sqrt{c^2x^2-1}}{384c^3} - \frac{3bd^2\operatorname{acosh}(cx)}{32c^4} - \frac{5bdex\sqrt{c^2x^2-1}}{48c^5} - \frac{35be^2x^3\sqrt{c^2x^2-1}}{1536c^5} - \frac{5bde\operatorname{acosh}(cx)}{48c^6} - \frac{35be^2x\sqrt{c^2x^2-1}}{1024c^7} - \frac{35be^2\operatorname{acosh}(cx)}{1024c^8} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)**2*(a+b*acosh(c*x)),x)

[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acosh(c*x)/4 + b*d*e*x**6*acosh(c*x)/3 + b*e**2*x**8*acosh(c*x)/8 - b*d**2*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*d*e*x**5*sqrt(c**2*x**2 - 1)/(18*c) - b*e**2*x**7*sqrt(c**2*x**2 - 1)/(64*c) - 3*b*d**2*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d*e*x**3*sqrt(c**2*x**2 - 1)/(72*c**3) - 7*b*e**2*x**5*sqrt(c**2*x**2 - 1)/(384*c**3) - 3*b*d**2*acosh(c*x)/(32*c**4) - 5*b*d*e*x*sqrt(c**2*x**2 - 1)/(48*c**5) - 35*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e*acosh(c*x)/(48*c**6) - 35*b*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*e**2*acosh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a + b\,\text{acosh}(c\,x))\,\left(e\,x^2 + d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

3.472 $\int x^2(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=260

$$\frac{b(35c^4d^2+42c^2de+15e^2)\,(1-c^2x^2)}{105c^7\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b(35c^4d^2+84c^2de+45e^2)\,(1-c^2x^2)^2}{315c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be(14c^2d+15e)\,(1-c^2x^2)^3}{175c^7\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/3*d^2*x^3*(a+b*arccosh(c*x))+2/5*d*e*x^5*(a+b*arccosh(c*x))+1/7*e^2*x^7*(
a+b*arccosh(c*x))+1/105*b*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)/c^7/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-1/315*b*(35*c^4*d^2+84*c^2*d*e+45*e^2)*(-c^2*x^2
+1)^2/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/175*b*e*(14*c^2*d+15*e)*(-c^2*x^2+1
)^3/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*e^2*(-c^2*x^2+1)^4/c^7/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {276, 5958, 12, 534, 1265, 785}

$$\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{be(1-c^2x^2)^3\,(14c^2d+15e)}{175c^7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be^2(1-c^2x^2)^4}{49c^7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b(1-c^2x^2)^2\,(35c^4d^2+84c^2de+45e^2)}{315c^7\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b(1-c^2x^2)\,(35c^4d^2+42c^2de+15e^2)}{105c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2))/(105*c^7*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^2)/(
315*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(14*c^2*d + 15*e)*(1 - c^2*x^2
)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(1 - c^2*x^2)^4)/(49*c
^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (2*d*
e*x^5*(a + b*ArcCosh[c*x]))/5 + (e^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
```

```
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+\right.$$

$$= \frac{b(35c^4d^2+42c^2de+15e^2)\left(1-c^2x^2\right)}{105c^7\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b(35c^4d^2+84c^2de+45e^2}{315c^7\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A]
time = 0.14, size = 163, normalized size = 0.63

$$\frac{105ax^3(35d^2+42dex^2+15e^2x^4)-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(720e^2+24c^2e(98d+15ex^2)+2c^4(1225d^2+588dex^2+135e^2x^4)+c^6(1225d^2x^2+882dex^4+225e^2x^6)\right)}{c^7}+105bx^3(35d^2+42dex^2+15e^2x^4)\cosh^{-1}(cx)}{11025}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(720*e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^2
+ 135*e^2*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 + 105
*b*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCosh[c*x])/11025
```

Maple [A]
time = 2.92, size = 195, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccosh}(cx)dc^7ex^5}{5}+\frac{\operatorname{arccosh}(cx)e^2c^7x^7}{7}-\frac{\sqrt{cx-1}\sqrt{cx+}}{}\right.}{}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccosh}(cx)dc^7ex^5}{5}+\frac{\operatorname{arccosh}(cx)e^2c^7x^7}{7}-\frac{\sqrt{cx-1}\sqrt{cx+}}{}\right.}{}}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^3*(a/c^4*(1/3*d^2*c^7*x^3+2/5*d*c^7*e*x^5+1/7*e^2*c^7*x^7)+b/c^4*(1/3*a
rccosh(c*x)*d^2*c^7*x^3+2/5*arccosh(c*x)*d*c^7*e*x^5+1/7*arccosh(c*x)*e^2*c
^7*x^7-1/11025*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(225*c^6*e^2*x^6+882*c^6*d*e*x^4
+1225*c^6*d^2*x^2+270*c^4*e^2*x^4+1176*c^4*d*e*x^2+2450*c^4*d^2+360*c^2*e^2
*x^2+2352*c^2*d*e+720*e^2)))

Maxima [A]
time = 0.26, size = 247, normalized size = 0.95

$$\frac{1}{7}ax^7e^2+\frac{2}{5}adx^5e+\frac{1}{3}ad^2x^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2+\frac{2}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bde+\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")

[Out] 1/7*a*x^7*e^2 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2 + 2/75*(15*x^
5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4
 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arccosh(c*x) - (5*sqrt
(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x
^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^2

Fricas [A]
time = 0.36, size = 385, normalized size = 1.48

$$\frac{1575ac^7x^7\cosh(1)^2+1575ac^7x^7\sinh(1)^2+4410ac^7dx^5\cosh(1)+3675ac^7d^2x^3+105\left(15bc^7x^7\cosh(1)^2+15bc^7x^7\sinh(1)^2+42bc^7dx^5\cosh(1)+35bc^7d^2x^3+6\left(5bc^7x^7\cosh(1)+7bc^7dx^5\right)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+630\left(5ac^7x^7\cosh(1)+7ac^7dx^5\right)\sinh(1)-\left(1225bc^6d^2x^2+2450bc^4d^2+45\left(5bc^6x^6+6bc^4x^4+8bc^2x^2+16b\right)\cosh(1)^2+45\left(5bc^6x^6+6bc^4x^4+8bc^2x^2+16b\right)\sinh(1)^2+294\left(3bc^6dx^4+4bc^4dx^2+8bc^2d\right)\cosh(1)+6\left(147bc^6dx^4+196bc^4dx^2+392bc^2d+15\left(5bc^6x^6+6bc^4x^4+8bc^2x^2+16b\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2-1}}{11025c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] 1/11025*(1575*a*c^7*x^7*cosh(1)^2 + 1575*a*c^7*x^7*sinh(1)^2 + 4410*a*c^7*d
*x^5*cosh(1) + 3675*a*c^7*d^2*x^3 + 105*(15*b*c^7*x^7*cosh(1)^2 + 15*b*c^7*
x^7*sinh(1)^2 + 42*b*c^7*d*x^5*cosh(1) + 35*b*c^7*d^2*x^3 + 6*(5*b*c^7*x^7*
cosh(1) + 7*b*c^7*d*x^5)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 630*(5*a*c
^7*x^7*cosh(1) + 7*a*c^7*d*x^5)*sinh(1) - (1225*b*c^6*d^2*x^2 + 2450*b*c^4*
d^2 + 45*(5*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*cosh(1)^2 + 45*(5
*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*sinh(1)^2 + 294*(3*b*c^6*d*x
^4 + 4*b*c^4*d*x^2 + 8*b*c^2*d)*cosh(1) + 6*(147*b*c^6*d*x^4 + 196*b*c^4*d*
x^2 + 392*b*c^2*d + 15*(5*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*cos
h(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^7

Sympy [C] Result contains complex when optimal does not.
time = 0.73, size = 340, normalized size = 1.31

$$\begin{cases} \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3\operatorname{acosh}(cx)}{3} + \frac{2bdex^5\operatorname{acosh}(cx)}{5} + \frac{be^2x^7\operatorname{acosh}(cx)}{7} - \frac{bd^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{2bdex^4\sqrt{c^2x^2-1}}{25c} - \frac{be^2x^6\sqrt{c^2x^2-1}}{49c} - \frac{2bd^2\sqrt{c^2x^2-1}}{9c^3} - \frac{8bdex^2\sqrt{c^2x^2-1}}{75c^3} - \frac{6be^2x^4\sqrt{c^2x^2-1}}{245c^3} - \frac{16bde\sqrt{c^2x^2-1}}{75c^5} - \frac{8be^2x^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16be^2\sqrt{c^2x^2-1}}{245c^7} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^3}{3} + \frac{2dex^5}{5} + \frac{e^2x^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*aco
sh(c*x)/3 + 2*b*d*e*x**5*acosh(c*x)/5 + b*e**2*x**7*acosh(c*x)/7 - b*d**2*x
**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**2*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*sqrt(c**2*x**2 - 1)/(9*c**
3) - 8*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e**2*x**4*sqrt(c**2*x
**2 - 1)/(245*c**3) - 16*b*d*e*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e**2*x**
2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*sqrt(c**2*x**2 - 1)/(245*c**7)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), Tr
ue))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\operatorname{acosh}(c\,x)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

3.473 $\int x(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=269

$$\frac{b(44c^4d^2+44c^2de+15e^2)\,x(1-c^2x^2)}{288c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5b(2c^2d+e)\,x(1-c^2x^2)\,(d+ex^2)}{144c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx(1-c^2x^2)\,(d+ex^2)^2}{36c\sqrt{-1+cx}\,\sqrt{1+cx}}+(d+$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccosh(c*x))/e+1/288*b*(44*c^4*d^2+44*c^2*d*e+15*e^2)
*x*(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/144*b*(2*c^2*d+e)*x*(-c^2
*x^2+1)*(e*x^2+d)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*x*(-c^2*x^2+1)*(e*
x^2+d)^2/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/96*b*(2*c^2*d+e)*(8*c^4*d^2+8*c^2*
d*e+5*e^2)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^6/e/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5957, 916, 427, 542, 396, 223, 212}

$$\frac{(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}+\frac{bx(1-c^2x^2)\,(d+ex^2)^2}{36c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bx(1-c^2x^2)\,(2c^2d+e)\,(d+ex^2)}{144c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\,(2c^2d+e)\,(8c^4d^2+8c^2de+5e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{96c^6e\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx(1-c^2x^2)\,(44c^4d^2+44c^2de+15e^2)}{288c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*(1 - c^2*x^2))/(288*c^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (5*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2))/(144*c
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(36*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/(6*e)
- (b*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*Sqrt[-1 + c^2*x^2]*ArcTa
nh[(c*x)/Sqrt[-1 + c^2*x^2]])/(96*c^6*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 916

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^FracPart[m]/(d*f
 + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int x(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6e} - \frac{(bc)\int\frac{(d+ex^2)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{6e}$$

$$= \frac{(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6e} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{(d+ex^2)^3}{\sqrt{-1+c^2x^2}}\,d}{6e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bx(1-c^2x^2)(d+ex^2)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)}{6e} - \frac{\left(b\sqrt{-1\right.}}{}$$

$$= \frac{5b(2c^2d+e)x(1-c^2x^2)(d+ex^2)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx(1-c^2x^2)(d+ex^2)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(d}{}$$

$$= \frac{b(44c^4d^2+44c^2de+15e^2)x(1-c^2x^2)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5b(2c^2d+e)x(1-c^2x^2)(d}{144c^3\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{b(44c^4d^2+44c^2de+15e^2)x(1-c^2x^2)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5b(2c^2d+e)x(1-c^2x^2)(d}{144c^3\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{b(44c^4d^2+44c^2de+15e^2)x(1-c^2x^2)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5b(2c^2d+e)x(1-c^2x^2)(d}{144c^3\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A]
time = 0.17, size = 189, normalized size = 0.70

$$\frac{cx\left(48ac^5x(3d^2+3dex^2+e^2x^4)-b\sqrt{-1+cx}\sqrt{1+cx}\,(15e^2+2c^2e(27d+5ex^2)+4c^4(18d^2+9dex^2+2e^2x^4))\right)+48bc^6x^2(3d^2+3dex^2+e^2x^4)\cosh^{-1}(cx)-3b(24c^4d^2+18c^2de+5e^2)\log\left(cx+\sqrt{-1+cx}\sqrt{1+cx}\right)}{288c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(15*e^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e^2
*x^4))) + 48*b*c^6*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCosh[c*x] - 3*b*(24
*c^4*d^2 + 18*c^2*d*e + 5*e^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(28
8*c^6)
```

Maple [A]
time = 2.80, size = 435, normalized size = 1.62

method	result

derivativedivides	$\frac{\frac{(c^2 e x^2+c^2 d)^3 a}{6c^4 e}+\frac{b c^2 \operatorname{arccosh}(cx) d^3}{6e}+\frac{d^2 b \operatorname{arccosh}(cx) c^2 x^2}{2}+\frac{b c^2 e \operatorname{arccosh}(cx) d x^4}{2}+\frac{b c^2 e^2 \operatorname{arccosh}(cx) x^6}{6}-\frac{b c^2 \sqrt{cx-1}\,\sqrt{c \ldots}}{6 \ldots}}{\ldots}$ [...]
default	$\frac{\frac{(c^2 e x^2+c^2 d)^3 a}{6c^4 e}+\frac{b c^2 \operatorname{arccosh}(cx) d^3}{6e}+\frac{d^2 b \operatorname{arccosh}(cx) c^2 x^2}{2}+\frac{b c^2 e \operatorname{arccosh}(cx) d x^4}{2}+\frac{b c^2 e^2 \operatorname{arccosh}(cx) x^6}{6}-\frac{b c^2 \sqrt{cx-1}\,\sqrt{c \ldots}}{6 \ldots}}{\ldots}$ [...]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*(c^2*e*x^2+c^2*d)^3*a/c^4/e+1/6*b*c^2/e*arccosh(c*x)*d^3+1/2*d^2
*b*arccosh(c*x)*c^2*x^2+1/2*b*c^2*e*arccosh(c*x)*d*x^4+1/6*b*c^2*e^2*arccos
h(c*x)*x^6-1/6*b*c^2/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln
(c*x+(c^2*x^2-1)^(1/2))-1/4*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/8*b*c*e
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^3-1/36*b*c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^5-1/4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)
^(1/2))*d^2-3/16*b*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^2*x^3*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c-3/16*b/c^2*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2
-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d-5/96*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c^3-5/96*b/c^4*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2)))
```

<u>Maxima</u> [A]
time = 0.26, size = 273, normalized size = 1.01

$$\frac{1}{6}ax^6e^2+\frac{1}{2}adx^4e+\frac{1}{2}ad^2x^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^2+\frac{1}{16}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bde+\frac{1}{288}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^2 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^
2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^
2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d*e + 1/288
*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)
*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1
)*c)/c^7)*c)*b*e^2
```

<u>Fricas</u> [A]
time = 0.36, size = 373, normalized size = 1.39

$$\frac{48\,ac^6x^6\cosh(1)^2+48\,ac^6x^6\sinh(1)^2+144\,ac^6dx^4\cosh(1)+144\,ac^6d^2x^2+3\left(48\,bc^6d^2x^2-24\,bc^4d^2+(16\,bc^6x^6-5\,b)\cosh(1)^2+(16\,bc^6x^6-5\,b)\sinh(1)^2+6\,(8\,bc^6dx^4-3\,bc^2d)\cosh(1)+2\,(24\,bc^6dx^4-9\,bc^2d+(16\,bc^6x^6-5\,b)\cosh(1))\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+48\,(2\,ac^6x^6\cosh(1)+3\,ac^6dx^4)\sinh(1)-\left(72\,bc^5d^2x+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\cosh(1)^2+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\sinh(1)^2+18\,(2\,bc^5dx^3+3\,bc^3dx)\cosh(1)+2\,(18\,bc^5dx^3+27\,bc^3dx+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\cosh(1))\sinh(1)\right)\sqrt{c^2x^2-1}}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*x^6*cosh(1)^2 + 48*a*c^6*x^6*sinh(1)^2 + 144*a*c^6*d*x^4*co
sh(1) + 144*a*c^6*d^2*x^2 + 3*(48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 + (16*b*c^6*
x^6 - 5*b)*cosh(1)^2 + (16*b*c^6*x^6 - 5*b)*sinh(1)^2 + 6*(8*b*c^6*d*x^4 -
3*b*c^2*d)*cosh(1) + 2*(24*b*c^6*d*x^4 - 9*b*c^2*d + (16*b*c^6*x^6 - 5*b)*c
osh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 48*(2*a*c^6*x^6*cosh(1) + 3
*a*c^6*d*x^4)*sinh(1) - (72*b*c^5*d^2*x + (8*b*c^5*x^5 + 10*b*c^3*x^3 + 15*
b*c*x)*cosh(1)^2 + (8*b*c^5*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*sinh(1)^2 + 18*(
2*b*c^5*d*x^3 + 3*b*c^3*d*x)*cosh(1) + 2*(18*b*c^5*d*x^3 + 27*b*c^3*d*x + (
8*b*c^5*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))
/c^6
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 306, normalized size = 1.14

$$\begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2\operatorname{acosh}(cx)}{2} + \frac{bdex^4\operatorname{acosh}(cx)}{2} + \frac{be^2x^6\operatorname{acosh}(cx)}{6} - \frac{bd^2x\sqrt{c^2x^2-1}}{4c} - \frac{bdex^3\sqrt{c^2x^2-1}}{8c} - \frac{be^2x^5\sqrt{c^2x^2-1}}{36c} - \frac{bd^2\operatorname{acosh}(cx)}{4c^2} - \frac{3bdex\sqrt{c^2x^2-1}}{16c^3} - \frac{5be^2x^3\sqrt{c^2x^2-1}}{144c^3} - \frac{3bde\operatorname{acosh}(cx)}{16c^4} - \frac{5be^2x\sqrt{c^2x^2-1}}{96c^5} - \frac{5be^2\operatorname{acosh}(cx)}{96c^6} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acosh
(c*x)/2 + b*d*e*x**4*acosh(c*x)/2 + b*e**2*x**6*acosh(c*x)/6 - b*d**2*x*sqr
t(c**2*x**2 - 1)/(4*c) - b*d*e*x**3*sqrt(c**2*x**2 - 1)/(8*c) - b*e**2*x**5
*sqrt(c**2*x**2 - 1)/(36*c) - b*d**2*acosh(c*x)/(4*c**2) - 3*b*d*e*x*sqrt(c
**2*x**2 - 1)/(16*c**3) - 5*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(144*c**3) - 3*
b*d*e*acosh(c*x)/(16*c**4) - 5*b*e**2*x*sqrt(c**2*x**2 - 1)/(96*c**5) - 5*b
*e**2*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**2/2 + d*e*x
**4/2 + e**2*x**6/6), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

3.474 $\int (d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=196

$$\frac{b(15c^4d^2+10c^2de+3e^2)(1-c^2x^2)}{15c^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2be(5c^2d+3e)(1-c^2x^2)^2}{45c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^2(1-c^2x^2)^3}{25c^5\sqrt{-1+cx}\sqrt{1+cx}}+d^2x(a+b\cosh$$

```
[Out] d^2*x*(a+b*arccosh(c*x))+2/3*d*e*x^3*(a+b*arccosh(c*x))+1/5*e^2*x^5*(a+b*ar
ccosh(c*x))+1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(-c^2*x^2+1)/c^5/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2/45*b*e*(5*c^2*d+3*e)*(-c^2*x^2+1)^2/c^5/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+1/25*b*e^2*(-c^2*x^2+1)^3/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {200, 5908, 12, 534, 1261, 712}

$$d^2x(a+b\cosh^{-1}(cx))+\frac{2}{3}dex^3(a+b\cosh^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\cosh^{-1}(cx))-\frac{2be(1-c^2x^2)^2(5c^2d+3e)}{45c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{be^2(1-c^2x^2)^3}{25c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)(15c^4d^2+10c^2de+3e^2)}{15c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*(1 - c^2*x^2))/(15*c^5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (2*b*e*(5*c^2*d + 3*e)*(1 - c^2*x^2)^2)/(45*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (b*e^2*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) + (2*d*e*x^3*(a + b*ArcCosh[c*x]))/3
+ (e^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int (d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right) dx = d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cos\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cos\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cos\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cos\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cos\right.$$

$$= \frac{b(15c^4d^2+10c^2de+3e^2)\left(1-c^2x^2\right)}{15c^5\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2be(5c^2d+3e)\left(1-c^2x^2\right)^2}{45c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

Mathematica [A]

time = 0.14, size = 130, normalized size = 0.66

$$\frac{1}{225}\left(15ax(15d^2+10dex^2+3e^2x^4)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5}+15bx(15d^2+10dex^2+3e^2x^4)\cosh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)))/c^5 + 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x])/225
```

Maple [A]
time = 1.96, size = 157, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)d^2c^5x+\frac{2\operatorname{arccosh}(cx)dc^5e\,x^3}{3}+\frac{\operatorname{arccosh}(cx)e^2c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9\dots}{\dots}\right.}{c^4}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arccosh}(cx)d^2c^5x+\frac{2\operatorname{arccosh}(cx)dc^5e\,x^3}{3}+\frac{\operatorname{arccosh}(cx)e^2c^5x^5}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9\dots}{\dots}\right.}{c^4}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b/c^4*(arccosh(c*x)*
d^2*c^5*x+2/3*arccosh(c*x)*d*c^5*e*x^3+1/5*arccosh(c*x)*e^2*c^5*x^5-1/225*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+225*c^4*d^2+12*c^2
*e^2*x^2+100*c^2*d*e+24*e^2)))
```

Maxima [A]
time = 0.28, size = 180, normalized size = 0.92

$$\frac{1}{5}ax^5e^2+\frac{2}{3}adx^3e+ad^2x+\frac{2}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^2}{c}+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^2 + 2/3*a*d*x^3*e + a*d^2*x + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt
(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e + (c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*d^2/c + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2
 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b
*e^2
```

Fricas [A]
time = 0.36, size = 321, normalized size = 1.64

$$\frac{45\,ac^5x^5\cosh\left(1\right)^2 + 45\,ac^5x^5\sinh\left(1\right)^2 + 150\,ac^5dx^3\cosh\left(1\right) + 225\,ac^5d^2x + 15\,\left(3\,bc^5x^5\cosh\left(1\right)^2 + 3\,bc^5x^5\sinh\left(1\right)^2 + 10\,bc^5dx^3\cosh\left(1\right) + 15\,bc^5d^2x + 2\,\left(3\,bc^5x^5\cosh\left(1\right) + 5\,bc^5dx^3\right)\sinh\left(1\right)\right)\log\left(cx + \sqrt{c^2x^2 - 1}\right) + 30\,\left(3\,ac^5x^5\cosh\left(1\right) + 5\,ac^5dx^3\right)\sinh\left(1\right) - \left(225\,bc^4d^2 + 3\,\left(3\,bc^4x^4 + 4\,bc^2x^2 + 8\,b\right)\cosh\left(1\right)^2 + 3\,\left(3\,bc^4x^4 + 4\,bc^2x^2 + 8\,b\right)\sinh\left(1\right)^2 + 50\,\left(bc^4dx^2 + 2\,bc^2d\right)\cosh\left(1\right) + 2\,\left(25\,bc^4dx^2 + 50\,bc^2d + 3\,\left(3\,bc^4x^4 + 4\,bc^2x^2 + 8\,b\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\sqrt{c^2x^2 - 1}}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*x^5*cosh(1)^2 + 45*a*c^5*x^5*sinh(1)^2 + 150*a*c^5*d*x^3*co
sh(1) + 225*a*c^5*d^2*x + 15*(3*b*c^5*x^5*cosh(1)^2 + 3*b*c^5*x^5*sinh(1)^2
 + 10*b*c^5*d*x^3*cosh(1) + 15*b*c^5*d^2*x + 2*(3*b*c^5*x^5*cosh(1) + 5*b*c
^5*d*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 30*(3*a*c^5*x^5*cosh(1) +
 5*a*c^5*d*x^3)*sinh(1) - (225*b*c^4*d^2 + 3*(3*b*c^4*x^4 + 4*b*c^2*x^2 + 8
*b)*cosh(1)^2 + 3*(3*b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sinh(1)^2 + 50*(b*c^4*d
*x^2 + 2*b*c^2*d)*cosh(1) + 2*(25*b*c^4*d*x^2 + 50*b*c^2*d + 3*(3*b*c^4*x^4
 + 4*b*c^2*x^2 + 8*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.35, size = 246, normalized size = 1.26

$$\begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{acosh}\left(cx\right) + \frac{2bdex^3\operatorname{acosh}\left(cx\right)}{3} + \frac{be^2x^5\operatorname{acosh}\left(cx\right)}{5} - \frac{bd^2\sqrt{c^2x^2-1}}{c} - \frac{2bdex^2\sqrt{c^2x^2-1}}{9c} - \frac{be^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{4bde\sqrt{c^2x^2-1}}{9c^3} - \frac{4be^2x^2\sqrt{c^2x^2-1}}{75c^3} - \frac{8be^2\sqrt{c^2x^2-1}}{75c^5} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acosh(c*x)
+ 2*b*d*e*x**3*acosh(c*x)/3 + b*e**2*x**5*acosh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 - 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 - 1)/(25*c) - 4*b*d*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e**2*x**2*sq
rt(c**2*x**2 - 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^2,x)
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

3.475 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=342

$$-\frac{be(16c^2d+3e)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{be(16c^2d+3e)\cosh^{-1}(cx)}{32c^4}+dex^2(a+$$

```
[Out] -1/32*b*e*(16*c^2*d+3*e)*arccosh(c*x)/c^4+d*e*x^2*(a+b*arccosh(c*x))+1/4*e^
2*x^4*(a+b*arccosh(c*x))+d^2*(a+b*arccosh(c*x))*ln(x)-1/32*b*e*(16*c^2*d+3*
e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/16*b*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c-1/2*I*b*d^2*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+b*d^2*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d^2*arcsin(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^
2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 369, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {272, 45, 5958, 6874, 92, 54, 102, 12, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$d^2\log(x)\left(a+b\cosh^{-1}(cx)\right)+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd^2\sqrt{1-c^2x^2}\,\text{Li}_2\left(e^{2i\text{ArcSin}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd^2\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)^2}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)\log\left(1-e^{2i\text{ArcSin}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd^2\sqrt{1-c^2x^2}\,\log(x)\text{ArcSin}(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3be^2\cosh^{-1}(cx)}{32c^4}-\frac{3be^2x\sqrt{cx-1}\,\sqrt{cx+1}}{32c^3}-\frac{bde\cosh^{-1}(cx)}{2c^2}-\frac{bdex\sqrt{cx-1}\,\sqrt{cx+1}}{2c}-\frac{be^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/2*(b*d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (3*b*e^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/(32*c^3) - (b*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) -
(b*d*e*ArcCosh[c*x])/(2*c^2) - (3*b*e^2*ArcCosh[c*x])/(32*c^4) + d*e*x^2*(a
 + b*ArcCosh[c*x]) + (e^2*x^4*(a + b*ArcCosh[c*x]))/4 - ((I/2)*b*d^2*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*Sqrt[1 -
 c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + d^2*(a + b*ArcCosh[c*x])*Log[x] - (b*d^2*Sqrt[1 - c^2*x^2]*ArcS
in[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*d^2*Sqrt[1 - c^2*
x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)}{x}\,dx &= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(\right. \\
&= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(\right. \\
&= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(\right. \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+dex^2\left(a+\right. \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{bde\cosh^{-}}{2c^2} \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-}}{} \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-}}{} \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-}}{} \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-}}{} \\
&= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 228, normalized size = 0.67

$$adex^2+\frac{1}{4}ae^2x^4+bdex^2\cosh^{-1}(cx)+\frac{1}{4}be^2x^4\cosh^{-1}(cx)-\frac{bde\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}+2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{2c^2}-\frac{be^2\left(cx\sqrt{\frac{-1+cx}{1+cx}}\,(3+3cx+2c^2x^2+2c^3x^3)+6\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{32c^4}+\frac{1}{2}bd^2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+ad^2\log(x)-\frac{1}{2}bd^2\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + b*d*e*x^2*ArcCosh[c*x] + (b*e^2*x^4*ArcCosh[c*x
])/4 - (b*d*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)
/(1 + c*x)]]))/(2*c^2) - (b*e^2*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3 + 3*c*x
+ 2*c^2*x^2 + 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(32*c^4)
```

```
+ (b*d^2*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]))/2 +
a*d^2*Log[x] - (b*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

Maple [A]
time = 2.94, size = 225, normalized size = 0.66

$$\frac{a\,e^{2}x^{4}}{4}+ade\,x^{2}+a\,d^{2}\ln\left(cx\right)-\frac{3b\,e^{2}\mathrm{arccosh}(cx)}{32c^{4}}-\frac{bde\,\mathrm{arccosh}(cx)}{2c^{2}}-\frac{bdex\sqrt{cx-1}\,\sqrt{cx+1}}{2c}-\frac{b\,e^{2}x^{3}\sqrt{cx-1}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/4*a*e^2*x^4+a*d*e*x^2+a*d^2*ln(c*x)-3/32*b*e^2*arccosh(c*x)/c^4-1/2*b*d*e
*arccosh(c*x)/c^2-1/2*b*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*b*e^2*x^3*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-
1/2*d^2*b*arccosh(c*x)^2+1/4*b*arccosh(c*x)*e^2*x^4+b*arccosh(c*x)*x^2*d*e+
d^2*b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d^2*poly
log(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^2 + a*d*x^2*e + a*d^2*log(x) + integrate(b*x^3*e^2*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)) + 2*b*d*x*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(d+ex^{2}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x, x)
```

3.476 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=160

$$\frac{be(6c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{be^2(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2(a+b\cosh^{-1}(cx))}{x}+2dex(a+b\cosh^{-1}(cx))+\frac{1}{3}e^2x^3$$

```
[Out] -d^2*(a+b*arccosh(c*x))/x+2*d*e*x*(a+b*arccosh(c*x))+1/3*e^2*x^3*(a+b*arcco
sh(c*x))+b*c*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b*e*(6*c^2*d+e)*(-
c^2*x^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*e^2*(-c^2*x^2+1)^2/c^3/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5958, 534, 1265, 911, 1167, 211}

$$-\frac{d^2(a+b\cosh^{-1}(cx))}{x}+2dex(a+b\cosh^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\cosh^{-1}(cx))+\frac{bcd^2\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{be(1-c^2x^2)(6c^2d+e)}{3c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{be^2(1-c^2x^2)^2}{9c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(6*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*e^2*(1 - c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*Ar
cCosh[c*x]))/x + 2*d*e*x*(a + b*ArcCosh[c*x]) + (e^2*x^3*(a + b*ArcCosh[c*x
]))/3 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
```

```
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^2}\,dx = -\frac{d^2(a+b\cosh^{-1}(cx))}{x} + 2dex(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\cos$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} + 2dex(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\cos$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} + 2dex(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\cos$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} + 2dex(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\cos$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{x} + 2dex(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\cos$$

$$= \frac{be(6c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2(a+b\cosh}{x}$$

$$= \frac{be(6c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2(a+b\cosh}{x}$$

Mathematica [A]
time = 0.15, size = 128, normalized size = 0.80

$$\frac{1}{3}\left(-\frac{3ad^2}{x} + 6adex + ae^2x^3 - \frac{be\sqrt{-1+cx}\sqrt{1+cx}\,(2e+c^2(18d+ex^2))}{3c^3} + \frac{b(-3d^2+6dex^2+e^2x^4)\cosh^{-1}(cx)}{x} - 3bcd^2\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-3*a*d^2)/x + 6*a*d*e*x + a*e^2*x^3 - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
2*e + c^2*(18*d + e*x^2)))/(3*c^3) + (b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcC
osh[c*x])/x - 3*b*c*d^2*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/3
```

Maple [A]
time = 1.92, size = 199, normalized size = 1.24

method	result

derivativedivides	$c\left(\frac{a\left(2c^{3}dex+\frac{e^{2}c^{3}x^{3}}{3}-\frac{c^{3}d^{2}}{x}\right)}{c^{4}}+\frac{2b\,\mathrm{arccosh}(cx)dex}{c}+\frac{b\,\mathrm{arccosh}(cx)e^{2}x^{3}}{3c}-\frac{b\,\mathrm{arccosh}(cx)d^{2}}{cx}-\frac{b\sqrt{cx-1}\,\sqrt{cx}}{}\right.$
default	$c\left(\frac{a\left(2c^{3}dex+\frac{e^{2}c^{3}x^{3}}{3}-\frac{c^{3}d^{2}}{x}\right)}{c^{4}}+\frac{2b\,\mathrm{arccosh}(cx)dex}{c}+\frac{b\,\mathrm{arccosh}(cx)e^{2}x^{3}}{3c}-\frac{b\,\mathrm{arccosh}(cx)d^{2}}{cx}-\frac{b\sqrt{cx-1}\,\sqrt{cx}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^4*(2*c^3*d*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+2*b/c*arccosh(c*x)*d*e*x+1
/3*b/c*arccosh(c*x)*e^2*x^3-b*arccosh(c*x)*d^2/c/x-b*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/(c^2*x^2-1)^(1/2)*d^2*arctan(1/(c^2*x^2-1)^(1/2))-2*b/c^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*d*e-1/9*b/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x^2-2/9*b/c^
4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2)
```

Maxima [A]
time = 0.48, size = 134, normalized size = 0.84

$$\frac{1}{3}\,ax^{3}e^{2}-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^{2}+2\,adxe+\frac{1}{9}\left(3\,x^{3}\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{2}}+\frac{2\,\sqrt{c^{2}x^{2}-1}}{c^{4}}\right)\right)be^{2}+\frac{2\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^{2}x^{2}-1}\right)bde}{c}-\frac{ad^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d^2 + 2*a*d*x*e
 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2
- 1)/c^4))*b*e^2 + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d*e/c - a*d^2
/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(141) = 282.
time = 0.37, size = 424, normalized size = 2.65

$$\frac{3\,ac^{3}x^{4}\cosh\left(1\right)^{2}+3\,ac^{3}x^{4}\sinh\left(1\right)^{2}+18\,bc^{4}d^{2}x\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+18\,ac^{3}dx^{2}\cosh\left(1\right)-9\,ac^{3}d^{2}+3\left(3\,bc^{3}d^{2}x-3\,bc^{3}d^{2}+\left(bc^{3}x^{4}-bc^{3}x\right)\cosh\left(1\right)^{2}+\left(bc^{3}x^{4}-bc^{3}x\right)\sinh\left(1\right)^{2}+6\left(bc^{3}dx^{2}-bc^{3}dx\right)\cosh\left(1\right)+2\left(3\,bc^{3}dx^{2}-3\,bc^{3}dx+\left(bc^{3}x^{4}-bc^{3}x\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)+3\left(3\,bc^{3}d^{2}x-6\,bc^{3}dx\cosh\left(1\right)-bc^{3}x\cosh\left(1\right)^{2}-bc^{3}x\sinh\left(1\right)^{2}-2\left(3\,bc^{3}dx+bc^{3}x\cosh\left(1\right)\right)\sinh\left(1\right)\right)\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+6\left(ac^{3}x^{4}\cosh\left(1\right)+3\,ac^{3}dx^{2}\right)\sinh\left(1\right)-\left(18\,bc^{2}dx\cosh\left(1\right)+\left(bc^{2}x^{3}+2\,bx\right)\cosh\left(1\right)^{2}+\left(bc^{2}x^{3}+2\,bx\right)\sinh\left(1\right)^{2}+2\left(9\,bc^{2}dx+\left(bc^{2}x^{3}+2\,bx\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\sqrt{c^{2}x^{2}-1}}{9\,c^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*x^4*cosh(1)^2 + 3*a*c^3*x^4*sinh(1)^2 + 18*b*c^4*d^2*x*arctan(
-c*x + sqrt(c^2*x^2 - 1)) + 18*a*c^3*d*x^2*cosh(1) - 9*a*c^3*d^2 + 3*(3*b*c
^3*d^2*x - 3*b*c^3*d^2 + (b*c^3*x^4 - b*c^3*x)*cosh(1)^2 + (b*c^3*x^4 - b*c
^3*x)*sinh(1)^2 + 6*(b*c^3*d*x^2 - b*c^3*d*x)*cosh(1) + 2*(3*b*c^3*d*x^2 -
```

```
3*b*c^3*d*x + (b*c^3*x^4 - b*c^3*x)*cosh(1))*sinh(1))*log(c*x + sqrt(c^2*x^
2 - 1)) + 3*(3*b*c^3*d^2*x - 6*b*c^3*d*x*cosh(1) - b*c^3*x*cosh(1)^2 - b*c^
3*x*sinh(1)^2 - 2*(3*b*c^3*d*x + b*c^3*x*cosh(1))*sinh(1))*log(-c*x + sqrt(
c^2*x^2 - 1)) + 6*(a*c^3*x^4*cosh(1) + 3*a*c^3*d*x^2)*sinh(1) - (18*b*c^2*d
*x*cosh(1) + (b*c^2*x^3 + 2*b*x)*cosh(1)^2 + (b*c^2*x^3 + 2*b*x)*sinh(1)^2
+ 2*(9*b*c^2*d*x + (b*c^2*x^3 + 2*b*x)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))
/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(cx))\,(d+ex^2)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2,x)
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2, x)
```

3.477 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=321

$$\frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\,\mathrm{co}\right.$$

```
[Out] -1/4*b*e^2*arccosh(c*x)/c^2-1/2*d^2*(a+b*arccosh(c*x))/x^2+1/2*e^2*x^2*(a+b
*arccosh(c*x))+2*d*e*(a+b*arccosh(c*x))*ln(x)+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/x-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-I*b*d*e*arcsin(c*x)^
2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*d*e*arcsin(c*x)*ln(1-(
I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-2*b*d*e*arcsin(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I
*b*d*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {272, 45, 5958, 12, 6874, 97, 92, 54, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+2de\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibde\sqrt{1-c^2x^2}\,\mathrm{Li}_2\left(e^{2i\mathrm{ArcSin}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibde\sqrt{1-c^2x^2}\,\mathrm{ArcSin}(cx)^2}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bde\sqrt{1-c^2x^2}\,\mathrm{ArcSin}(cx)\log\left(1-e^{2i\mathrm{ArcSin}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bde\sqrt{1-c^2x^2}\,\log(x)\mathrm{ArcSin}(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be^2\cosh^{-1}(cx)}{4c^2}+\frac{bcd^2\sqrt{cx-1}\,\sqrt{cx+1}}{2x}-\frac{be^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*e^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(4*c) - (b*e^2*ArcCosh[c*x])/(4*c^2) - (d^2*(a + b*ArcCosh[c*x])
)/(2*x^2) + (e^2*x^2*(a + b*ArcCosh[c*x]))/2 - (I*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 2*d*e*(a + b*ArcCosh[c*x])*Log[x] - (2*b*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]
*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*e*Sqrt[1 - c^2*x^2]*PolyLo
g[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^3}\,dx = -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh$$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\cosh^{-1}(cx)) + 2de(a+b\cosh$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{d^2(a+b\cos}{2x}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{be^2x\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{be^2\cosh^{-1}(}{4c^2}$$

Mathematica [A]
time = 0.27, size = 173, normalized size = 0.54

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2} + 2ae^2x^2 + \frac{2bd^2\left(cx\sqrt{-1+cx}\sqrt{1+cx} - \cosh^{-1}(cx)\right)}{x^2} + \frac{be^2\left(-cx\sqrt{-1+cx}\sqrt{1+cx} + 2c^2x^2\cosh^{-1}(cx) - 2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{c^2} + 8ade\log(x) + 4bde\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx) + 2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right) - \text{PolyLog}\left(2, -e^{-2\cosh^{-1}(cx)}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 + (2*b*d^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
- ArcCosh[c*x]))/x^2 + (b*e^2*(-(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 2*c^2*
```

```
x^2*ArcCosh[c*x] - 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2 + 8*a*d*e*Lo
g[x] + 4*b*d*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]
) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/4
```

Maple [A]
time = 7.72, size = 225, normalized size = 0.70

method	result
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}-\frac{b\operatorname{arccosh}(cx)^2de}{c^2}+\frac{b\operatorname{arccosh}(cx)x^2e^2}{2c^2}-\frac{b\,e^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3}-\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}-\frac{b\operatorname{arccosh}(cx)^2de}{c^2}+\frac{b\operatorname{arccosh}(cx)x^2e^2}{2c^2}-\frac{b\,e^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/c^2*x^2*e^2-1/2*a*d^2/c^2/x^2+2*a/c^2*d*e*ln(c*x)-b/c^2*arccosh(
c*x)^2*d*e+1/2*b/c^2*arccosh(c*x)*x^2*e^2-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^3-1/4*b*e^2*arccosh(c*x)/c^4+1/2*d^2*b/c/x*(c*x+1)^(1/2)*(c*x-1)^(
1/2)-1/2*d^2*b-1/2*d^2*b*arccosh(c*x)/c^2/x^2+2*b/c^2*d*e*arccosh(c*x)*ln(1
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+b/c^2*d*e*polylog(2,-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + 1/2*a*x^2*e^2 + 2*a*
d*e*log(x) - 1/2*a*d^2/x^2 + integrate(b*x*e^2*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 2*b*d*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccosh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx)) (d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c x)) (e x^2 + d)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3, x)
```

3.478 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=184

$$\frac{be^2(1-c^2x^2)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\right.$$

```
[Out] -1/3*d^2*(a+b*arccosh(c*x))/x^3-2*d*e*(a+b*arccosh(c*x))/x+e^2*x*(a+b*arcco
sh(c*x))+b*e^2*(-c^2*x^2+1)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c*d^2*(-c^2
*x^2+1)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/6*b*c*d*(c^2*d+12*e)*arctan((c^2*
x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {276, 5958, 534, 1265, 911, 1171, 396, 211}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)(c^2d+12e)}{6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be^2(1-c^2x^2)}{c\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*e^2*(1 - c^2*x^2))/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^2*
x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(3*
x^3) - (2*d*e*(a + b*ArcCosh[c*x]))/x + e^2*x*(a + b*ArcCosh[c*x]) + (b*c*d
*(c^2*d + 12*e)*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\cosh^{-1}(cx))}{x^4} dx = -\frac{d^2(a+b\cosh^{-1}(cx))}{3x^3} - \frac{2de(a+b\cosh^{-1}(cx))}{x} + e^2x(a+b\cosh^{-1}($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{3x^3} - \frac{2de(a+b\cosh^{-1}(cx))}{x} + e^2x(a+b\cosh^{-1}($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{3x^3} - \frac{2de(a+b\cosh^{-1}(cx))}{x} + e^2x(a+b\cosh^{-1}($$

$$= -\frac{d^2(a+b\cosh^{-1}(cx))}{3x^3} - \frac{2de(a+b\cosh^{-1}(cx))}{x} + e^2x(a+b\cosh^{-1}($$

$$= -\frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2(a+b\cosh^{-1}(cx))}{3x^3} - \frac{2de(a+b\cosh^{-}}{x}$$

$$= \frac{be^2(1-c^2x^2)}{c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2(a+b\cosh^{-1}(}{3x^3}$$

$$= \frac{be^2(1-c^2x^2)}{c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2(a+b\cosh^{-1}(}{3x^3}$$

Mathematica [A]
time = 0.16, size = 133, normalized size = 0.72

$$-\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x + b\left(-\frac{e^2}{c} + \frac{cd^2}{6x^2}\right)\sqrt{-1+cx}\sqrt{1+cx} - \frac{b(d^2+6dex^2-3e^2x^4)\cosh^{-1}(cx)}{3x^3} - \frac{1}{6}bcd(c^2d+12e)\operatorname{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d^2)/x^3 - (2*a*d*e)/x + a*e^2*x + b*(-(e^2/c) + (c*d^2)/(6*x^2))*S
qrt[-1 + c*x]*Sqrt[1 + c*x] - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCosh[c*x]
)/(3*x^3) - (b*c*d*(c^2*d + 12*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])
/6
```

Maple [A]
time = 1.90, size = 216, normalized size = 1.17

method	result

derivativedivides	$c^3 \left(\frac{a\left(e^2 c x - \frac{c\,d^2}{3x^3} - \frac{2cde}{x}\right)}{c^4} + \frac{b \operatorname{arccosh}(cx) e^2 x}{c^3} - \frac{b \operatorname{arccosh}(cx) d^2}{3c^3 x^3} - \frac{2b \operatorname{arccosh}(cx) de}{c^3 x} - \frac{b\sqrt{cx-1}\,\sqrt{cx+}}{6\sqrt{}} \right.$
default	$c^3 \left(\frac{a\left(e^2 c x - \frac{c\,d^2}{3x^3} - \frac{2cde}{x}\right)}{c^4} + \frac{b \operatorname{arccosh}(cx) e^2 x}{c^3} - \frac{b \operatorname{arccosh}(cx) d^2}{3c^3 x^3} - \frac{2b \operatorname{arccosh}(cx) de}{c^3 x} - \frac{b\sqrt{cx-1}\,\sqrt{cx+}}{6\sqrt{}} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^4*(e^2*c*x-1/3*c*d^2/x^3-2*c*d*e/x)+b/c^3*arccosh(c*x)*e^2*x-1/3*b
*arccosh(c*x)*d^2/c^3/x^3-2*b/c^3*arccosh(c*x)*d*e/x-1/6*b*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*arctan(1/(c^2*x^2-1)^(1/2))-2*b/c^2*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*d*e+1/
6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/x^2*d^2-b/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*e^2)
```

Maxima [A]
time = 0.47, size = 126, normalized size = 0.68

$$-\frac{1}{6}\left(\left(c^2 \arcsin\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2}\right)c + \frac{2\,\operatorname{arcosh}(cx)}{x^3}\right)bd^2 - 2\left(c \arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bde + axe^2 + \frac{\left(cx\operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)be^2}{c} - \frac{2\,ade}{x} - \frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)
/x^3)*b*d^2 - 2*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d*e + a*x*e^2 +
 (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(161) = 322.
time = 0.43, size = 390, normalized size = 2.12

$$\frac{6\,acx^4\cosh(1)^2 + 6\,acx^4\sinh(1)^2 - 12\,acdx^2\cosh(1) - 2\,acd^2 + 2\left(bc^4d^2x^3 + 12\,bc^2dx^3\cosh(1) + 12\,bc^2dx^3\sinh(1)\right)\arctan\left(-cx + \sqrt{c^2x^2-1}\right) + 2\left(bcd^2x^3 - bcd^2 + 3\left(bcx^4 - bcx^3\right)\cosh(1)^2 + 3\left(bcx^4 - bcx^3\right)\sinh(1)^2 + 6\left(bcdx^3 - bcdx^2\right)\cosh(1) + 6\left(bcdx^3 - bcdx^2 + \left(bcx^4 - bcx^3\right)\cosh(1)\right)\sinh(1)\right)\log\left(cx + \sqrt{c^2x^2-1}\right) + 2\left(bcd^2x^3 + 6\,bcdx^3\cosh(1) - 3\,bcx^3\cosh(1)^2 - 3\,bcx^3\sinh(1)^2 + 6\left(bcdx^3 - bcx^3\cosh(1)\right)\sinh(1)\right)\log\left(-cx + \sqrt{c^2x^2-1}\right) + 12\left(acx^4\cosh(1) - acdx^2\right)\sinh(1) + \left(bc^2d^2x - 6\,bx^3\cosh(1)^2 - 12\,bx^3\cosh(1)\sinh(1) - 6\,bx^3\sinh(1)^2\right)\sqrt{c^2x^2-1}}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c*x^4*cosh(1)^2 + 6*a*c*x^4*sinh(1)^2 - 12*a*c*d*x^2*cosh(1) - 2*a
*c*d^2 + 2*(b*c^4*d^2*x^3 + 12*b*c^2*d*x^3*cosh(1) + 12*b*c^2*d*x^3*sinh(1)
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c*d^2*x^3 - b*c*d^2 + 3*(b*c*x^4
- b*c*x^3)*cosh(1)^2 + 3*(b*c*x^4 - b*c*x^3)*sinh(1)^2 + 6*(b*c*d*x^3 - b*c
```

```
*d*x^2)*cosh(1) + 6*(b*c*d*x^3 - b*c*d*x^2 + (b*c*x^4 - b*c*x^3)*cosh(1))*s
inh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c*d^2*x^3 + 6*b*c*d*x^3*cosh(1)
 - 3*b*c*x^3*cosh(1)^2 - 3*b*c*x^3*sinh(1)^2 + 6*(b*c*d*x^3 - b*c*x^3*cosh(
1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) + 12*(a*c*x^4*cosh(1) - a*c*d*x^
2)*sinh(1) + (b*c^2*d^2*x - 6*b*x^3*cosh(1)^2 - 12*b*x^3*cosh(1)*sinh(1) -
6*b*x^3*sinh(1)^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\,(d + ex^2)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{acosh}(c\,x))\,\left(e\,x^2 + d\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4,x)
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4, x)
```

3.479 $\int x^4(d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=435

$$\frac{b(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3)(1-c^2x^2)}{1155c^{11}\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b(462c^6d^3+1485c^4d^2e+1540c^2de^2+525e^3)(1-c^2}{3465c^{11}\sqrt{-1+cx}\sqrt{1+cx}}$$

```
[Out] 1/5*d^3*x^5*(a+b*arccosh(c*x))+3/7*d^2*e*x^7*(a+b*arccosh(c*x))+1/3*d*e^2*x
^9*(a+b*arccosh(c*x))+1/11*e^3*x^11*(a+b*arccosh(c*x))+1/1155*b*(231*c^6*d^
3+495*c^4*d^2*e+385*c^2*d*e^2+105*e^3)*(-c^2*x^2+1)/c^11/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/3465*b*(462*c^6*d^3+1485*c^4*d^2*e+1540*c^2*d*e^2+525*e^3)*(-c^
2*x^2+1)^2/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1925*b*(77*c^6*d^3+495*c^4*d^
2*e+770*c^2*d*e^2+350*e^3)*(-c^2*x^2+1)^3/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/2)-
1/1617*b*e*(99*c^4*d^2+308*c^2*d*e+210*e^2)*(-c^2*x^2+1)^4/c^11/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+1/297*b*e^2*(11*c^2*d+15*e)*(-c^2*x^2+1)^5/c^11/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-1/121*b*e^3*(-c^2*x^2+1)^6/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A]
time = 0.44, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {276, 5958, 12, 1624, 1813, 1634}

$$\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{11}e^3x^{11}\left(a+b\cosh^{-1}(cx)\right)+\frac{be^2(1-c^2x^2)^5(11c^2d+15e)}{297c^{11}\sqrt{cx-1}\sqrt{cx+1}}-\frac{be^3(1-c^2x^2)^6}{121c^{11}\sqrt{cx-1}\sqrt{cx+1}}-\frac{be(1-c^2x^2)^4(99c^4d^2+308c^2de+210e^2)}{1617c^{11}\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)^3(77c^6d^3+495c^4d^2e+770c^2de^2+350e^3)}{1925c^{11}\sqrt{cx-1}\sqrt{cx+1}}-\frac{b(1-c^2x^2)^2(462c^6d^3+1485c^4d^2e+1540c^2de^2+525e^3)}{3465c^{11}\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3)}{1155c^{11}\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*(1 - c^2*x^2))/(
1155*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(462*c^6*d^3 + 1485*c^4*d^2*e
+ 1540*c^2*d*e^2 + 525*e^3)*(1 - c^2*x^2)^2)/(3465*c^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + (b*(77*c^6*d^3 + 495*c^4*d^2*e + 770*c^2*d*e^2 + 350*e^3)*(1 -
 c^2*x^2)^3)/(1925*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(99*c^4*d^2 +
308*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^4)/(1617*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*e^2*(11*c^2*d + 15*e)*(1 - c^2*x^2)^5)/(297*c^11*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*e^3*(1 - c^2*x^2)^6)/(121*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCosh[c*x]
))/7 + (d*e^2*x^9*(a + b*ArcCosh[c*x]))/3 + (e^3*x^11*(a + b*ArcCosh[c*x]))
/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^4\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a\right.$$

$$=\frac{b(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3)\left(1-c^2x^2\right)}{1155c^{11}\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b(462c^6d^3}{}$$

Mathematica [A]
time = 0.25, size = 276, normalized size = 0.63

$$\frac{3465ax^5(231d^3+495d^2ex^2+385de^2x^4+105e^3x^6)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(134400e^3+4480c^2e^2(121d+15ex^2)+80c^4e(9801d^2+3388dex^2+630e^2x^4)+24c^6(17787d^3+16335d^2ex^2+8470de^2x^4+1750e^3x^6)+c^{10}x^4(160083d^3+245025d^2ex^2+148225de^2x^4+33075e^3x^6)+2c^8(106722d^3x^2+147015d^2ex^4+84700de^2x^6+18375e^3x^8))}{c^{11}}+3465bx^5(231d^3+495d^2ex^2+385de^2x^4+105e^3x^6)\cosh^{-1}(cx)}{4002075}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2) +
80*c^4*e*(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 1633
5*d^2*e*x^2 + 8470*d*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 24502
5*d^2*e*x^2 + 148225*d*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^2 + 1
47015*d^2*e*x^4 + 84700*d*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^5*(231
*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcCosh[c*x])/4002075
```

Maple [A]
time = 3.51, size = 335, normalized size = 0.77

method	result
derivativedivides	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5+\frac{3}{7}d^2c^{11}e\,x^7+\frac{1}{3}d\,c^{11}e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^3c^{11}x^5}{5}+\frac{3\operatorname{arccosh}(cx)d^2c^{11}e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)d\,c^{11}e^2x^9}{3}\right.}{}$

default	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5+\frac{3}{7}d^2c^{11}e\,x^7+\frac{1}{3}d\,c^{11}e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^3c^{11}x^5}{5}+\frac{3\operatorname{arccosh}(cx)d^2c^{11}e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)d\,c^{11}e^2x^9}{3}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/c^6*(1/5*d^3*c^11*x^5+3/7*d^2*c^11*e*x^7+1/3*d*c^11*e^2*x^9+1/11*e
^3*c^11*x^11)+b/c^6*(1/5*arccosh(c*x)*d^3*c^11*x^5+3/7*arccosh(c*x)*d^2*c^1
1*e*x^7+1/3*arccosh(c*x)*d*c^11*e^2*x^9+1/11*arccosh(c*x)*e^3*c^11*x^11-1/4
002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^10*e^3*x^10+148225*c^10*d*e^2*x
^8+245025*c^10*d^2*e*x^6+36750*c^8*e^3*x^8+160083*c^10*d^3*x^4+169400*c^8*d
*e^2*x^6+294030*c^8*d^2*e*x^4+42000*c^6*e^3*x^6+213444*c^8*d^3*x^2+203280*c
^6*d*e^2*x^4+392040*c^6*d^2*e*x^2+50400*c^4*e^3*x^4+426888*c^6*d^3+271040*c
^4*d*e^2*x^2+784080*c^4*d^2*e+67200*c^2*e^3*x^2+542080*c^2*d*e^2+134400*e^3
)))
```

Maxima [A]
time = 0.31, size = 449, normalized size = 1.03

$$\frac{1}{11}ax^{11}e^3+\frac{1}{3}adx^9e^2+\frac{3}{7}ad^2x^7e+\frac{1}{5}ad^3x^5+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^3+\frac{3}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bd^2e+\frac{1}{945}\left(315x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)bde^2+\frac{1}{7623}\left(693x^{11}\operatorname{arcosh}(cx)-\left(\frac{63\sqrt{c^2x^2-1}\,x^{10}}{c^2}+\frac{70\sqrt{c^2x^2-1}\,x^8}{c^4}+\frac{80\sqrt{c^2x^2-1}\,x^6}{c^6}+\frac{96\sqrt{c^2x^2-1}\,x^4}{c^8}+\frac{128\sqrt{c^2x^2-1}\,x^2}{c^{10}}+\frac{256\sqrt{c^2x^2-1}}{c^{12}}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*x^11*e^3 + 1/3*a*d*x^9*e^2 + 3/7*a*d^2*x^7*e + 1/5*a*d^3*x^5 + 1/75*
(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x
^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arccosh(c*x) - (
5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2
- 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d^2*e + 1/945*(315*x^9*arccos
h(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*
sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2
 - 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 -
 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6
+ 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(
c^2*x^2 - 1)/c^12)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(381) = 762$.
time = 0.39, size = 822, normalized size = 1.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

[Out] 1/4002075*(363825*a*c^11*x^11*cosh(1)^3 + 363825*a*c^11*x^11*sinh(1)^3 + 13
34025*a*c^11*d*x^9*cosh(1)^2 + 1715175*a*c^11*d^2*x^7*cosh(1) + 800415*a*c^
11*d^3*x^5 + 121275*(9*a*c^11*x^11*cosh(1) + 11*a*c^11*d*x^9)*sinh(1)^2 + 3
465*(105*b*c^11*x^11*cosh(1)^3 + 105*b*c^11*x^11*sinh(1)^3 + 385*b*c^11*d*x
^9*cosh(1)^2 + 495*b*c^11*d^2*x^7*cosh(1) + 231*b*c^11*d^3*x^5 + 35*(9*b*c^
11*x^11*cosh(1) + 11*b*c^11*d*x^9)*sinh(1)^2 + 5*(63*b*c^11*x^11*cosh(1)^2
+ 154*b*c^11*d*x^9*cosh(1) + 99*b*c^11*d^2*x^7)*sinh(1))*log(c*x + sqrt(c^2
x^2 - 1)) + 17325(63*a*c^11*x^11*cosh(1)^2 + 154*a*c^11*d*x^9*cosh(1) + 9
9*a*c^11*d^2*x^7)*sinh(1) - (160083*b*c^10*d^3*x^4 + 213444*b*c^8*d^3*x^2 +
 426888*b*c^6*d^3 + 525*(63*b*c^10*x^10 + 70*b*c^8*x^8 + 80*b*c^6*x^6 + 96*
b*c^4*x^4 + 128*b*c^2*x^2 + 256*b)*cosh(1)^3 + 525*(63*b*c^10*x^10 + 70*b*c
^8*x^8 + 80*b*c^6*x^6 + 96*b*c^4*x^4 + 128*b*c^2*x^2 + 256*b)*sinh(1)^3 + 4
235*(35*b*c^10*d*x^8 + 40*b*c^8*d*x^6 + 48*b*c^6*d*x^4 + 64*b*c^4*d*x^2 + 1
28*b*c^2*d)*cosh(1)^2 + 35*(4235*b*c^10*d*x^8 + 4840*b*c^8*d*x^6 + 5808*b*c
^6*d*x^4 + 7744*b*c^4*d*x^2 + 15488*b*c^2*d + 45*(63*b*c^10*x^10 + 70*b*c^8
*x^8 + 80*b*c^6*x^6 + 96*b*c^4*x^4 + 128*b*c^2*x^2 + 256*b)*cosh(1))*sinh(1
)^2 + 49005*(5*b*c^10*d^2*x^6 + 6*b*c^8*d^2*x^4 + 8*b*c^6*d^2*x^2 + 16*b*c^
4*d^2)*cosh(1) + 5*(49005*b*c^10*d^2*x^6 + 58806*b*c^8*d^2*x^4 + 78408*b*c^
6*d^2*x^2 + 156816*b*c^4*d^2 + 315*(63*b*c^10*x^10 + 70*b*c^8*x^8 + 80*b*c^
6*x^6 + 96*b*c^4*x^4 + 128*b*c^2*x^2 + 256*b)*cosh(1)^2 + 1694*(35*b*c^10*d
*x^8 + 40*b*c^8*d*x^6 + 48*b*c^6*d*x^4 + 64*b*c^4*d*x^2 + 128*b*c^2*d)*cosh
(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^11

Sympy [C] Result contains complex when optimal does not.
time = 2.81, size = 638, normalized size = 1.47

$$\begin{cases} \frac{ad^3x^5}{5} + \frac{3ad^2ex^7}{7} + \frac{ade^2x^9}{3} + \frac{ae^3x^{11}}{11} + \frac{bd^3x^5\operatorname{acosh}(cx)}{5} + \frac{3bd^2ex^7\operatorname{acosh}(cx)}{7} + \frac{bde^2x^9\operatorname{acosh}(cx)}{3} + \frac{be^3x^{11}\operatorname{acosh}(cx)}{11} - \frac{bd^3x^4\sqrt{c^2x^2-1}}{25c} - \frac{3bd^2ex^6\sqrt{c^2x^2-1}}{49c} - \frac{bde^2x^8\sqrt{c^2x^2-1}}{27c} - \frac{be^3x^{10}\sqrt{c^2x^2-1}}{121c} - \frac{4bd^3x^2\sqrt{c^2x^2-1}}{75c^3} - \frac{18bd^2ex^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8bde^2x^6\sqrt{c^2x^2-1}}{189c^3} - \frac{10be^3x^8\sqrt{c^2x^2-1}}{1089c^3} - \frac{8bd^3\sqrt{c^2x^2-1}}{75c^5} - \frac{24bd^2ex^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16bde^2x^4\sqrt{c^2x^2-1}}{315c^5} - \frac{80be^3x^6\sqrt{c^2x^2-1}}{7623c^5} - \frac{48bd^2e\sqrt{c^2x^2-1}}{245c^7} - \frac{64bde^2x^2\sqrt{c^2x^2-1}}{945c^7} - \frac{32be^3x^4\sqrt{c^2x^2-1}}{2541c^7} - \frac{128bde^2\sqrt{c^2x^2-1}}{945c^9} - \frac{128be^3x^2\sqrt{c^2x^2-1}}{7623c^9} - \frac{256be^3\sqrt{c^2x^2-1}}{7623c^{11}} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^5}{5} + \frac{3d^2ex^7}{7} + \frac{de^2x^9}{3} + \frac{e^3x^{11}}{11}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**2+d)**3*(a+b*acosh(c*x)),x)

[Out] Piecewise((a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x**
11/11 + b*d**3*x**5*acosh(c*x)/5 + 3*b*d**2*e*x**7*acosh(c*x)/7 + b*d*e**2*
x**9*acosh(c*x)/3 + b*e**3*x**11*acosh(c*x)/11 - b*d**3*x**4*sqrt(c**2*x**2
 - 1)/(25*c) - 3*b*d**2*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*d*e**2*x**8*s
qrt(c**2*x**2 - 1)/(27*c) - b*e**3*x**10*sqrt(c**2*x**2 - 1)/(121*c) - 4*b*
d**3*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 18*b*d**2*e*x**4*sqrt(c**2*x**2 -
 1)/(245*c**3) - 8*b*d*e**2*x**6*sqrt(c**2*x**2 - 1)/(189*c**3) - 10*b*e**3
*x**8*sqrt(c**2*x**2 - 1)/(1089*c**3) - 8*b*d**3*sqrt(c**2*x**2 - 1)/(75*c*
*5) - 24*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*d*e**2*x**4*sq
rt(c**2*x**2 - 1)/(315*c**5) - 80*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(7623*c**
5) - 48*b*d**2*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(945*c**7) - 32*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(2541*c**7) - 1
28*b*d*e**2*sqrt(c**2*x**2 - 1)/(945*c**9) - 128*b*e**3*x**2*sqrt(c**2*x**2
 - 1)/(7623*c**9) - 256*b*e**3*sqrt(c**2*x**2 - 1)/(7623*c**11), Ne(c, 0)),

```
((a + I*pi*b/2)*(d**3*x**5/5 + 3*d**2*e*x**7/7 + d*e**2*x**9/3 + e**3*x**1
1/11), True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

3.480 $\int x^3(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=494

$$-\frac{b(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4)\,x(1-c^2x^2)}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b(136c^6d^3-1096c^4d^2e-1617}{38400c^7e\sqrt{}}$$

```
[Out] -1/8*d*(e*x^2+d)^4*(a+b*arccosh(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arccosh(c*x
))/e^2-1/76800*b*(1232*c^8*d^4-2536*c^6*d^3*e-7758*c^4*d^2*e^2-6615*c^2*d*e
^3-1890*e^4)*x*(-c^2*x^2+1)/c^9/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/38400*b*(13
6*c^6*d^3-1096*c^4*d^2*e-1617*c^2*d*e^2-630*e^3)*x*(-c^2*x^2+1)*(e*x^2+d)/c
^7/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9600*b*(26*c^4*d^2+201*c^2*d*e+126*e^2)*
x*(-c^2*x^2+1)*(e*x^2+d)^2/c^5/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1600*b*(11*c
^2*d+18*e)*x*(-c^2*x^2+1)*(e*x^2+d)^3/c^3/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1
00*b*x*(-c^2*x^2+1)*(e*x^2+d)^4/c/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5120*b*(1
28*c^10*d^5-480*c^6*d^3*e^2-800*c^4*d^2*e^3-525*c^2*d*e^4-126*e^5)*arctanh(
c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^10/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A]

time = 0.45, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {272, 45, 5958, 12, 580, 542, 396, 223, 212}

$$\frac{(d+ex^2)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{d(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{bx(1-c^2x^2)(d+ex^2)^4}{100ce\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx(1-c^2x^2)(11c^2d+18e)(d+ex^2)^3}{1600c^3e\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx(1-c^2x^2)(26c^4d^2+201c^2de+126e^2)(d+ex^2)^2}{9600c^5e\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\sqrt{c^2x^2-1}\,(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{5120c^{10}e^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bx(1-c^2x^2)(136c^6d^3-1096c^4d^2e-1617c^2de^2-630e^3)(d+ex^2)}{38400c^7e\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bx(1-c^2x^2)(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4)}{76800c^9e\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/76800*(b*(1232*c^8*d^4 - 2536*c^6*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*
e^3 - 1890*e^4)*x*(1 - c^2*x^2))/(c^9*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
(136*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3)*x*(1 - c^2*x^2)*(
d + e*x^2))/(38400*c^7*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(26*c^4*d^2 + 2
01*c^2*d*e + 126*e^2)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(9600*c^5*e*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (b*(11*c^2*d + 18*e)*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(
1600*c^3*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^4
)/(100*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(d + e*x^2)^4*(a + b*ArcCosh[
c*x]))/(8*e^2) + ((d + e*x^2)^5*(a + b*ArcCosh[c*x]))/(10*e^2) + (b*(128*c^
10*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*e^4 - 126*e^5)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*c^10*e^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 580

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
```

```
Dist[(e1 + f1*x^(n/2))^FracPart[r]*((e2 + f2*x^(n/2))^FracPart[r]/(e1*e2 +
f1*f2*x^n)^FracPart[r]), Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{d(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{(d+ex^2)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-$$

$$= -\frac{d(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{(d+ex^2)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-$$

$$= -\frac{d(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{(d+ex^2)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-$$

$$= \frac{bx(1-c^2x^2)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+(d$$

$$= \frac{b(11c^2d+18e)\,x(1-c^2x^2)\left(d+ex^2\right)^3}{1600c^3e\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx(1-c^2x^2)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{b(26c^4d^2+201c^2de+126e^2)\,x(1-c^2x^2)\left(d+ex^2\right)^2}{9600c^5e\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b(11c^2d+18}{1600c^3e}$$

$$= -\frac{b(136c^6d^3-1096c^4d^2e-1617c^2de^2-630e^3)\,x(1-c^2x^2)\left(d+ex^2\right.}{38400c^7e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{b(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4)\,x(1}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{b(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4)\,x(1}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{b(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4)\,x(1}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A]
time = 0.15, size = 310, normalized size = 0.63

$$\frac{1}{4}ad^3x^4+\frac{1}{2}ad^2ex^6+\frac{3}{8}ade^2x^8+\frac{1}{10}ae^3x^{10}-\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\,(1890e^3+315c^2e^2(25d+4ex^2)+6c^4e(2000d^2+875dex^2+168e^2x^4)+8c^6(900d^3+1000d^2ex^2+525de^2x^4+108e^3x^6)+16c^8(300d^3x^2+400d^2ex^4+225de^2x^6+48e^3x^8))}{76800c^9}+\frac{1}{40}bx^4(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6)\cosh^{-1}(cx)-\frac{b(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3)\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{5120c^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*d^3*x^4)/4 + (a*d^2*e*x^6)/2 + (3*a*d*e^2*x^8)/8 + (a*e^3*x^10)/10 - (b*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6
```

```
*c^4*e*(2000*d^2 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e
*x^2 + 525*d*e^2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 +
 225*d*e^2*x^6 + 48*e^3*x^8)))/(76800*c^9) + (b*x^4*(10*d^3 + 20*d^2*e*x^2
+ 15*d*e^2*x^4 + 4*e^3*x^6)*ArcCosh[c*x])/40 - (b*(480*c^6*d^3 + 800*c^4*d^
2*e + 525*c^2*d*e^2 + 126*e^3)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(51
20*c^10)
```

Maple [A]
time = 3.40, size = 745, normalized size = 1.51

method	result
derivativedivides	$-\frac{a\left(\frac{c^2 d\left(c^2 e\,x^2+c^2 d\right)^4}{4}-\frac{\left(c^2 e\,x^2+c^2 d\right)^5}{5}\right)}{2c^6e^2}-\frac{b\,c^3 e\sqrt{cx-1}\,\sqrt{cx+1}\,d^2x^5}{12}-\frac{7bc\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,d\,x^5}{128}+$
default	$-\frac{a\left(\frac{c^2 d\left(c^2 e\,x^2+c^2 d\right)^4}{4}-\frac{\left(c^2 e\,x^2+c^2 d\right)^5}{5}\right)}{2c^6e^2}-\frac{b\,c^3 e\sqrt{cx-1}\,\sqrt{cx+1}\,d^2x^5}{12}-\frac{7bc\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,d\,x^5}{128}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*a/c^6/e^2*(1/4*c^2*d*(c^2*e*x^2+c^2*d)^4-1/5*(c^2*e*x^2+c^2*d)^
5)-1/12*b*c^3*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*x^5-7/128*b*c*e^2*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*d*x^5+1/2*b*c^4*e*arccosh(c*x)*d^2*x^6+3/8*b*c^4*e^2*arc
cosh(c*x)*d*x^8-5/48*b*c*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*x^3-63/2560*b/c^
6*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2
))-3/64*b*c^3*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^7+1/40*b*c^4/e^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^5*ln(c*x+(c^2*x^2-1)^(1/2))-5/32*b/
c^2*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2
))*d^2-105/1024*b/c^4*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(
c*x+(c^2*x^2-1)^(1/2))*d-5/32*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-105/1
024*b*d*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-35/512*b*d*e^2*x^3*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c+1/10*b*c^4*e^3*arccosh(c*x)*x^10-1/100*b*c^3*e^3*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*x^9-9/800*b*c*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^7-63/
2560*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-21/1280*b*e^3*x^3*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^3-21/1600*b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/40*b*
c^4/e^2*arccosh(c*x)*d^5+1/4*b*arccosh(c*x)*d^3*c^4*x^4-1/16*b*(c*x-1)^(1/2
)*(c*x+1)^(1/2)*d^3*c^3*x^3-3/32*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^
(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^3-3/32*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1
/2))
```

Maxima [A]
time = 0.28, size = 485, normalized size = 0.98

$$\frac{1}{10}\,ax^{10}e^3+\frac{3}{8}\,adx^8e^2+\frac{1}{2}\,ad^2x^6e+\frac{1}{4}\,ad^3x^4+\frac{1}{32}\left(8\,x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd^3+\frac{1}{96}\left(48\,x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bd^2e+\frac{1}{1024}\left(384\,x^8\operatorname{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)bde^2+\frac{1}{12800}\left(1280\,x^{10}\operatorname{arcosh}(cx)-\left(\frac{128\sqrt{c^2x^2-1}\,x^9}{c^2}+\frac{144\sqrt{c^2x^2-1}\,x^7}{c^4}+\frac{168\sqrt{c^2x^2-1}\,x^5}{c^6}+\frac{210\sqrt{c^2x^2-1}\,x^3}{c^8}+\frac{315\sqrt{c^2x^2-1}\,x}{c^{10}}+\frac{315\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^{11}}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*x^10*e^3 + 3/8*a*d*x^8*e^2 + 1/2*a*d^2*x^6*e + 1/4*a*d^3*x^4 + 1/32*
(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/
c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3 + 1/96*(48*x^6*a
rccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 +
 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*
c)*b*d^2*e + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 +
 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2
*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*d*e^2
+ 1/12800*(1280*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sq
rt(c^2*x^2 - 1)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2
- 1)*x^3/c^8 + 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*c)/c^11)*c)*b*e^3
```

Fricas [A]
time = 0.36, size = 815, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*x^10*cosh(1)^3 + 7680*a*c^10*x^10*sinh(1)^3 + 28800*a*
c^10*d*x^8*cosh(1)^2 + 38400*a*c^10*d^2*x^6*cosh(1) + 19200*a*c^10*d^3*x^4
+ 5760*(4*a*c^10*x^10*cosh(1) + 5*a*c^10*d*x^8)*sinh(1)^2 + 15*(1280*b*c^10
*d^3*x^4 - 480*b*c^6*d^3 + 2*(256*b*c^10*x^10 - 63*b)*cosh(1)^3 + 2*(256*b*
c^10*x^10 - 63*b)*sinh(1)^3 + 15*(128*b*c^10*d*x^8 - 35*b*c^2*d)*cosh(1)^2
+ 3*(640*b*c^10*d*x^8 - 175*b*c^2*d + 2*(256*b*c^10*x^10 - 63*b)*cosh(1))*s
inh(1)^2 + 160*(16*b*c^10*d^2*x^6 - 5*b*c^4*d^2)*cosh(1) + 2*(1280*b*c^10*d
^2*x^6 - 400*b*c^4*d^2 + 3*(256*b*c^10*x^10 - 63*b)*cosh(1)^2 + 15*(128*b*c
^10*d*x^8 - 35*b*c^2*d)*cosh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 38
40*(6*a*c^10*x^10*cosh(1)^2 + 15*a*c^10*d*x^8*cosh(1) + 10*a*c^10*d^2*x^6)*
sinh(1) - (4800*b*c^9*d^3*x^3 + 7200*b*c^7*d^3*x + 6*(128*b*c^9*x^9 + 144*b
*c^7*x^7 + 168*b*c^5*x^5 + 210*b*c^3*x^3 + 315*b*c*x)*cosh(1)^3 + 6*(128*b*
c^9*x^9 + 144*b*c^7*x^7 + 168*b*c^5*x^5 + 210*b*c^3*x^3 + 315*b*c*x)*sinh(1
)^3 + 75*(48*b*c^9*d*x^7 + 56*b*c^7*d*x^5 + 70*b*c^5*d*x^3 + 105*b*c^3*d*x)
*cosh(1)^2 + 3*(1200*b*c^9*d*x^7 + 1400*b*c^7*d*x^5 + 1750*b*c^5*d*x^3 + 26
25*b*c^3*d*x + 6*(128*b*c^9*x^9 + 144*b*c^7*x^7 + 168*b*c^5*x^5 + 210*b*c^3
*x^3 + 315*b*c*x)*cosh(1))*sinh(1)^2 + 800*(8*b*c^9*d^2*x^5 + 10*b*c^7*d^2*
x^3 + 15*b*c^5*d^2*x)*cosh(1) + 2*(3200*b*c^9*d^2*x^5 + 4000*b*c^7*d^2*x^3
+ 6000*b*c^5*d^2*x + 9*(128*b*c^9*x^9 + 144*b*c^7*x^7 + 168*b*c^5*x^5 + 210
*b*c^3*x^3 + 315*b*c*x)*cosh(1)^2 + 75*(48*b*c^9*d*x^7 + 56*b*c^7*d*x^5 + 7
0*b*c^5*d*x^3 + 105*b*c^3*d*x)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^10
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 604, normalized size = 1.22

$$\begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\operatorname{acosh}(cx)}{4} + \frac{bd^2ex^6\operatorname{acosh}(cx)}{2} + \frac{3bde^2x^8\operatorname{acosh}(cx)}{8} + \frac{be^3x^{10}\operatorname{acosh}(cx)}{10} - \frac{bd^3x^3\sqrt{c^2x^2-1}}{16c} - \frac{bd^2ex^5\sqrt{c^2x^2-1}}{12c} - \frac{3bde^2x^7\sqrt{c^2x^2-1}}{64c} - \frac{be^3x^9\sqrt{c^2x^2-1}}{100c} - \frac{3bd^3x\sqrt{c^2x^2-1}}{32c^3} - \frac{5bd^2ex^3\sqrt{c^2x^2-1}}{48c^3} - \frac{7bde^2x^5\sqrt{c^2x^2-1}}{128c^3} - \frac{9be^3x^7\sqrt{c^2x^2-1}}{800c^3} - \frac{3bd^3\operatorname{acosh}(cx)}{32c^4} - \frac{5bd^2ex\sqrt{c^2x^2-1}}{32c^5} - \frac{35bde^2x^3\sqrt{c^2x^2-1}}{512c^5} - \frac{21be^3x^5\sqrt{c^2x^2-1}}{1600c^5} - \frac{5bd^2e\operatorname{acosh}(cx)}{32c^6} - \frac{105bde^2x\sqrt{c^2x^2-1}}{1024c^7} - \frac{21be^3x^3\sqrt{c^2x^2-1}}{1280c^7} - \frac{105bde^2\operatorname{acosh}(cx)}{1024c^8} - \frac{63be^3x\sqrt{c^2x^2-1}}{2560c^9} - \frac{63be^3\operatorname{acosh}(cx)}{2560c^{10}} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**
10/10 + b*d**3*x**4*acosh(c*x)/4 + b*d**2*e*x**6*acosh(c*x)/2 + 3*b*d*e**2*
x**8*acosh(c*x)/8 + b*e**3*x**10*acosh(c*x)/10 - b*d**3*x**3*sqrt(c**2*x**2
 - 1)/(16*c) - b*d**2*e*x**5*sqrt(c**2*x**2 - 1)/(12*c) - 3*b*d*e**2*x**7*s
qrt(c**2*x**2 - 1)/(64*c) - b*e**3*x**9*sqrt(c**2*x**2 - 1)/(100*c) - 3*b*d
**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(
48*c**3) - 7*b*d*e**2*x**5*sqrt(c**2*x**2 - 1)/(128*c**3) - 9*b*e**3*x**7*s
qrt(c**2*x**2 - 1)/(800*c**3) - 3*b*d**3*acosh(c*x)/(32*c**4) - 5*b*d**2*e*
x*sqrt(c**2*x**2 - 1)/(32*c**5) - 35*b*d*e**2*x**3*sqrt(c**2*x**2 - 1)/(512
*c**5) - 21*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(1600*c**5) - 5*b*d**2*e*acosh(
c*x)/(32*c**6) - 105*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 21*b*e**3
*x**3*sqrt(c**2*x**2 - 1)/(1280*c**7) - 105*b*d*e**2*acosh(c*x)/(1024*c**8)
 - 63*b*e**3*x*sqrt(c**2*x**2 - 1)/(2560*c**9) - 63*b*e**3*acosh(c*x)/(2560
*c**10), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**4/4 + d**2*e*x**6/2 + 3*d*e**2
*x**8/8 + e**3*x**10/10), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(e\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

3.481 $\int x^2(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=365

$$\frac{b(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)\,(1-c^2x^2)}{315c^9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b(105c^6d^3+378c^4d^2e+405c^2de^2+140e^3)\,(1-c^2x^2)^2}{945c^9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/3*d^3*x^3*(a+b*arccosh(c*x))+3/5*d^2*e*x^5*(a+b*arccosh(c*x))+3/7*d*e^2*x
^7*(a+b*arccosh(c*x))+1/9*e^3*x^9*(a+b*arccosh(c*x))+1/315*b*(105*c^6*d^3+1
89*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)/c^9/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/945*b*(105*c^6*d^3+378*c^4*d^2*e+405*c^2*d*e^2+140*e^3)*(-c^2*x^2+1)
^2/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*e*(63*c^4*d^2+135*c^2*d*e+70*e^2
)*(-c^2*x^2+1)^3/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/441*b*e^2*(27*c^2*d+28*e
)*(-c^2*x^2+1)^4/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*e^3*(-c^2*x^2+1)^5/
c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {276, 5958, 12, 1624, 1813, 1634}

$$\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{be^2(1-c^2x^2)^4(27c^2d+28e)}{441c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{be^3(1-c^2x^2)^5}{81c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{be(1-c^2x^2)^3(63c^4d^2+135c^2de+70e^2)}{525c^9\sqrt{cx-1}\sqrt{cx+1}}-\frac{b(1-c^2x^2)^2(105c^6d^3+378c^4d^2e+405c^2de^2+140e^3)}{945c^9\sqrt{cx-1}\sqrt{cx+1}}+\frac{b(1-c^2x^2)(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)}{315c^9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2))/(3
15*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(105*c^6*d^3 + 378*c^4*d^2*e + 40
5*c^2*d*e^2 + 140*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^4)/(44
1*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 + (3*d^2*e*x^5*
(a + b*ArcCosh[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcCosh[c*x]))/7 + (e^3*x^9*(
a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a\right.$$

$$= \frac{b(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b(105c^6d^3+}{}$$

Mathematica [A]

time = 0.21, size = 236, normalized size = 0.65

$$\frac{315ax^3(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(4480e^3+80c^2e^2\left(243d+28ex^2\right)+24c^4e\left(1323d^2+405dex^2+70e^2x^4\right)+2c^6\left(11025d^3+7938d^2ex^2+3645de^2x^4+700e^3x^6\right)+c^8\left(11025d^3x^2+11907d^2ex^4+6075de^2x^6+1225e^3x^8\right)\right)}{c^9}+315bx^3(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)\cosh^{-1}(cx)}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c^4
*e*(1323*d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e*x^
2 + 3645*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^4 +
6075*d*e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*x^2 +
 135*d*e^2*x^4 + 35*e^3*x^6)*ArcCosh[c*x])/99225
```

Maple [A]

time = 2.75, size = 289, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^3c^9x^3}{3}+\frac{3\operatorname{arccosh}(cx)d^2c^9e\,x^5}{5}+\frac{3\operatorname{arccosh}(cx)d\,c^9e^2x^7}{7}+\frac{\operatorname{arccosh}(c\right.}{9}}{}$

default	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)d^3c^9x^3}{3}+\frac{3\operatorname{arccosh}(cx)d^2c^9e\,x^5}{5}+\frac{3\operatorname{arccosh}(cx)d\,c^9e^2x^7}{7}+\operatorname{arccosh}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^6*(1/3*d^3*c^9*x^3+3/5*d^2*c^9*e*x^5+3/7*d*c^9*e^2*x^7+1/9*e^3*c
^9*x^9)+b/c^6*(1/3*arccosh(c*x)*d^3*c^9*x^3+3/5*arccosh(c*x)*d^2*c^9*e*x^5+
3/7*arccosh(c*x)*d*c^9*e^2*x^7+1/9*arccosh(c*x)*e^3*c^9*x^9-1/99225*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^3*x^8+6075*c^8*d*e^2*x^6+11907*c^8*d^2*e*x
^4+1400*c^6*e^3*x^6+11025*c^8*d^3*x^2+7290*c^6*d*e^2*x^4+15876*c^6*d^2*e*x^
2+1680*c^4*e^3*x^4+22050*c^6*d^3+9720*c^4*d*e^2*x^2+31752*c^4*d^2*e+2240*c^
2*e^3*x^2+19440*c^2*d*e^2+4480*e^3)))
```

Maxima [A]
time = 0.28, size = 372, normalized size = 1.02

$$\frac{1}{9}ax^9e^3+\frac{3}{7}adx^7e^2+\frac{3}{5}ad^2x^5e+\frac{1}{3}ad^3x^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3+\frac{1}{25}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2e+\frac{3}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bde^2+\frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9*e^3 + 3/7*a*d*x^7*e^2 + 3/5*a*d^2*x^5*e + 1/3*a*d^3*x^3 + 1/9*(3*
x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))
*b*d^3 + 1/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(
c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e + 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^2 + 1/283
5*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 -
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(319) = 638.
time = 0.37, size = 720, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*x^9*cosh(1)^3 + 11025*a*c^9*x^9*sinh(1)^3 + 42525*a*c^
9*d*x^7*cosh(1)^2 + 59535*a*c^9*d^2*x^5*cosh(1) + 33075*a*c^9*d^3*x^3 + 472
5*(7*a*c^9*x^9*cosh(1) + 9*a*c^9*d*x^7)*sinh(1)^2 + 315*(35*b*c^9*x^9*cosh(
```

```
1)^3 + 35*b*c^9*x^9*sinh(1)^3 + 135*b*c^9*d*x^7*cosh(1)^2 + 189*b*c^9*d^2*x
^5*cosh(1) + 105*b*c^9*d^3*x^3 + 15*(7*b*c^9*x^9*cosh(1) + 9*b*c^9*d*x^7)*s
inh(1)^2 + 3*(35*b*c^9*x^9*cosh(1)^2 + 90*b*c^9*d*x^7*cosh(1) + 63*b*c^9*d^
2*x^5)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 945*(35*a*c^9*x^9*cosh(1)^2
+ 90*a*c^9*d*x^7*cosh(1) + 63*a*c^9*d^2*x^5)*sinh(1) - (11025*b*c^8*d^3*x^2
 + 22050*b*c^6*d^3 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4 + 64*b*
c^2*x^2 + 128*b)*cosh(1)^3 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4
 + 64*b*c^2*x^2 + 128*b)*sinh(1)^3 + 1215*(5*b*c^8*d*x^6 + 6*b*c^6*d*x^4 +
8*b*c^4*d*x^2 + 16*b*c^2*d)*cosh(1)^2 + 15*(405*b*c^8*d*x^6 + 486*b*c^6*d*x
^4 + 648*b*c^4*d*x^2 + 1296*b*c^2*d + 7*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b
*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1))*sinh(1)^2 + 3969*(3*b*c^8*d^2*x^4
 + 4*b*c^6*d^2*x^2 + 8*b*c^4*d^2)*cosh(1) + 3*(3969*b*c^8*d^2*x^4 + 5292*b*
c^6*d^2*x^2 + 10584*b*c^4*d^2 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*
x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1)^2 + 810*(5*b*c^8*d*x^6 + 6*b*c^6*d*x^4
+ 8*b*c^4*d*x^2 + 16*b*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [C] Result contains complex when optimal does not.
time = 1.47, size = 532, normalized size = 1.46

$$\begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3\operatorname{acosh}(cx)}{3} + \frac{3bd^2ex^5\operatorname{acosh}(cx)}{5} + \frac{3bde^2x^7\operatorname{acosh}(cx)}{7} + \frac{be^3x^9\operatorname{acosh}(cx)}{9} - \frac{bd^3x^2\sqrt{c^2x^2-1}}{9c} - \frac{3bd^2ex^4\sqrt{c^2x^2-1}}{25c} - \frac{3bde^2x^6\sqrt{c^2x^2-1}}{49c} - \frac{be^3x^8\sqrt{c^2x^2-1}}{81c} - \frac{2bd^3\sqrt{c^2x^2-1}}{9c^3} - \frac{4bd^2ex^2\sqrt{c^2x^2-1}}{25c^3} - \frac{18bde^2x^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8be^3x^6\sqrt{c^2x^2-1}}{567c^3} - \frac{8bd^2e\sqrt{c^2x^2-1}}{25c^5} - \frac{24bde^2x^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16be^3x^4\sqrt{c^2x^2-1}}{945c^5} - \frac{48bde^2\sqrt{c^2x^2-1}}{245c^7} - \frac{64be^3x^2\sqrt{c^2x^2-1}}{2835c^7} - \frac{128be^3\sqrt{c^2x^2-1}}{2835c^9} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x
**9/9 + b*d**3*x**3*acosh(c*x)/3 + 3*b*d**2*e*x**5*acosh(c*x)/5 + 3*b*d*e**
2*x**7*acosh(c*x)/7 + b*e**3*x**9*acosh(c*x)/9 - b*d**3*x**2*sqrt(c**2*x**2
 - 1)/(9*c) - 3*b*d**2*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 3*b*d*e**2*x**6*
sqrt(c**2*x**2 - 1)/(49*c) - b*e**3*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 2*b*d
**3*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(25*
c**3) - 18*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**3*x**6*sqr
t(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*e*sqrt(c**2*x**2 - 1)/(25*c**5) - 24
*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*x**4*sqrt(c**2*x*
*2 - 1)/(945*c**5) - 48*b*d*e**2*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**3
*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**3*sqrt(c**2*x**2 - 1)/(283
5*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e*
*2*x**7/7 + e**3*x**9/9), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2 + d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

3.482 $\int x(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=358

$$\frac{5b(2c^2d+e)\left(40c^4d^2+40c^2de+21e^2\right)x(1-c^2x^2)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b\left(104c^4d^2+104c^2de+35e^2\right)x(1-c^2x^2)\,(d+ex^2)}{1536c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{7b(2$$

```
[Out] 1/8*(e*x^2+d)^4*(a+b*arccosh(c*x))/e+5/3072*b*(2*c^2*d+e)*(40*c^4*d^2+40*c^
2*d*e+21*e^2)*x*(-c^2*x^2+1)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1536*b*(104*
c^4*d^2+104*c^2*d*e+35*e^2)*x*(-c^2*x^2+1)*(e*x^2+d)/c^5/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+7/384*b*(2*c^2*d+e)*x*(-c^2*x^2+1)*(e*x^2+d)^2/c^3/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+1/64*b*x*(-c^2*x^2+1)*(e*x^2+d)^3/c/(c*x-1)^(1/2)/(c*x+1)^(1/
2)-1/1024*b*(128*c^8*d^4+256*c^6*d^3*e+288*c^4*d^2*e^2+160*c^2*d*e^3+35*e^4
)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^8/e/(c*x-1)^(1/2)/(c*x
+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5957, 916, 427, 542, 396, 223, 212}

$$\frac{(d+ex^2)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}+\frac{bx(1-c^2x^2)(d+ex^2)^3}{64c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{7bx(1-c^2x^2)(2c^2d+e)(d+ex^2)^2}{384c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bx(1-c^2x^2)(2c^2d+e)\left(40c^4d^2+40c^2de+21e^2\right)}{3072c^7\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx(1-c^2x^2)\left(104c^4d^2+104c^2de+35e^2\right)(d+ex^2)}{1536c^5\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\left(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{1024c^8e\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (5*b*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e + 21*e^2)*x*(1 - c^2*x^2))/(307
2*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(104*c^4*d^2 + 104*c^2*d*e + 35*e^
2)*x*(1 - c^2*x^2)*(d + e*x^2))/(1536*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
7*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(384*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(64*c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + ((d + e*x^2)^4*(a + b*ArcCosh[c*x]))/(8*e) - (b*(128*c^8*d^4 +
256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*Sqrt[-1 + c^2*x^2
]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(1024*c^8*e*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 916

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^FracPart[m]/(d*f
 + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{(bc)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{8e}$$

$$= \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+c^2x^2}}\,d}{8e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{\left(b\sqrt{-1\right.}{}$$

$$= \frac{7b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{384c^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(d}{}$$

$$= \frac{b\left(104c^4d^2+104c^2de+35e^2\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7b\left(2c^2d+e\right)x}{384c^3\sqrt{-1}}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{153}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{153}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{153}$$

Mathematica [A]
time = 0.25, size = 262, normalized size = 0.73

$$\frac{cx\left(384ac^7x\left(4d^3+6d^2ex^2+4de^2x^4+e^3x^6\right)-b\sqrt{-1+cx}\,\sqrt{1+cx}\left(105e^3+10c^2e^2\left(48d+7ex^2\right)+8c^4e\left(108d^2+40dex^2+7e^2x^4\right)+16c^6\left(48d^3+36d^2ex^2+16de^2x^4+3e^3x^6\right)\right)\right)+384bc^8x^2\left(4d^3+6d^2ex^2+4de^2x^4+e^3x^6\right)\cosh^{-1}(cx)-3b\left(256c^6d^3+288c^4d^2e+160c^2de^2+35e^3\right)\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{3072c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(108
*d^2 + 40*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^2*x
^4 + 3*e^3*x^6))) + 384*b*c^8*x^2*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*
x^6)*ArcCosh[c*x] - 3*b*(256*c^6*d^3 + 288*c^4*d^2*e + 160*c^2*d*e^2 + 35*e
^3)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(3072*c^8)
```

Maple [A]
time = 3.39, size = 614, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{(c^2 e\,x^2+c^2 d)^4 a}{8c^6 e}+\frac{b\,c^2 \operatorname{arccosh}(cx) d^4}{8e}+\frac{b \operatorname{arccosh}(cx) d^3 c^2 x^2}{2}+\frac{3b\,c^2 e \operatorname{arccosh}(cx) d^2 x^4}{4}+\frac{b\,c^2 e^2 \operatorname{arccosh}(cx) d\,x^6}{2}+\frac{b\,c^2 e^3 \operatorname{arccosh}(cx) x}{8}}{}$
default	$\frac{\frac{(c^2 e\,x^2+c^2 d)^4 a}{8c^6 e}+\frac{b\,c^2 \operatorname{arccosh}(cx) d^4}{8e}+\frac{b \operatorname{arccosh}(cx) d^3 c^2 x^2}{2}+\frac{3b\,c^2 e \operatorname{arccosh}(cx) d^2 x^4}{4}+\frac{b\,c^2 e^2 \operatorname{arccosh}(cx) d\,x^6}{2}+\frac{b\,c^2 e^3 \operatorname{arccosh}(cx) x}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/8*(c^2*e*x^2+c^2*d)^4*a/c^6/e+1/8*b*c^2/e*arccosh(c*x)*d^4+1/2*b*a
rccosh(c*x)*d^3*c^2*x^2+3/4*b*c^2*e*arccosh(c*x)*d^2*x^4+1/2*b*c^2*e^2*arcc
osh(c*x)*d*x^6+1/8*b*c^2*e^3*arccosh(c*x)*x^8-1/8*b*c^2/e*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-1/4*b*c*d^3*x*(c
*x-1)^(1/2)*(c*x+1)^(1/2)-3/16*b*c*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*x^3-1/
12*b*c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^5-1/64*b*c*e^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*x^7-1/4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(
c^2*x^2-1)^(1/2))*d^3-9/32*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/48*b*d
*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-7/384*b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-9/32*b/c^2*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2))*d^2-5/32*b*d*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-35/1
536*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-5/32*b/c^4*e^2*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d-35/1024*b*e^3*x
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-35/1024*b/c^6*e^3*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [A]
time = 0.27, size = 407, normalized size = 1.14

$$\frac{1}{8}ax^8e^3+\frac{1}{2}adx^6e^2+\frac{3}{4}ad^2x^4e+\frac{1}{2}ad^3x^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^3+\frac{3}{32}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bd^2e+\frac{1}{96}\left(48x^6\operatorname{arcosh}(cx)-\left(\frac{8\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^7}\right)c\right)bde^2+\frac{1}{3072}\left(384x^8\operatorname{arcosh}(cx)-\left(\frac{48\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105\sqrt{c^2x^2-1}\,x}{c^8}+\frac{105\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^9}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^3 + 1/2*a*d*x^6*e^2 + 3/4*a*d^2*x^4*e + 1/2*a*d^3*x^2 + 1/4*(2*
x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^
2 - 1)*c)/c^3))*b*d^3 + 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3
/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c
^5)*c)*b*d^2*e + 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 +
 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x
 + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e^2 + 1/3072*(384*x^8*arccosh(c*x) -
(48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*
```

```
x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*c)/c^9)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(320) = 640$.
time = 0.40, size = 705, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*x^8*cosh(1)^3 + 384*a*c^8*x^8*sinh(1)^3 + 1536*a*c^8*d*x^
6*cosh(1)^2 + 2304*a*c^8*d^2*x^4*cosh(1) + 1536*a*c^8*d^3*x^2 + 384*(3*a*c^
8*x^8*cosh(1) + 4*a*c^8*d*x^6)*sinh(1)^2 + 3*(512*b*c^8*d^3*x^2 - 256*b*c^6
*d^3 + (128*b*c^8*x^8 - 35*b)*cosh(1)^3 + (128*b*c^8*x^8 - 35*b)*sinh(1)^3
+ 32*(16*b*c^8*d*x^6 - 5*b*c^2*d)*cosh(1)^2 + (512*b*c^8*d*x^6 - 160*b*c^2*
d + 3*(128*b*c^8*x^8 - 35*b)*cosh(1))*sinh(1)^2 + 96*(8*b*c^8*d^2*x^4 - 3*b
*c^4*d^2)*cosh(1) + (768*b*c^8*d^2*x^4 - 288*b*c^4*d^2 + 3*(128*b*c^8*x^8 -
 35*b)*cosh(1)^2 + 64*(16*b*c^8*d*x^6 - 5*b*c^2*d)*cosh(1))*sinh(1))*log(c*
x + sqrt(c^2*x^2 - 1)) + 384*(3*a*c^8*x^8*cosh(1)^2 + 8*a*c^8*d*x^6*cosh(1)
 + 6*a*c^8*d^2*x^4)*sinh(1) - (768*b*c^7*d^3*x + (48*b*c^7*x^7 + 56*b*c^5*x
^5 + 70*b*c^3*x^3 + 105*b*c*x)*cosh(1)^3 + (48*b*c^7*x^7 + 56*b*c^5*x^5 + 7
0*b*c^3*x^3 + 105*b*c*x)*sinh(1)^3 + 32*(8*b*c^7*d*x^5 + 10*b*c^5*d*x^3 + 1
5*b*c^3*d*x)*cosh(1)^2 + (256*b*c^7*d*x^5 + 320*b*c^5*d*x^3 + 480*b*c^3*d*x
 + 3*(48*b*c^7*x^7 + 56*b*c^5*x^5 + 70*b*c^3*x^3 + 105*b*c*x)*cosh(1))*sinh
(1)^2 + 288*(2*b*c^7*d^2*x^3 + 3*b*c^5*d^2*x)*cosh(1) + (576*b*c^7*d^2*x^3
+ 864*b*c^5*d^2*x + 3*(48*b*c^7*x^7 + 56*b*c^5*x^5 + 70*b*c^3*x^3 + 105*b*c
*x)*cosh(1)^2 + 64*(8*b*c^7*d*x^5 + 10*b*c^5*d*x^3 + 15*b*c^3*d*x)*cosh(1))
*sinh(1))*sqrt(c^2*x^2 - 1))/c^8
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 490, normalized size = 1.37

$$\begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{acosh}(cx)}{2} + \frac{3bd^2ex^4\operatorname{acosh}(cx)}{4} + \frac{bde^2x^6\operatorname{acosh}(cx)}{2} + \frac{be^3x^8\operatorname{acosh}(cx)}{8} - \frac{bd^3x\sqrt{c^2x^2-1}}{4c} - \frac{3bd^2ex^3\sqrt{c^2x^2-1}}{16c} - \frac{bde^2x^5\sqrt{c^2x^2-1}}{12c} - \frac{be^3x^7\sqrt{c^2x^2-1}}{64c} - \frac{bd^3\operatorname{acosh}(cx)}{4c^2} - \frac{9bd^2ex\sqrt{c^2x^2-1}}{32c^3} - \frac{5bde^2x^3\sqrt{c^2x^2-1}}{48c^3} - \frac{7be^3x^5\sqrt{c^2x^2-1}}{384c^3} - \frac{9bd^2e\operatorname{acosh}(cx)}{32c^4} - \frac{5bde^2x\sqrt{c^2x^2-1}}{32c^5} - \frac{35be^3x^3\sqrt{c^2x^2-1}}{1536c^5} - \frac{5bde^2\operatorname{acosh}(cx)}{32c^6} - \frac{35be^3x\sqrt{c^2x^2-1}}{1024c^7} - \frac{35be^3\operatorname{acosh}(cx)}{1024c^8} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**
8/8 + b*d**3*x**2*acosh(c*x)/2 + 3*b*d**2*e*x**4*acosh(c*x)/4 + b*d*e**2*x*
*6*acosh(c*x)/2 + b*e**3*x**8*acosh(c*x)/8 - b*d**3*x*sqrt(c**2*x**2 - 1)/(
4*c) - 3*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*d*e**2*x**5*sqrt(c**2
*x**2 - 1)/(12*c) - b*e**3*x**7*sqrt(c**2*x**2 - 1)/(64*c) - b*d**3*acosh(c
*x)/(4*c**2) - 9*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d*e**2*x**3
*sqrt(c**2*x**2 - 1)/(48*c**3) - 7*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(384*c**
```

```
3) - 9*b*d**2*e*acosh(c*x)/(32*c**4) - 5*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(32
*c**5) - 35*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e**2*acosh(
c*x)/(32*c**6) - 35*b*e**3*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*e**3*ac
osh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**2/2 + 3*d**2*e*x*
*4/4 + d*e**2*x**6/2 + e**3*x**8/8), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

3.483 $\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=287

$$\frac{b(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3)\left(1-c^2x^2\right)}{35c^7\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{be(35c^4d^2+42c^2de+15e^2)\left(1-c^2x^2\right)^2}{105c^7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3be^2(7c^2d+5e)\,(1}{175c^7\sqrt{-1+cx}}$$

```
[Out] d^3*x*(a+b*arccosh(c*x))+d^2*e*x^3*(a+b*arccosh(c*x))+3/5*d*e^2*x^5*(a+b*ar
ccosh(c*x))+1/7*e^3*x^7*(a+b*arccosh(c*x))+1/35*b*(35*c^6*d^3+35*c^4*d^2*e+
21*c^2*d*e^2+5*e^3)*(-c^2*x^2+1)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/105*b*e*
(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^2/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+3/175*b*e^2*(7*c^2*d+5*e)*(-c^2*x^2+1)^3/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-1/49*b*e^3*(-c^2*x^2+1)^4/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {200, 5908, 12, 1624, 1813, 1864}

$$d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3be^2\left(1-c^2x^2\right)^3\left(7c^2d+5e\right)}{175c^7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be^3\left(1-c^2x^2\right)^4}{49c^7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be\left(1-c^2x^2\right)^2\left(35c^4d^2+42c^2de+15e^2\right)}{105c^7\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\left(1-c^2x^2\right)\left(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3\right)}{35c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*(1 - c^2*x^2))/(35*c^
7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(
1 - c^2*x^2)^2)/(105*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(7*c^2*d
+ 5*e)*(1 - c^2*x^2)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1
- c^2*x^2)^4)/(49*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[
c*x]) + d^2*e*x^3*(a + b*ArcCosh[c*x]) + (3*d*e^2*x^5*(a + b*ArcCosh[c*x]))
/5 + (e^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
```

```
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= \frac{b(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3)\left(1-c^2x^2\right)}{35c^7\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{be(35c^4d^2+42c^2\mathrm{c}}{105c^7\sqrt{-1}}$$

Mathematica [A]

time = 0.20, size = 193, normalized size = 0.67

$$a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6))}{3675c^7}+\frac{1}{35}bx\left(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6\right)\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 2
94*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
 75*e^3*x^6)))/(3675*c^7) + (b*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*
e^3*x^6)*ArcCosh[c*x])/35
```

Maple [A]

time = 1.85, size = 235, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arccosh}(cx)d^3c^7x+\operatorname{arccosh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arccosh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arccosh}(cx)e^3c^7x^7}{7}\right.}{}}{}$
default	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\operatorname{arccosh}(cx)d^3c^7x+\operatorname{arccosh}(cx)d^2c^7e\,x^3+\frac{3\operatorname{arccosh}(cx)d\,c^7e^2x^5}{5}+\frac{\operatorname{arccosh}(cx)e^3c^7x^7}{7}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b/c^
6*(arccosh(c*x)*d^3*c^7*x+arccosh(c*x)*d^2*c^7*e*x^3+3/5*arccosh(c*x)*d*c^7
*e^2*x^5+1/7*arccosh(c*x)*e^3*c^7*x^7-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(7
5*c^6*e^3*x^6+441*c^6*d*e^2*x^4+1225*c^6*d^2*e*x^2+90*c^4*e^3*x^4+3675*c^6*
d^3+588*c^4*d*e^2*x^2+2450*c^4*d^2*e+120*c^2*e^3*x^2+1176*c^2*d*e^2+240*e^3
)))
```

Maxima [A]

time = 0.27, size = 285, normalized size = 0.99

$$\frac{1}{7}\,ax^7e^3+\frac{3}{5}\,adx^5e^2+ad^2x^3e+ad^3x+\frac{1}{3}\left(3\,x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^2e+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^3}{c}+\frac{1}{25}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bde^2+\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x + 1/3*(3*x^3*arccos
h(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2*e +
```

```
 (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c + 1/25*(15*x^5*arccosh(c*x)
 - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 - 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1
)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*
sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(253) = 506$.
time = 0.36, size = 611, normalized size = 2.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*x^7*cosh(1)^3 + 525*a*c^7*x^7*sinh(1)^3 + 2205*a*c^7*d*x^
5*cosh(1)^2 + 3675*a*c^7*d^2*x^3*cosh(1) + 3675*a*c^7*d^3*x + 315*(5*a*c^7*
x^7*cosh(1) + 7*a*c^7*d*x^5)*sinh(1)^2 + 105*(5*b*c^7*x^7*cosh(1)^3 + 5*b*c
^7*x^7*sinh(1)^3 + 21*b*c^7*d*x^5*cosh(1)^2 + 35*b*c^7*d^2*x^3*cosh(1) + 35
*b*c^7*d^3*x + 3*(5*b*c^7*x^7*cosh(1) + 7*b*c^7*d*x^5)*sinh(1)^2 + (15*b*c^
7*x^7*cosh(1)^2 + 42*b*c^7*d*x^5*cosh(1) + 35*b*c^7*d^2*x^3)*sinh(1))*log(c
*x + sqrt(c^2*x^2 - 1)) + 105*(15*a*c^7*x^7*cosh(1)^2 + 42*a*c^7*d*x^5*cosh
(1) + 35*a*c^7*d^2*x^3)*sinh(1) - (3675*b*c^6*d^3 + 15*(5*b*c^6*x^6 + 6*b*c
^4*x^4 + 8*b*c^2*x^2 + 16*b)*cosh(1)^3 + 15*(5*b*c^6*x^6 + 6*b*c^4*x^4 + 8*
b*c^2*x^2 + 16*b)*sinh(1)^3 + 147*(3*b*c^6*d*x^4 + 4*b*c^4*d*x^2 + 8*b*c^2*
d)*cosh(1)^2 + 3*(147*b*c^6*d*x^4 + 196*b*c^4*d*x^2 + 392*b*c^2*d + 15*(5*b
*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*cosh(1))*sinh(1)^2 + 1225*(b*c
^6*d^2*x^2 + 2*b*c^4*d^2)*cosh(1) + (1225*b*c^6*d^2*x^2 + 2450*b*c^4*d^2 +
45*(5*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*cosh(1)^2 + 294*(3*b*c^
6*d*x^4 + 4*b*c^4*d*x^2 + 8*b*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c
^7
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 396, normalized size = 1.38

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{acosh}(cx) + bd^2ex^3\operatorname{acosh}(cx) + \frac{3bde^2x^5\operatorname{acosh}(cx)}{5} + \frac{be^3x^7\operatorname{acosh}(cx)}{7} - \frac{bd^3\sqrt{c^2x^2-1}}{c} - \frac{bd^2ex^2\sqrt{c^2x^2-1}}{3c} - \frac{3bde^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{be^3x^6\sqrt{c^2x^2-1}}{49c} - \frac{2bd^2e\sqrt{c^2x^2-1}}{3c^3} - \frac{4bde^2x^2\sqrt{c^2x^2-1}}{25c^3} - \frac{6be^3x^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8bde^2\sqrt{c^2x^2-1}}{25c^5} - \frac{8be^3x^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16be^3\sqrt{c^2x^2-1}}{245c^7} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*acosh(c*x) + b*d**2*e*x**3*acosh(c*x) + 3*b*d*e**2*x**5*acosh(c*x)/
5 + b*e**3*x**7*acosh(c*x)/7 - b*d**3*sqrt(c**2*x**2 - 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 - 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c
```

```
**3) - 4*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(25*c**3) - 6*b*e**3*x**4*sqrt(c
**2*x**2 - 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5
 + e**3*x**7/7), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

3.484 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=509

$$-\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{96c^5}-\frac{3bde^2x^3\sqrt{-1+c}}{16c}$$

```
[Out] -3/4*b*d^2*e*arccosh(c*x)/c^2-9/32*b*d*e^2*arccosh(c*x)/c^4-5/96*b*e^3*arcc
osh(c*x)/c^6+3/2*d^2*e*x^2*(a+b*arccosh(c*x))+3/4*d*e^2*x^4*(a+b*arccosh(c*
x))+1/6*e^3*x^6*(a+b*arccosh(c*x))+d^3*(a+b*arccosh(c*x))*ln(x)-3/4*b*d^2*e
*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3-5/96*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-3/16*b*d*e^2*x^3*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*
b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*I*b*d^3*arcsin(c*x)^2*(-c^2*x^2
+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^3*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x
^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d^3*arcsin
(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d^3*poly
log(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {272, 45, 5958, 12, 6874, 92, 54, 102, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$d^3\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd^3\sqrt{1-c^2x^2}\,\text{Li}_2\left(e^{2i\text{ArcSin}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd^3\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)^2}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^3\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)\log\left(1-e^{2i\text{ArcSin}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd^3\sqrt{1-c^2x^2}\,\log(x)\text{ArcSin}(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5be^3\cosh^{-1}(cx)}{96c^6}-\frac{5be^3x\sqrt{cx-1}\,\sqrt{cx+1}}{96c^5}-\frac{9bde^2\cosh^{-1}(cx)}{32c^4}-\frac{9bde^2x\sqrt{cx-1}\,\sqrt{cx+1}}{32c^3}-\frac{5be^3x^3\sqrt{cx-1}\,\sqrt{cx+1}}{144c^3}-\frac{3bd^2e\cosh^{-1}(cx)}{4c^2}-\frac{3bd^2ex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{3bde^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{be^3x^5\sqrt{cx-1}\,\sqrt{cx+1}}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (9*b*d*e^2*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(32*c^3) - (5*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96
*c^5) - (3*b*d*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (5*b*e^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e^3*x^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(36*c) - (3*b*d^2*e*ArcCosh[c*x])/(4*c^2) - (9*b*d*e^2*ArcCosh[c*x]
)/(32*c^4) - (5*b*e^3*ArcCosh[c*x])/(96*c^6) + (3*d^2*e*x^2*(a + b*ArcCosh[
c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/4 + (e^3*x^6*(a + b*ArcCosh[c
*x]))/6 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[
c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*(a + b*ArcCosh[c*x])*Log[x] -
(b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
```

```
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$= \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{be^3x^5}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{be^3x^5}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{3bde^2}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{3bde^2}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{5be^3x}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{5be^3x}{}$$

$$= -\frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{5be^3x}{}$$

Mathematica [A]

time = 0.72, size = 344, normalized size = 0.68

$$\frac{3}{2}ad^2ex^2+\frac{3}{4}ade^2x^4+\frac{1}{6}ae^3x^6-\frac{3bd^2e\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}-2c^2x^2\cosh^{-1}(cx)+2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{4c^2}-\frac{3bde^2\left(cx\sqrt{\frac{-1+cx}{1+cx}}\,(3+3cx+2c^2x^2+2c^3x^3)-8c^4x^4\cosh^{-1}(cx)+6\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{32c^4}-\frac{be^3\left(cx\sqrt{\frac{-1+cx}{1+cx}}\,(15+15cx+10c^2x^2+10c^3x^3+8c^4x^4+8c^5x^5)-48c^6x^6\cosh^{-1}(cx)+30\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{288c^6}+ad^3\log(x)+\frac{1}{2}bd^3\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)-\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 + (a*e^3*x^6)/6 - (3*b*d^2*e*(c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x] - 2*c^2*x^2*ArcCosh[c*x] + 2*ArcTanh[Sqrt[(-1 + c
*x)/(1 + c*x)]]))/(4*c^2) - (3*b*d*e^2*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3 +
 3*c*x + 2*c^2*x^2 + 2*c^3*x^3) - 8*c^4*x^4*ArcCosh[c*x] + 6*ArcTanh[Sqrt[(
-1 + c*x)/(1 + c*x)]]))/(32*c^4) - (b*e^3*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(
15 + 15*c*x + 10*c^2*x^2 + 10*c^3*x^3 + 8*c^4*x^4 + 8*c^5*x^5) - 48*c^6*x^6
*ArcCosh[c*x] + 30*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(288*c^6) + a*d^3*
Log[x] + (b*d^3*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])
]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Maple [A]
time = 1.21, size = 351, normalized size = 0.69

$$\frac{a\,e^3x^6}{6}+\frac{3ad\,e^2x^4}{4}+\frac{3a\,d^2e\,x^2}{2}+a\,d^3\ln(cx)-\frac{5b\,e^3\mathrm{arccosh}(cx)}{96c^6}-\frac{3bd\,e^2x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{9bd\,e^2x\sqrt{cx-1}}{32c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/6*a*e^3*x^6+3/4*a*d*e^2*x^4+3/2*a*d^2*e*x^2+a*d^3*ln(c*x)-5/96*b*e^3*arcc
osh(c*x)/c^6-3/16*b*d*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/4*b*d^2*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
-3/4*b*d^2*e*arccosh(c*x)/c^2+3/4*b*arccosh(c*x)*d*e^2*x^4+3/2*b*arccosh(c*
x)*d^2*e*x^2-1/36*b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-5/96*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5
+b*d^3*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d^3*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/6*b*arccosh(c*x)*e^3*x^6-1/2
*b*arccosh(c*x)^2*d^3-9/32*b*d*e^2*arccosh(c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^3 + 3/4*a*d*x^4*e^2 + 3/2*a*d^2*x^2*e + a*d^3*log(x) + integrat
e(b*x^5*e^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*x^3*e^2*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d^2*x*e*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)) + b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccosh(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx)) (d + ex^2)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\, (e\,x^2 + d)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x, x)
```

3.485 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=265

$$\frac{be(15c^4d^2+5c^2de+e^2)(1-c^2x^2)}{5c^5\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{be^2(5c^2d+2e)(1-c^2x^2)^2}{15c^5\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3(1-c^2x^2)^3}{25c^5\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3(a+b\cosh^{-}}{x}$$

```
[Out] -d^3*(a+b*arccosh(c*x))/x+3*d^2*e*x*(a+b*arccosh(c*x))+d*e^2*x^3*(a+b*arcco
sh(c*x))+1/5*e^3*x^5*(a+b*arccosh(c*x))+1/5*b*e*(15*c^4*d^2+5*c^2*d*e+e^2)*
(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/15*b*e^2*(5*c^2*d+2*e)*(-c^2
*x^2+1)^2/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/25*b*e^3*(-c^2*x^2+1)^3/c^5/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d^3*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5958, 1624, 1813, 1634, 65, 211}

$$-\frac{d^3(a+b\cosh^{-1}(cx))}{x}+3d^2ex(a+b\cosh^{-1}(cx))+de^2x^3(a+b\cosh^{-1}(cx))+\frac{1}{5}e^3x^5(a+b\cosh^{-1}(cx))+\frac{bcd^3\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be^2(1-c^2x^2)^2(5c^2d+2e)}{15c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be^3(1-c^2x^2)^3}{25c^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be(1-c^2x^2)(15c^4d^2+5c^2de+e^2)}{5c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*(1 - c^2*x^2))/(5*c^5*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*e^2*(5*c^2*d + 2*e)*(1 - c^2*x^2)^2)/(15*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x + 3*d^2*e*x*(a + b*ArcCosh[c*x]) + d*
e^2*x^3*(a + b*ArcCosh[c*x]) + (e^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*
Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} + 3d^2ex\left(a+b\cosh^{-1}(cx)\right) + de^2x^3\left(a+b\cos\right.$$

$$= \frac{be(15c^4d^2+5c^2de+e^2)(1-c^2x^2)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2(5c^2d+2e)(1-c^2x^2)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{}{2}$$

$$= \frac{be(15c^4d^2+5c^2de+e^2)(1-c^2x^2)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2(5c^2d+2e)(1-c^2x^2)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{}{2}$$

$$= \frac{be(15c^4d^2+5c^2de+e^2)(1-c^2x^2)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^2(5c^2d+2e)(1-c^2x^2)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{}{2}$$

Mathematica [A]
time = 0.21, size = 182, normalized size = 0.69

$$-\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{1}{5}ae^3x^5 - \frac{be\sqrt{-1+cx}\sqrt{1+cx}\,(8e^2+2c^2e(25d+2ex^2)+c^4(225d^2+25dex^2+3e^2x^4))}{75c^5} + \frac{b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\cosh^{-1}(cx)}{5x} - bcd^3\,\text{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 - (b*e*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d*e
*x^2 + 3*e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 + e^
3*x^6)*ArcCosh[c*x])/(5*x) - b*c*d^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)]
```

Maple [A]
time = 2.11, size = 309, normalized size = 1.17

method	result

derivativedivides	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{3b\,\mathrm{arccosh}(cx)d^2ex}{c}+\frac{b\,\mathrm{arccosh}(cx)d\,e^2x^3}{c}+\frac{b\,\mathrm{arccosh}(cx)e^3x^5}{5c}-\frac{b\,\mathrm{ar}}{}\right.$
default	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{3b\,\mathrm{arccosh}(cx)d^2ex}{c}+\frac{b\,\mathrm{arccosh}(cx)d\,e^2x^3}{c}+\frac{b\,\mathrm{arccosh}(cx)e^3x^5}{5c}-\frac{b\,\mathrm{ar}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^6*(3*c^5*d^2*e*x+c^5*d*e^2*x^3+1/5*e^3*c^5*x^5-c^5*d^3/x)+3*b/c*arcc
osh(c*x)*d^2*e*x+b/c*arccosh(c*x)*d*e^2*x^3+1/5*b/c*arccosh(c*x)*e^3*x^5-b*
arccosh(c*x)*d^3/c/x-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ar
ctan(1/(c^2*x^2-1)^(1/2))-3*b/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*e-1/3*b/c
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2*x^2-1/25*b/c^2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*e^3*x^4-2/3*b/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2-4/75*b/c^4*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*e^3*x^2-8/75*b/c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3)
```

Maxima [A]
time = 0.47, size = 220, normalized size = 0.83

$$\frac{1}{5}\,ax^5e^3+adx^3e^2-\left(c\,\mathrm{arcsin}\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^3+3\,ad^2xe+\frac{1}{3}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bde^2+\frac{3\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bd^2e}{c}-\frac{ad^3}{x}+\frac{1}{75}\left(15\,x^5\,\mathrm{arcosh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^3 + a*d*x^3*e^2 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d
^3 + 3*a*d^2*x*e + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e^2 + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1
))*b*d^2*e/c - a*d^3/x + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x
^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(233) = 466.
time = 0.41, size = 771, normalized size = 2.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*c^5*x^6*cosh(1)^3 + 15*a*c^5*x^6*sinh(1)^3 + 75*a*c^5*d*x^4*cosh
(1)^2 + 150*b*c^6*d^3*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 225*a*c^5*d^2*x^
```

```
2*cosh(1) - 75*a*c^5*d^3 + 15*(3*a*c^5*x^6*cosh(1) + 5*a*c^5*d*x^4)*sinh(1)
^2 + 15*(5*b*c^5*d^3*x - 5*b*c^5*d^3 + (b*c^5*x^6 - b*c^5*x)*cosh(1)^3 + (b
*c^5*x^6 - b*c^5*x)*sinh(1)^3 + 5*(b*c^5*d*x^4 - b*c^5*d*x)*cosh(1)^2 + (5*
b*c^5*d*x^4 - 5*b*c^5*d*x + 3*(b*c^5*x^6 - b*c^5*x)*cosh(1))*sinh(1)^2 + 15
*(b*c^5*d^2*x^2 - b*c^5*d^2*x)*cosh(1) + (15*b*c^5*d^2*x^2 - 15*b*c^5*d^2*x
 + 3*(b*c^5*x^6 - b*c^5*x)*cosh(1)^2 + 10*(b*c^5*d*x^4 - b*c^5*d*x)*cosh(1)
)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 15*(5*b*c^5*d^3*x - 15*b*c^5*d^2*
x*cosh(1) - 5*b*c^5*d*x*cosh(1)^2 - b*c^5*x*cosh(1)^3 - b*c^5*x*sinh(1)^3 -
 (5*b*c^5*d*x + 3*b*c^5*x*cosh(1))*sinh(1)^2 - (15*b*c^5*d^2*x + 10*b*c^5*d
*x*cosh(1) + 3*b*c^5*x*cosh(1)^2)*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) +
15*(3*a*c^5*x^6*cosh(1)^2 + 10*a*c^5*d*x^4*cosh(1) + 15*a*c^5*d^2*x^2)*sinh
(1) - (225*b*c^4*d^2*x*cosh(1) + (3*b*c^4*x^5 + 4*b*c^2*x^3 + 8*b*x)*cosh(1
)^3 + (3*b*c^4*x^5 + 4*b*c^2*x^3 + 8*b*x)*sinh(1)^3 + 25*(b*c^4*d*x^3 + 2*b
*c^2*d*x)*cosh(1)^2 + (25*b*c^4*d*x^3 + 50*b*c^2*d*x + 3*(3*b*c^4*x^5 + 4*b
*c^2*x^3 + 8*b*x)*cosh(1))*sinh(1)^2 + (225*b*c^4*d^2*x + 3*(3*b*c^4*x^5 +
4*b*c^2*x^3 + 8*b*x)*cosh(1)^2 + 50*(b*c^4*d*x^3 + 2*b*c^2*d*x)*cosh(1))*si
nh(1))*sqrt(c^2*x^2 - 1))/(c^5*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))(d + ex^2)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))\,(e\,x^2 + d)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2,x)
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2, x)
```

3.486 $\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=476

$$-\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3be^2(8c^2d+e)\,x(1-c^2x^2)}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3x^3(1-c^2x^2)}{16c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}d$$

```
[Out] -1/2*d^3*(a+b*arccosh(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arccosh(c*x))+1/4*e^3*x^
4*(a+b*arccosh(c*x))+3*d^2*e*(a+b*arccosh(c*x))*ln(x)-1/2*b*c*d^3*(-c^2*x^2
+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/32*b*e^2*(8*c^2*d+e)*x*(-c^2*x^2+1)/c^3
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*e^3*x^3*(-c^2*x^2+1)/c/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-3/2*I*b*d^2*e*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+3*b*d^2*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*d^2*e*arcsin(c*x)*ln(x)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*d^2*e*polylog(2,(I*c*x+(-c
^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/32*b*e
^2*(8*c^2*d+e)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^4/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 1.17, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {272, 45, 5958, 12, 6874, 1624, 1821, 1598, 470, 327, 223, 212, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+3d^2e\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{3ibd^2e\sqrt{1-c^2x^2}\,\text{Li}_2\left(e^{2i\text{ArcSin}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3ibd^2e\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)^2}{2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bd^2e\sqrt{1-c^2x^2}\,\text{ArcSin}(cx)\log\left(1-e^{2i\text{ArcSin}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bd^2e\sqrt{1-c^2x^2}\,\log(x)\text{ArcSin}(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^3(1-c^2x^2)}{2x\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be^3x^3(1-c^2x^2)}{16c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3be^2\sqrt{c^2x^2-1}\,(8c^2d+e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{32c^4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3be^2x(1-c^2x^2)(8c^2d+e)}{32c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^3*(1 - c^2*x^2))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(8
*c^2*d + e)*x*(1 - c^2*x^2))/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3
*x^3*(1 - c^2*x^2))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCo
sh[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a + b*ArcCosh[c*x]))/2 + (e^3*x^4*(a + b*
ArcCosh[c*x]))/4 - (((3*I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*e^2*(8*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/(32*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + 3*d^2*e*(a + b*ArcCosh[c*x])*Log[x] - (3*b*d^2*e
*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((
3*I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
```

```
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\,\mathrm{cos}\right.$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\,\mathrm{cos}\right.$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3x^3(1-c^2x^2)}{16c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh\right.}{2x^2}$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3be^2(8c^2d+e)\,x(1-c^2x^2)}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3x^3(1}{16c\sqrt{-1+}}$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3be^2(8c^2d+e)\,x(1-c^2x^2)}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3x^3(1}{16c\sqrt{-1+}}$$

$$= -\frac{bcd^3(1-c^2x^2)}{2x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3be^2(8c^2d+e)\,x(1-c^2x^2)}{32c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^3x^3(1}{16c\sqrt{-1+}}$$

<u>Mathematica</u> [A]
time = 0.49, size = 278, normalized size = 0.58

$$\frac{1}{4}\left(-\frac{2ad^3}{x^2}+6ade^2x^2+ae^3x^4+\frac{2bd^3\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}-\cosh^{-1}(cx)\right)}{x^2}+6bde^2x^2\cosh^{-1}(cx)+be^3x^4\cosh^{-1}(cx)-\frac{3bde^2\left(cx\sqrt{-1+cx}\,\sqrt{1+cx}+2\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{c^2}-\frac{be^3\left(cx\sqrt{\frac{-1+cx}{1+cx}}\,(3+3cx+2c^2x^2+2c^3x^3)+6\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)}{8c^4}+6bd^2e\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(1+e^{-2\cosh^{-1}(cx)}\right)\right)+12ad^2e\log(x)-6bd^2e\,\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^3)/x^2 + 6*a*d*e^2*x^2 + a*e^3*x^4 + (2*b*d^3*(c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x] - ArcCosh[c*x]))/x^2 + 6*b*d*e^2*x^2*ArcCosh[c*x] + b*e^3*x^4
```

```
*ArcCosh[c*x] - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sq
rt[(-1 + c*x)/(1 + c*x)]]))/c^2 - (b*e^3*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(3
 + 3*c*x + 2*c^2*x^2 + 2*c^3*x^3) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))
/(8*c^4) + 6*b*d^2*e*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c
*x])]) + 12*a*d^2*e*Log[x] - 6*b*d^2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/4
```

Maple [A]
time = 2.49, size = 296, normalized size = 0.62

$$\frac{a\,e^3x^4}{4}+\frac{3ad\,e^2x^2}{2}-\frac{a\,d^3}{2x^2}+3a\,d^2e\ln\left(cx\right)-\frac{3b\,\mathrm{arccosh}(cx)\,e^3}{32c^4}+\frac{cb\,d^3\sqrt{cx+1}\,\sqrt{cx-1}}{2x}+\frac{3b\,\mathrm{arccosh}(cx)\,x^2d\,e^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/4*a*e^3*x^4+3/2*a*d*e^2*x^2-1/2*a*d^3/x^2+3*a*d^2*e*ln(c*x)-3/32/c^4*b*ar
ccosh(c*x)*e^3+1/2*c*b*d^3/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)+3/2*b*arccosh(c*x)
*x^2*d*e^2+3/2*b*d^2*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/4/
c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e^2+1/4*b*arccosh(c*x)*e^3*x^4+3*b*d^2*
e*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/2*b*d^2*e*arccos
h(c*x)^2-1/2*b*arccosh(c*x)*d^3/x^2-1/16/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^
3*e^3-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e^3-3/4/c^2*b*arccosh(c*x)*d
*e^2-1/2*c^2*d^3*b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 + 1/2*b*d^3*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + 3/2*
a*d*x^2*e^2 + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate(b*x^3*e^3*log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*x*e^2*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 3*b*d^2*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccosh(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\,(d + ex^2)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acosh}(c\,x))\,\left(e\,x^2 + d\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3, x)
```

3.487 $\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^4} \, dx$

Optimal. Leaf size=260

$$\frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e}{}$$

```
[Out] -1/3*d^3*(a+b*arccosh(c*x))/x^3-3*d^2*e*(a+b*arccosh(c*x))/x+3*d*e^2*x*(a+b
*arccosh(c*x))+1/3*e^3*x^3*(a+b*arccosh(c*x))+1/3*b*e^2*(9*c^2*d+e)*(-c^2*x
^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c*d^3*(-c^2*x^2+1)/x^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-1/9*b*e^3*(-c^2*x^2+1)^2/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+1/6*b*c*d^2*(c^2*d+18*e)*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {276, 5958, 12, 1624, 1813, 1635, 911, 1167, 211}

$$-\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(cx)) + \frac{1}{3}e^3x^3(a+b\cosh^{-1}(cx)) + \frac{bcd^2\sqrt{c^2x^2-1}\,\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)(c^2d+18e)}{6\sqrt{cx-1}\sqrt{cx+1}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{cx-1}\sqrt{cx+1}} + \frac{be^2(1-c^2x^2)(9c^2d+e)}{3c^3\sqrt{cx-1}\sqrt{cx+1}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*e^2*(9*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(b*c*d^3*(1 - c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1 -
c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]
))/(3*x^3) - (3*d^2*e*(a + b*ArcCosh[c*x]))/x + 3*d*e^2*x*(a + b*ArcCosh[c*
x]) + (e^3*x^3*(a + b*ArcCosh[c*x]))/3 + (b*c*d^2*(c^2*d + 18*e)*Sqrt[-1 +
c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^3\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x} + 3de^2x\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x} + 3de^2x\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x} + 3de^2x\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x} + 3de^2x\left(a+b\cos\right.$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cos\right.}{x}$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cos\right.}{x}$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{3d^2e\left(a+b\cos\right.}{x}$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{be^3(1-}{9c^3\sqrt{-1+cx}}$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{be^3(1-}{9c^3\sqrt{-1+cx}}$$

Mathematica [A]
time = 0.25, size = 184, normalized size = 0.71

$$\frac{1}{6}\left(-\frac{2ad^3}{x^3} - \frac{18ad^2e}{x} + 18ade^2x + 2ae^3x^3 - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\left(-3c^4d^3+4e^3x^2+2c^2e^2x^2(27d+ex^2)\right)}{3c^3x^2} + \frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\cosh^{-1}(cx)}{x^3} - bcd^2(c^2d+18e)\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 - (b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^2)
))/(3*c^3*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcCosh[
c*x])/x^3 - b*c*d^2*(c^2*d + 18*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]
)/6
```

Maple [A]
time = 1.94, size = 309, normalized size = 1.19

method	result
derivativedivides	$c^{3}\left(\frac{a\left(3c^{3}d\,e^{2}x+\frac{e^{3}c^{3}x^{3}}{3}-\frac{3c^{3}d^{2}e}{x}-\frac{c^{3}d^{3}}{3x^{3}}\right)}{c^{6}}+\frac{3b\,\mathrm{arccosh}(cx)d\,e^{2}x}{c^{3}}+\frac{b\,\mathrm{arccosh}(cx)e^{3}x^{3}}{3c^{3}}-\frac{3b\,\mathrm{arccosh}(cx)d^{2}e}{c^{3}x}-\frac{b\,\mathrm{arc}}{}\right.$
default	$c^{3}\left(\frac{a\left(3c^{3}d\,e^{2}x+\frac{e^{3}c^{3}x^{3}}{3}-\frac{3c^{3}d^{2}e}{x}-\frac{c^{3}d^{3}}{3x^{3}}\right)}{c^{6}}+\frac{3b\,\mathrm{arccosh}(cx)d\,e^{2}x}{c^{3}}+\frac{b\,\mathrm{arccosh}(cx)e^{3}x^{3}}{3c^{3}}-\frac{3b\,\mathrm{arccosh}(cx)d^{2}e}{c^{3}x}-\frac{b\,\mathrm{arc}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^6*(3*c^3*d*e^2*x+1/3*e^3*c^3*x^3-3*c^3*d^2*e/x-1/3*c^3*d^3/x^3)+3*
b/c^3*arccosh(c*x)*d*e^2*x+1/3*b/c^3*arccosh(c*x)*e^3*x^3-3*b/c^3*arccosh(c
*x)*d^2*e/x-1/3*b*arccosh(c*x)*d^3/c^3/x^3-1/6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/(c^2*x^2-1)^(1/2)*d^3*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^2/x^2*d^3-3*b/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2
)*arctan(1/(c^2*x^2-1)^(1/2))*d^2*e-3*b/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e
^2-1/9*b/c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^2-2/9*b/c^6*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*e^3)
```

Maxima [A]
time = 0.48, size = 195, normalized size = 0.75

$$-\frac{1}{6}\left(\left(c^{2}\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^{2}x^{2}-1}}{x^{2}}\right)c+\frac{2\,\mathrm{arcosh}\left(cx\right)}{x^{3}}\right)bd^{3}+\frac{1}{3}\,ax^{3}e^{3}-3\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\left(cx\right)}{x}\right)bd^{2}e+3\,adxe^{2}+\frac{1}{9}\left(3\,x^{3}\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{2}}+\frac{2\sqrt{c^{2}x^{2}-1}}{c^{4}}\right)\right)be^{3}+\frac{3\left(cx\,\mathrm{arcosh}\left(cx\right)-\sqrt{c^{2}x^{2}-1}\right)bde^{2}}{c}-\frac{3\,ad^{2}e}{x}-\frac{ad^{3}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)
/x^3)*b*d^3 + 1/3*a*x^3*e^3 - 3*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b
*d^2*e + 3*a*d*x*e^2 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c
^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*e^3 + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 -
 1))*b*d*e^2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(224) = 448$.
time = 0.42, size = 767, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*x^6*cosh(1)^3 + 6*a*c^3*x^6*sinh(1)^3 + 54*a*c^3*d*x^4*cosh(1
)^2 - 54*a*c^3*d^2*x^2*cosh(1) - 6*a*c^3*d^3 + 18*(a*c^3*x^6*cosh(1) + 3*a*
c^3*d*x^4)*sinh(1)^2 + 6*(b*c^6*d^3*x^3 + 18*b*c^4*d^2*x^3*cosh(1) + 18*b*c
^4*d^2*x^3*sinh(1))*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^3*d^3*x^3 - b
*c^3*d^3 + (b*c^3*x^6 - b*c^3*x^3)*cosh(1)^3 + (b*c^3*x^6 - b*c^3*x^3)*sinh
(1)^3 + 9*(b*c^3*d*x^4 - b*c^3*d*x^3)*cosh(1)^2 + 3*(3*b*c^3*d*x^4 - 3*b*c^
3*d*x^3 + (b*c^3*x^6 - b*c^3*x^3)*cosh(1))*sinh(1)^2 + 9*(b*c^3*d^2*x^3 - b
*c^3*d^2*x^2)*cosh(1) + 3*(3*b*c^3*d^2*x^3 - 3*b*c^3*d^2*x^2 + (b*c^3*x^6 -
 b*c^3*x^3)*cosh(1)^2 + 6*(b*c^3*d*x^4 - b*c^3*d*x^3)*cosh(1))*sinh(1))*log
(c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^3*d^3*x^3 + 9*b*c^3*d^2*x^3*cosh(1) - 9*
b*c^3*d*x^3*cosh(1)^2 - b*c^3*x^3*cosh(1)^3 - b*c^3*x^3*sinh(1)^3 - 3*(3*b*
c^3*d*x^3 + b*c^3*x^3*cosh(1))*sinh(1)^2 + 3*(3*b*c^3*d^2*x^3 - 6*b*c^3*d*x
^3*cosh(1) - b*c^3*x^3*cosh(1)^2)*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) +
18*(a*c^3*x^6*cosh(1)^2 + 6*a*c^3*d*x^4*cosh(1) - 3*a*c^3*d^2*x^2)*sinh(1)
+ (3*b*c^4*d^3*x - 54*b*c^2*d*x^3*cosh(1)^2 - 2*(b*c^2*x^5 + 2*b*x^3)*cosh(
1)^3 - 2*(b*c^2*x^5 + 2*b*x^3)*sinh(1)^3 - 6*(9*b*c^2*d*x^3 + (b*c^2*x^5 +
2*b*x^3)*cosh(1))*sinh(1)^2 - 6*(18*b*c^2*d*x^3*cosh(1) + (b*c^2*x^5 + 2*b*
x^3)*cosh(1)^2)*sinh(1))*sqrt(c^2*x^2 - 1))/(c^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\,(d + ex^2)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4, x)
```

3.488 $\int \left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=395

$$\frac{b(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4be(105c^6d^3+189c^4d^2e+135c^2de^2+35}{945c^9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] d^4*x*(a+b*arccosh(c*x))+4/3*d^3*e*x^3*(a+b*arccosh(c*x))+6/5*d^2*e^2*x^5*(
a+b*arccosh(c*x))+4/7*d*e^3*x^7*(a+b*arccosh(c*x))+1/9*e^4*x^9*(a+b*arccosh
(c*x))+1/315*b*(315*c^8*d^4+420*c^6*d^3*e+378*c^4*d^2*e^2+180*c^2*d*e^3+35*
e^4)*(-c^2*x^2+1)/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/945*b*e*(105*c^6*d^3+18
9*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)^2/c^9/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+2/525*b*e^2*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)^3/c^9/(c*x-1)
^(1/2)/(c*x+1)^(1/2)-4/441*b*e^3*(9*c^2*d+7*e)*(-c^2*x^2+1)^4/c^9/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/81*b*e^4*(-c^2*x^2+1)^5/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2
)
```

Rubi [A]
time = 0.33, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {200, 5908, 12, 1624, 1813, 1864}

$$d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{4be^3\left(1-c^2x^2\right)^4\left(9c^2d+7e\right)}{441c^9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be^4\left(1-c^2x^2\right)^5}{81c^9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2be^2\left(1-c^2x^2\right)^3\left(63c^4d^2+90c^2de+35e^2\right)}{525c^9\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{4be\left(1-c^2x^2\right)^2\left(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3\right)}{945c^9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\left(1-c^2x^2\right)\left(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4\right)}{315c^9\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 + 35*e^4)
*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e*(105*c^6*d^
3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (2*b*e^2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 -
 c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^4*(1 - c
^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^4*x*(a + b*ArcCosh[c*x
]) + (4*d^3*e*x^3*(a + b*ArcCosh[c*x]))/3 + (6*d^2*e^2*x^5*(a + b*ArcCosh[c
*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCosh[c*x]))/7 + (e^4*x^9*(a + b*ArcCosh[c*
x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\right.$$

$$= \frac{b(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4)\,(1-c^2x^2)}{315c^9\sqrt{-1+cx}\,\sqrt{1+cx}} -$$

Mathematica [A]
time = 0.28, size = 265, normalized size = 0.67

$$\frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4480e^4+320c^2e^3(81d+7ex^2)+48c^4e^2(1323d^2+270dex^2+35e^2x^4)+8c^6e(11025d^3+3969d^2ex^2+1215de^2x^4+175e^3x^6)+c^8(99225d^4+44100d^3ex^2+23814d^2e^2x^4+8100de^3x^6+1225e^4x^8))}{c^9}+315bx(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)\cosh^{-1}(cx)}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^4 + 320*c^2*e^3*(81*d + 7*
e*x^2) + 48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) + 8*c^6*e*(11025*
d^3 + 3969*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 441
00*d^3*e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 3
15*b*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*
x^8)*ArcCosh[c*x])/99225
```

Maple [A]
time = 1.86, size = 331, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\operatorname{arccosh}(cx)d^4c^9x+\frac{4\operatorname{arccosh}(cx)d^3c^9e\,x^3}{3}+\frac{6\operatorname{arccosh}(cx)d^2c^9e^2x^5}{5}\right.}{}}{}$

default	$\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\operatorname{arccosh}(cx)d^4c^9x+\frac{4\operatorname{arccosh}(cx)d^3c^9e\,x^3}{3}+\frac{6\operatorname{arccosh}(cx)d^2c^9e^2x^5}{5}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^8*(d^4*c^9*x+4/3*d^3*c^9*e*x^3+6/5*d^2*c^9*e^2*x^5+4/7*d*c^9*e^3*x
^7+1/9*e^4*c^9*x^9)+b/c^8*(arccosh(c*x)*d^4*c^9*x+4/3*arccosh(c*x)*d^3*c^9*
e*x^3+6/5*arccosh(c*x)*d^2*c^9*e^2*x^5+4/7*arccosh(c*x)*d*c^9*e^3*x^7+1/9*a
rccosh(c*x)*e^4*c^9*x^9-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^4*x
^8+8100*c^8*d*e^3*x^6+23814*c^8*d^2*e^2*x^4+1400*c^6*e^4*x^6+44100*c^8*d^3*
e*x^2+9720*c^6*d*e^3*x^4+99225*c^8*d^4+31752*c^6*d^2*e^2*x^2+1680*c^4*e^4*x
^4+88200*c^6*d^3*e+12960*c^4*d*e^3*x^2+63504*c^4*d^2*e^2+2240*c^2*e^4*x^2+2
5920*c^2*d*e^3+4480*e^4)))
```

Maxima [A]
time = 0.27, size = 411, normalized size = 1.04

$$\frac{1}{9}ax^9e^4+\frac{4}{7}adx^7e^3+\frac{6}{5}ad^2x^5e^2+\frac{4}{3}ad^3x^3e+ad^4x+\frac{4}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3e+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd^4}{c}+\frac{2}{25}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2e^2+\frac{4}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2x^2-1}}{c^8}\right)c\right)bde^3+\frac{1}{2835}\left(315x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\frac{64\sqrt{c^2x^2-1}\,x^2}{c^8}+\frac{128\sqrt{c^2x^2-1}}{c^{10}}\right)c\right)be^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9*e^4 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*a*d^3*x^3*e + a*d
^4*x + 4/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*
x^2 - 1)/c^4))*b*d^3*e + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^4/c + 2
/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 -
1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e^2 + 4/245*(35*x^7*arccosh(
c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(
c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^3 + 1/2835*(315*x
^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/
c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqr
t(c^2*x^2 - 1)/c^10)*c)*b*e^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1039 vs. 2(346) = 692.
time = 0.36, size = 1039, normalized size = 2.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*x^9*cosh(1)^4 + 11025*a*c^9*x^9*sinh(1)^4 + 56700*a*c^
9*d*x^7*cosh(1)^3 + 119070*a*c^9*d^2*x^5*cosh(1)^2 + 132300*a*c^9*d^3*x^3*c
osh(1) + 99225*a*c^9*d^4*x + 6300*(7*a*c^9*x^9*cosh(1) + 9*a*c^9*d*x^7)*sin
h(1)^3 + 1890*(35*a*c^9*x^9*cosh(1)^2 + 90*a*c^9*d*x^7*cosh(1) + 63*a*c^9*d
^2*x^5)*sinh(1)^2 + 315*(35*b*c^9*x^9*cosh(1)^4 + 35*b*c^9*x^9*sinh(1)^4 +
180*b*c^9*d*x^7*cosh(1)^3 + 378*b*c^9*d^2*x^5*cosh(1)^2 + 420*b*c^9*d^3*x^3
*cosh(1) + 315*b*c^9*d^4*x + 20*(7*b*c^9*x^9*cosh(1) + 9*b*c^9*d*x^7)*sinh(
1)^3 + 6*(35*b*c^9*x^9*cosh(1)^2 + 90*b*c^9*d*x^7*cosh(1) + 63*b*c^9*d^2*x^
5)*sinh(1)^2 + 4*(35*b*c^9*x^9*cosh(1)^3 + 135*b*c^9*d*x^7*cosh(1)^2 + 189*
b*c^9*d^2*x^5*cosh(1) + 105*b*c^9*d^3*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2
- 1)) + 1260*(35*a*c^9*x^9*cosh(1)^3 + 135*a*c^9*d*x^7*cosh(1)^2 + 189*a*c^
9*d^2*x^5*cosh(1) + 105*a*c^9*d^3*x^3)*sinh(1) - (99225*b*c^8*d^4 + 35*(35*
b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1)^4 +
 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*sin
h(1)^4 + 1620*(5*b*c^8*d*x^6 + 6*b*c^6*d*x^4 + 8*b*c^4*d*x^2 + 16*b*c^2*d)*
cosh(1)^3 + 20*(405*b*c^8*d*x^6 + 486*b*c^6*d*x^4 + 648*b*c^4*d*x^2 + 1296*
b*c^2*d + 7*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4 + 64*b*c^2*x^2 + 12
8*b)*cosh(1))*sinh(1)^3 + 7938*(3*b*c^8*d^2*x^4 + 4*b*c^6*d^2*x^2 + 8*b*c^4
*d^2)*cosh(1)^2 + 6*(3969*b*c^8*d^2*x^4 + 5292*b*c^6*d^2*x^2 + 10584*b*c^4*
d^2 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b
)*cosh(1)^2 + 810*(5*b*c^8*d*x^6 + 6*b*c^6*d*x^4 + 8*b*c^4*d*x^2 + 16*b*c^2
*d)*cosh(1))*sinh(1)^2 + 44100*(b*c^8*d^3*x^2 + 2*b*c^6*d^3)*cosh(1) + 4*(1
1025*b*c^8*d^3*x^2 + 22050*b*c^6*d^3 + 35*(35*b*c^8*x^8 + 40*b*c^6*x^6 + 48
*b*c^4*x^4 + 64*b*c^2*x^2 + 128*b)*cosh(1)^3 + 1215*(5*b*c^8*d*x^6 + 6*b*c^
6*d*x^4 + 8*b*c^4*d*x^2 + 16*b*c^2*d)*cosh(1)^2 + 3969*(3*b*c^8*d^2*x^4 + 4
*b*c^6*d^2*x^2 + 8*b*c^4*d^2)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^9
```

Sympy [C] Result contains complex when optimal does not.
time = 1.50, size = 600, normalized size = 1.52

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{acosh}(cx) + \frac{4bd^3ex^3\operatorname{acosh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{acosh}(cx)}{5} + \frac{4bde^3x^7\operatorname{acosh}(cx)}{7} + \frac{be^4x^9\operatorname{acosh}(cx)}{9} - \frac{bd^4\sqrt{c^2x^2-1}}{c} - \frac{4bd^3ex^2\sqrt{c^2x^2-1}}{9c} - \frac{6bd^2e^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{4bde^3x^6\sqrt{c^2x^2-1}}{49c} - \frac{be^4x^8\sqrt{c^2x^2-1}}{81c} - \frac{8bd^3e\sqrt{c^2x^2-1}}{9c^3} - \frac{8bd^2e^2x^2\sqrt{c^2x^2-1}}{25c^3} - \frac{24bde^3x^4\sqrt{c^2x^2-1}}{245c^3} - \frac{8be^4x^6\sqrt{c^2x^2-1}}{567c^3} - \frac{16bd^2e^2\sqrt{c^2x^2-1}}{25c^5} - \frac{32bde^3x^2\sqrt{c^2x^2-1}}{245c^5} - \frac{16be^4x^4\sqrt{c^2x^2-1}}{945c^5} - \frac{64bde^3\sqrt{c^2x^2-1}}{245c^7} - \frac{64be^4x^2\sqrt{c^2x^2-1}}{2835c^7} - \frac{128be^4\sqrt{c^2x^2-1}}{2835c^9} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*acosh(c*x) + 4*b*d**3*e*x**3*acosh(c*x)/
3 + 6*b*d**2*e**2*x**5*acosh(c*x)/5 + 4*b*d*e**3*x**7*acosh(c*x)/7 + b*e**4
*x**9*acosh(c*x)/9 - b*d**4*sqrt(c**2*x**2 - 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 - 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 - 1)/(81*
c) - 8*b*d**3*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 - 1)/(25*c**3) - 24*b*d*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*
b*e**4*x**6*sqrt(c**2*x**2 - 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
- 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**
```

```
4*x**4*sqrt(c**2*x**2 - 1)/(945*c**5) - 64*b*d*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7) - 64*b*e**4*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 - 1)/(2835*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**4*x + 4*d**3*e*x
**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^4,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^4, x)
```

3.489 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=627

$$-\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{-1}(cx)}{e^2}+\frac{x^3(a$$

[Out] -a*d*x/e^2-b*d*x*arccosh(c*x)/e^2+1/3*x^3*(a+b*arccosh(c*x))/e+1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*b*(-d)^(3/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*b*(-d)^(3/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)+b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2-2/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/e-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e

Rubi [A]
time = 0.75, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5959, 5879, 75, 5883, 102, 12, 5909, 5962, 5681, 2221, 2317, 2438}

$$\frac{(-d)^{3/2}(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{5/2}}+\frac{(-d)^{3/2}(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^{5/2}}+\frac{x^3(a+b\cosh^{-1}(cx))}{3e}-\frac{adx}{e^2}-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}}{9c^3e}-\frac{b(-d)^{3/2}\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{5/2}}-\frac{b(-d)^{3/2}\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2e^{5/2}}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}}{ce^2}-\frac{bdx\cosh^{-1}(cx)}{e^2}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{9ce}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]

[Out] -((a*d*x)/e^2) + (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3*e) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*e) - (b*d*x*ArcCosh[c*x])/e^2 + (x^3*(a + b*ArcCosh[c*x]))/(3*e) + ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) + ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcC

```
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*(-d)^(3/2)*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^2} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{e} + \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)$$

$$= -\frac{d\int\left(a+b\cosh^{-1}(cx)\right)\,dx}{e^2} + \frac{d^2\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2} + \frac{\int x^2\left(a+b\cosh^{-1}(cx)\right)\,d}{e}$$

$$= -\frac{adx}{e^2} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e} - \frac{(bd)\int\cosh^{-1}(cx)\,dx}{e^2} + \frac{d^2\int\left(\frac{\sqrt{-d}\,(a+b\cosh^{-}}{2d\left(\sqrt{-d}-\sqrt{e}\right.}\right.}{}$$

$$= -\frac{adx}{e^2} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce} - \frac{bdx\cosh^{-1}(cx)}{e^2} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e} -$$

$$= -\frac{adx}{e^2} + \frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce} - \frac{bdx\cosh^{-1}(cx)}{e^2}$$

$$= -\frac{adx}{e^2} + \frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{}}{9ce}$$

$$= -\frac{adx}{e^2} + \frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{}}{9ce}$$

$$= -\frac{adx}{e^2} + \frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{}}{9ce}$$

$$= -\frac{adx}{e^2} + \frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{}}{9ce}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.04, size = 524, normalized size = 0.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^
(5/2) + (b*((4*d*Sqrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCos
h[c*x]))/c - (4*e^(3/2)*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2) - 3*c^3
*x^3*ArcCosh[c*x]))/(9*c^3) - I*d^(3/2)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(L
```

```
og[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] + Log[1 +
(I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyLog[2,
(I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*PolyLog[2,
 ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + I*d^(3/2)*
(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*S
qrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 + Sqrt[c^2*d + e])])) + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 - Sqrt[c^2*d + e])] + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
 Sqrt[c^2*d + e])])))/(4*e^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 79.09, size = 364, normalized size = 0.58

$$\frac{a\,x^3}{3e}-\frac{adx}{e^2}+\frac{a\,d^2\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^2\sqrt{de}}+\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}}{c\,e^2}-\frac{bdx\,\mathrm{arccosh}(cx)}{e^2}+\frac{b\,\mathrm{arccosh}(cx)\,x^3}{3e}-\frac{2b\sqrt{cx-1}}{9c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] 1/3*a*x^3/e-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+b*d*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2-b*d*x*arccosh(c*x)/e^2+1/3*b/e*arccosh(c*x)*
x^3-2/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/e-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/e+1/2*c*b*d^2/e^2*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/2*c*b*d^2/e^2*sum
(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*
_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2) + (x^3*e - 3*d*x)*e^(-2))
*a + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{acosh}(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a + b\,\operatorname{acosh}(c\,x)\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

3.490 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=521

$$-\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce}-\frac{b\cosh^{-1}(cx)}{4c^2e}+\frac{x^2(a+b\cosh^{-1}(cx))}{2e}+\frac{d(a+b\cosh^{-1}(cx))^2}{2be^2}-\frac{d(a+b\cosh^{-1}(cx)}{}$$

```
[Out] -1/4*b*arccosh(c*x)/c^2/e+1/2*x^2*(a+b*arccosh(c*x))/e+1/2*d*(a+b*arccosh(c
*x))^2/b/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(
1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/
2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1
/2*b*d*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e
)^(1/2)))/e^2-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e
```

Rubi [A]
time = 0.62, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5959, 5883, 92, 54, 5962, 5681, 2221, 2317, 2438}

$$-\frac{d(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{d(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}-\frac{d(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}-\frac{d(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^2}+\frac{d(a+b\cosh^{-1}(cx))^2}{2be^2}+\frac{x^2(a+b\cosh^{-1}(cx))}{2e}-\frac{bd\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bd\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{b\cosh^{-1}(cx)}{4c^2e}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/4*(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) - (b*ArcCosh[c*x])/(4*c^2*e)
+ (x^2*(a + b*ArcCosh[c*x]))/(2*e) + (d*(a + b*ArcCosh[c*x])^2)/(2*b*e^2) -
 (d*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Arc
Cosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[
c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/
(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) - (b*d*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^
2) - (b*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e])])/(2*e^2)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
```

```
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e} - \frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int x\left(a+b\cosh^{-1}(cx)\right)\,dx}{e} - \frac{d\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2e} - \frac{d\int\left(-\frac{a+b\cosh^{-1}\ldots}{2\sqrt{e}\left(\sqrt{-d}\ldots\right.}\ldots\right.}{\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}} - \frac{d\int\ldots}{\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\,\mathrm{co}\ldots\right.}{2\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\,\mathrm{co}\ldots\right.}{2\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\,\mathrm{co}\ldots\right.}{2\ldots}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\,\mathrm{co}\ldots\right.}{2\ldots}$$

Mathematica [A]
time = 0.40, size = 512, normalized size = 0.98

$$-\frac{-2ac^2ex^2 + bcex\sqrt{-1+cx}\,\sqrt{1+cx} - 2bc^2ex^2\cosh^{-1}(cx) - 2bc^2d\cosh^{-1}(cx)^2 + 2be\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right) + 2bc^2d\cosh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right) + 2bc^2d\cosh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right) + 2bc^2d\cosh^{-1}(cx)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right) + 2bc^2d\cosh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right) + 2ac^2d\log\left(d+ex^2\right) + 2bc^2d\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right) + 2bc^2d\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right) + 2bc^2d\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right) + 2bc^2d\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{4c^2e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/4*(-2*a*c^2*e*x^2 + b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*e*x^2
*ArcCosh[c*x] - 2*b*c^2*d*ArcCosh[c*x]^2 + 2*b*e*ArcTanh[Sqrt[(-1 + c*x)/(1
 + c*x)]] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt
[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^Arc
```

```
Cosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*L
og[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + 2*b*c^
2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e])] + 2*a*c^2*d*Log[d + e*x^2] + 2*b*c^2*d*PolyLog[2, (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] + 2*b*c^
2*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(c^2*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.77, size = 2970, normalized size = 5.70

method	result	size
derivativedivides	Expression too large to display	2970
default	Expression too large to display	2970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/4*b*c^2/e*arccosh(c*x)+1/2*a*c^4/e*x^2+b*c^4*((c^2*d+e)*c^2*d)^(1
/2)/e^2*d/(c^2*d+e)*arccosh(c*x)^2+1/4*b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2*d/
(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2
*d+e)*c^2*d)^(1/2)-e))+3*b*c^6/e^3*d^2/(c^2*d+e)*arccosh(c*x)^2*((c^2*d+e)*
c^2*d)^(1/2)+1/4*b*c^2*((c^2*d+e)*c^2*d)^(1/2)/e/(c^2*d+e)*arccosh(c*x)*ln(
1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)
-e))-1/4*b*c^2/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c
^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)+2*b
*c^8/e^4*d^3/(c^2*d+e)*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)+4*b*c^8/e^3*d
^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d
+e)*c^2*d)^(1/2)-e))*arccosh(c*x)-1/4*b*c^3/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
+b*c^4/e^3*d*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d
+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)+2*b*c^6/e^4*d^2*l
n(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/
2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)-4*b*c^8/e^3*d^3/(c^2*d+e)*arcco
sh(c*x)^2+1/2*b*c^4/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/
(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*d+2*b*c^10/e^4*d^4/(c^
2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^
2*d)^(1/2)-e))*arccosh(c*x)+5/2*b*c^6/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*d^
2-3/4*b*c^4/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(
-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d*((c^2*d+e)*c^2*d)^(1/2)-b*c^8/e^4*
d^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(
(c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-3/2*b*c^6/e^3*d^2/(c^2*d
+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*
```

```
c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-1/8*b*c^2/e/(c^2*d+e)*polylog(2,e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))
*((c^2*d+e)*c^2*d)^(1/2)+5/4*b*c^6/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d^2+1/4*b*c^4
/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((
c^2*d+e)*c^2*d)^(1/2)-e))*d-2*b*c^6/e^3*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)-1/2*b*c^4/
e^2*d*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2
*d)^(1/2)-e))*arccosh(c*x)-2*b*c^8/e^4*d^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)+b*c^6/e^4*d
^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c
^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-1/2*b*c^4/e/(c^2*d+e)*arccosh(c*x)^
2*d-2*b*c^6/e^4*d^2*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)+1/8*b*c^2*((c^2*
d+e)*c^2*d)^(1/2)/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+1/2*b*c^4*arccosh(c*x)/e*x^2-5/2
*b*c^6/e^2*d^2/(c^2*d+e)*arccosh(c*x)^2+2*b*c^8/e^3*d^3/(c^2*d+e)*polylog(2
,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-
e))+1/2*b*c^4/e^3*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2
*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-b*c^4/e^3*d*arccos
h(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)+b*c^10/e^4*d^4/(c^2*d+e)*polylog(2,e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-2*b*
c^10/e^4*d^4/(c^2*d+e)*arccosh(c*x)^2-1/2*a*c^4*d/e^2*ln(c^2*e*x^2+c^2*d)-b
*c^8/e^4*d^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((
c^2*d+e)*c^2*d)^(1/2)-e))-b*c^6/e^3*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-1/4*b*c^4/e^2*d*polylo
g(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/
2)-e))+2*b*c^6/e^3*d^2*arccosh(c*x)^2+2*b*c^8/e^4*d^3*arccosh(c*x)^2-1/2*b*
c^4*d/e^2*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_
R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+b*c^4*d*arccosh(c*
x)^2/e^2-2*b*c^8/e^4*d^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(
1/2)-3*b*c^6/e^3*d^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(
-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)
-3/2*b*c^4/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2
*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*d*((c^2*d+e)*c^2*d)^(1/2)+1/2
*b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2*d/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e^(-1) - d*e^(-2)*log(x^2*e + d))*a + b*integrate(x^3*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(x^2*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acosh}(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

3.491 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=544

$$\frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\ \cdots}{}$$

[Out] a*x/e+b*x*arccosh(c*x)/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e

Rubi [A]
time = 0.63, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5959, 5879, 75, 5909, 5962, 5681, 2221, 2317, 2438}

$$\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^{3/2}}+\frac{ax}{e}-\frac{b\sqrt{-d}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]

[Out] (a*x)/e - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) + (b*x*ArcCosh[c*x])/e + (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^(3/2

```
)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*e^(3/2))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{e}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \left(a+b\cosh^{-1}(cx)\right)dx}{e}-\frac{d\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e}+\frac{b\int \cosh^{-1}(cx)\,dx}{e}-\frac{d\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)}{e}$$

$$= \frac{ax}{e}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{e}-\frac{\sqrt{-d}\int \frac{a+b\cosh^{-1}(cx}{\sqrt{-d}-\sqrt{e}}}{2e}$$

$$= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{\sqrt{-d}\,\text{Subst}\left(\int \frac{(a+bx)\sinh(}{c\sqrt{-d}-\sqrt{e}}\right.}{2e}$$

$$= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}-\frac{\sqrt{-d}\,\text{Subst}\left(\int \frac{e^x(}{c\sqrt{-d}-\sqrt{-}}\right.}{}$$

$$= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

$$= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

$$= \frac{ax}{e}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce}+\frac{bx\cosh^{-1}(cx)}{e}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log}{2e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 457, normalized size = 0.84

$$\frac{4ac\sqrt{e}\,x-4ac\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+ib\left(4i\sqrt{e}\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)-cx\cosh^{-1}(cx)\right)-c\sqrt{d}\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-2\left(\log\left(1+\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}-\sqrt{c^2d+e}}\right)+\log\left(1+\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}+\sqrt{c^2d+e}}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{d}+\sqrt{c^2d+e}}\right)-2\text{PolyLog}\left(2,-\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}+\sqrt{c^2d+e}}\right)\right)+c\sqrt{d}\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-2\left(\log\left(1+\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{d}+\sqrt{c^2d+e}}\right)+\log\left(1-\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}+\sqrt{c^2d+e}}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}-\sqrt{c^2d+e}}\right)-2\text{PolyLog}\left(2,\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{d}+\sqrt{c^2d+e}}\right)\right)\right)}{4ce^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*((4*I)*S
qrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCosh[c*x]) - c*Sqrt[d
]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[d] - Sqrt[c^2*d + e])] + Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
```

```
Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]
) + Sqrt[c^2*d + e])] - 2*PolyLog[2, ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
d] + Sqrt[c^2*d + e])]) + c*Sqrt[d]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1
+ (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] - 2*PolyLog[2, (I*S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])))/(4*c*e^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.79, size = 301, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{a c^3 x}{e} - \frac{a c^3 d \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b c^3 \operatorname{arccosh}(cx) x}{e} - \frac{b c^2 \sqrt{cx-1}\,\sqrt{cx+1}}{e} - \frac{b c^4 d \left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2 \ldots\right.}\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{a c^3 x}{e} - \frac{a c^3 d \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b c^3 \operatorname{arccosh}(cx) x}{e} - \frac{b c^2 \sqrt{cx-1}\,\sqrt{cx+1}}{e} - \frac{b c^4 d \left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2 \ldots\right.}\right.}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a*c^3/e*x-a*c^3*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+b*c^3*arccos
h(c*x)/e*x-b*c^2/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*b*c^4*d/e*sum(_R1/(_R1^2
*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+d
ilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d
+2*e)*_Z^2+e))+1/2*b*c^4*d/e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln
((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1))*a + b*integrate(x^
2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acosh}(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a + b\,\mathrm{acosh}(c\,x)\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

3.492 $\int \frac{x(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=449

$$-\frac{(a+b\cosh^{-1}(cx))^2}{2be}+\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{(a+b\cosh^{-1}(cx))\log\left(1+\frac{}{c\sqrt{}}\right.}{2e}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2/b/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccos
h(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*
d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1+(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+
1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-
c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e
```

Rubi [A]
time = 0.50, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5959, 5962, 5681, 2221, 2317, 2438}

$$\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e}+\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e}-\frac{(a+b\cosh^{-1}(cx))^2}{2be}+\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/2*(a + b*ArcCosh[c*x])^2/(b*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2, -((S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) +
 (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
)])/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/(2*e)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{e}}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{e}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x}\right.}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, \cosh^{-1}(cx)\right.}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+$$

Mathematica [A]
time = 0.10, size = 447, normalized size = 1.00

$$-\frac{b\cosh^{-1}(cx)^2}{2e}+\frac{b\cosh^{-1}(cx)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\cosh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\cosh^{-1}(cx)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\cosh^{-1}(cx)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{a\log(d+ex^2)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/2*(b*ArcCosh[c*x]^2)/e + (b*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[
c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(
2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*e) + (a*Log[d + e*x^2])/(2*e) + (b*PolyLog[2, -((Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
```

```
2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e])])/(2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.04, size = 2845, normalized size = 6.34

method	result	size
derivativedivides	Expression too large to display	2845
default	Expression too large to display	2845

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*b*c^2/(c^2*d+e)*arccosh(c*x)^2-1/4*b*c^2/(c^2*d+e)*polylog(2,e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))+
1/2*b*c^2/e*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln(
(_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b*c^2*arccosh(c*
x)^2/e+1/2*a*c^2/e*ln(c^2*e*x^2+c^2*d)-b*c^2*((c^2*d+e)*c^2*d)^(1/2)/e/(c^2
*d+e)*arccosh(c*x)^2-b*c^2/e^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(
-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)
-1/4*b*((c^2*d+e)*c^2*d)^(1/2)/d/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))-3*b*c^4*((c
^2*d+e)*c^2*d)^(1/2)/e^2*d/(c^2*d+e)*arccosh(c*x)^2-2*b*c^6/e^3*d^2/(c^2*d+
e)*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)-1/2*b*c^2*((c^2*d+e)*c^2*d)^(1/2)
/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^
2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+3/2*b*c^2/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x
)*((c^2*d+e)*c^2*d)^(1/2)-2*b*c^8/e^3*d^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)-2*
b*c^4/e^3*d*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+
e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)-1/2*b*c^2/(c^2*d+e
)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^
(1/2)-e))*arccosh(c*x)+b*c^2/e^2*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)-1/2
*b*c^2/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^
2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-1/8*b*((c^2*d+e)*c^2*d)^(1/
2)/d/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*
((c^2*d+e)*c^2*d)^(1/2)-e))+1/8*b/d/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d
)^(1/2)+2*b*c^8/e^3*d^3/(c^2*d+e)*arccosh(c*x)^2+1/4*b/d/(c^2*d+e)*ln(1-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*
arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)-5/2*b*c^4/e/(c^2*d+e)*ln(1-e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(
c*x)*d-4*b*c^6/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*d^2+3/2*b*c^4/e^2/(c^2*d+
```

```
e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c
^2*d)^(1/2)-e))*d*((c^2*d+e)*c^2*d)^(1/2)+b*c^6/e^3*d^2/(c^2*d+e)*polylog(2
,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-
e))*((c^2*d+e)*c^2*d)^(1/2)+3/4*b*c^2/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c
^2*d)^(1/2)-2*b*c^6/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d^2-5/4*b*c^4/e/(c^2*d+e)*po
lylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)
^(1/2)-e))*d+2*b*c^6/e^3*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)+2*b*c^4/e^2*d*ln(1-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arc
cosh(c*x)+5/2*b*c^4/e/(c^2*d+e)*arccosh(c*x)^2*d-1/4*b*c^2*((c^2*d+e)*c^2*d
)^(1/2)/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+4*b*c^6/e^2*d^2/(c^2*d+e)*arccosh(c*x)^2-b
*c^8/e^3*d^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*
c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-b*c^4/e^3*d*polylog(2,e*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2
*d)^(1/2)+2*b*c^4/e^3*d*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)+1/2*b*c^2/e*
ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1
/2)-e))*arccosh(c*x)+b*c^6/e^3*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))+b*c^4/e^2*d*polylog(2,e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-2*
b*c^6/e^3*d^2*arccosh(c*x)^2-2*b*c^4*d*arccosh(c*x)^2/e^2+1/4*b*c^2/e*polyl
og(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1
/2)-e))+2*b*c^6/e^3*d^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1
/2)+3*b*c^4/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^
2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*d*((c^2*d+e)*c^2*d)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(-1)*log(x^2*e + d) + b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))/(x^2*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccosh(c*x) + a*x)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acosh}(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

3.493 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=501

$$\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\cosh^{-1}(cx))\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a$$

[Out] 1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)

Rubi [A]

time = 0.53, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5909, 5962, 5681, 2221, 2317, 2438}

$$\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2),x]

[Out] ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\,\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\,\cosh(x)}\,dx\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x}{c\sqrt{-d}+\sqrt{-}}\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}\,\sqrt{}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}\,\sqrt{}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\right.}{2\sqrt{-d}\,\sqrt{}}$$

<u>Mathematica</u> [A]
time = 0.25, size = 397, normalized size = 0.79

$$\frac{-\left(\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)\right)+\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)+\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)-\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)+b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)-b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)-b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)+b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh[c*x])
*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] - b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - b
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
```

```
+ b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.95, size = 241, normalized size = 0.48

method	result
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc^{2}\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{4}+\left(4c^{2}d+2e\right)\_Z^{2}+e\right)}\frac{\_R1\left(\mathrm{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\,\sqrt{c}}{\_R1}\right.\right.}{\_R}\right.}{2}}{}$
default	$\frac{\frac{ac\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{bc^{2}\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{4}+\left(4c^{2}d+2e\right)\_Z^{2}+e\right)}\frac{\_R1\left(\mathrm{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\,\sqrt{c}}{\_R1}\right.\right.}{\_R}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*b*c^2*sum(_R1/(_R1^2*e+2*c
^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*
_Z^2+e))-1/2*b*c^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + b*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\, x)}{e\, x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2), x)
```

3.494 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=489

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)}{d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}}}{c\sqrt{-d}-\sqrt{-c}}\right.}{2d}$$

```
[Out] (a+b*arccosh(c*x))^2/b/d+(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/d-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-
1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(2
,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*b*polylog
(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/d-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d
```

Rubi [A]

time = 0.66, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5959, 5882, 3799, 2221, 2317, 2438, 5962, 5681}

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd}+\frac{\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d}-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(b*d) + ((a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[
c*x])])/d - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh
[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*d) - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d)
 - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e])])/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d(d+ex^2)}\right) dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d} - \frac{e\int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{}{2\sqrt{e}\ldots}\right.}{d}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{d} + \frac{\sqrt{e}\int \frac{a+b\cosh^{-1}\ldots}{\sqrt{-d}-\ldots}}{2d}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\text{Subst}\left(\int \log \ldots\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(cx)}\right)}{2d} + \ldots$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1 \ldots\right.}{2\ldots}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1 \ldots\right.}{2\ldots}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1 \ldots\right.}{2\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.58, size = 418, normalized size = 0.85

$$\frac{4a\log(x)-2a\log(d+ex^2)+b\left(-4i\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\tanh^{-1}\left(\frac{cd\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{\sqrt{c^2d(c^2d+e)}\,x}\right)+4\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)-2\cosh^{-1}(cx)\log\left(1+\frac{\left(2c^2d+e-2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+2i\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\log\left(1+\frac{\left(2c^2d+e-2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-2\cosh^{-1}(cx)\log\left(1+\frac{\left(2c^2d+e+2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-2i\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\log\left(1+\frac{\left(2c^2d+e+2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-2\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)+\text{PolyLog}\left(2,-\frac{\left(2c^2d+e-2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+\text{PolyLog}\left(2,-\frac{\left(2c^2d+e+2\sqrt{c^2d(c^2d+e)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)\right)}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*((-4*I)*ArcSin[Sqrt[1 + (c^2*d)/e]]*Ar
cTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c^2*d*(c^2*d + e)]*x
)] + 4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + (
2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)*Ar
cSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])
/(e*E^(2*ArcCosh[c*x]))] - 2*ArcCosh[c*x]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2
*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] - (2*I)*ArcSin[Sqrt[1 + (c^2*d)/e]
]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))]
 - 2*PolyLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, -((2*c^2*d + e - 2*Sqrt[
c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))] + PolyLog[2, -((2*c^2*d + e +
2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))]))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.26, size = 393, normalized size = 0.80

method	result
derivativedivides	$-\frac{a\ln\left(c^{2}e\,x^{2}+c^{2}d\right)}{2d}+\frac{a\ln\left(cx\right)}{d}-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{4}+\left(4c^{2}d+2e\right)\_Z^{2}+e\right)}\frac{\left(\_R1^{2}+1\right)\left(\mathrm{arccosh}\left(cx\right)\ln\left(\frac{\_R1-cx-}{}\right.\right.}{}\right.}{4d}$
default	$-\frac{a\ln\left(c^{2}e\,x^{2}+c^{2}d\right)}{2d}+\frac{a\ln\left(cx\right)}{d}-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{4}+\left(4c^{2}d+2e\right)\_Z^{2}+e\right)}\frac{\left(\_R1^{2}+1\right)\left(\mathrm{arccosh}\left(cx\right)\ln\left(\frac{\_R1-cx-}{}\right.\right.}{}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*ln(c*x)-1/4*b*sum((_R1^2+1)/(_R1^2*e+2*c^2
*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z
^2+e))*e/d+b/d*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)-1/4*b*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c
*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(x^2*e + d)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/(x^3*e + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^3*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\, x)}{x\, (e\, x^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)), x)
```

3.495 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=543

$$-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d}+\frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right.}{2(-d)^{3/2}}$$

```
[Out] (-a-b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+1/2*(a+b*
arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(
-d)^(3/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(
1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccosh(c
*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e
)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*poly
log(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1
/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(
2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2))
)*e^(1/2)/(-d)^(3/2)
```

Rubi [A]
time = 0.65, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5959, 5883, 94, 211, 5909, 5962, 5681, 2221, 2317, 2438}

$$\frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2(-d)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d}-\frac{b\sqrt{e}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2(-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh[c*x])*
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d
)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (b*S
qrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
```

```
e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
```

```
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d} - \frac{e\int \left(\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{(bc^2)\,\mathrm{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} - \frac{e\int \frac{a+b\,c}{\sqrt{-}}}{2(}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} - \frac{e\mathrm{Subst}\left(\int \frac{(a+bx)\sinh(}{c\sqrt{-d}-\sqrt{e}\,c}\right.}{2(-}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} - \frac{e\mathrm{Subst}\left(\int \frac{e^x(}{c\sqrt{-d}-\sqrt{-c}}\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\,\mathrm{l}}{2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\,\mathrm{l}}{2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\,\mathrm{l}}{2}$$

<u>Mathematica</u> [A]
time = 1.01, size = 549, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{dx} + \frac{2bc\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{d\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{d\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}} + \frac{bd\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{bd\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}} + \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{3/2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x) + (2*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d*Sqrt[e]*(a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
```

```
(-d)^(5/2) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])
/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(3/2) + (Sqrt[e]*(a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(3/2) + (d*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2,
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)^(3/2) +
(b*d*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^
2*d) - e])])/(-d)^(5/2) + (b*d*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(3/2))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 17.03, size = 339, normalized size = 0.62

method	result
derivativedivides	$c\left(-\frac{ae\arctan\left(\frac{xe}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{a}{dcx}-\frac{b\,\mathrm{arccosh}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2e+4c^2d+e\right.}{}\right.}{}\right.$
default	$c\left(-\frac{ae\arctan\left(\frac{xe}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{a}{dcx}-\frac{b\,\mathrm{arccosh}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2e+4c^2d+e\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/c*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-a/d/c/x-b*arccosh(c*x)/d/c/
x+1/8*b/c^2/d^2*e*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(
c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*b/d*ar
ctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*b/c^2/d^2*e*sum((4*_R1^2*c^2*d+_R
1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Roo
tOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x)) + b*integrate(log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^4*e + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^2 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\, x)}{x^2\, (e\, x^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)), x)
```

3.496 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=550

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{bd^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)}{d^2}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d/x^2-e*(a+b*arccosh(c*x))^2/b/d^2-e*(a+b*arccosh(c
*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+1/2*e*(a+b*arccosh(c*x
))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^
(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,-1/(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)/d^2+1/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*
b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^
2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1
/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2+1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/x
```

Rubi [A]

time = 0.71, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5959, 5883, 97, 5882, 3799, 2221, 2317, 2438, 5962, 5681}

$$\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{bd^2}-\frac{e\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{be\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2d^2}+\frac{be\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{2d^2}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 - (e*(a + b*ArcCosh[c*x])^2)/(b*d^2) - (e*(a + b*ArcCosh[c*x])*Log[1 + E^(
-2*ArcCosh[c*x])])/d^2 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x]
)*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d
^2) + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog[
2, -E^(-2*ArcCosh[c*x])])/(2*d^2) + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*PolyLog[2, (Sqrt[e]
```

```
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^2)
 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*d^2)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)}\,dx=\int \left(\frac{a+b\cosh^{-1}(cx)}{dx^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)\,dx$$

$$=\frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2}+\frac{e^2\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d}-\frac{e\text{Subst}\left(\int (a+bx)\tanh(\right.}{d^2}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2}-\frac{(2e)\text{Subst}\left(\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2}-\frac{e\left(a+b\cos\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}}\right.}{d^2}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{}}{c\sqrt{-}}\right.}{2d^2}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{}}{c\sqrt{-}}\right.}{2d^2}$$

$$=\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{}}{c\sqrt{-}}\right.}{2d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 479, normalized size = 0.87

$$\frac{-\frac{2ad}{x^2}-4ae\log(x)+2ae\log\left(d+ex^2\right)+b\left(\frac{2cd\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{x}-\frac{2d\cosh^{-1}(cx)}{x^2}+4ie\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\tanh^{-1}\left(\frac{cd\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}{\sqrt{c^2d\left(c^2d+e\right)}\,x}\right)-4e\cosh^{-1}(cx)\log\left(1+e^{-2\cosh^{-1}(cx)}\right)+2e\cosh^{-1}(cx)\log\left(1+\frac{\left(2c^2d+e-2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-2ie\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\log\left(1+\frac{\left(2c^2d+e-2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+2e\cosh^{-1}(cx)\log\left(1+\frac{\left(2c^2d+e+2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+2ie\text{ArcSin}\left(\sqrt{1+\frac{c^2d}{e}}\right)\log\left(1+\frac{\left(2c^2d+e+2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+2e\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(cx)}\right)-e\text{PolyLog}\left(2,-\frac{\left(2c^2d+e-2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)-e\text{PolyLog}\left(2,-\frac{\left(2c^2d+e+2\sqrt{c^2d\left(c^2d+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)\right)}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*a*d)/x^2 - 4*a*e*Log[x] + 2*a*e*Log[d + e*x^2] + b*((2*c*d*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x))/x - (2*d*ArcCosh[c*x])/x^2 + (4*I)*e*ArcSin[Sqrt
[1 + (c^2*d)/e]]*ArcTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c
^2*d*(c^2*d + e)]*x)] - 4*e*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 2*e
```

```
*ArcCosh[c*x]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*Arc
Cosh[c*x]))] - (2*I)*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2
*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*ArcCosh[c*x]*Log[1
+ (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)
*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d +
 e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])] - e*Po
lyLog[2, -((2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))
)] - e*PolyLog[2, -((2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCo
sh[c*x]))])])/(4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.61, size = 495, normalized size = 0.90

method	result
derivativedivides	$c^2\left(\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2dcx}-\frac{b}{2d}-\frac{b\,\mathrm{arccosh}(cx)}{2d\,c^2x^2}+\frac{b\,e^2\left(\_R\right.}{}\right.$
default	$c^2\left(\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+\frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2dcx}-\frac{b}{2d}-\frac{b\,\mathrm{arccosh}(cx)}{2d\,c^2x^2}+\frac{b\,e^2\left(\_R\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/c^2*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*a/d/c^2/x^2-a/c^2/d^2*e*ln(c*x
)+1/2*b/d/c/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2*b/d-1/2*b*arccosh(c*x)/d/c^2/
x^2+1/4*b/c^2/d^2*e^2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_
R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b/c^2/d^2*e*arccos
h(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-b/c^2/d^2*e*arccosh(c*x)*l
n(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-b/c^2/d^2*e*dilog(1+I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))-b/c^2/d^2*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))+1/4*b/c^2/d^2*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c
*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
[Out] 1/2*a*(e*log(x^2*e + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^5*e + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(x^5*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^3 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d),x)
[Out] Integral((a + b*acosh(c*x))/(x**3*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x^3\,(e\,x^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)),x)
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)), x)
```

3.497 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d+ex^2)}\,dx$

Optimal. Leaf size=624

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\text{ArcTan}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{6d}-\frac{bce\text{Ar}}{}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3+e*(a+b*arccosh(c*x))/d^2/x+1/6*b*c^3*arctan((
c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c*e*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2
+1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1
/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+b*arccosh(c*
x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)
^(1/2)))/(-d)^(5/2)+1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*e^(3
/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+
1/2*b*e^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(
1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/2*
b*e^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2
```

Rubi [A]
time = 0.67, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5959, 5883, 105, 12, 94, 211, 5909, 5962, 5681, 2221, 2317, 2438}

$$\frac{e^{3/2}(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{5/2}}+\frac{e^{3/2}(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2(-d)^{5/2}}+\frac{e(a+b\cosh^{-1}(cx))}{d^2x}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{bc^3\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{6d}-\frac{bce\text{ArcTan}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d^2}-\frac{be^{3/2}\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}-\frac{be^{3/2}\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2(-d)^{5/2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) + (e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
 + c*x]])/(6*d) - (b*c*e*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (e^(3/
2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) +
 (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(
5/2)) - (b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
```

```
[-(c^2*d) - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (b*e^(3/2)*Pol
yLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*
(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d+ex^2\right)}\,dx=\int\left(\frac{a+b\cosh^{-1}(cx)}{dx^4}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x^2}+\frac{e^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{a+b\cosh^{-1}(cx)}{x^4}\,dx}{d}-\frac{e\int\frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2}+\frac{e^2\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{(bc)\int\frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3d}-$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{(bc)\int\frac{}{x\sqrt{-}}}{}$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}-\frac{bce\tan^{-1}\left(\sqrt{}\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}-\frac{bce\tan^{-1}\left(\sqrt{}\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{}\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{}\right.}{}$$

$$=\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}+\frac{bc^3\tan^{-1}\left(\sqrt{}\right.}{}$$

Mathematica [A]
time = 1.12, size = 641, normalized size = 1.03

$$\frac{1}{6}\left(-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{dx^3}+\frac{6e\left(a+b\cosh^{-1}(cx)\right)}{d^2x}-\frac{6bce\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)\right)}{dx^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}-\frac{3be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}+\frac{3be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{(-d)^{5/2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x^3) + (6*e*(a + b*ArcCosh[c*x]))/(d^2*x) - (
6*b*c*e*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(d^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (b*c*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqr
```

```
t[-1 + c^2*x^2]]))/(d*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*e^(3/2)*(a + b
*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 - e])])/(-d)^(5/2) + (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Ar
cCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*e^(3/2)*
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(
c^2*d) - e])])/(-d)^(5/2) - (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-
d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2))/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.65, size = 430, normalized size = 0.69

method	result
derivativedivides	$c^3\left(\frac{a\,e^2\arctan\left(\frac{xe}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,c^2x^2}-\frac{b\,\mathrm{arccosh}(cx)}{3d\,c^3x^3}+\frac{b\,\mathrm{arccosh}(cx)e}{c^3d^2x}\right.$
default	$c^3\left(\frac{a\,e^2\arctan\left(\frac{xe}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,c^2x^2}-\frac{b\,\mathrm{arccosh}(cx)}{3d\,c^3x^3}+\frac{b\,\mathrm{arccosh}(cx)e}{c^3d^2x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^3*e^2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/3*a/d/c^3/x^3+a/c^
3/d^2*e/x+1/6*b/d/c^2/x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/3*b*arccosh(c*x)/d/
c^3/x^3+b/c^3*arccosh(c*x)/d^2*e/x-2*b/c^2/d^2*e*arctan(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))-1/8*b/c^4/d^3*e^2*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*
d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^
2+e))+1/3*b/d*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/8*b/c^4/d^3*e^2*sum
((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*
```

```
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/d^(5/2) + (3*x^2*e - d)/(d^2*x^3
)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^6*e + d*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^6*e + d*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{x^{4} \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d + e*x^2)),x)
[Out] int((a + b*acosh(c*x))/(x^4*(d + e*x^2)), x)
```

3.498 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=562

$$\frac{d(a+b\cosh^{-1}(cx))}{2e^2(d+ex^2)}-\frac{(a+b\cosh^{-1}(cx))^2}{2be^2}-\frac{bc\sqrt{d}\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(a+b\cosh^{-1}}{}$$

```
[Out] 1/2*d*(a+b*arccosh(c*x))/e^2/(e*x^2+d)-1/2*(a+b*arccosh(c*x))^2/b/e^2+1/2*(
a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*(a+b*arccosh
(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d
-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*b*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))/e^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c
*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*c*arctanh(x*(c
^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)*(c^2*x^2-1)^(1/2)/e^2/(c^2
*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5959, 5957, 533, 385, 214, 5962, 5681, 2221, 2317, 2438}

$$\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}+\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^2}+\frac{d(a+b\cosh^{-1}(cx))}{2e^2(d+ex^2)}-\frac{(a+b\cosh^{-1}(cx))^2}{2be^2}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^2}-\frac{bc\sqrt{d}\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2e^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (d*(a + b*ArcCosh[c*x]))/(2*e^2*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*
e^2) - (b*c*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e
^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqr
```

```
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
```

```
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{(bcd)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}\,dx}{2e^2} + \frac{\int \left(-\frac{a+b\,c}{2\sqrt{e}\,\left(\sqrt{\right.}\right.}\right.}{}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2e^{3/2}} - \frac{\left(bcd\sqrt{-1+\right.}{2e}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2e^{3/2}} + \frac{\text{Subs}}{}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}}$$

$$= \frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2} - \frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 693, normalized size = 1.23

$$\frac{\frac{2ad}{d+ex^2}+2a\log\left(d+ex^2\right)+b\left(-2\cosh^{-1}(cx)^2+2\cosh^{-1}(cx)\left(\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)+2\cosh^{-1}(cx)\left(\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)-i\sqrt{d}\left(\frac{\cosh^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}\,x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}\,x-i\sqrt{-c^2d-e}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right)}{\sqrt{-c^2d-e}}\right)-i\sqrt{d}\left(-\frac{\cosh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\sqrt{d}\,x+\sqrt{-c^2d-e}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right)}{\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] + b*(-2*ArcCosh[c*x]^2 + 2*ArcCos
h[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
```

```
e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
) + 2*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]) - I*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2
*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e
]) - I*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqr
t[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/
(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Po
lyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]
 + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]))/(4*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 10.67, size = 3019, normalized size = 5.37

method	result	size
derivativedivides	Expression too large to display	3019
default	Expression too large to display	3019

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-5/4*b*c^6*d/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))+1/2*b*c^6*arccosh(c*x)*d/e^2
/(c^2*e*x^2+c^2*d)-b*c^10*d^3/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-2*b*c^8*d^2/e^3/(c
^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d
+e)*c^2*d)^(1/2)-e))+2*b*c^10*d^3/e^4/(c^2*d+e)*arccosh(c*x)^2+4*b*c^8*d^2/
e^3/(c^2*d+e)*arccosh(c*x)^2+2*b*c^8/e^4*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d^2*arccosh(c*x)+3/4*b*c^4
/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*
((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-b*c^6/e^4*polylog(2,e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*
d*((c^2*d+e)*c^2*d)^(1/2)-1/2*b*c^4/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)+2*b*c^
6/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2
*d)^(1/2)-e))*d*arccosh(c*x)-1/4*b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2/(c^2*d+e
)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^
2*d)^(1/2)-e))-b*c^4/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2
*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)+5/2*b
*c^6*d/e^2/(c^2*d+e)*arccosh(c*x)^2-b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2/(c^2*
d+e)*arccosh(c*x)^2+1/2*b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2/(c^2*d+e)*arctanh
```

(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-1/2*b*c^4*((c^2*d+e)*c^2*d)^(1/2)/e^2/(c^2*d+e)*arccosh(c*x)*ln(1- e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+2*b*c^6/e^4*d*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)+1/2*a*c^6*d/e^2/(c^ 2*e*x^2+c^2*d)-1/4*b*c^4/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1) ^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))-2*b*c^6/e^3*arccosh(c*x)^ 2*d+b*c^8/e^4*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d^2+b*c^6/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c* x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d+1/2*b*c^4/e/(c^2*d+ e)*arccosh(c*x)^2+b*c^4/e^3*arccosh(c*x)^2*((c^2*d+e)*c^2*d)^(1/2)-2*b*c^8/ e^4*d^2*arccosh(c*x)^2-1/2*b*c^4/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1) ^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)+1 /2*b*c^4/e^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d +e)*c^2*d)^(1/2)-e))*arccosh(c*x)+3*b*c^6*d/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x- 1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c *x)*((c^2*d+e)*c^2*d)^(1/2)+1/4*b*c^2/d/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/ 2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^(1/2)+2*b*c^8*d^2/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2* d+e)*c^2*d)^(1/2)-1/4*b*c^2*((c^2*d+e)*c^2*d)^(1/2)/d/e/(c^2*d+e)*arccosh(c *x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+3/2*b*c^4/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)*((c^2*d+e)*c^2*d)^ (1/2)-1/8*b*c^2*((c^2*d+e)*c^2*d)^(1/2)/d/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x -1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e))+3/2*b*c^ 6*d/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d -2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)-5/2*b*c^6*d/e^2/(c^2 *d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2 *d)^(1/2)-e))*arccosh(c*x)-2*b*c^10*d^3/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)- 4*b*c^8*d^2/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^ 2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*arccosh(c*x)-2*b*c^6/e^4*ln(1-e*(c*x+(c*x -1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*d*arccos h(c*x)*((c^2*d+e)*c^2*d)^(1/2)-2*b*c^8*d^2/e^4/(c^2*d+e)*arccosh(c*x)^2*((c ^2*d+e)*c^2*d)^(1/2)-3*b*c^6*d/e^3/(c^2*d+e)*arccosh(c*x)^2*((c^2*d+e)*c^2* d)^(1/2)+1/8*b*c^2/d/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/ 2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)+b*c^8 *d^2/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2* d-2*((c^2*d+e)*c^2*d)^(1/2)-e))*((c^2*d+e)*c^2*d)^(1/2)+1/2*a*c^4/e^2*ln(c^ 2*e*x^2+c^2*d)+1/2*b*c^4/e^2*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)* (arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x- (c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)) -b*c^4*arccosh(c*x)^2/e^2+1/4*b*c^4/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x +1)^(1/2))^2/(-2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)-e)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*(e^(-2)*log(x^2*e + d) + d/(x^2*e^3 + d*e^2))*a + b*integrate(x^3*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acosh}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(a + b\operatorname{acosh}(c\,x))}{(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

3.499 $\int \frac{x(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex^2)}+\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2\sqrt{d}\,e\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/e/(e*x^2+d)+1/2*b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/
2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/e/d^(1/2)/(c^2*d+e)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5957, 533, 385, 214}

$$\frac{bc\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}\,e\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d+e}}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCosh[c*x])/(e*(d + e*x^2)) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh
[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d
 + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
```

```
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(d+ex^2)^2}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{2e\,(d+ex^2)}+\frac{(bc)\int\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)}\,dx}{2e}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\,(d+ex^2)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\,(d+ex^2)}\,dx}{2e\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\,(d+ex^2)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{d-(c^2d+e)x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{2e\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{a+b\cosh^{-1}(cx)}{2e\,(d+ex^2)}+\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2\sqrt{d}\,e\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 123, normalized size = 1.09

$$-\frac{\frac{a}{d+ex^2}+\frac{b\cosh^{-1}(cx)}{d+ex^2}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\text{ArcTan}\left(\frac{\sqrt{-c^2d-e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\,\sqrt{-c^2d-e}\,\sqrt{-1+c^2x^2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a/(d + e*x^2) + (b*ArcCosh[c*x])/(d + e*x^2) - (b*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(S
qrt[d]*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c^2*x^2]))/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(96) = 192.
time = 7.67, size = 646, normalized size = 5.72

method	result
derivativedivides	$\frac{-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccosh}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{b c^6 \sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(\frac{2\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2x^2-1}\,e-2\sqrt{-c^2de}\,c}{ecx+\sqrt{-c^2de}}\right.}{4\sqrt{c^2x^2-1}\,\left(\sqrt{-c^2de}+e\right)\left(e-\sqrt{-c^2de}\right)\sqrt{-c^2de}\,\sqrt{-\frac{c^2d+}{e}}}}{}$
default	$\frac{-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccosh}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{b c^6 \sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(\frac{2\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2x^2-1}\,e-2\sqrt{-c^2de}\,c}{ecx+\sqrt{-c^2de}}\right.}{4\sqrt{c^2x^2-1}\,\left(\sqrt{-c^2de}+e\right)\left(e-\sqrt{-c^2de}\right)\sqrt{-c^2de}\,\sqrt{-\frac{c^2d+}{e}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a*c^4/e/(c^2*e*x^2+c^2*d)-1/2*b*c^4/e/(c^2*e*x^2+c^2*d)*arccosh
(c*x)+1/4*b*c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-c^2*d*e)^(
1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)*ln(2*((-
(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2
*d*e)^(1/2)))*d-1/4*b*c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-
c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2
)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e
*c*x-(-c^2*d*e)^(1/2)))*d+1/4*b*c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
^(1/2)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+
e)/e)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)
*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*e-1/4*b*c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(
c^2*x^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)
/(-(c^2*d+e)/e)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

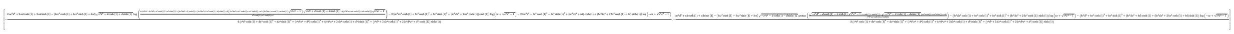
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c*integrate(1/2/(c^3*x^5*e^2 + (c^3*d*e - c*e^2)*x^3 - c*d*x*e + (c
^2*x^4*e^2 + (c^2*d*e - e^2)*x^2 - d*e)*e^(1/2*log(c*x + 1) + 1/2*log(c*x -
 1))), x) + c^2*log(x^2*e + d)/(c^2*d*e + e^2) + (2*(c^2*d + e)*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1)) - (c^2*x^2*e + c^2*d)*log(c*x + 1) - (c^2*x^2*e
```

```
 + c^2*d)*log(c*x - 1))/(c^2*d^2*e + (c^2*d*e^2 + e^3)*x^2 + d*e^2))*b - 1/
2*a/(x^2*e^2 + d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.
time = 0.41, size = 978, normalized size = 8.65



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*cosh(1) + 2*a*d*sinh(1) - (b*c*x^2*cosh(1) + b*c
*x^2*sinh(1) + b*c*d)*sqrt(c^2*d^2 + d*cosh(1) + d*sinh(1))*log((4*c^4*d^2*
x^2 - 2*c^2*d^2 + x^2*cosh(1)^2 + x^2*sinh(1)^2 + (4*c^2*d*x^2 - d)*cosh(1)
 + (4*c^2*d*x^2 + 2*x^2*cosh(1) - d)*sinh(1) + 2*(2*c^3*d*x^2 + c*x^2*cosh(
1) + c*x^2*sinh(1) - c*d + (2*c^2*d*x + x*cosh(1) + x*sinh(1))*sqrt(c^2*x^2
 - 1))*sqrt(c^2*d^2 + d*cosh(1) + d*sinh(1)) + 4*(c^3*d^2*x + c*d*x*cosh(1)
 + c*d*x*sinh(1))*sqrt(c^2*x^2 - 1))/(x^2*cosh(1) + x^2*sinh(1) + d)) - 2*(
b*c^2*d*x^2*cosh(1) + b*x^2*cosh(1)^2 + b*x^2*sinh(1)^2 + (b*c^2*d*x^2 + 2*
b*x^2*cosh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^2 + b*x^2
*cosh(1)^2 + b*x^2*sinh(1)^2 + (b*c^2*d*x^2 + b*d)*cosh(1) + (b*c^2*d*x^2 +
 2*b*x^2*cosh(1) + b*d)*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*co
sh(1) + d*x^2*cosh(1)^3 + d*x^2*sinh(1)^3 + (c^2*d^2*x^2 + d^2)*cosh(1)^2 +
 (c^2*d^2*x^2 + 3*d*x^2*cosh(1) + d^2)*sinh(1)^2 + (c^2*d^3 + 3*d*x^2*cosh(
1)^2 + 2*(c^2*d^2*x^2 + d^2)*cosh(1))*sinh(1)), -1/2*(a*c^2*d^2 + a*d*cosh(
1) + a*d*sinh(1) - (b*c*x^2*cosh(1) + b*c*x^2*sinh(1) + b*c*d)*sqrt(-c^2*d^
2 - d*cosh(1) - d*sinh(1))*arctan((sqrt(-c^2*d^2 - d*cosh(1) - d*sinh(1))*s
qrt(c^2*x^2 - 1)*(x*cosh(1) + x*sinh(1)) - sqrt(-c^2*d^2 - d*cosh(1) - d*si
nh(1))*(c*x^2*cosh(1) + c*x^2*sinh(1) + c*d))/(c^2*d^2 + d*cosh(1) + d*sinh
(1))) - (b*c^2*d*x^2*cosh(1) + b*x^2*cosh(1)^2 + b*x^2*sinh(1)^2 + (b*c^2*d
*x^2 + 2*b*x^2*cosh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^2
+ b*x^2*cosh(1)^2 + b*x^2*sinh(1)^2 + (b*c^2*d*x^2 + b*d)*cosh(1) + (b*c^2*
d*x^2 + 2*b*x^2*cosh(1) + b*d)*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2
*d^3*cosh(1) + d*x^2*cosh(1)^3 + d*x^2*sinh(1)^3 + (c^2*d^2*x^2 + d^2)*cosh
(1)^2 + (c^2*d^2*x^2 + 3*d*x^2*cosh(1) + d^2)*sinh(1)^2 + (c^2*d^3 + 3*d*x^
2*cosh(1)^2 + 2*(c^2*d^2*x^2 + d^2)*cosh(1))*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{acosh}\,(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

3.500 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=598

$$\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd^2}-\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{d^2}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/d/(e*x^2+d)+(a+b*arccosh(c*x))^2/b/d^2+(a+b*arccosh(
c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*(a+b*arccosh(c*x)
)*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(
1/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^
(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-
1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/
d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/d^2-1/2*b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-
1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(3/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5959, 5882, 3799, 2221, 2317, 2438, 5957, 533, 385, 214, 5962, 5681}

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^2}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd^2}+\frac{\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{bc\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{2d^{3/2}\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d+e}}-\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2d^2}-\frac{b\,\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(2*d*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^2) - (
b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a + b*A
rcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^2 - ((a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) -
((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2)
```

```
 - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, -((Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) - (b*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - (b
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])
/(2*d^2) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e])])/(2*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^2} - \frac{(bc)\int \frac{}{\sqrt{-1+}}}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{d^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right.}{d^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(a+b\cosh^{-1}\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)} - \frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}\right.}{}$$

Mathematica [F]
time = 2.88, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.83, size = 529, normalized size = 0.88

method	result
derivativedivides	$\frac{a\,c^2}{2d(c^2 e\,x^2+c^2 d)}-\frac{a\ln(c^2 e\,x^2+c^2 d)}{2d^2}+\frac{a\ln(cx)}{d^2}+\frac{b\,c^2\mathrm{arccosh}(cx)}{2d(c^2 e\,x^2+c^2 d)}+\frac{b\sqrt{(c^2 d+e)\,c^2 d}\ \mathrm{arctanh}\left(\frac{4c^2 d+2e\left(cx+\right.}{}\right.}{2d^2(c^2 d+e)}$
default	$\frac{a\,c^2}{2d(c^2 e\,x^2+c^2 d)}-\frac{a\ln(c^2 e\,x^2+c^2 d)}{2d^2}+\frac{a\ln(cx)}{d^2}+\frac{b\,c^2\mathrm{arccosh}(cx)}{2d(c^2 e\,x^2+c^2 d)}+\frac{b\sqrt{(c^2 d+e)\,c^2 d}\ \mathrm{arctanh}\left(\frac{4c^2 d+2e\left(cx+\right.}{}\right.}{2d^2(c^2 d+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)+1
/2*b*c^2*arccosh(c*x)/d/(c^2*e*x^2+c^2*d)+1/2*b*((c^2*d+e)*c^2*d)^(1/2)/d^2
/(c^2*d+e)*arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e
)/(c^4*d^2+c^2*d*e)^(1/2))-1/4*b/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2
*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z
^2+e))+b/d^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1+I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-1/4*b/d^2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*x^2*e + d^2) - log(x^2*e + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**2,x)
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)^2),x)
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)^2), x)
```

3.501 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)^2}\,dx$

Optimal. Leaf size=634

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)}-\frac{2e\left(a+b\cosh^{-1}(cx)\right)^2}{bd^3}+\frac{bce\sqrt{-1+c^2x^2}\ \mathrm{ta}}{2d^{5/2}\sqrt{c^2d+}}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d^2/x^2-1/2*e*(a+b*arccosh(c*x))/d^2/(e*x^2+d)-2*e*
(a+b*arccosh(c*x))^2/b/d^3-2*e*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/d^3+e*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2
)))/d^3+e*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+b*e*pol
ylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+b*e*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+b*e*p
olylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)
^(1/2)))/d^3+b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+1/2*b*c*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/x+1/2*b*c*e*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1
/2))*(c^2*x^2-1)^(1/2)/d^(5/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5959, 5883, 97, 5882, 3799, 2221, 2317, 2438, 5957, 533, 385, 214, 5962, 5681}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^2*x) - (a + b*ArcCosh[c*x])/(2*d^2*
x^2) - (e*(a + b*ArcCosh[c*x]))/(2*d^2*(d + e*x^2)) - (2*e*(a + b*ArcCosh[c
*x])^2)/(b*d^3) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sq
rt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (2*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^3 + (
e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 + (e*(
```

```
a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/d^3 + (b*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/d^3 + (b*e*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/d^3 +
 (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])
])/d^3 + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^
2*d) - e]))])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/d^3
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^3} - \frac{2e\left(a+b\cosh^{-1}(cx)\right)}{d^3x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} + \frac{2e^2x(a \ldots}{d^3 \ldots}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} + \frac{e^2\int \frac{x(a+b \ldots}{(\ldots}}{\ldots}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2d^2} - \frac{(2 \ldots}{\ldots}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1} \ldots\right.}{bd^3}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1} \ldots\right.}{bd^3}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2 \ldots}}{2d^{5/2}\sqrt{c^2 \ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2 \ldots}}{2d^{5/2}\sqrt{c^2 \ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2 \ldots}}{2d^{5/2}\sqrt{c^2 \ldots}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2 \ldots}}{2d^{5/2}\sqrt{c^2 \ldots}}$$

Mathematica [F]
time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.53, size = 744, normalized size = 1.17

method	result
derivativedivides	$c^{2}\left(-\frac{ae}{2d^{2}(c^{2}e\,x^{2}+c^{2}d)}+\frac{ae\ln(c^{2}e\,x^{2}+c^{2}d)}{c^{2}d^{3}}-\frac{a}{2d^{2}c^{2}x^{2}}-\frac{2ae\ln(cx)}{c^{2}d^{3}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2x(c^{2}e\,x^{2}+c^{2}d)d}+\frac{bcx\sqrt{}\right.$
default	$c^{2}\left(-\frac{ae}{2d^{2}(c^{2}e\,x^{2}+c^{2}d)}+\frac{ae\ln(c^{2}e\,x^{2}+c^{2}d)}{c^{2}d^{3}}-\frac{a}{2d^{2}c^{2}x^{2}}-\frac{2ae\ln(cx)}{c^{2}d^{3}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2x(c^{2}e\,x^{2}+c^{2}d)d}+\frac{bcx\sqrt{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a*e/d^2/(c^2*e*x^2+c^2*d)+a/c^2*e/d^3*ln(c^2*e*x^2+c^2*d)-1/2*a/d
^2/c^2/x^2-2*a/c^2/d^3*e*ln(c*x)+1/2*b*c/x/(c^2*e*x^2+c^2*d)/d*(c*x-1)^(1/2
)*(c*x+1)^(1/2)+1/2*b*c*x/(c^2*e*x^2+c^2*d)/d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*e-1/2*b*c^2/(c^2*e*x^2+c^2*d)/d-1/2*b*c^2*x^2/(c^2*e*x^2+c^2*d)/d^2*e-1/2*
b/x^2/(c^2*e*x^2+c^2*d)/d*arccosh(c*x)-b*arccosh(c*x)*e/d^2/(c^2*e*x^2+c^2*
d)-1/2*b/c^2*((c^2*d+e)*c^2*d)^(1/2)/d^3/(c^2*d+e)*e*arctanh(1/4*(4*c^2*d+2
*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+1/2*b/
c^2/d^3*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-2*b/c^2/d^3*e*arccos
h(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*b/c^2/d^3*e*arccosh(c*x)
*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*b/c^2/d^3*e*dilog(1+I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))-2*b/c^2/d^3*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))+1/2*b/c^2/d^3*e^2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x
)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*x^2*e + d)/(d^2*x^4*e + d^3*x^2) - 2*e*log(x^2*e + d)/d^3 + 4*e*
log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^7*e^2 +
 2*d*x^5*e + d^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^7*e^2 + 2*d*x^5*e + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^3 (d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(d + e*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x^3\,(e\,x^2 + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2), x)
```

3.502 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=839

$$\frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{bcd\tanh^{-1}\Bigg(}{2\sqrt{c\sqrt{-d}}}$$

```
[Out] a*x/e^2+b*x*arccosh(c*x)/e^2+3/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)
-3/4*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccosh(c*x))*ln(1
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*po
lylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)
^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylo
g(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/
2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arcco
sh(c*x))/e^(5/2)/((-d)^(1/2)-x*e^(1/2))+1/4*d*(a+b*arccosh(c*x))/e^(5/2)/((
-d)^(1/2)+x*e^(1/2))-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2+1/2*b*c*d*arctanh(
(c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1
/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/
2)-1/2*b*c*d*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/
2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-
d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A]
time = 1.59, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5959, 5879, 75, 5909, 5963, 95, 214, 5962, 5681, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) + (b*x*ArcCosh[c*x])/e
^2 - (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a +
b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sqrt[
```

```
c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 +
c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) -
 (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d]
 + Sqrt[e]]*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*Sqrt[-d]
*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*S
qrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[
-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
```

```
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{e^2}+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}-\frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)\,dx$$

$$= \frac{\int \left(a+b\cosh^{-1}(cx)\right)\,dx}{e^2} - \frac{(2d)\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2} + \frac{d^2\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e^2}$$

$$= \frac{ax}{e^2} + \frac{b\int \cosh^{-1}(cx)\,dx}{e^2} - \frac{(2d)\int \left(\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right.}{e^2}$$

$$= \frac{ax}{e^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{e^2} - \frac{\sqrt{-d}\,\int \frac{a+b\cosh^{-1}(cx}{\sqrt{-d}-\sqrt{e}}}{e^2}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

$$= \frac{ax}{e^2} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2} + \frac{bx\cosh^{-1}(cx)}{e^2} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{d}{4e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.35, size = 777, normalized size = 0.93

$$\frac{8a\sqrt{e}x+\frac{4ad\sqrt{e}x}{d+ex^2}-12a\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+b\left(\frac{8\sqrt{e}\left(-\sqrt{\frac{-1+cx}{1+cx}}(1+cx)+cx\cosh^{-1}(cx)\right)}{c}+2d\left(\frac{\cosh^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)+2d\left(\frac{\cosh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\sqrt{d}x+\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)-3i\sqrt{d}\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)+3i\sqrt{d}\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)}{8e^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt[
e]*x)/Sqrt[d]] + b*((8*Sqrt[e]*(-(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + c
*x*ArcCosh[c*x]))/c + 2*d*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log
[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
- e]) + 2*d*(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(-Sqrt[e] -
 I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqr
t[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt
[d]*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)
*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[d]*(ArcCosh[c*x]*(-ArcC
osh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*
d) - e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])])))/(8*e^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 75.66, size = 1749, normalized size = 2.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))+c^5*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^3
*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)
^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)+c^3*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2
)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*
((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c^3*b*((2*c^2*d+2*((c^2*
d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((
2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))*d^2/e^5-1/2*c*b*((2*c^2*d+2*
((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))*d/e^4-c^3*b*(-(2*c^2*d
```

```
-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))*d^2/e^5-1/2*c*b*(
-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))*d/e^4+c^
3*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))
/e^5/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)+1/2*c*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*
d)^(1/2)+e)*e)^(1/2)*d*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)*((c^2*d+e)*c^2*d)^(
1/2)-c^3*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1
/2))/e^5/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)-1/2*c*b*((2*c^2*d+2*((c^2*d+e)*c
^2*d)^(1/2)+e)*e)^(1/2)*d*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^
2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*((c^2*d+e)*c^2*d)^
(1/2)+b*x*arccosh(c*x)/e^2-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2-c*b*(-(2*c^2
*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))*d/e^5*((c^2*d+e
)*c^2*d)^(1/2)+c^5*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^3*ar
ctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2
)+e)*e)^(1/2))/e^5/(c^2*d+e)+c^3*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e
)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+
e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)+c*b*((2*c^2*d+2*((c^2*d+e)*c^2*d
)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*
((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))*d/e^5*((c^2*d+e)*c^2*d)^(1/2)+3/4*c*b*
d/e^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=
RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/4*c*b*d/e^2*sum(_R1/(_R1^2*e+2*c^2*d
+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))+1/2*c^2*b*d*arccosh(c*x)*x/e^2/(c^2*e*x^2+c^2*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2) - 2*x*e^(-2) - d*x/(x^2*
e^3 + d*e^2))*a + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x
^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b \operatorname{acosh}(cx))}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a + b\,\operatorname{acosh}(c\,x)\right)}{{\left(e\,x^2 + d\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

3.503 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=792

$$\frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}}+\frac{bc\tanh^{-1}\left(\cdots\right.}{2\sqrt{c\sqrt{-d}\cdots}}$$

```
[Out] 1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/
e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*
arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1
/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b
*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-
e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccosh(c*x))/e^(3/2)/((-d)^(1/2)-x*
e^(1/2))+1/4*(-a-b*arccosh(c*x))/e^(3/2)/((-d)^(1/2)+x*e^(1/2))-1/2*b*c*arc
tanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)
+e^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2)
)^(1/2)+1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^
(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c
*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A]
time = 1.41, antiderivative size = 792, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5959, 5909, 5963, 95, 214, 5962, 5681, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCosh[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCosh[c*
x])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sq
rt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) + (b*c*ArcTanh[(
```

```
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/
2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]
*e^(3/2)) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2,
-((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*Sqrt[-d]
*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/(4*Sqrt[-d]*e^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)}\right)\,dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)\,dx}{e} - \frac{d\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}-ex\right.}\right.}{}$$

$$= \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx + \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx + \frac{1}{2}\int \frac{a+b\cosh^{-1}}{-de-e^2}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{1}{2}\int \left(-\frac{\sqrt{-d}\,\left(a+b\cosh\right.}{2de\left(\sqrt{-d}-\right.}\right.$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{(bc)\text{Subst}\left(\int \frac{}{c\sqrt{-d}\,\sqrt{e}+}\right.}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-}}{\sqrt{c\sqrt{-d}+}}\right.}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{}}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-}}{\sqrt{c\sqrt{-d}+}}\right.}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{}}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-}}{\sqrt{c\sqrt{-d}+}}\right.}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{}}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-}}{\sqrt{c\sqrt{-d}+}}\right.}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{}}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-}}{\sqrt{c\sqrt{-d}+}}\right.}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.06, size = 720, normalized size = 0.91

$$\frac{-\frac{4a\sqrt{e}\,x}{d+ex^2}+\frac{4a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+b\left(-\frac{2\cosh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}\,x}-2\left(\frac{\cosh^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}\,x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}\,x-i\sqrt{-c^2d-e}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\,\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right)}{\sqrt{-c^2d-e}}\right)-\frac{2c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\sqrt{d}\,x+\sqrt{-c^2d-e}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\,\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right)}{\sqrt{-c^2d-e}}+\frac{i\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)}{\sqrt{d}}+\frac{i\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-2\left(\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)-2\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)}{\sqrt{d}}\right)}{8e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((-2*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x) - 2*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]) - (2*c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x +
 Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I
*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (I*(ArcCosh[c*x]*(-ArcCosh[c*
x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])
] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))
+ 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e]))]))/Sqrt[d] + (I*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (Sqrt[e]*E^
ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) - 2*PolyLog[2, (Sqrt[e]*
E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] - 2*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/Sqrt[d]))/(8*e^(3/2)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 31.33, size = 1689, normalized size = 2.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)-1/2*c^2*b*arccosh(c*x)*x/e/(c^2*e*x^2+c^2*d)-c^5*b*(-(2*c^2*d-2*((c^2*d+e
)*c^2*d)^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/
((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c^3*b*(-(2*
c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d*arctanh((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d
+e)*((c^2*d+e)*c^2*d)^(1/2)-c^3*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e
)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)
*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*d-1/2*c*b*(-(2*c^2*d-2*((c^2*d+e)*
c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c
^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*((c^2*d+e)*c^2*d)
```

```
^(1/2)+c^3*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(
1/2))*d/e^4+c*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e
)^(1/2))/e^4*((c^2*d+e)*c^2*d)^(1/2)+1/2*c*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)
^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2
*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3-c^5*b*((2*c^2*d+2*((c^2*d+e)*c^2*
d)^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)+c^3*b*((2*c^2*d+2*(
(c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*((c^2*d
+e)*c^2*d)^(1/2)-c^3*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arct
an((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+
e)*e)^(1/2))/e^3/(c^2*d+e)*d+1/2*c*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)
*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)
*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)+c^3*b*((2*
c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))*d/e^4-c*b*((2*
c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4*((c^2*d+e)
*c^2*d)^(1/2)+1/2*c*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arcta
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e
)*e)^(1/2))/e^3+1/4*c*b/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*c*b/e*sum(1/_R1/
(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) - x/(x^2*e^2 + d*e))*a + b*
integrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e +
 d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acosh}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a + b\operatorname{acosh}(c\,x))}{(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

3.504 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=804

$$-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{e}}-\frac{bc\tanh^{-1}}{2d\sqrt{c}}$$

```
[Out] -1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccosh(c*x))*ln(1
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))
/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b
*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)-
1/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c
^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*
b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d
-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-a-b*arccosh(c*x))/d/e^(1/2)/((-d)^(1/2
)-x*e^(1/2))+1/4*(a+b*arccosh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))+1/2*b*
c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^
(1/2)+e^(1/2))^(1/2))/d/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+
e^(1/2))^(1/2)-1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(
c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))
^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5909, 5963, 95, 214, 5962, 5681, 2221, 2317, 2438}

$$-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{e}\,x+\sqrt{-d}\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{e}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{e}}+\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcCosh[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCo
sh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-
d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/
(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) - (b*c*
```

```
ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[
e]]*Sqrt[-1 + c*x])])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqr
t[e]]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCosh[c
*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(
4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*Ar
cCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sq
rt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e])
```

<u>Rule 95</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)$$

$$= -\frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,dx}{2d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{(bc)\int \frac{}{\sqrt{-1+cx}\,\sqrt{1+}}}{4d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}\,-\sqrt{e}\,x}\,dx}{4(-d)^{3/2}}+$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}\,-}}{\sqrt{c\sqrt{-d}\,+}}\right)}{2d\sqrt{c\sqrt{-d}\,-\sqrt{e}}\,\sqrt{c}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}\,-}}{\sqrt{c\sqrt{-d}\,+}}\right)}{2d\sqrt{c\sqrt{-d}\,-\sqrt{e}}\,\sqrt{c}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}\,-}}{\sqrt{c\sqrt{-d}\,+}}\right)}{2d\sqrt{c\sqrt{-d}\,-\sqrt{e}}\,\sqrt{c}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}\,-}}{\sqrt{c\sqrt{-d}\,+}}\right)}{2d\sqrt{c\sqrt{-d}\,-\sqrt{e}}\,\sqrt{c}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}\,-}}{\sqrt{c\sqrt{-d}\,+}}\right)}{2d\sqrt{c\sqrt{-d}\,-\sqrt{e}}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.47, size = 734, normalized size = 0.91

$$\frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}+\frac{b\left(2\sqrt{d}\left(\frac{\cosh^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)-2\sqrt{d}\left(-\frac{\cosh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x}-\frac{c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\sqrt{d}x+\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)+i\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)-i\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)}{4d^{3/2}\sqrt{e}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(2*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*S
qrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
)/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - 2*
Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] -
I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt
[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + I*(ArcCosh[
c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sq
rt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] +
Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]))]) - I*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + 2*PolyLog[2,
(Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])))/(4*d^(3/2)*
Sqrt[e]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.93, size = 1716, normalized size = 2.13

method	result	size
derivativedivides	Expression too large to display	1716
default	Expression too large to display	1716

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*a*c^3*x/d/(c^2*e*x^2+c^2*d)+1/2*a*c/d/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))+1/2*b*c^3*arccosh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*b*c^2/d*sum(_R1/(_
R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c
^2*d+2*e)*_Z^2+e))+b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*a
rctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(
1/2)-e)*e)^(1/2))*d/(c^2*d+e)/e^3+b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2
)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^
```

```
2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)+b*c^
4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*
d+e)/e^2+1/2*b*c^2*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e
)*e)^(1/2))/d/(c^2*d+e)/e^2*((c^2*d+e)*c^2*d)^(1/2)-b*c^4*(-(2*c^2*d-2*((c^
2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3-b*c^2*(-(2*c^2*d-2*(
(c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/e^3*((c^2*d+e)*c^2*
d)^(1/2)-1/2*b*c^2*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e
)*e)^(1/2))/d/e^2+b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arc
tan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)
+e)*e)^(1/2))*d/(c^2*d+e)/e^3-b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*
e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*
c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)+b*c^4*((2*c
^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2-1
/2*b*c^2*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d
/(c^2*d+e)/e^2*((c^2*d+e)*c^2*d)^(1/2)-b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^
(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((
c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3+b*c^2*((2*c^2*d+2*((c^2*d+e)*c^2*d)^
(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((
c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/e^3*((c^2*d+e)*c^2*d)^(1/2)-1/2*b*c^2*
((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/e^2-1/4*
b*c^2/d*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) + x/(d*x^2*e + d^2)) + b*
integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e + d^2
), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**2,x)
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{(e\,x^2 + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^2,x)
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^2, x)
```

3.505 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=846

$$-\frac{a+b\cosh^{-1}(cx)}{d^2x}+\frac{\sqrt{e}\,(a+b\cosh^{-1}(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{\sqrt{e}\,(a+b\cosh^{-1}(cx))}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{bc\text{ArcTan}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2}-\cdots$$

```
[Out] (-a-b*arccosh(c*x))/d^2/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-3/4*(
a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccosh(c*x))*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/
2)/(-d)^(5/2)-3/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arcco
sh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2
*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*poly
log(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1
/2)))*e^(1/2)/(-d)^(5/2)+1/4*(a+b*arccosh(c*x))*e^(1/2)/d^2/((-d)^(1/2)-x*e
^(1/2))-1/4*(a+b*arccosh(c*x))*e^(1/2)/d^2/((-d)^(1/2)+x*e^(1/2))-1/2*b*c*a
rctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/
2)+e^(1/2))^(1/2))*e^(1/2)/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e
^(1/2))^(1/2)+1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c
*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))*e^(1/2)/d^2/(c*(-d)^(1/2)-e^(1/2)
)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A]
time = 1.47, antiderivative size = 846, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5959, 5883, 94, 211, 5909, 5963, 95, 214, 5962, 5681, 2221, 2317, 2438}

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)(a+b\cosh^{-1}(cx))}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)(a+b\cosh^{-1}(cx))}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)(a+b\cosh^{-1}(cx))}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}+1\right)(a+b\cosh^{-1}(cx))}{4(-d)^{5/2}}-\frac{a+b\cosh^{-1}(cx)}{d^2x}+\frac{\sqrt{e}\,(a+b\cosh^{-1}(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{\sqrt{e}\,(a+b\cosh^{-1}(cx))}{4d^2\left(\sqrt{e}\,x+\sqrt{-d}\right)}+\frac{bc\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{d^2}-\frac{bc\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\sqrt{cx-1}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}\,c+\sqrt{e}}}+\frac{bc\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{cx-1}}\right)}{2d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{\sqrt{-d}\,c+\sqrt{e}}}+\frac{3b\sqrt{e}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\,\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{4(-d)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sq
rt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sqrt[-d] + Sq
rt[e]*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 - (b*c*Sqrt[e]*A
```

```
rcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e
]]*Sqrt[-1 + c*x])]])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sq
rt[e]]) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(
Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[
e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2))
+ (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(
-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d
)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*(-d)^(5/2)) - (3*b*Sq
rt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(4*(-d)^(5/2))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)\,dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d^2} - \frac{e\int \left(\frac{\sqrt{-d}\,(a+b\cosh^{-1}(cx))}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{(bc^2)\,\mathrm{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\,\sqrt{1+cx}\right)}{d^2} + \frac{e\int \frac{a+b\,\mathrm{c}}{\sqrt{-}}}{2(-}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.71, size = 821, normalized size = 0.97

$$\frac{-\frac{8a\sqrt{d}}{x}-\frac{4a\sqrt{d}ex}{d+ex^2}-12a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+b\left(8\sqrt{d}\left(-\frac{\cosh^{-1}(cx)}{x}+\frac{c\sqrt{-1+c^2x^2}\,\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right)-2\sqrt{d}\sqrt{e}\left(\frac{\cosh^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}x}+\frac{c\log\left(\frac{2e\left(i\sqrt{e}+c^2\sqrt{d}x-i\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)+2\sqrt{d}\sqrt{e}\left(-\frac{\cosh^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x}-\frac{c\log\left(\frac{2e\left(-\sqrt{e}-ic^2\sqrt{d}x+\sqrt{-c^2d-e}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c\sqrt{-c^2d-e}\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)-3i\sqrt{e}\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)+3i\sqrt{e}\left(\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{-ic\sqrt{d}+\sqrt{-c^2d-e}}\right)+\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}-\sqrt{-c^2d-e}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{ic\sqrt{d}+\sqrt{-c^2d-e}}\right)\right)\right)}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*Sqrt[d]*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - 2*Sqrt[d]*Sq
rt[e]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c
^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[
-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Sqrt[d]*Sq
rt[e]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*
c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-
(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt[e]
*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sq
rt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d
] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*
Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I
*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[e]*(ArcCosh[c*x]*(-ArcCosh
[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d)
- e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e])])))/(8*d^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 89.48, size = 1821, normalized size = 2.15

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2*a*e/d^2*x*c^2/(c^2*e*x^2+c^2*d)-3/2*a*e/d^2/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))-3/2*b*x*c^2*arccosh(c*x)/d^2/(c^2*e*x^2+c^2*d)*e-b*c^2/x*
arccosh(c*x)/d/(c^2*e*x^2+c^2*d)-b*c^5*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)
+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2
*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^2-b*c^3*(-(2*c^2*d-2*((c^2*d+e)
*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)/e^2*((c^2*d+e)*c^2
*d)^(1/2)-b*c^3*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e
```

```
)^(1/2))/d/(c^2*d+e)/e-1/2*c*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^
2*d)^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)/e*((c^2*d+e)*c^2*d)^(1/2)+b*c^3*(-(2*
c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/e^2+c*b*(-
(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d^2/e^2*(
(c^2*d+e)*c^2*d)^(1/2)+1/2*c*b*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^
2*d)^(1/2)-e)*e)^(1/2))/d^2/e-b*c^5*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*
e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*
c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2+b*c^3*((2*c^2*d+2*((c^2*d+e)*c^2*d)
^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(
(c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2*((c^2*d+e)*c^2*d)^(1/2)
-b*c^3*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/(
c^2*d+e)/e+1/2*c*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e
)^(1/2))/d^2/(c^2*d+e)/e*((c^2*d+e)*c^2*d)^(1/2)+b*c^3*((2*c^2*d+2*((c^2*d+
e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/e^2-c*b*((2*c^2*d+2*((c^2*d+
e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d^2/e^2*((c^2*d+e)*c^2*d)^(1/2
)+1/2*c*b*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/
d^2/e+3/16*b/c/d^3*e*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arcco
sh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*c*b
/d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/16*b/c/d^3*e*sum((4*_R1^2*c^
2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_
R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*x^2*e + 2*d)/(d^2*x^3*e + d^3*x) + 3*arctan(x*e^(1/2)/sqrt(d))*e
^(1/2)/d^(5/2)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^6*e
^2 + 2*d*x^4*e + d^2*x^2), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(x^6*e^2 + 2*d*x^4*e + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x^2\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x^2\,\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2), x)
```

3.506 $\int \frac{x^5(a+b\cosh^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=737

$$\frac{bcdx(1-c^2x^2)}{8e^2(c^2d+e)\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex^2)}-\frac{d^2(a+b\cosh^{-1}(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\cosh^{-1}(cx))}{e^3(d+ex^2)}-\frac{(a+b\cosh^{-1}(c}{2be^3}$$

```
[Out] -1/4*d^2*(a+b*arccosh(c*x))/e^3/(e*x^2+d)^2+d*(a+b*arccosh(c*x))/e^3/(e*x^2
+d)-1/2*(a+b*arccosh(c*x))^2/b/e^3+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b
*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*
x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)
^(1/2)))/e^3+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+
1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c
^2*d-e)^(1/2)))/e^3+1/8*b*c*d*x*(-c^2*x^2+1)/e^2/(c^2*d+e)/(e*x^2+d)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)
/(c^2*x^2-1)^(1/2))*d^(1/2)*(c^2*x^2-1)^(1/2)/e^3/(c^2*d+e)^(3/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))
*d^(1/2)*(c^2*x^2-1)^(1/2)/e^3/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.84, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5959, 5957, 533, 390, 385, 214, 5962, 5681, 2221, 2317, 2438}

$$\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^3}+\frac{(a+b\cosh^{-1}(cx))\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^3}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2e^3}-\frac{d^2(a+b\cosh^{-1}(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\cosh^{-1}(cx))}{e^3(d+ex^2)}-\frac{(a+b\cosh^{-1}(cx))^2}{2be^3}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2e^3}+\frac{bc\sqrt{d}\sqrt{c^2x^2-1}(2c^2d+e)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8e^3\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)^{3/2}}-\frac{bc\sqrt{d}\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^3\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}+\frac{bcdx(1-c^2x^2)}{8e^2\sqrt{cx-1}\sqrt{cx+1}(c^2d+e)(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*x*(1 - c^2*x^2))/(8*e^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
+ e*x^2)) - (d^2*(a + b*ArcCosh[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*Ar
cCosh[c*x]))/(e^3*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*e^3) - (b*c*Sq
rt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(e^3*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d]*
(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])])/(8*e^3*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
```

```
c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Lo
g[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
+ ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[
2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = \int \left(\frac{d^2x\left(a+b\cosh^{-1}(cx)\right)}{e^2\,(d+ex^2)^3} - \frac{2dx\left(a+b\cosh^{-1}(cx)\right)}{e^2\,(d+ex^2)^2} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{e^2\,(d+ex^2)}\right)$$

$$= \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(d+ex^2)^2}\,dx}{e^2} + \frac{d^2\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(d+ex^2)^3}\,dx}{e^2}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\,(d+ex^2)} - \frac{(bcd)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(}}{e^3}$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\,(d+ex^2)} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{5/2}} + \frac{\int \frac{a+b\,\mathrm{co}}{\sqrt{-d}}}{2}$$

$$= \frac{bcdx(1-c^2x^2)}{8e^2\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d(a+b}{e^3\,(}$$

$$= \frac{bcdx(1-c^2x^2)}{8e^2\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d(a+b}{e^3\,(}$$

$$= \frac{bcdx(1-c^2x^2)}{8e^2\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d(a+b}{e^3\,(}$$

$$= \frac{bcdx(1-c^2x^2)}{8e^2\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d(a+b}{e^3\,(}$$

$$= \frac{bcdx(1-c^2x^2)}{8e^2\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\,(d+ex^2)^2} + \frac{d(a+b}{e^3\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.55, size = 1097, normalized size = 1.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a*d^2)/(d + e*x^2)^2 + (16*a*d)/(d + e*x^2) + 8*a*Log[d + e*x^2] + b*(
-((c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + S
```

```
qrt[e]*x))) - (c*d*Sqrt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sq
rt[d] + Sqrt[e]*x)) + (7*Sqrt[d]*ArcCosh[c*x])/(Sqrt[d] - I*Sqrt[e]*x) - (d
*ArcCosh[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (7*Sqrt[d]*ArcCosh[c*x])/(Sqrt[d
] + I*Sqrt[e]*x) + (d*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x)^2 - 8*ArcCosh[c
*x]^2 + 8*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt
[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*
Sqrt[d] + Sqrt[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[
c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 8*ArcCosh[c*x]*Log[1 + (Sqrt[e]
*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] - ((7*I)*c*Sqrt[d]*Log
[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
- e] + ((7*I)*c*Sqrt[d]*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d
) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sq
rt[e]*x))])/Sqrt[-(c^2*d) - e] - (c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*((-I)*
Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c
^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2) + (c^3*d^(3/2)*Log[(e*Sqr
t[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2) + 8*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + 8*P
olyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] +
 8*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])
)] + 8*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])]))/(16*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 30.06, size = 5257, normalized size = 7.13

method	result	size
derivativedivides	Expression too large to display	5257
default	Expression too large to display	5257

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*e^(-3)*log(x^2*e + d) + (4*d*x^2*e + 3*d^2)/(x^4*e^5 + 2*d*x^2*e^4 +
 d^2*e^3))*a + b*integrate(x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^6*
e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccosh(c*x) + a*x^5)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\operatorname{acosh}(cx))}{(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a + b\operatorname{acosh}(c\,x)\right)}{\left(e\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

3.507 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{bcx(1-c^2x^2)}{8e\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\,(d+ex^2)^2}-\frac{b\sqrt{1-c^2x^2}\,\mathrm{ArcSin}(cx)}{4de^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc(2c^2d+}{8\sqrt{}}$$

```
[Out] 1/4*x^4*(a+b*arccosh(c*x))/d/(e*x^2+d)^2-1/8*b*c*x*(-c^2*x^2+1)/e/(c^2*d+e)
/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*arcsin(c*x)*(-c^2*x^2+1)^(1/2)
/d/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+3*e)*arctan(x*(c^2*d+e)
^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)^(3/2)/d
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {270, 5958, 12, 533, 481, 537, 223, 212, 385, 214}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\,(d+ex^2)^2}-\frac{b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{4de^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc\sqrt{c^2x^2-1}\,(2c^2d+3e)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{8\sqrt{d}\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2d+e)^{3/2}}-\frac{bcx(1-c^2x^2)}{8e\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2d+e)\,(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*(1 - c^2*x^2))/(e*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d +
 e*x^2)) + (x^4*(a + b*ArcCosh[c*x]))/(4*d*(d + e*x^2)^2) - (b*Sqrt[-1 + c^
2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*Sqrt[d]*e^2*(c^2*d + e)^(3/2)*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-(bc)\int \frac{x^4}{4d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^2}\,dx$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{(bc)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x^4}{\sqrt{-1+c^2x^2}\,\left(d+ex^2\right)^2}\,dx}{4d\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcx(1-c^2x^2)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{\left(bc\sqrt{\right.}}{}$$

$$= -\frac{bcx(1-c^2x^2)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{\left(bc\sqrt{\right.}}{}$$

$$= -\frac{bcx(1-c^2x^2)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{\left(bc\sqrt{\right.}}{}$$

$$= -\frac{bcx(1-c^2x^2)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{-}}{}$$

Mathematica [A]
time = 0.53, size = 192, normalized size = 0.83

$$\frac{\frac{\frac{bcex\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)}{c^2d+e}-2a(d+2ex^2)}{(d+ex^2)^2}-\frac{2b(d+2ex^2)\cosh^{-1}(cx)}{(d+ex^2)^2}-\frac{bc(2c^2d+3e)\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{ArcTan}\left(\frac{\sqrt{-c^2d-e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\,(-c^2d-e)^{3/2}\sqrt{-1+c^2x^2}}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (((b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(c^2*d + e) - 2*a*(d +
 2*e*x^2))/(d + e*x^2)^2 - (2*b*(d + 2*e*x^2)*ArcCosh[c*x])/(d + e*x^2)^2 -
 (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) -
e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*(-(c^2*d) - e)^(3/2)*Sqrt[-1
+ c^2*x^2]))/(8*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2515 vs. $2(197) = 394$.
time = 10.16, size = 2516, normalized size = 10.89

method	result	size
derivativedivides	Expression too large to display	2516
default	Expression too large to display	2516

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^6*(-1/2/e^2/(c^2*e*x^2+c^2*d)+1/4*d*c^2/e^2/(c^2*e*x^2+c^2*d)^2)
-1/2*b*c^6*arccosh(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*b*c^8*arccosh(c*x)*d/e^2/
(c^2*e*x^2+c^2*d)^2-1/8*b*c^12*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d
*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(
e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2
*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e
)^(1/2)))*d^3-1/8*b*c^12*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^
(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-
c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e
)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1
/2)))*d^2*x^2+1/8*b*c^12*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1
/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^
2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/
e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2
)))*d^3+1/8*b*c^12*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))
/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*
e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(
1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*
d^2*x^2+1/8*b*c^9*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/
(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*d*x-
5/16*b*c^10*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*
d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2
))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c
^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*d^2-5/1
6*b*c^10*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e
)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^
2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*
x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*d*x^2+5/16
```

```
*b*c^10*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)
^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2
/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x
^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d^2+5/16*b*
c^10*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1
/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((
-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-
1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d*x^2+1/8*b*c^
7*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e
)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x-3/16*b*c^8*e^3*(c*
x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d
+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1
/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(
-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*d-3/16*b*c^8*e^4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)
^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)
^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d
*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*x^2+3/16*b*c^8*e^3*(c*x+1)^(1/2)
*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/
2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(
c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^
(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d+3/16*b*c^8*e^4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*
c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^
2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*
c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*((c^4*d + 2*c^2*e)*log(x^2*e + d)/(c^4*d^2*e^2 + 2*c^2*d*e^3 + e^4)
+ (c^4*d^3 + c^2*d^2*e + (c^4*d^2*e + c^2*d*e^2)*x^2 + 2*(c^4*d^3 + 2*c^2*d
^2*e + 2*(c^4*d^2*e + 2*c^2*d*e^2 + e^3)*x^2 + d*e^2)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)) - (c^4*d^3 + (c^4*d*e^2 + 2*c^2*e^3)*x^4 + 2*c^2*d^2*e +
2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x + 1) - (c^4*d^3 + (c^4*d*e^2 + 2*c
^2*e^3)*x^4 + 2*c^2*d^2*e + 2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x - 1))/
(c^4*d^4*e^2 + 2*c^2*d^3*e^3 + (c^4*d^2*e^4 + 2*c^2*d*e^5 + e^6)*x^4 + 2*(c
^4*d^3*e^3 + 2*c^2*d^2*e^4 + d*e^5)*x^2 + d^2*e^4) + 8*integrate(1/4*(2*c*x
^2*e + c*d)/(c^3*x^7*e^4 + (2*c^3*d*e^3 - c*e^4)*x^5 - c*d^2*x*e^2 + (c^3*d
^2*e^2 - 2*c*d*e^3)*x^3 + (c^2*x^6*e^4 + (2*c^2*d*e^3 - e^4)*x^4 + (c^2*d^2
```

```
*e^2 - 2*d*e^3)*x^2 - d^2*e^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)
) - 1/4*(2*x^2*e + d)*a/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(199) = 398$.
time = 0.51, size = 3548, normalized size = 15.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(2*a - b)*c^4*d^4 - 2*(b*c^2*d*x^4 - 4*a*d*x^2)*cosh(1)^3 - 2*(b*
c^2*d*x^4 - 4*a*d*x^2)*sinh(1)^3 - 2*(b*c^4*d^2*x^4 - 2*(4*a - b)*c^2*d^2*x
^2 - 2*a*d^2)*cosh(1)^2 - 2*(b*c^4*d^2*x^4 - 2*(4*a - b)*c^2*d^2*x^2 - 2*a*
d^2 + 3*(b*c^2*d*x^4 - 4*a*d*x^2)*cosh(1))*sinh(1)^2 - (3*b*c*x^4*cosh(1)^3
 + 3*b*c*x^4*sinh(1)^3 + 2*b*c^3*d^3 + 2*(b*c^3*d*x^4 + 3*b*c*d*x^2)*cosh(1
)^2 + (2*b*c^3*d*x^4 + 9*b*c*x^4*cosh(1) + 6*b*c*d*x^2)*sinh(1)^2 + (4*b*c^
3*d^2*x^2 + 3*b*c*d^2)*cosh(1) + (4*b*c^3*d^2*x^2 + 9*b*c*x^4*cosh(1)^2 + 3
*b*c*d^2 + 4*(b*c^3*d*x^4 + 3*b*c*d*x^2)*cosh(1))*sinh(1))*sqrt(c^2*d^2 + d
*cosh(1) + d*sinh(1))*log((4*c^4*d^2*x^2 - 2*c^2*d^2 + x^2*cosh(1)^2 + x^2*
sinh(1)^2 + (4*c^2*d*x^2 - d)*cosh(1) + (4*c^2*d*x^2 + 2*x^2*cosh(1) - d)*s
inh(1) + 2*(2*c^3*d*x^2 + c*x^2*cosh(1) + c*x^2*sinh(1) - c*d + (2*c^2*d*x
+ x*cosh(1) + x*sinh(1))*sqrt(c^2*x^2 - 1))*sqrt(c^2*d^2 + d*cosh(1) + d*si
nh(1)) + 4*(c^3*d^2*x + c*d*x*cosh(1) + c*d*x*sinh(1))*sqrt(c^2*x^2 - 1))/(
x^2*cosh(1) + x^2*sinh(1) + d)) + 2*(2*(2*a - b)*c^4*d^3*x^2 + (4*a - b)*c^
2*d^3)*cosh(1) - 4*(b*c^4*d^2*x^4*cosh(1)^2 + 2*b*c^2*d*x^4*cosh(1)^3 + b*x
^4*cosh(1)^4 + b*x^4*sinh(1)^4 + 2*(b*c^2*d*x^4 + 2*b*x^4*cosh(1))*sinh(1)^
3 + (b*c^4*d^2*x^4 + 6*b*c^2*d*x^4*cosh(1) + 6*b*x^4*cosh(1)^2)*sinh(1)^2 +
 2*(b*c^4*d^2*x^4*cosh(1) + 3*b*c^2*d*x^4*cosh(1)^2 + 2*b*x^4*cosh(1)^3)*si
nh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - 4*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^
4*sinh(1)^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^4 + 2*b*x^
4*cosh(1) + b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*
cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 +
6*(b*c^2*d*x^4 + b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3
)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 + b*c^2*d^3 + 3*(b*c^2*d*x
^4 + b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)
)*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(2*(2*a - b)*c^4*d^3*x^2 + (4*
a - b)*c^2*d^3 - 3*(b*c^2*d*x^4 - 4*a*d*x^2)*cosh(1)^2 - 2*(b*c^4*d^2*x^4 -
 2*(4*a - b)*c^2*d^2*x^2 - 2*a*d^2)*cosh(1))*sinh(1) - 2*(b*c^3*d^3*x*cosh(
1) + b*c*d*x^3*cosh(1)^3 + b*c*d*x^3*sinh(1)^3 + (b*c^3*d^2*x^3 + b*c*d^2*x
)*cosh(1)^2 + (b*c^3*d^2*x^3 + 3*b*c*d*x^3*cosh(1) + b*c*d^2*x)*sinh(1)^2 +
 (b*c^3*d^3*x + 3*b*c*d*x^3*cosh(1)^2 + 2*(b*c^3*d^2*x^3 + b*c*d^2*x)*cosh(
1))*sinh(1))*sqrt(c^2*x^2 - 1))/(c^4*d^5*cosh(1)^2 + d*x^4*cosh(1)^6 + d*x^
4*sinh(1)^6 + 2*(c^2*d^2*x^4 + d^2*x^2)*cosh(1)^5 + 2*(c^2*d^2*x^4 + 3*d*x^
```

```
4*cosh(1) + d^2*x^2)*sinh(1)^5 + (c^4*d^3*x^4 + 4*c^2*d^3*x^2 + d^3)*cosh(1
)^4 + (c^4*d^3*x^4 + 4*c^2*d^3*x^2 + 15*d*x^4*cosh(1)^2 + d^3 + 10*(c^2*d^2
*x^4 + d^2*x^2)*cosh(1))*sinh(1)^4 + 2*(c^4*d^4*x^2 + c^2*d^4)*cosh(1)^3 +
2*(c^4*d^4*x^2 + 10*d*x^4*cosh(1)^3 + c^2*d^4 + 10*(c^2*d^2*x^4 + d^2*x^2)*
cosh(1)^2 + 2*(c^4*d^3*x^4 + 4*c^2*d^3*x^2 + d^3)*cosh(1))*sinh(1)^3 + (c^4
*d^5 + 15*d*x^4*cosh(1)^4 + 20*(c^2*d^2*x^4 + d^2*x^2)*cosh(1)^3 + 6*(c^4*d
^3*x^4 + 4*c^2*d^3*x^2 + d^3)*cosh(1)^2 + 6*(c^4*d^4*x^2 + c^2*d^4)*cosh(1)
)*sinh(1)^2 + 2*(c^4*d^5*cosh(1) + 3*d*x^4*cosh(1)^5 + 5*(c^2*d^2*x^4 + d^2
*x^2)*cosh(1)^4 + 2*(c^4*d^3*x^4 + 4*c^2*d^3*x^2 + d^3)*cosh(1)^3 + 3*(c^4*
d^4*x^2 + c^2*d^4)*cosh(1)^2)*sinh(1)), -1/8*((2*a - b)*c^4*d^4 - (b*c^2*d*
x^4 - 4*a*d*x^2)*cosh(1)^3 - (b*c^2*d*x^4 - 4*a*d*x^2)*sinh(1)^3 - (b*c^4*d
^2*x^4 - 2*(4*a - b)*c^2*d^2*x^2 - 2*a*d^2)*cosh(1)^2 - (b*c^4*d^2*x^4 - 2*
(4*a - b)*c^2*d^2*x^2 - 2*a*d^2 + 3*(b*c^2*d*x^4 - 4*a*d*x^2)*cosh(1))*sinh
(1)^2 - (3*b*c*x^4*cosh(1)^3 + 3*b*c*x^4*sinh(1)^3 + 2*b*c^3*d^3 + 2*(b*c^3
*d*x^4 + 3*b*c*d*x^2)*cosh(1)^2 + (2*b*c^3*d*x^4 + 9*b*c*x^4*cosh(1) + 6*b*
c*d*x^2)*sinh(1)^2 + (4*b*c^3*d^2*x^2 + 3*b*c*d^2)*cosh(1) + (4*b*c^3*d^2*x
^2 + 9*b*c*x^4*cosh(1)^2 + 3*b*c*d^2 + 4*(b*c^3*d*x^4 + 3*b*c*d*x^2)*cosh(1
))*sinh(1))*sqrt(-c^2*d^2 - d*cosh(1) - d*sinh(1))*arctan((sqrt(-c^2*d^2 -
d*cosh(1) - d*sinh(1))*sqrt(c^2*x^2 - 1)*(x*cosh(1) + x*sinh(1)) - sqrt(-c^
2*d^2 - d*cosh(1) - d*sinh(1))*(c*x^2*cosh(1) + c*x^2*sinh(1) + c*d))/(c^2*
d^2 + d*cosh(1) + d*sinh(1))) + (2*(2*a - b)*c^4*d^3*x^2 + (4*a - b)*c^2*d^
3)*cosh(1) - 2*(b*c^4*d^2*x^4*cosh(1)^2 + 2*b*c^2*d*x^4*cosh(1)^3 + b*x^4*c
osh(1)^4 + b*x^4*sinh(1)^4 + 2*(b*c^2*d*x^4 + 2*b*x^4*cosh(1))*sinh(1)^3 +
(b*c^4*d^2*x^4 + 6*b*c^2*d*x^4*cosh(1) + 6*b*x^4*cosh(1)^2)*sinh(1)^2 + 2*(
b*c^4*d^2*x^4*cosh(1) + 3*b*c^2*d*x^4*cosh(1)^2 + 2*b*x^4*cosh(1)^3)*sinh(1
))*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*si
nh(1)^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^4 + 2*b*x^4*co
sh(1) + b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*cosh
(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 + 6*(b
*c^2*d*x^4 + b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*co
sh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 + b*c^2*d^3 + 3*(b*c^2*d*x^4 +
 b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*si
nh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*(2*a - b)*c^4*d^3*x^2 + (4*a - b)
*c^2*d^3 - 3*(b*c^2*d*x^4 - 4*a*d*x^2)*cosh(1)^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{acosh}\,(cx))}{(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

3.508 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=177

$$\frac{bcx(1-c^2x^2)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc(2c^2d+e)\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2 \cdots}}{\sqrt{d}\,\sqrt{- \cdots}}\right.}{8d^{3/2}e\left(c^2d+e\right)^{3/2}\sqrt{-1+cx}\,\sqrt{1+ \cdots}}$$

```
[Out] 1/4*(-a-b*arccosh(c*x))/e/(e*x^2+d)^2+1/8*b*c*x*(-c^2*x^2+1)/d/(c^2*d+e)/(e
*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+e)*arctanh(x*(c^2*d+e)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(3/2)/e/(c^2*d+e)^(3/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5957, 533, 390, 385, 214}

$$-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\sqrt{c^2x^2-1}\,(2c^2d+e)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{8d^{3/2}e\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)^{3/2}}+\frac{bcx(1-c^2x^2)}{8d\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*(1 - c^2*x^2))/(8*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*
x^2)) - (a + b*ArcCosh[c*x])/(4*e*(d + e*x^2)^2) + (b*c*(2*c^2*d + e)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8
*d^(3/2)*e*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
```

```
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^2}\,dx}{4e}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\,(d+ex^2)^2}\,dx}{4e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bcx(1-c^2x^2)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc(2c^2d+e\right.}{8de}$$

$$= \frac{bcx(1-c^2x^2)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc(2c^2d+e\right.}{}$$

$$= \frac{bcx(1-c^2x^2)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc(2c^2d+e)}{8d^{3/2}}$$

Mathematica [A]

time = 0.63, size = 183, normalized size = 1.03

$$\frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex^2)}{d(c^2d+e)}}{(d+ex^2)^2}-\frac{2b\cosh^{-1}(cx)}{e\,(d+ex^2)^2}-\frac{bc(2c^2d+e)\sqrt{-1+cx}\sqrt{1+cx}\,\text{ArcTan}\left(\frac{\sqrt{-c^2d-e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{d^{3/2}\,(-c^2d-e)^{3/2}\,e\sqrt{-1+c^2x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (-(((2*a)/e + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d +
e)))/(d + e*x^2)^2) - (2*b*ArcCosh[c*x])/(e*(d + e*x^2)^2) - (b*c*(2*c^2*d
+ e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])])/(d^(3/2)*(-(c^2*d) - e)^(3/2)*e*Sqrt[-1 + c^2*x^2]))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2458 vs. $2(154) = 308$.
time = 10.17, size = 2459, normalized size = 13.89

method	result	size
derivativedivides	Expression too large to display	2459
default	Expression too large to display	2459

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*a*c^6/e/(c^2*e*x^2+c^2*d)^2-1/4*b*c^6/e/(c^2*e*x^2+c^2*d)^2*arc
cosh(c*x)-1/8*b*c^10*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2
))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*
d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)
^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2))
)*d^2-1/8*b*c^10*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(
-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)
^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/
2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*d*
x^2+1/8*b*c^10*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c
^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(
1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)
*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d^2+
1/8*b*c^10*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d
*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2)
)^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^
2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d*x^2-3/
16*b*c^8*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e
)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^
2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*
```

```
x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*d-3/16*b*c
^8*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2
)/(-(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c
^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)
^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))*x^2+3/16*b*c^8*e
^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-
(c^2*d+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d
*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/
2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*d+3/16*b*c^8*e^4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d
+e)/e)^(1/2)/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1
/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(
-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*x^2-1/8*b*c^7*e^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/(e-(-
c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x-1/16*b*c^6*e^4*(c*x+1)^(1/2)*(c*
x-1)^(1/2)/(e*c*x+(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)
^(1/2)/(-c^2*d*e)^(1/2)/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*
x^2-1)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2
)*c*x-e)/(e*c*x-(-c^2*d*e)^(1/2)))-1/16*b*c^6*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/
2)/(e*c*x+(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(
-c^2*d*e)^(1/2)/d/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)
^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-
e)/(e*c*x-(-c^2*d*e)^(1/2)))*x^2+1/16*b*c^6*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)
/(e*c*x+(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(-c
^2*d*e)^(1/2)/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/
2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(
e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^6*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+
(-c^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(-c^2*d*e)^
(1/2)/d/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(
2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+
(-c^2*d*e)^(1/2)))*x^2-1/8*b*c^5*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(e*c*x+(-c
^2*d*e)^(1/2))/(e*c*x-(-c^2*d*e)^(1/2))/d/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e
)^(1/2)+e)^2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(x^2*e + d)/(c^4*d^2*e + 2*c^2*d*e^2 + e^3) + 8*c*integrate(1/
4/(c^3*x^7*e^3 + (2*c^3*d*e^2 - c*e^3)*x^5 - c*d^2*x*e + (c^3*d^2*e - 2*c*d
*e^2)*x^3 + (c^2*x^6*e^3 + (2*c^2*d*e^2 - e^3)*x^4 + (c^2*d^2*e - 2*d*e^2)*
```

```
x^2 - d^2*e)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x) - (c^4*d^2 + c^2*
d*e + (c^4*d*e + c^2*e^2)*x^2 - 2*(c^4*d^2 + 2*c^2*d*e + e^2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + (c^4*x^4*e^2 + 2*c^4*d*x^2*e + c^4*d^2)*log(c*x
 + 1) + (c^4*x^4*e^2 + 2*c^4*d*x^2*e + c^4*d^2)*log(c*x - 1))/(c^4*d^4*e +
2*c^2*d^3*e^2 + (c^4*d^2*e^3 + 2*c^2*d*e^4 + e^5)*x^4 + 2*(c^4*d^3*e^2 + 2*
c^2*d^2*e^3 + d*e^4)*x^2 + d^2*e^3))*b - 1/4*a/(x^4*e^3 + 2*d*x^2*e^2 + d^2
*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(154) = 308$.
time = 0.48, size = 3391, normalized size = 19.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*b*c^2*d*x^4*cosh(1)^3 + 2*b*c^2*d*x^4*sinh(1)^3 + 2*(2*a + b)*c^4
*d^4 + 2*(b*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + 2*a*d^2)*cosh(1)^2 + 2*(b*c^4*d
^2*x^4 + 3*b*c^2*d*x^4*cosh(1) + 2*b*c^2*d^2*x^2 + 2*a*d^2)*sinh(1)^2 - (b*
c*x^4*cosh(1)^3 + b*c*x^4*sinh(1)^3 + 2*b*c^3*d^3 + 2*(b*c^3*d*x^4 + b*c*d*
x^2)*cosh(1)^2 + (2*b*c^3*d*x^4 + 3*b*c*x^4*cosh(1) + 2*b*c*d*x^2)*sinh(1)^
2 + (4*b*c^3*d^2*x^2 + b*c*d^2)*cosh(1) + (4*b*c^3*d^2*x^2 + 3*b*c*x^4*cosh
(1)^2 + b*c*d^2 + 4*(b*c^3*d*x^4 + b*c*d*x^2)*cosh(1))*sinh(1))*sqrt(c^2*d^
2 + d*cosh(1) + d*sinh(1))*log((4*c^4*d^2*x^2 - 2*c^2*d^2 + x^2*cosh(1)^2 +
 x^2*sinh(1)^2 + (4*c^2*d*x^2 - d)*cosh(1) + (4*c^2*d*x^2 + 2*x^2*cosh(1) -
 d)*sinh(1) + 2*(2*c^3*d*x^2 + c*x^2*cosh(1) + c*x^2*sinh(1) - c*d + (2*c^2
*d*x + x*cosh(1) + x*sinh(1))*sqrt(c^2*x^2 - 1))*sqrt(c^2*d^2 + d*cosh(1) +
 d*sinh(1)) + 4*(c^3*d^2*x + c*d*x*cosh(1) + c*d*x*sinh(1))*sqrt(c^2*x^2 -
1))/(x^2*cosh(1) + x^2*sinh(1) + d)) + 2*(2*b*c^4*d^3*x^2 + (4*a + b)*c^2*d
^3)*cosh(1) - 4*(2*b*c^4*d^3*x^2*cosh(1) + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^
4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^4 + 2*b*x^4*cosh(1)
+ b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2)*cosh(1)^2 + (b*c^4
*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + 6*(b*c^2*d*x^4 + b*d*x^2)*
cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 + 3*(b*c^2*d*x^4
+ b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2)*cosh(1))*sinh(1))*
log(c*x + sqrt(c^2*x^2 - 1)) - 4*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(
1)^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^4 + 2*b*x^4*cosh(
1) + b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)
^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 + 6*(b*c^
2*d*x^4 + b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*cosh(
1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 + b*c^2*d^3 + 3*(b*c^2*d*x^4 + b*
d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(
1))*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(2*b*c^4*d^3*x^2 + 3*b*c^2*d*x^4*cosh
(1)^2 + (4*a + b)*c^2*d^3 + 2*(b*c^4*d^2*x^4 + 2*b*c^2*d^2*x^2 + 2*a*d^2)*c
```

```
osh(1))*sinh(1) + 2*(b*c^3*d^3*x*cosh(1) + b*c*d*x^3*cosh(1)^3 + b*c*d*x^3*
sinh(1)^3 + (b*c^3*d^2*x^3 + b*c*d^2*x)*cosh(1)^2 + (b*c^3*d^2*x^3 + 3*b*c*
d*x^3*cosh(1) + b*c*d^2*x)*sinh(1)^2 + (b*c^3*d^3*x + 3*b*c*d*x^3*cosh(1)^2
 + 2*(b*c^3*d^2*x^3 + b*c*d^2*x)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/(c^4*
d^6*cosh(1) + d^2*x^4*cosh(1)^5 + d^2*x^4*sinh(1)^5 + 2*(c^2*d^3*x^4 + d^3*
x^2)*cosh(1)^4 + (2*c^2*d^3*x^4 + 5*d^2*x^4*cosh(1) + 2*d^3*x^2)*sinh(1)^4
+ (c^4*d^4*x^4 + 4*c^2*d^4*x^2 + d^4)*cosh(1)^3 + (c^4*d^4*x^4 + 4*c^2*d^4*
x^2 + 10*d^2*x^4*cosh(1)^2 + d^4 + 8*(c^2*d^3*x^4 + d^3*x^2)*cosh(1))*sinh(
1)^3 + 2*(c^4*d^5*x^2 + c^2*d^5)*cosh(1)^2 + (2*c^4*d^5*x^2 + 10*d^2*x^4*co
sh(1)^3 + 2*c^2*d^5 + 12*(c^2*d^3*x^4 + d^3*x^2)*cosh(1)^2 + 3*(c^4*d^4*x^4
 + 4*c^2*d^4*x^2 + d^4)*cosh(1))*sinh(1)^2 + (c^4*d^6 + 5*d^2*x^4*cosh(1)^4
 + 8*(c^2*d^3*x^4 + d^3*x^2)*cosh(1)^3 + 3*(c^4*d^4*x^4 + 4*c^2*d^4*x^2 + d
^4)*cosh(1)^2 + 4*(c^4*d^5*x^2 + c^2*d^5)*cosh(1))*sinh(1)), -1/8*(b*c^2*d*
x^4*cosh(1)^3 + b*c^2*d*x^4*sinh(1)^3 + (2*a + b)*c^4*d^4 + (b*c^4*d^2*x^4
+ 2*b*c^2*d^2*x^2 + 2*a*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 3*b*c^2*d*x^4*cos
h(1) + 2*b*c^2*d^2*x^2 + 2*a*d^2)*sinh(1)^2 - (b*c*x^4*cosh(1)^3 + b*c*x^4*
sinh(1)^3 + 2*b*c^3*d^3 + 2*(b*c^3*d*x^4 + b*c*d*x^2)*cosh(1)^2 + (2*b*c^3*
d*x^4 + 3*b*c*x^4*cosh(1) + 2*b*c*d*x^2)*sinh(1)^2 + (4*b*c^3*d^2*x^2 + b*c
*d^2)*cosh(1) + (4*b*c^3*d^2*x^2 + 3*b*c*x^4*cosh(1)^2 + b*c*d^2 + 4*(b*c^3
*d*x^4 + b*c*d*x^2)*cosh(1))*sinh(1))*sqrt(-c^2*d^2 - d*cosh(1) - d*sinh(1)
)*arctan((sqrt(-c^2*d^2 - d*cosh(1) - d*sinh(1))*sqrt(c^2*x^2 - 1)*(x*cosh(
1) + x*sinh(1)) - sqrt(-c^2*d^2 - d*cosh(1) - d*sinh(1))*(c*x^2*cosh(1) + c
*x^2*sinh(1) + c*d))/(c^2*d^2 + d*cosh(1) + d*sinh(1))) + (2*b*c^4*d^3*x^2
+ (4*a + b)*c^2*d^3)*cosh(1) - 2*(2*b*c^4*d^3*x^2*cosh(1) + b*x^4*cosh(1)^4
 + b*x^4*sinh(1)^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^4 +
 2*b*x^4*cosh(1) + b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2)*c
osh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + 6*(b*c^2*
d*x^4 + b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3
+ 3*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2)*c
osh(1))*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^4*d^4 + b*x^4*cosh(1
)^4 + b*x^4*sinh(1)^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^3 + 2*(b*c^2*d*x^
4 + 2*b*x^4*cosh(1) + b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2
 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2
+ b*d^2 + 6*(b*c^2*d*x^4 + b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 +
 b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 + b*c^2*d^3 + 3*
(b*c^2*d*x^4 + b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^
2)*cosh(1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^4*d^3*x^2 + 3*b
*c^2*d*x^4*cosh(1)^2 + (4*a + b)*c^2*d^3 + 2*(b*c^4*d^2*x^4 + 2*b*c^2*d^2*x
^2 + 2*a*d^2)*cosh(1))*sinh(1) + (b*c^3*d^3*x*c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{acosh}\,(cx))}{\left(d + ex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

3.509 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=772

$$-\frac{bcex(1-c^2x^2)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd^3}-\ldots$$

```
[Out] 1/4*(a+b*arccosh(c*x))/d/(e*x^2+d)^2+1/2*(a+b*arccosh(c*x))/d^2/(e*x^2+d)+(
a+b*arccosh(c*x))^2/b/d^3+(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)/d^3-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3-1/2*(a+b*arccosh(c*x))*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/d^3-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*
b*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^
3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^
(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/8*b*c*e*x*
(-c^2*x^2+1)/d^2/(c^2*d+e)/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*(2
*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^
(1/2)/d^(5/2)/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*c*arctanh(x
*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(5/2)/(c^2*
d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5959, 5882, 3799, 2221, 2317, 2438, 5957, 533, 390, 385, 214, 5962, 5681}

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^3}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2d^3}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{bd^3}+\frac{\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{d^3}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2}-\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{5/2}\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}-\frac{bc\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2d^{5/2}\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d+e}}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{-dc^2-e}}\right)}{2d^3}-\frac{bcex(1-c^2x^2)}{8d^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\left(d+ex^2\right)}-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -1/8*(b*c*e*x*(1 - c^2*x^2))/(d^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(d + e*x^2)) + (a + b*ArcCosh[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCosh[c*
x])/(2*d^2*(d + e*x^2)) + (a + b*ArcCosh[c*x])^2/(b*d^3) - (b*c*Sqrt[-1 + c
^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/
2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(2*c^2*d + e)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8
```

```
*d^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((a + b*ArcCosh[
c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^3 - ((a + b*ArcCosh[c*x])*Log[1 - (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b
*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*P
olyLog[2, -E^(-2*ArcCosh[c*x])])/(2*d^3) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*PolyLog[2, (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^3
) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
)])/(2*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\,(d+ex^2)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x} - \frac{ex(a+b\cosh^{-1}(cx))}{d\,(d+ex^2)^3} - \frac{ex(a+b\cosh^{-1}(cx))}{d^2\,(d+ex^2)^2} - \frac{ex(a+b}{d^3\,(c}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e\int \frac{x(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{x(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx}{d^2} - \frac{e\int \frac{x(a+b\cosh^{-1}}{(d+ex^2)}}{d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\,(d+ex^2)} + \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^3}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\,(d+ex^2)} - \frac{(a+b\cosh^{-1}(cx))^2}{2bd^3} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,d\right.}{d^3}$$

$$= -\frac{bcex(1-c^2x^2)}{8d^2\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} + \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(c}{2d^2\,(d+ex^2}$$

$$= -\frac{bcex(1-c^2x^2)}{8d^2\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} + \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(c}{2d^2\,(d+ex^2}$$

$$= -\frac{bcex(1-c^2x^2)}{8d^2\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} + \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(c}{2d^2\,(d+ex^2}$$

$$= -\frac{bcex(1-c^2x^2)}{8d^2\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} + \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(c}{2d^2\,(d+ex^2}$$

$$= -\frac{bcex(1-c^2x^2)}{8d^2\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)} + \frac{a+b\cosh^{-1}(cx)}{4d\,(d+ex^2)^2} + \frac{a+b\cosh^{-1}(c}{2d^2\,(d+ex^2}$$

<u>Mathematica</u> [F]
time = 4.72, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x\,(d+ex^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.75, size = 1478, normalized size = 1.91

method	result	size
derivativedivides	Expression too large to display	1478
default	Expression too large to display	1478

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)-1/4*b/d^3/(c^2*d+e)*e*
sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+b/d^3/(c^2*d+e)*e*dilog(1+I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^3/(c^2*d+e)*e*dilog(1-I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))-1/4*b/d^3/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*
(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
*e^2-1/4*b*c^2/d^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(a
rccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+b
/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b*c
^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b*c
^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4
*b*c^2/d^2/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e+1/2*b*c^6/d/(c^2*
d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e*x^2+1/2*b*c^4/d^2/(c^2*d+e)/(c^2*e*
x^2+c^2*d)^2*arccosh(c*x)*e^2*x^2+5/8*b*((c^2*d+e)*c^2*d)^(1/2)/d^3/(c^2*d+
e)^2*e*arctanh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c
^4*d^2+c^2*d*e)^(1/2))+a/d^3*ln(c*x)-1/4*b*c^6/d/(c^2*d+e)/(c^2*e*x^2+c^2*d
)^2*e*x^2-1/8*b*c^6/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e^2*x^4-1/2*a/d^3*ln(
c^2*e*x^2+c^2*d)+1/8*b*c^5/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*e*x+1/8*b*c^5/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*e^2*x^3+3/4*b*c^4/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e*arccosh(c*
x)+3/4*b*c^2*((c^2*d+e)*c^2*d)^(1/2)/d^2/(c^2*d+e)^2*arctanh(1/4*(4*c^2*d+2
*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+b/d^3/
(c^2*d+e)*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b*c^2/d^
2/(c^2*d+e)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b*c^2/d^2/(c^2*d+e
)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/4*a*c^4/d/(c^2*e*x^2+c^2*d
)^2+1/2*a*c^2/d^2/(c^2*e*x^2+c^2*d)-1/8*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((2*x^2*e + 3*d)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) - 2*log(x^2*e + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(
x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{(cx)}}{x \left(d + ex^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**3,x)
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x\,\left(e\,x^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)^3),x)
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)^3), x)
```

3.510 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)^3}\,dx$

Optimal. Leaf size=834

$$\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x}+\frac{bce^2x(1-c^2x^2)}{8d^3\,(c^2d+e)\,\sqrt{-1+cx}\,\sqrt{1+cx}\,\,(d+ex^2)}-\frac{a+b\cosh^{-1}(cx)}{2d^3x^2}-\frac{e\big(a+b\cosh^{-1}(c\ldots}{4d^2\,(d+ex^2)^2}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d^3/x^2-1/4*e*(a+b*arccosh(c*x))/d^2/(e*x^2+d)^2-e*
(a+b*arccosh(c*x))/d^3/(e*x^2+d)-3*e*(a+b*arccosh(c*x))^2/b/d^4-3*e*(a+b*ar
ccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4+3/2*e*(a+b*arcc
osh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x
))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^
(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,-1/(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4+3/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d
^4+3/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4+1/8*b*c*e^2*x*(-c^2*x^2+1)/
d^3/(c^2*d+e)/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^3/x+1/8*b*c*e*(2*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/
(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(7/2)/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b*c*e*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c
^2*x^2-1)^(1/2)/d^(7/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 834, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5959, 5883, 97, 5882, 3799, 2221, 2317, 2438, 5957, 533, 390, 385, 214, 5962, 5681}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^3*x) + (b*c*e^2*x*(1 - c^2*x^2))/(8
*d^3*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2)) - (a + b*ArcCosh
[c*x])/(2*d^3*x^2) - (e*(a + b*ArcCosh[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a
 + b*ArcCosh[c*x]))/(d^3*(d + e*x^2)) - (3*e*(a + b*ArcCosh[c*x])^2)/(b*d^4
```

```
) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1
+ c^2*x^2])])/(d^(7/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])])/(8*d^(7/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*e*(a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])])/d^4 + (3*e*(a
 + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -E^(-2*ArcCosh[c*x]
)])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E
^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLo
g[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x^3} - \frac{3e\left(a+b\cosh^{-1}(cx)\right)}{d^4x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^3} + \frac{2e^2x(a}{d^3}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^4} + \frac{(3e^2)\int \frac{x(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx}{d^4} + \frac{(2e^2)\int}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+ex^2\right)} + \frac{(bc)\int \frac{}{x^2\sqrt{-1}}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh\right.}{d^3\left(d+\right.}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x(1-c^2x^2)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} - \frac{a+b\cos}{2d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x(1-c^2x^2)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} - \frac{a+b\cos}{2d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x(1-c^2x^2)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} - \frac{a+b\cos}{2d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x(1-c^2x^2)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} - \frac{a+b\cos}{2d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x(1-c^2x^2)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} - \frac{a+b\cos}{2d}$$

Mathematica [F]
time = 7.25, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.18, size = 1938, normalized size = 2.32

method	result	size
derivativedivides	Expression too large to display	1938
default	Expression too large to display	1938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3/4*b/d^3/(c^2*d+e)*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arc
cosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3*b
/d^3/(c^2*d+e)*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3*b/d^3/(c^2*
d+e)*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+3/4*b/d^3/(c^2*d+e)*sum
((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x
+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootO
f(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e^2+1/2*b*c^3/x/d/(c^2*e*x^2+c^2*d)^2/(c^2*
d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e+7/8*b*c^3*x/d^2/(c^2*e*x^2+c^2*d)^2/(c^2
*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2-3*b/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1
-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3/8*b*c^4*x^4/d^3/(c^2*e*x^2+c^2*d)^2
/(c^2*d+e)*e^3-9/4*b*c^2/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*e^2
-9/8*b/c^2*((c^2*d+e)*c^2*d)^(1/2)/d^4/(c^2*d+e)^2*e^2*arctanh(1/4*(4*c^2*d
+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-3*b/
c^2/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)-3*b/c^2/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))-1/2*b*c^4/x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)+3/8*b*c^3*
x^3/d^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3-3/2*b
*c^4/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e^2*x^2-3/4*b*c^4*x^2/d
^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*e^2-3/2*b*c^2*x^2/d^3/(c^2*e*x^2+c^2*d)^2/
(c^2*d+e)*arccosh(c*x)*e^3+1/2*b*c^5/x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)-5/4*b*((c^2*d+e)*c^2*d)^(1/2)/d^3/(c^2*d+e)^2*e*arcta
nh(1/4*(4*c^2*d+2*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d
*e)^(1/2))-b*c^6/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e*x^2-1/2*b*c^6/d^2/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2*e^2*x^4-a*e/d^3/(c^2*e*x^2+c^2*d)-1/2*a/d^3/c^2/x^2
+b*c^5/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x+1/2*
b*c^5/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x^3
-9/4*b*c^4/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e*arccosh(c*x)-3*b/d^3/(c^2*d+e)
*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3/8*b*c^4/d/(c^2*
e*x^2+c^2*d)^2/(c^2*d+e)*e-3*b/c^2/d^4*e^2/(c^2*d+e)*dilog(1+I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))-3*b/c^2/d^4*e^2/(c^2*d+e)*dilog(1-I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))+3/4*b/c^2/d^4*e^3/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2
*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
```

```
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z
^2+e))+3/4*b/c^2/d^4*e^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d
+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))-1/2*b*c^2/x^2/d/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*e*arccosh(c*x)-3*a/c^2/d
^4*e*ln(c*x)+3/2*a/c^2*e/d^4*ln(c^2*e*x^2+c^2*d)-1/4*a*c^2*e/d^2/(c^2*e*x^2
+c^2*d)^2-1/2*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*x^4*e^2 + 9*d*x^2*e + 2*d^2)/(d^3*x^6*e^2 + 2*d^4*x^4*e + d^5*x^
2) - 6*e*log(x^2*e + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2*x^5*e + d^3*x^3), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2*x^5*e + d^3*x^
3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3), x)
```

3.511 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1224

$$-\frac{bc\sqrt{-d}\,\sqrt{-1+cx}\,\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\,\sqrt{-1+cx}\,\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2}+\frac{5\left(a+b\right.}{16e^{5/2}\left(\sqrt{\right.}}$$

```
[Out] 3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccosh(c*x))*ln(
1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(
a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1
/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2
)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2
)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-1/8*b*c^3*d*arctanh((c*x+1)^(1/2)*(c*
(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/e^(5/
2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/8*b*c^3*d*ar
ctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2
)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2
))^(3/2)-1/16*(a+b*arccosh(c*x))*(-d)^(1/2)/e^(5/2)/((-d)^(1/2)-x*e^(1/2))^
2+5/16*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh(
c*x))*(-d)^(1/2)/e^(5/2)/((-d)^(1/2)+x*e^(1/2))^2-5/16*(a+b*arccosh(c*x))/e
^(5/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/e^2/(c^2*d+e)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)+x*e^(1/2))-5/8*b*c*arctanh((c*x+1)^(
1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2
))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)+5/8*b*
c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^
(1/2)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^
(1/2))^(1/2)
```

Rubi [A]

time = 2.99, antiderivative size = 1224, normalized size of antiderivative = 1.00, number of steps used = 80, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5959, 5909, 5963, 98, 95, 214, 5962, 5681, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(e^2*(c^2*d + e)*(Sqrt[-d
] - Sqrt[e]*x)) - (b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*
d + e)*(Sqrt[-d] + Sqrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2
)*(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d
] - Sqrt[e]*x)) + (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + S
qrt[e]*x)^2) - (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x))
 - (b*c^3*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt
[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-
d] + Sqrt[e])^(3/2)*e^(5/2)) - (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*S
qrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[
-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (b*c^3*d*ArcTanh[(Sqrt
[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 +
 c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(5/
2)) + (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqr
t[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqr
t[-d] + Sqrt[e]]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*(a
 + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)
) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) +
(3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]
)/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/
2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
```

```
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3}-\frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{e^2\left(d+ex^2\right)}\right)\,dx$$

$$=\frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2}-\frac{(2d)\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e^2}+\frac{d^2\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$=\frac{\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)\,dx}{e^2}-\frac{(2d)\int \left(-\frac{e\left(a+b\cosh^{-1}\right.}{4d\left(\sqrt{-d}\sqrt{\right.}}\right.}{}$$

$$=-\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}\,e^2}-\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}\,e^2}-\frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{16e}-\frac{3\int \cdots}{}$$

$$=-\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2}+\frac{5\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}\right.}{16e^{5/2}\left(\sqrt{-d}+\sqrt{\right.}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{16e^{5/}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{16e^{5/}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{16e^{5/}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{16e^{5/}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{16e^{5/}}$$

$$=-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{\sqrt{-d}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.05, size = 1185, normalized size = 0.97

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((-5*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + (5*(-(ArcCosh[c*x]/(I*Sqrt[d]
 + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) -
e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e
]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/
(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt
[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/
2))))/e^2 - ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 - (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d
]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sq
rt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/e^2 + (((3*I)/32)*(ArcCosh[c*x]*
(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sq
rt[-(c^2*d) - e]))]))/(Sqrt[d]*e^(5/2)) - (((3*I)/32)*(ArcCosh[c*x]*(-ArcCo
sh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
 e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])]))/(Sqrt[d]*e^(5/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 69.81, size = 3148, normalized size = 2.57

method	result	size
derivativedivides	Expression too large to display	3148
default	Expression too large to display	3148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^5*(-3/8*b*c^9/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*d*x-5/8*b*c^
11/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*d*x^3-3/8*b*c^11/e^2/(c^2*e
*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*d^2*x-5/8*b*c^6*(-(2*c^2*d-2*((c^2*d+e
)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2
*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*((c^2*d+e)*c^
2*d)^(1/2)-5/8*b*c^9/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x^3-b*c^10*
(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^5
/(c^2*d+e)^2*((c^2*d+e)*c^2*d)^(1/2)+b*c^8*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(
1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(
(c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))*d/e^5/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)
-7/4*b*c^8*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/
2))/e^4/(c^2*d+e)^2*((c^2*d+e)*c^2*d)^(1/2)*d+b*c^10*((2*c^2*d+2*((c^2*d+e)
*c^2*d)^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((
2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)^2*((c^2*d+e)*c
^2*d)^(1/2)-b*c^8*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)
^(1/2))*d/e^5/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)+7/4*b*c^8*((2*c^2*d+2*((c^2
*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/(
(2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)^2*((c^2*d+e)*
c^2*d)^(1/2)*d-1/8*b*c^10/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*d*x^2-9/4*b*c^10*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)
*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(
1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)^2*d^2-b*c^12*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)
^(1/2)+e)*e)^(1/2)*d^3*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)^2+b*c^10*(-(2*c^2*d
-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))*d^2/e^5/(c^2*d+e)
-3/8*a*c^9/(c^2*e*x^2+c^2*d)^2*d/e^2*x+7/4*b*c^8*(-(2*c^2*d-2*((c^2*d+e)*c^
2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)*d-1/8*b*c^10/e^2/(c
^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-b*c^12*((2*c^2*
d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*d^3*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)^
2+b*c^10*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))*d
^2/e^5/(c^2*d+e)-5/4*b*c^8*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*
arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1
/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+5/8*b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^
(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((
c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*((c^2*d+e)*c^2*d)^(1/2)-
5/4*b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-
```

```
1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/
e^4/(c^2*d+e)*((c^2*d+e)*c^2*d)^(1/2)+7/4*b*c^8*((2*c^2*d+2*((c^2*d+e)*c^2*
d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2
*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*d+5/4*b*c^6*(-(2*c^2*d-
2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)*((c^
2*d+e)*c^2*d)^(1/2)-9/4*b*c^10*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^
2*d)^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)^2*d^2-5/4*b*c^8*(-(2*c^2*d-2*((c^2*d+
e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-
2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+3/8*a*c^5/
e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-3/16*b*c^6/e/(c^2*d+e)*sum(1/_R1/(_
R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c
^2*d+2*e)*_Z^2+e))+3/16*b*c^6/e/(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arcc
osh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+5/8*
b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/
(c^2*d+e)+5/8*b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan
h((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-
e)*e)^(1/2))/e^3/(c^2*d+e)-3/16*b*c^8/e^2/(c^2*d+e)*d*sum(1/_R1/(_R1^2*e+2*
c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog(
(_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) - (5*x^3*e + 3*d*x)/(x^4*
e^4 + 2*d*x^2*e^3 + d^2*e^2))*a + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{acosh}(cx))}{(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a + b\,\operatorname{acosh}(c\,x))}{(e\,x^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

3.512 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1234

$$-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\,(c^2d+e)\left(\sqrt{-d}\,-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\,(c^2d+e)\left(\sqrt{-d}\,+\sqrt{e}\,x\right)}-\frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}\,e^{3/2}\left(\sqrt{-d}\,-\sqrt{e}\,x\right.}$$

```
[Out] -1/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccosh(c*x))*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*
(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(
1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(
3/2)-1/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/
2)-1/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(
-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/e^(3/2
)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)-1/8*b*c^3*arcta
nh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e
^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^
(3/2)+1/16*(-a-b*arccosh(c*x))/e^(3/2)/(-d)^(1/2)/((-d)^(1/2)-x*e^(1/2))^2+
1/16*(-a-b*arccosh(c*x))/d/e^(3/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh
(c*x))/e^(3/2)/(-d)^(1/2)/((-d)^(1/2)+x*e^(1/2))^2+1/16*(a+b*arccosh(c*x))/
d/e^(3/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e/(c^
2*d+e)/(-d)^(1/2)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/e/(c^2*d+e)/(-d)^(1/2)/((-d)^(1/2)+x*e^(1/2))+1/8*b*c*arctanh((c*x+1)^(1
/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
)/d/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)-1/8*b
*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)
^(1/2)-e^(1/2))^(1/2))/d/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)
+e^(1/2))^(1/2)
```

Rubi [A]

time = 2.27, antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5959, 5909, 5963, 98, 95, 214, 5962, 5681, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d]
- Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d +
e)*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqr
t[-d] - Sqrt[e]*x)^2) - (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt
[e]*x)) + (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^
2) + (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*Ar
cTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]
]*Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^
(3/2)*e^(3/2)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(S
qrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]
*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] + Sq
rt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*(c*S
qrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(3/2)) - (b*c*ArcTa
nh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*S
qrt[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]
*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCosh[c*x])
*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(
-d)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcC
osh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
)])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e
^(3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
```

```
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^3}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e}-\frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e}$$

$$= \frac{\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx}{e}-\frac{d\int \Bigg(\cdots}{\cdots}$$

$$= \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx}{16d}+\frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx}{16d}-\frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx}{4d}-\cdots$$

$$= -\frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}\,e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2}-\frac{a+b\cosh^{-1}(cx)}{16de^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\,\mathrm{c}\cdots}{16\sqrt{-d}\,e^{3/2}\left(\cdots\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right.}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.04, size = 1193, normalized size = 0.97

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*((ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt
[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])
/Sqrt[-(c^2*d) - e])/(16*d*e^(3/2)) - (-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*
x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqr
t[-(c^2*d) - e])/(16*d*e^(3/2)) - ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqr
t[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*
Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c
^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e)
+ ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[
e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(L
og[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d +
 e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt
[e]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x]
 + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])]
+ Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) +
2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])
] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
e]))]))/(d^(3/2)*e^(3/2)) - ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1
- (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2
*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/
(d^(3/2)*e^(3/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 60.57, size = 2292, normalized size = 1.86

method	result	size
derivativedivides	Expression too large to display	2292
default	Expression too large to display	2292

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/8*b*c^9/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*d*x+1/8*a*c^
7/(c^2*e*x^2+c^2*d)^2/d*x^3+1/4*b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e
)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e
)*c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2+1/4*b*c^6*(-(2*c^2*d-2*((c^2*d+
e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-
2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/e^2+1/8*a*c^3/d/
e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4*b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2
*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*
d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*((c^2*d+e)*c^2*d)^
(1/2)+1/8*b*c^9/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x^3-1/8*a*c^7/(c
^2*e*x^2+c^2*d)^2/e*x-1/8*b*c^7/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*
x-1/8*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1
/2))/d/(c^2*d+e)/e^2-1/8*b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1
/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d
)^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2+1/8*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*
d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d
+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/d/e^2*((c^2*d+e)*c^2*d)
^(1/2)-1/4*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*
e)^(1/2))/d/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)-1/8*b*c^4*((2*c^2*d+2*((c
^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/d/e^2*((c^2*d
+e)*c^2*d)^(1/2)+1/4*b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*
arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1
/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)+1/8*b*c^8/e/(c^2*e
*x^2+c^2*d)^2*d/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/8*b*c^8/(c^2*e*x^2+
c^2*d)^2/(c^2*d+e)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/4*b*c^8*((2*c^2*d+2*((
c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d-1/4*b*
c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c
^2*d+e)^2*((c^2*d+e)*c^2*d)^(1/2)+1/4*b*c^8*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^
(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*
((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d-1/16*b*c^6/e/(c^2*d+
e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Roo
tOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/16*b*c^6/e/(c^2*d+e)*sum(_R1/(_R1^2*e+2
*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog
((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e
)*_Z^2+e))+1/16*b*c^4/d/(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)
*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/16*b*c^4/d
```

```
/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^
(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1))
,_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*b*c^6*((2*c^2*d+2*((c^2*d+e)*
c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)-1/4*b*c^6*(-(2*c^2*
d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)+1/
8*b*c^7*e/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*arccosh(c*x)*x^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(3/2) + (x^3*e - d*x)/(d*x^4*e^
3 + 2*d^2*x^2*e^2 + d^3*e)) + b*integrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{acosh}\,(cx))}{\left(d + ex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a+b\,\mathrm{acosh}(c\,x))}{{(e\,x^2+d)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

3.513 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^3}\, dx$

Optimal. Leaf size=1234

$$-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\,\left(\sqrt{-d}-\cdots\right.}$$

```
[Out] 3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(3/2)/e^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(3/2)/e^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/16*(-a-b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)-x*e^(1/2))^2-3/16*(a+b*arccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)+x*e^(1/2))^2+3/16*(a+b*arccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)+x*e^(1/2))+3/8*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/d^2/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)-3/8*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d^2/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A]

time = 1.16, antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5909, 5963, 98, 95, 214, 5962, 5681, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]
- Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d +
e)*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(S
qrt[-d] - Sqrt[e]*x)^2) - (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d
] - Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + S
qrt[e]*x)^2) + (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]
*x)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sq
rt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sq
rt[-d] + Sqrt[e])^(3/2)*Sqrt[e]) + (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e
]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (b*c^3*ArcTanh
[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/
2)*Sqrt[e]) - (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sq
rt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]
]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sq
rt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] - Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*
(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*Poly
Log[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))]])/(16*
(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e])]])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]])/(16*(-d)^(5/2)*Sqrt[
e]) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
- e])]])/(16*(-d)^(5/2)*Sqrt[e])
```

<u>Rule 95</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 98</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \left(-\frac{e^{3/2}(a+b\cosh^{-1}(cx))}{8(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}-ex\right)^3} - \frac{3e(a+b\cosh^{-1}(cx))}{16d^2\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2} - \frac{e^{3/2}(a+b\,\mathrm{c}}{8(-d)^{3/2}\left(\sqrt{-\right.}}\right.$$

$$= -\frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx}{16d^2} - \frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx}{16d^2} - \frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,d}{8d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2} - \frac{3(a+b\cosh^{-1}(cx))}{16d^2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{a+b}{16(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}+\sqrt{e}\,x\right)} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.63, size = 1162, normalized size = 0.94

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

```
[Out] ((8*a*d^(3/2)*x)/(d + e*x^2)^2 + (12*a*Sqrt[d]*x)/(d + e*x^2) + (12*a*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (6*b*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d]
+ Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e
]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]
*x))])/Sqrt[-(c^2*d) - e]))/Sqrt[e] - (6*b*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[
d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]))/Sqrt[e] + (2*I)*b*d*((c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*
((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d +
e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))) - (2
*I)*b*d*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]
*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log
[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e
]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e
]*(c^2*d + e)^(3/2))) + ((3*I)*b*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqr
t[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[
2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]))/Sqrt[e
] + ((3*I)*b*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*
x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x
])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) - 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[
c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] - 2*PolyLog[2, (Sqrt[e]*E^ArcCosh
[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/Sqrt[e])/(32*d^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.67, size = 3149, normalized size = 2.55

method	result	size
derivativedivides	Expression too large to display	3149
default	Expression too large to display	3149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/c*(3/8*a*c^3/d^2*x/(c^2*e*x^2+c^2*d)+7/4*b*c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2+7/4*b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/e^2+3/4*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)^2/e-3/8*b*c^2*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)/e+3/4*b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)^2/e-3/8*b*c^2*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d^2/(c^2*d+e)/e+b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*((c^2*d+e)*c^2*d)^(1/2)+1/4*a*c^5*x/d/(c^2*e*x^2+c^2*d)^2-3/16*b*c^2/d^2/(c^2*d+e)*e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/8*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3/16*b*c^2/d^2/(c^2*d+e)*e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+5/8*b*c^5/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e*x+3/8*b*c^5/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e^2*x^3-1/8*b*c^6/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*e*x^2+5/8*b*c^7/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x-5/4*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)/e^2-5/4*b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2+5/4*b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/d/e^2*((c^2*d+e)*c^2*d)^(1/2)-b*c^4*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)-5/4*b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/d/e^2*((c^2*d+e)*c^2*d)^(1/2)+b*c^4*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^3*((c^2*d+e)*c^2*d)^(1/2)+3/8*b*c^2*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)^2/e*((c^2*d+e)*c^2*d)^(1/2)-3/4*b*c^2*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)

```
^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)-e)*e)^(1/2))/d
^2/(c^2*d+e)/e^2*((c^2*d+e)*c^2*d)^(1/2)-3/8*b*c^2*((2*c^2*d+2*((c^2*d+e)*c
^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*
d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/d^2/(c^2*d+e)^2/e*((c^2*d+e)*c^2*d
)^(1/2)+3/4*b*c^2*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)
^(1/2))/d^2/(c^2*d+e)/e^2*((c^2*d+e)*c^2*d)^(1/2)+b*c^8*((2*c^2*d+2*((c^2*d
+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2
*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d-b*c^6*((2*c
^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2
*((c^2*d+e)*c^2*d)^(1/2)+b*c^8*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*((c^2*d+e)*c^
2*d)^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+3/16*b*c^4/d/(c^2*d+e)*sum(_R1/(_
R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c
^2*d+2*e)*_Z^2+e))-3/16*b*c^4/d/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(ar
ccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b*
c^6*((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*((c^2*d+e)*c^2*d)^(1/2)+e)*e)^(1/2))/e^3/(c
^2*d+e)-b*c^6*(-(2*c^2*d-2*((c^2*d+e)*c^2*d)^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*x^3*e + 5*d*x)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) + 3*arctan(x*e^(
1/2)/sqrt(d))*e^(-1/2)/d^(5/2)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{(e\,x^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^3, x)
```

3.514 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 3.64, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}(cx)\right)\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + sqrt(x^2*e + d)*x)*a + b*integ
rate(sqrt(x^2*e + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acosh}(cx)) \sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b\,\mathrm{acosh}(c\,x))\,\sqrt{e\,x^2 + d}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^(1/2), x)
```

3.515 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccosh}(cx)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + b*integrate(log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(1/2), x)
```

3.516 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] -b*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))*(c^2*x^2-1)^(1/2)/d
/e^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {197, 5908, 12, 533, 455, 65, 223, 212}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\,\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*Sqrt[-1 + c^2*x^2]*ArcTan
h[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx &= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - (bc)\int \frac{x}{d\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}}\,dx \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}}\,dx}{d} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x}{\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}\,dx}{d\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,\sqrt{d+ex}}\,dx, x, x^2\right.}{2d\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{\left(b\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+\right.}{cd\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{\left(b\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd\sqrt{-1+cx}\,\sqrt{1+cx}} \\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}
$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 12.38, size = 556, normalized size = 5.50

$$
\frac{ax+bx\cosh^{-1}(cx)+\frac{2b(-1+cx)^{3/2}\sqrt{\frac{\left(c\sqrt{d}-i\sqrt{e}\right)(1+cx)}{\left(c\sqrt{d}+i\sqrt{e}\right)(-1+cx)}}\left(\frac{c\left(-ic\sqrt{d}+\sqrt{e}\right)\left(i\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}}\,F\left(\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{-1+cx}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{(c^2d+e)(d+ex^2)}{de(-1+cx)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}\,\Pi\left(\frac{2c\sqrt{d}}{c\sqrt{d}+i\sqrt{e}};\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)\right)}{c(c^2d+e)\sqrt{1+cx}\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}}}{d\sqrt{d+ex^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (a*x + b*x*ArcCosh[c*x] + (2*b*(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e
])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(-1 + c*x))]*((c*((-I)*c*Sqrt[d] + S
qrt[e])*(I*Sqrt[d] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*
Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqr
```

```
t[e])/(c*Sqrt[d] + I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*
Sqrt[e])*Sqrt[((c^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I
*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(
2*c*Sqrt[d])/(c*Sqrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqr
t[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e
])/(c*Sqrt[d] + I*Sqrt[e])^2]))/(c*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (
I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(d*Sqrt[d
 + e*x^2])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccosh}(cx)}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(85) = 170.
time = 0.39, size = 333, normalized size = 3.30

$$\frac{(bx^2\cosh(1)+bx^2\sinh(1)+bd)\sqrt{\cosh(1)+\sinh(1)}\,\log\left(c^4d^2+(8\,c^4x^4-8\,c^2x^2+1)\cosh(1)^2+(8\,c^4x^4-8\,c^2x^2+1)\sinh(1)^2-4\,(c^3d+(2\,c^3x^2-c)\cosh(1)+(2\,c^3x^2-c)\sinh(1))\sqrt{c^2x^2-1}\,\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\,\sqrt{\cosh(1)+\sinh(1)}+2\,(4\,c^4dx^2-3\,c^2d)\cosh(1)+2\,(4\,c^4dx^2-3\,c^2d+(8\,c^4x^4-8\,c^2x^2+1)\cosh(1))\sinh(1)\right)+4\,(bx\cosh(1)+bx\sinh(1))\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+4\,(ax\cosh(1)+ax\sinh(1))\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}}{4\,(dx^2\cosh(1)^2+dx^2\sinh(1)^2+d^2\cosh(1)+(2\,dx^2\cosh(1)+d^2)\sinh(1))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(cosh(1) + sinh(1))*log(c^4*
d^2 + (8*c^4*x^4 - 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 - 8*c^2*x^2 + 1)*s
inh(1)^2 - 4*(c^3*d + (2*c^3*x^2 - c)*cosh(1) + (2*c^3*x^2 - c)*sinh(1))*sq
rt(c^2*x^2 - 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) + sinh(1))
```

```
 + 2*(4*c^4*d*x^2 - 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 - 3*c^2*d + (8*c^4*x^
4 - 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4*(b*x*cosh(1) + b*x*sinh(1))*sqrt(x
^2*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(a*x*cosh(1)
 + a*x*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d*x^2*cosh(1)^2 + d*x
^2*sinh(1)^2 + d^2*cosh(1) + (2*d*x^2*cosh(1) + d^2)*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{(e\,x^2 + d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(3/2), x)
```

3.517 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{2b\sqrt{1-c^2x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{1-c}}{c\sqrt{d+ex}}\right.}{3d^2\sqrt{e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(3/2)+2/3*b*arctan(e^(1/2)*(-c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))*(-c^2*x^2+1)^(1/2)/d^2/e^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2/3*x*(a+b*arccosh(c*x))/d^2/(e*x^2+d)^(1/2)-1/3*b*c*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 190, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {198, 197, 5908, 12, 533, 585, 79, 65, 223, 212}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}-\frac{2b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc(1-c^2x^2)}{3d\sqrt{cx-1}\,\sqrt{cx+1}\,(c^2d+e)\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*(1 - c^2*x^2))/(3*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*Ar
cCosh[c*x]))/(3*d^2*Sqrt[d + e*x^2]) - (2*b*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqr
t[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(3*d^2*Sqrt[e]*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x(3d+2ex^2)}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(d$$

$$= \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x(3d+2ex^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)}}{3d^2}$$

$$= \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x(3d+2e}{\sqrt{-1+c^2x^2}}}{3d^2\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\cosh^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.}{6d^2\sqrt{-1+cx}}$$

$$= \frac{bc(1-c^2x^2)}{3d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\,\mathrm{co}}{3d^2\sqrt{d}}$$

$$= \frac{bc(1-c^2x^2)}{3d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\,\mathrm{co}}{3d^2\sqrt{d}}$$

$$= \frac{bc(1-c^2x^2)}{3d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\,\mathrm{co}}{3d^2\sqrt{d}}$$

$$= \frac{bc(1-c^2x^2)}{3d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x(a+b\cosh^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\,\mathrm{co}}{3d^2\sqrt{d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 1.88, size = 633, normalized size = 3.48

$$\frac{-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)}{d(c^2d+e)}+\frac{ax(3d+2ex^2)}{d^2}+\frac{bx(3d+2ex^2)\cosh^{-1}(cx)}{d^2}+\frac{4b(-1+cx)^{3/2}\sqrt{\frac{\left(c\sqrt{d}-i\sqrt{e}\right)(1+cx)}{\left(c\sqrt{d}+i\sqrt{e}\right)(-1+cx)}}\,(d+ex^2)\left(\frac{c\left(-ic\sqrt{d}+\sqrt{e}\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}\,x}{\sqrt{d}}}{1-cx}}\,F\left(\mathrm{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}\,x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\,\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{-1+cx}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{(c^2d+e)(d+ex^2)}{de(-1+cx)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{e}\,x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}\,\Pi\left(\frac{2c\sqrt{d}}{c\sqrt{d}+i\sqrt{e}};\mathrm{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}\,x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\,\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)\right)}{cd^2(c^2d+e)\sqrt{1+cx}\,\sqrt{-\frac{-1+\frac{i\sqrt{e}\,x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}}}{3\,(d+ex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d + e))) + (a*x*(
3*d + 2*e*x^2))/d^2 + (b*x*(3*d + 2*e*x^2)*ArcCosh[c*x])/d^2 + (4*b*(-1 + c
*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])
*(-1 + c*x))]*(d + e*x^2)*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d] + Sqrt[
e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c
*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/S
qrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[
e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c^2*d + e)
*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((
I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*Sqrt[d] +
I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt
[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[e])
^2]))/(c*d^2*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] +
 c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(3*(d + e*x^2)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccosh}(cx)}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(153) = 306$.
time = 0.46, size = 955, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3 + b*c^2*d^3 + (b*c^2*d*x^4 + 2*b*d*
x^2)*cosh(1)^2 + (b*c^2*d*x^4 + 3*b*x^4*cosh(1) + 2*b*d*x^2)*sinh(1)^2 + (2
*b*c^2*d^2*x^2 + b*d^2)*cosh(1) + (2*b*c^2*d^2*x^2 + 3*b*x^4*cosh(1)^2 + b*
d^2 + 2*(b*c^2*d*x^4 + 2*b*d*x^2)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))
*log(c^4*d^2 + (8*c^4*x^4 - 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 - 8*c^2*x
^2 + 1)*sinh(1)^2 - 4*(c^3*d + (2*c^3*x^2 - c)*cosh(1) + (2*c^3*x^2 - c)*si
nh(1))*sqrt(c^2*x^2 - 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) +
 sinh(1)) + 2*(4*c^4*d*x^2 - 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 - 3*c^2*d +
(8*c^4*x^4 - 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 2*(3*b*c^2*d^2*x*cosh(1) +
2*b*x^3*cosh(1)^3 + 2*b*x^3*sinh(1)^3 + (2*b*c^2*d*x^3 + 3*b*d*x)*cosh(1)^2
 + (2*b*c^2*d*x^3 + 6*b*x^3*cosh(1) + 3*b*d*x)*sinh(1)^2 + (3*b*c^2*d^2*x +
 6*b*x^3*cosh(1)^2 + 2*(2*b*c^2*d*x^3 + 3*b*d*x)*cosh(1))*sinh(1))*sqrt(x^2
*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(3*a*c^2*d^2*x
*cosh(1) + 2*a*x^3*cosh(1)^3 + 2*a*x^3*sinh(1)^3 + (2*a*c^2*d*x^3 + 3*a*d*x
)*cosh(1)^2 + (2*a*c^2*d*x^3 + 6*a*x^3*cosh(1) + 3*a*d*x)*sinh(1)^2 + (3*a*
c^2*d^2*x + 6*a*x^3*cosh(1)^2 + 2*(2*a*c^2*d*x^3 + 3*a*d*x)*cosh(1))*sinh(1
) - (b*c*d*x^2*cosh(1)^2 + b*c*d*x^2*sinh(1)^2 + b*c*d^2*cosh(1) + (2*b*c*d
*x^2*cosh(1) + b*c*d^2)*sinh(1))*sqrt(c^2*x^2 - 1))*sqrt(x^2*cosh(1) + x^2*
sinh(1) + d))/(d^2*x^4*cosh(1)^4 + d^2*x^4*sinh(1)^4 + c^2*d^5*cosh(1) + (c
^2*d^3*x^4 + 2*d^3*x^2)*cosh(1)^3 + (c^2*d^3*x^4 + 4*d^2*x^4*cosh(1) + 2*d^
3*x^2)*sinh(1)^3 + (2*c^2*d^4*x^2 + d^4)*cosh(1)^2 + (2*c^2*d^4*x^2 + 6*d^2
*x^4*cosh(1)^2 + d^4 + 3*(c^2*d^3*x^4 + 2*d^3*x^2)*cosh(1))*sinh(1)^2 + (4*
d^2*x^4*cosh(1)^3 + c^2*d^5 + 3*(c^2*d^3*x^4 + 2*d^3*x^2)*cosh(1)^2 + 2*(2*
c^2*d^4*x^2 + d^4)*cosh(1))*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(5/2), x)
```

3.518 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=284

$$\frac{bc(1-c^2x^2)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}}+\frac{2bc(3c^2d+2e)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}}+\frac{x\left(a+b\,\mathrm{c}\right.}{5d\left(d+\right.}$$

```
[Out] 1/5*x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arccosh(c*x))/d^2/(e
*x^2+d)^(3/2)+1/15*b*c*(-c^2*x^2+1)/d/(c^2*d+e)/(e*x^2+d)^(3/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2)
)*(c^2*x^2-1)^(1/2)/d^3/e^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/15*x*(a+b*arc
cosh(c*x))/d^3/(e*x^2+d)^(1/2)+2/15*b*c*(3*c^2*d+2*e)*(-c^2*x^2+1)/d^2/(c^2
*d+e)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {198, 197, 5908, 12, 533, 6847, 963, 79, 65, 223, 212}

$$\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{8b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bc(1-c^2x^2)\left(3c^2d+2e\right)}{15d^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)^2\sqrt{d+ex^2}}+\frac{bc(1-c^2x^2)}{15d\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (b*c*(1 - c^2*x^2))/(15*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x
^2)^(3/2)) + (2*b*c*(3*c^2*d + 2*e)*(1 - c^2*x^2))/(15*d^2*(c^2*d + e)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(5*d
*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCosh[c*x]))/(15*d^2*(d + e*x^2)^(3/2))
 + (8*x*(a + b*ArcCosh[c*x]))/(15*d^3*Sqrt[d + e*x^2]) - (8*b*Sqrt[-1 + c^2
*x^2]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*d^3*Sq
rt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - (bc)\int \frac{}{15d}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{}{\sqrt{-}}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1-}\right.}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\,(d+ex^2)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1-}\right.}{}$$

$$= \frac{bc(1-c^2x^2)}{15d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\,(d+ex^2)^{5/2}} + \frac{4x\left(a+b\right.}{15d^2\,(d}$$

$$= \frac{bc(1-c^2x^2)}{15d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^{3/2}} + \frac{2bc(3c^2d+2e)\,(1-c^2}{15d^2\,(c^2d+e)^2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bc(1-c^2x^2)}{15d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^{3/2}} + \frac{2bc(3c^2d+2e)\,(1-c^2}{15d^2\,(c^2d+e)^2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bc(1-c^2x^2)}{15d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^{3/2}} + \frac{2bc(3c^2d+2e)\,(1-c^2}{15d^2\,(c^2d+e)^2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{bc(1-c^2x^2)}{15d\,(c^2d+e)\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)^{3/2}} + \frac{2bc(3c^2d+2e)\,(1-c^2}{15d^2\,(c^2d+e)^2\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.95, size = 685, normalized size = 2.41

$$\frac{\frac{ax(15d^2+20dex^2+8e^2x^4)}{d^3} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex^2)\left(e(5d+4ex^2)+c^2d(7d+6ex^2)\right)}{d^2(c^2d+e)^2} + \frac{bx(15d^2+20dex^2+8e^2x^4)\cosh^{-1}(cx)}{d^3} + \frac{16b(-1+cx)^{3/2}\sqrt{\frac{\left(c\sqrt{d}-i\sqrt{e}\right)(1+cx)}{\left(c\sqrt{d}+i\sqrt{e}\right)(-1+cx)}}\,(d+ex^2)^2\left(\frac{c\left(-ic\sqrt{d}+\sqrt{e}\right)\left(i\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}}\,F\left(\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)}{-1+cx}+c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{(c^2d+e)(d+ex^2)}{de(-1+cx)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}\,\Pi\left(\frac{2c\sqrt{d}}{c\sqrt{d}+i\sqrt{e}};\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{2-2cx}}\right)\middle|\frac{4ic\sqrt{d}\sqrt{e}}{\left(c\sqrt{d}+i\sqrt{e}\right)^2}\right)\right)}{cd^3(c^2d+e)\sqrt{1+cx}\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}}}{15\,(d+ex^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] ((a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4))/d^3 - (b*c*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(d + e*x^2)*(e*(5*d + 4*e*x^2) + c^2*d*(7*d + 6*e*x^2)))/(d^2*(c^2*d
+ e)^2) + (b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcCosh[c*x])/d^3 + (16*b*
(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*S
qrt[e])*(-1 + c*x))]*(d + e*x^2)^2*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d
] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d
])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*S
qrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d]
+ I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c
^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[
d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*S
qrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt
[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I
*Sqrt[e])^2]))/(c*d^3*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(15*(d + e*x^2)^(5/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccosh}(cx)}{(e\,x^2+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(x^2*e + d)*d^3) + 4*x/((x^2*e + d)^(3/2)*d^2) + 3*x/((x^2
*e + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2
*e + d)^(7/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2519 vs. 2(245) = 490.
time = 0.49, size = 2519, normalized size = 8.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")

[Out] 1/15*(2*(b*x^6*cosh(1)^5 + b*x^6*sinh(1)^5 + b*c^4*d^5 + (2*b*c^2*d*x^6 + 3*b*d*x^4)*cosh(1)^4 + (2*b*c^2*d*x^6 + 5*b*x^6*cosh(1) + 3*b*d*x^4)*sinh(1)^4 + (b*c^4*d^2*x^6 + 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1)^3 + (b*c^4*d^2*x^6 + 6*b*c^2*d^2*x^4 + 10*b*x^6*cosh(1)^2 + 3*b*d^2*x^2 + 4*(2*b*c^2*d*x^6 + 3*b*d*x^4)*cosh(1))*sinh(1)^3 + (3*b*c^4*d^3*x^4 + 6*b*c^2*d^3*x^2 + b*d^3)*cosh(1)^2 + (3*b*c^4*d^3*x^4 + 10*b*x^6*cosh(1)^3 + 6*b*c^2*d^3*x^2 + b*d^3 + 6*(2*b*c^2*d*x^6 + 3*b*d*x^4)*cosh(1)^2 + 3*(b*c^4*d^2*x^6 + 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1))*sinh(1)^2 + (3*b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1) + (3*b*c^4*d^4*x^2 + 5*b*x^6*cosh(1)^4 + 2*b*c^2*d^4 + 4*(2*b*c^2*d*x^6 + 3*b*d*x^4)*cosh(1)^3 + 3*(b*c^4*d^2*x^6 + 6*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*cosh(1)^2 + 2*(3*b*c^4*d^3*x^4 + 6*b*c^2*d^3*x^2 + b*d^3)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 - 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 - 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^3*d + (2*c^3*x^2 - c)*cosh(1) + (2*c^3*x^2 - c)*sinh(1))*sqrt(c^2*x^2 - 1)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 - 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 - 3*c^2*d + (8*c^4*x^4 - 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + (15*b*c^4*d^4*x*cosh(1) + 8*b*x^5*cosh(1)^5 + 8*b*x^5*sinh(1)^5 + 4*(4*b*c^2*d*x^5 + 5*b*d*x^3)*cosh(1)^4 + 4*(4*b*c^2*d*x^5 + 10*b*x^5*cosh(1) + 5*b*d*x^3)*sinh(1)^4 + (8*b*c^4*d^2*x^5 + 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1)^3 + (8*b*c^4*d^2*x^5 + 40*b*c^2*d^2*x^3 + 80*b*x^5*cosh(1)^2 + 15*b*d^2*x + 16*(4*b*c^2*d*x^5 + 5*b*d*x^3)*cosh(1))*sinh(1)^3 + 10*(2*b*c^4*d^3*x^3 + 3*b*c^2*d^3*x)*cosh(1)^2 + (20*b*c^4*d^3*x^3 + 80*b*x^5*cosh(1)^3 + 30*b*c^2*d^3*x + 24*(4*b*c^2*d*x^5 + 5*b*d*x^3)*cosh(1)^2 + 3*(8*b*c^4*d^2*x^5 + 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1))*sinh(1)^2 + (15*b*c^4*d^4*x + 40*b*x^5*cosh(1)^4 + 16*(4*b*c^2*d*x^5 + 5*b*d*x^3)*cosh(1)^3 + 3*(8*b*c^4*d^2*x^5 + 40*b*c^2*d^2*x^3 + 15*b*d^2*x)*cosh(1)^2 + 20*(2*b*c^4*d^3*x^3 + 3*b*c^2*d^3*x)*cosh(1))*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (15*a*c^4*d^4*x*cosh(1) + 8*a*x^5*cosh(1)^5 + 8*a*x^5*sinh(1)^5 + 4*(4*a*c^2*d*x^5 + 5*a*d*x^3)*cosh(1)^4 + 4*(4*a*c^2*d*x^5 + 10*a*x^5*cosh(1) + 5*a*d*x^3)*sinh(1)^4 + (8*a*c^4*d^2*x^5 + 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1)^3 + (8*a*c^4*d^2*x^5 + 40*a*c^2*d^2*x^3 + 80*a*x^5*cosh(1)^2 + 15*a*d^2*x + 16*(4*a*c^2*d*x^5 + 5*a*d*x^3)*cosh(1))*sinh(1)^3 + 10*(2*a*c^4*d^3*x^3 + 3*a*c^2*d^3*x)*cosh(1)^2 + (20*a*c^4*d^3*x^3 + 80*a*x^5*cosh(1)^3 + 30*a*c^2*d^3*x + 24*(4*a*c^2*d*x^5 + 5*a*d*x^3)*cosh(1)^2 + 3*(8*a*c^4*d^2*x^5 + 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1))*sinh(1)^2 + (15*a*c^4*d^4*x + 40*a*x^5*cosh(1)^4 + 16*(4*a*c^2*d*x^5 + 5*a*d*x^3)*cosh(1)^3 + 3*(8*a*c^4*d^2*x^5 + 40*a*c^2*d^2*x^3 + 15*a*d^2*x)*cosh(1)^2 + 20*(2*a*c^4*d^3*x^3 + 3*a*c^2*d^3*x)*cosh(1))*sinh(1) - (4*b*c*d*x^4*cosh(1)^4 + 4*b*c*d*x^4*sinh(1)^4 + 7*b*c^3*d^4*cosh(1) + 3*(2*b*c^3*d^2*x^4 + 3*b*c*d^2*x^2)*cosh(1)^3 + (6*b*c^3*d^2*x^4 + 16*b*c*d*x^4*cosh(1) + 9*b*c*d^2*x^2)*sinh(1)^3 + (13*b*c^3*d^3*x^2 + 5*b*c*d^3)*cosh(1)^2 + (13*b*c^3*d^3

```
*x^2 + 24*b*c*d*x^4*cosh(1)^2 + 5*b*c*d^3 + 9*(2*b*c^3*d^2*x^4 + 3*b*c*d^2*
x^2)*cosh(1))*sinh(1)^2 + (16*b*c*d*x^4*cosh(1)^3 + 7*b*c^3*d^4 + 9*(2*b*c^
3*d^2*x^4 + 3*b*c*d^2*x^2)*cosh(1)^2 + 2*(13*b*c^3*d^3*x^2 + 5*b*c*d^3)*cos
h(1))*sinh(1))*sqrt(c^2*x^2 - 1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d^3
*x^6*cosh(1)^6 + d^3*x^6*sinh(1)^6 + c^4*d^8*cosh(1) + (2*c^2*d^4*x^6 + 3*d
^4*x^4)*cosh(1)^5 + (2*c^2*d^4*x^6 + 6*d^3*x^6*cosh(1) + 3*d^4*x^4)*sinh(1)
^5 + (c^4*d^5*x^6 + 6*c^2*d^5*x^4 + 3*d^5*x^2)*cosh(1)^4 + (c^4*d^5*x^6 + 6
*c^2*d^5*x^4 + 15*d^3*x^6*cosh(1)^2 + 3*d^5*x^2 + 5*(2*c^2*d^4*x^6 + 3*d^4*
x^4)*cosh(1))*sinh(1)^4 + (3*c^4*d^6*x^4 + 6*c^2*d^6*x^2 + d^6)*cosh(1)^3 +
 (3*c^4*d^6*x^4 + 20*d^3*x^6*cosh(1)^3 + 6*c^2*d^6*x^2 + d^6 + 10*(2*c^2*d^
4*x^6 + 3*d^4*x^4)*cosh(1)^2 + 4*(c^4*d^5*x^6 + 6*c^2*d^5*x^4 + 3*d^5*x^2)*
cosh(1))*sinh(1)^3 + (3*c^4*d^7*x^2 + 2*c^2*d^7)*cosh(1)^2 + (3*c^4*d^7*x^2
 + 15*d^3*x^6*cosh(1)^4 + 2*c^2*d^7 + 10*(2*c^2*d^4*x^6 + 3*d^4*x^4)*cosh(1
)^3 + 6*(c^4*d^5*x^6 + 6*c^2*d^5*x^4 + 3*d^5*x^2)*cosh(1)^2 + 3*(3*c^4*d^6*
x^4 + 6*c^2*d^6*x^2 + d^6)*cosh(1))*sinh(1)^2 + (6*d^3*x^6*cosh(1)^5 + c^4*
d^8 + 5*(2*c^2*d^4*x^6 + 3*d^4*x^4)*cosh(1)^4 + 4*(c^4*d^5*x^6 + 6*c^2*d^5*
x^4 + 3*d^5*x^2)*cosh(1)^3 + 3*(3*c^4*d^6*x^4 + 6*c^2*d^6*x^2 + d^6)*cosh(1
)^2 + 2*(3*c^4*d^7*x^2 + 2*c^2*d^7)*cosh(1))*sinh(1))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{{\left(e\,x^2+d\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(7/2),x)
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(7/2), x)
```

3.519 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=558

$$\frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+342m+119m^2+18m^3+m^4\right)\right)(fx}{c^5f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccosh
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arccosh(c*x))/f^7/(7+m)+b*e*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c
^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*m^3+119*m^2+342*m+360))*(f*x)^(2+m)*(-c^
2*x^2+1)/c^5/f^2/(3+m)^2/(5+m)^2/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*e^2*
(3*c^2*d*(7+m)^2+e*(m^2+11*m+30))*(f*x)^(4+m)*(-c^2*x^2+1)/c^3/f^4/(5+m)^2/
(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*e^3*(f*x)^(6+m)*(-c^2*x^2+1)/c/f^6/(7
+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*(c^6*d^3*(3+m)*(5+m)*(7+m)/(1+m)+e*(2+m
)*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*m^3
+119*m^2+342*m+360))/(m^3+15*m^2+71*m+105))*(f*x)^(2+m)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f^2/(2+m)/(3+m)/(5+m)/(7+m)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.78, antiderivative size = 529, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {276, 5958, 12, 1624, 1823, 1281, 470, 372, 371}

$$\frac{d^3(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7)}+\frac{be^3\left(1-c^2x^2\right)(fx)^{m+6}}{cf^6(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be^2\left(1-c^2x^2\right)(fx)^{m+4}\left(3c^2d(m+7)^2+e\left(m^2+11m+30\right)\right)}{c^3f^4(m+5)^2(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc\sqrt{1-c^2x^2}\,(fx)^{m+2}\left(\frac{e\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^6(m+3)^2(m+5)^2(m+7)^2}+\frac{d^3}{m^2+3m+2}\right){}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be\left(1-c^2x^2\right)(fx)^{m+2}\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^5f^2(m+3)^2(m+5)^2(m+7)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2
+ e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^5*f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*(1 - c^2*x^2))/(
c^3*f^4*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(f*x)^(6
 + m)*(1 - c^2*x^2))/(c*f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a
 + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c
*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)
) - (b*c*(d^3/(2 + 3*m + m^2) + (e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) +
3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))
)/(c^6*(3 + m)^2*(5 + m)^2*(7 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
```

```
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(c\right.}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(c\right.}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(c\right.}{f^3(3+m)}$$

$$= \frac{be^3(fx)^{6+m}\left(1-c^2x^2\right)}{cf^6(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(\right.}{f(1+m)}$$

$$= \frac{be^2\left(3c^2d(7+m)^2+e\left(30+11m+m^2\right)\right)(fx)^{4+m}\left(1-c^2x^2\right)}{c^3f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

$$= \frac{be\Big(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2}{c^5f^2(3+m)^2(5+m)^2(7}$$

$$= \frac{be\Big(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2}{c^5f^2(3+m)^2(5+m)^2(7}$$

$$= \frac{be\Big(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2}{c^5f^2(3+m)^2(5+m)^2(7}$$

Mathematica [F]
time = 0.80, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]), x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^3 \left(a+b\,\mathrm{arccosh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^7*e^(m*log(x) + 3)/(m + 7) + 3*a*d*f^m*x^5*e^(m*log(x) + 2)/(m + 5)
 + 3*a*d^2*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1
)) + ((m^3 + 9*m^2 + 23*m + 15)*b*f^m*x^7*e^3 + 3*(m^3 + 11*m^2 + 31*m + 21
)*b*d*f^m*x^5*e^2 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*f^m*x^3*e + (m^3 + 1
5*m^2 + 71*m + 105)*b*d^3*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + integrate(((m^3 + 9*m^2 + 23*m + 1
5)*b*c*f^m*x^7*e^3 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c*d*f^m*x^5*e^2 + 3*(m^
3 + 13*m^2 + 47*m + 35)*b*c*d^2*f^m*x^3*e + (m^3 + 15*m^2 + 71*m + 105)*b*c
*d^3*f^m*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*
m^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^
2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)),
x) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^2*f^m*x^8*e^3 + 3*(m^3 + 11*m
^2 + 31*m + 21)*b*c^2*d*f^m*x^6*e^2 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^
2*f^m*x^4*e + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16
*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\operatorname{acosh}(cx)) (d + ex^2)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,\left(e\,x^2 + d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3, x)
```

3.520 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=353

$$\frac{be(2c^2d(5+m)^2+e(12+7m+m^2))\,(fx)^{2+m}\,(1-c^2x^2)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{be^2(fx)^{4+m}\,(1-c^2x^2)}{cf^4(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{d^2(fx)^{1+m}\,(a}{f(1}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccosh(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)+b*e*(2*c^2*d*(5
+m)^2+e*(m^2+7*m+12))*(f*x)^(2+m)*(-c^2*x^2+1)/c^3/f^2/(3+m)^2/(5+m)^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)+b*e^2*(f*x)^(4+m)*(-c^2*x^2+1)/c/f^4/(5+m)^2/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-b*(c^4*d^2*(3+m)*(5+m)/(1+m)+e*(2+m)*(2*c^2*d*(5+m)^2
+e*(m^2+7*m+12))/(3+m)/(5+m))*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m
],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f^2/(2+m)/(3+m)/(5+m)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 332, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {276, 5958, 12, 534, 1281, 470, 372, 371}

$$\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{be^2(1-c^2x^2)\,(fx)^{m+4}}{cf^4(m+5)^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc\sqrt{1-c^2x^2}\,(fx)^{m+2}\left(\frac{e(2c^2d(m+5)^2+e(m^2+7m+12))}{c^4(m+3)^2(m+5)^2}+\frac{d^2}{m^2+3m+2}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{f^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{be(1-c^2x^2)\,(fx)^{m+2}\,(2c^2d(m+5)^2+e(m^2+7m+12))}{c^3f^2(m+3)^2(m+5)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^3*f^2*(3 + m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*(f*x)^(
4 + m)*(1 - c^2*x^2))/(c*f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a
+ b*ArcCosh[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))
/(f^5*(5 + m)) - (b*c*(d^2/(2 + 3*m + m^2) + (e*(2*c^2*d*(5 + m)^2 + e*(12
+ 7*m + m^2)))/(c^4*(3 + m)^2*(5 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$=\frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$=\frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$=\frac{be^2(fx)^{4+m}\left(1-c^2x^2\right)}{cf^4(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}$$

$$=\frac{be(2c^2d(5+m)^2+e(12+7m+m^2))\,(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

$$=\frac{be(2c^2d(5+m)^2+e(12+7m+m^2))\,(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

$$=\frac{be(2c^2d(5+m)^2+e(12+7m+m^2))\,(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

Mathematica [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]), x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(e\,x^{2}+d\right)^{2} \left(a+b\,\mathrm{arccosh}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] a*f^m*x^5*e^(m*log(x) + 2)/(m + 5) + 2*a*d*f^m*x^3*e^(m*log(x) + 1)/(m + 3)
 + (f*x)^(m + 1)*a*d^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*f^m*x^5*e^2 + 2*(m^
2 + 6*m + 5)*b*d*f^m*x^3*e + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*
m + 3)*b*c*f^m*x^5*e^2 + 2*(m^2 + 6*m + 5)*b*c*d*f^m*x^3*e + (m^2 + 8*m + 1
5)*b*c*d^2*f^m*x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m^2 + 2
3*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 1
5)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^2*f^m*
x^6*e^2 + 2*(m^2 + 6*m + 5)*b*c^2*d*f^m*x^4*e + (m^2 + 8*m + 15)*b*c^2*d^2*
f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(a+b\,\mathrm{acosh}\,(cx)\right) \left(d+ex^{2}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))\,(f\,x)^m\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2, x)
```

3.521 $\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right)\, dx$

Optimal. Leaf size=198

$$-\frac{be(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{cf^2(3+m)^2}+\frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}-\frac{b(e(1+m$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccosh(c*x))/f
^3/(3+m)-b*e*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/f^2/(3+m)^2-b*(e*(1+
m)*(2+m)+c^2*d*(3+m)^2)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*
x^2)*(-c^2*x^2+1)^(1/2)/c/f^2/(1+m)/(2+m)/(3+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5956, 471, 127, 372, 371}

$$\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}-\frac{b\sqrt{1-c^2x^2}\,(fx)^{m+2}\left(\frac{c^2d}{m^2+3m+2}+\frac{e}{(m+3)^2}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{cf^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{be\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}}{cf^2(m+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*f^2*(3 + m)^2)) + (d*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a + b*A
rcCosh[c*x]))/(f^3*(3 + m)) - (b*(e/(3 + m)^2 + (c^2*d)/(2 + 3*m + m^2))*(f
*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
c^2*x^2])/(c*f^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5956

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*ArcCosh[c*x])/(f*(m + 1)))
, x] + (-Dist[b*(c/(f*(m + 1)*(m + 3))), Int[(f*x)^(m + 1)*((d*(m + 3) + e*
(m + 1)*x^2)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Simp[e*(f*x)^(m + 3)
*((a + b*ArcCosh[c*x])/(f^3*(m + 3))), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx = \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m} \left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} -$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right.}{f(1+m)}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right.}{f(1+m)}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right.}{f(1+m)}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m} \left(a+b\cosh^{-1}(cx)\right.}{f(1+m)}$$

Mathematica [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]), x]
```

Maple [F]
time = 10.90, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(e\,x^{2}+d\right) (a+b\,\mathrm{arccosh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (b*f^m*(m + 1)*x^3*e + b*d*f^m*(m + 3)
*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m +
 1)*a*d/(f*(m + 1)) + integrate((b*c*f^m*(m + 1)*x^3*e + b*c*d*f^m*(m + 3)*
x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^
2*x^2 - m^2 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate((b*c^2*
f^m*(m + 1)*x^4*e + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 -
 m^2 - 4*m - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccosh(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{m} (a+b\operatorname{acosh}(cx)) \left(d+ex^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^m\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2), x)
```

3.522 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A]
time = 8.39, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arccosh}(cx)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a + b\operatorname{acosh}\left(cx\right)\right)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\,(f\,x)^m}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2), x)
```

3.523 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A]
time = 8.28, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(x^2*e + d)^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^m}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2,x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2, x)
```

3.524 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(d+ex^2)^3} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3},x\right)$$

[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3} dx$$

Verification is not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3} dx = \int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3} dx$$

Mathematica [A]
time = 19.31, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3} dx$$

Verification is not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]

[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccosh}(cx)\right)}{\left(e\,x^2+d\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(x^2*e + d)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\,(f\,x)^m}{(e\,x^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3,x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3, x)
```

3.525 $\int (d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=609

$$2b^2d^3x+\frac{4b^2d^2ex}{3c^2}+\frac{16b^2de^2x}{25c^4}+\frac{32b^2e^3x}{245c^6}+\frac{2}{9}b^2d^2ex^3+\frac{8b^2de^2x^3}{75c^2}+\frac{16b^2e^3x^3}{735c^4}+\frac{6}{125}b^2de^2x^5+\frac{12b^2e^3x^5}{1225c^2}+\frac{2}{343}b^2e^3x^7-$$

```
[Out] 2*b^2*d^3*x+4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4+32/245*b^2*e^3*x/c^6+
2/9*b^2*d^2*e*x^3+8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2*d
*e^2*x^5+12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arccosh(c*x))
^2+d^2*e*x^3*(a+b*arccosh(c*x))^2+3/5*d*e^2*x^5*(a+b*arccosh(c*x))^2+1/7*e^
3*x^7*(a+b*arccosh(c*x))^2-2*b*d^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c-4/3*b*d^2*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-16/
25*b*d*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-32/245*b*e^3*
(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arc
cosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/25*b*d*e^2*x^2*(a+b*arccosh(c*x)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-16/245*b*e^3*x^2*(a+b*arccosh(c*x))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arccosh(c*x))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c-12/245*b*e^3*x^4*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^3-2/49*b*e^3*x^6*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 1.38, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5909, 5879, 5915, 8, 5883, 5939, 30}

$$-\frac{32be^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{245c^7}-\frac{16bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^5}-\frac{16be^3x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{245c^5}-\frac{4bd^2e\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{8bde^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^3}-\frac{12be^3x^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{245c^3}+d^3x\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd^2ex^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c}+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)^2-\frac{6bde^2x^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c}+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2be^3x^6\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{49c}+\frac{32b^2e^3x}{245c^6}+\frac{16b^2de^2x}{25c^4}+\frac{16b^2e^3x^3}{735c^4}+\frac{4b^2d^2ex}{3c^2}+\frac{8b^2de^2x^3}{75c^2}+\frac{12b^2e^3x^5}{1225c^2}+2b^2d^3x+\frac{2}{9}b^2d^2ex^3+\frac{6}{125}b^2de^2x^5+\frac{2}{343}b^2e^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x + (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) + (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 + (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 + (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (4*b*d^2*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(3*c^3) - (16*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(25*c^5) - (32*b*e^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(24
5*c^7) - (2*b*d^2*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(3*c) - (8*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(
25*c^3) - (16*b*e^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(245*c^5) - (6*b*d*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(25*c) - (12*b*e^3*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(245*c^3) - (2*b*e^3*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(49*c) + d^3*x*(a + b*ArcCosh[c*x])^2 + d^2*e*x^3*(a + b*ArcCosh[c*x])^2
```

```
 + (3*d*e^2*x^5*(a + b*ArcCosh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcCosh[c*x])^2
)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
```

```
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \int \left(d^3\left(a+b\cosh^{-1}(cx)\right)^2 + 3d^2ex^2\left(a+b\cosh^{-1}(cx)\right)^2 + 3de^2x^4\left(a -\right.\right.$$

$$= d^3 \int \left(a+b\cosh^{-1}(cx)\right)^2 \, dx + \left(3d^2e\right) \int x^2\left(a+b\cosh^{-1}(cx)\right)^2 \, dx +$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)^2 + d^2ex^3\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5\left(a+b\,c\right.$$

$$= -\frac{2bd^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{-1+cx}}{}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{-1+cx}\,\sqrt{1}}{}$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 + \frac{12b^2e^3x^5}{1225c^2}$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} +$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \cdot$$

Mathematica [A]
time = 0.48, size = 453, normalized size = 0.74

$$\frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6) - 210ab\sqrt{-1+cx}\,\sqrt{1+cx}\,(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6)) + 2b^2cx(25200e^3+840c^2e^2(147d+5ex^2)+210c^4e(1225d^2+98dex^2+9e^2x^4)+c^6(385875d^3+42875d^2ex^2+9261de^2x^4+1125e^3x^6)) - 210b\left(-105ac^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6))\right)\cosh^{-1}(cx) + 11025b^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)\cosh^{-1}(cx)^2}{385875c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*
c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^
```

```
2 + 441*d*e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(25200*e^3 + 840*c^2*e^2*(147*
d + 5*e*x^2) + 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*
d^3 + 42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7
*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 29
4*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
75*e^3*x^6)))*ArcCosh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*
e^2*x^4 + 5*e^3*x^6)*ArcCosh[c*x]^2)/(385875*c^7)
```

Maple [A]
time = 1.57, size = 683, normalized size = 1.12

$$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(\frac{3c^2d\,e^2\left(25\operatorname{arccosh}(cx)^2\cosh(5\operatorname{arccosh}(cx))-10\operatorname{arccosh}(cx)\sinh(5\operatorname{arccosh}(cx))+2\cosh(5\operatorname{arccosh}(cx))\right)}{2000}+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+1/
c^6*b^2*(3/2000*c^2*d*e^2*(25*arccosh(c*x)^2*cosh(5*arccosh(c*x))-10*arccos
h(c*x)*sinh(5*arccosh(c*x))+2*cosh(5*arccosh(c*x)))+1/1600*e^3*(25*arccosh(
c*x)^2*cosh(5*arccosh(c*x))-10*arccosh(c*x)*sinh(5*arccosh(c*x))+2*cosh(5*a
rccosh(c*x)))+1/36*c^4*d^2*e*(9*arccosh(c*x)^2*cosh(3*arccosh(c*x))-6*arcco
sh(c*x)*sinh(3*arccosh(c*x))+2*cosh(3*arccosh(c*x)))+1/48*c^2*d*e^2*(9*arcc
osh(c*x)^2*cosh(3*arccosh(c*x))-6*arccosh(c*x)*sinh(3*arccosh(c*x))+2*cosh(
3*arccosh(c*x)))+1/192*e^3*(9*arccosh(c*x)^2*cosh(3*arccosh(c*x))-6*arccosh
(c*x)*sinh(3*arccosh(c*x))+2*cosh(3*arccosh(c*x)))+d^3*c^6*(arccosh(c*x)^2*
x*c-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x)+3/4*c^4*d^2*e*(arccos
h(c*x)^2*x*c-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x)+3/8*c^2*d*e^
2*(arccosh(c*x)^2*x*c-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x)+5/6
4*e^3*(arccosh(c*x)^2*x*c-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x)
+1/21952*e^3*(49*arccosh(c*x)^2*cosh(7*arccosh(c*x))-14*arccosh(c*x)*sinh(7
*arccosh(c*x))+2*cosh(7*arccosh(c*x))))+2*a*b/c^6*(arccosh(c*x)*d^3*c^7*x+a
rccosh(c*x)*d^2*c^7*e*x^3+3/5*arccosh(c*x)*d*c^7*e^2*x^5+1/7*arccosh(c*x)*e
^3*c^7*x^7-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e^3*x^6+441*c^6*d*e^2
*x^4+1225*c^6*d^2*e*x^2+90*c^4*e^3*x^4+3675*c^6*d^3+588*c^4*d*e^2*x^2+2450*
c^4*d^2*e+120*c^2*e^3*x^2+1176*c^2*d*e^2+240*e^3)))
```

Maxima [A]
time = 0.30, size = 680, normalized size = 1.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7*arccosh(c*x)^2*e^3 + 3/5*b^2*d*x^5*arccosh(c*x)^2*e^2 + 1/7*a^2
*x^7*e^3 + b^2*d^2*x^3*arccosh(c*x)^2*e + 3/5*a^2*d*x^5*e^2 + b^2*d^3*x*arc
cosh(c*x)^2 + a^2*d^2*x^3*e + 2*b^2*d^3*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)
/c) + a^2*d^3*x + 2/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) - (c^2*x^3 + 6*x)/c^2)*b^2*d^2*e + 2
*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^3/c + 2/25*(15*x^5*arccosh(c*
x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 - 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*s
qrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c*arccosh(c*x) - (9*c^4
*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2 + 2/245*(35*x^7*arccosh(c*x) - (5
*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 -
 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(c
^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2
/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c*arccosh(c*x) - (75*c^6*x^7 + 126*c^4*x^5
 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(518) = 1036$.
time = 0.43, size = 1528, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(385875*(a^2 + 2*b^2)*c^7*d^3*x + 15*(75*(49*a^2 + 2*b^2)*c^7*x^7
+ 252*b^2*c^5*x^5 + 560*b^2*c^3*x^3 + 3360*b^2*c*x)*cosh(1)^3 + 15*(75*(49*
a^2 + 2*b^2)*c^7*x^7 + 252*b^2*c^5*x^5 + 560*b^2*c^3*x^3 + 3360*b^2*c*x)*si
nh(1)^3 + 1029*(9*(25*a^2 + 2*b^2)*c^7*d*x^5 + 40*b^2*c^5*d*x^3 + 240*b^2*c
^3*d*x)*cosh(1)^2 + 11025*(5*b^2*c^7*x^7*cosh(1)^3 + 5*b^2*c^7*x^7*sinh(1)^
3 + 21*b^2*c^7*d*x^5*cosh(1)^2 + 35*b^2*c^7*d^2*x^3*cosh(1) + 35*b^2*c^7*d^
3*x + 3*(5*b^2*c^7*x^7*cosh(1) + 7*b^2*c^7*d*x^5)*sinh(1)^2 + (15*b^2*c^7*x
^7*cosh(1)^2 + 42*b^2*c^7*d*x^5*cosh(1) + 35*b^2*c^7*d^2*x^3)*sinh(1))*log(
c*x + sqrt(c^2*x^2 - 1))^2 + 3*(3087*(25*a^2 + 2*b^2)*c^7*d*x^5 + 13720*b^2
*c^5*d*x^3 + 82320*b^2*c^3*d*x + 15*(75*(49*a^2 + 2*b^2)*c^7*x^7 + 252*b^2*
c^5*x^5 + 560*b^2*c^3*x^3 + 3360*b^2*c*x)*cosh(1))*sinh(1)^2 + 42875*((9*a^
2 + 2*b^2)*c^7*d^2*x^3 + 12*b^2*c^5*d^2*x)*cosh(1) + 210*(525*a*b*c^7*x^7*c
osh(1)^3 + 525*a*b*c^7*x^7*sinh(1)^3 + 2205*a*b*c^7*d*x^5*cosh(1)^2 + 3675*
a*b*c^7*d^2*x^3*cosh(1) + 3675*a*b*c^7*d^3*x + 315*(5*a*b*c^7*x^7*cosh(1) +
 7*a*b*c^7*d*x^5)*sinh(1)^2 + 105*(15*a*b*c^7*x^7*cosh(1)^2 + 42*a*b*c^7*d*
x^5*cosh(1) + 35*a*b*c^7*d^2*x^3)*sinh(1) - (3675*b^2*c^6*d^3 + 15*(5*b^2*c
^6*x^6 + 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 + 16*b^2)*cosh(1)^3 + 15*(5*b^2*c^6*
x^6 + 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 + 16*b^2)*sinh(1)^3 + 147*(3*b^2*c^6*d*
```

```
x^4 + 4*b^2*c^4*d*x^2 + 8*b^2*c^2*d)*cosh(1)^2 + 3*(147*b^2*c^6*d*x^4 + 196
*b^2*c^4*d*x^2 + 392*b^2*c^2*d + 15*(5*b^2*c^6*x^6 + 6*b^2*c^4*x^4 + 8*b^2*
c^2*x^2 + 16*b^2)*cosh(1))*sinh(1)^2 + 1225*(b^2*c^6*d^2*x^2 + 2*b^2*c^4*d^
2)*cosh(1) + (1225*b^2*c^6*d^2*x^2 + 2450*b^2*c^4*d^2 + 45*(5*b^2*c^6*x^6 +
 6*b^2*c^4*x^4 + 8*b^2*c^2*x^2 + 16*b^2)*cosh(1)^2 + 294*(3*b^2*c^6*d*x^4 +
 4*b^2*c^4*d*x^2 + 8*b^2*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))*log(c*
x + sqrt(c^2*x^2 - 1)) + (42875*(9*a^2 + 2*b^2)*c^7*d^2*x^3 + 514500*b^2*c^
5*d^2*x + 45*(75*(49*a^2 + 2*b^2)*c^7*x^7 + 252*b^2*c^5*x^5 + 560*b^2*c^3*x
^3 + 3360*b^2*c*x)*cosh(1)^2 + 2058*(9*(25*a^2 + 2*b^2)*c^7*d*x^5 + 40*b^2*
c^5*d*x^3 + 240*b^2*c^3*d*x)*cosh(1))*sinh(1) - 210*(3675*a*b*c^6*d^3 + 15*
(5*a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 + 16*a*b)*cosh(1)^3 + 15*(5*
a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 + 16*a*b)*sinh(1)^3 + 147*(3*a*
b*c^6*d*x^4 + 4*a*b*c^4*d*x^2 + 8*a*b*c^2*d)*cosh(1)^2 + 3*(147*a*b*c^6*d*x
^4 + 196*a*b*c^4*d*x^2 + 392*a*b*c^2*d + 15*(5*a*b*c^6*x^6 + 6*a*b*c^4*x^4
+ 8*a*b*c^2*x^2 + 16*a*b)*cosh(1))*sinh(1)^2 + 1225*(a*b*c^6*d^2*x^2 + 2*a*
b*c^4*d^2)*cosh(1) + (1225*a*b*c^6*d^2*x^2 + 2450*a*b*c^4*d^2 + 45*(5*a*b*c
^6*x^6 + 6*a*b*c^4*x^4 + 8*a*b*c^2*x^2 + 16*a*b)*cosh(1)^2 + 294*(3*a*b*c^6
*d*x^4 + 4*a*b*c^4*d*x^2 + 8*a*b*c^2*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1)
)/c^7
```

Sympy [C] Result contains complex when optimal does not.
time = 1.18, size = 996, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*acosh(c*x) + 2*a*b*d**2*e*x**3*acosh(c*x) + 6*a*b*d
*e**2*x**5*acosh(c*x)/5 + 2*a*b*e**3*x**7*acosh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 - 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*
c) - 4*a*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c**3) - 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(25*c**3) - 12*a*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 - 1)/(245*c**5) - 32*a*b*e**3*sqrt(c**2*x**2 - 1)/(245*c**7) + b**2*d**3*
x*acosh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*acosh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*acosh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*acosh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 - 1)*acosh(c*x)/(49*c) + 4*b**2*d**2*e*x/(3*c*
*2) + 8*b**2*d*e**2*x**3/(75*c**2) + 12*b**2*e**3*x**5/(1225*c**2) - 4*b**2
*d**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 8*b**2*d*e**2*x**2*sqrt(c
```

```
**2*x**2 - 1)*acosh(c*x)/(25*c**3) - 12*b**2*e**3*x**4*sqrt(c**2*x**2 - 1)*
acosh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**5) + 32*b**2*e**3*x/(245*c
**6) - 32*b**2*e**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**7), Ne(c, 0)), (
(a + I*pi*b/2)**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), T
rue))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^3, x)
```

3.526 $\int (d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=359

$$2b^2d^2x+\frac{8b^2dex}{9c^2}+\frac{16b^2e^2x}{75c^4}+\frac{4}{27}b^2dex^3+\frac{8b^2e^2x^3}{225c^2}+\frac{2}{125}b^2e^2x^5-\frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-$$

```
[Out] 2*b^2*d^2*x+8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3+8/225*b^
2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arccosh(c*x))^2+2/3*d*e*x^3*(a+b
*arccosh(c*x))^2+1/5*e^2*x^5*(a+b*arccosh(c*x))^2-2*b*d^2*(a+b*arccosh(c*x)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/9*b*d*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^3-16/75*b*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c^5-4/9*b*d*e*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/75*b
*e^2*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/25*b*e^2*x^4*
(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]

time = 0.84, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5909, 5879, 5915, 8, 5883, 5939, 30}

$$-\frac{16be^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{75c^5}-\frac{8bde\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{8be^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{75c^3}+d^2x\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)^2-\frac{4bdex^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c}+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2be^2x^4\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{25c}+\frac{16b^2e^2x}{75c^4}+\frac{8b^2dex}{9c^2}+\frac{8b^2e^2x^3}{225c^2}+2b^2d^2x+\frac{4}{27}b^2dex^3+\frac{2}{125}b^2e^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x + (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 + (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (8*b*d*e*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(75*c^5) - (4*b*d*e*x^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (8*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(25*c) + d^2*x*(a + b*ArcCosh[c*x])^2 + (2*d*e*x
^3*(a + b*ArcCosh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCosh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \int \left(d^2\left(a+b\cosh^{-1}(cx)\right)^2 + 2dex^2\left(a+b\cosh^{-1}(cx)\right)^2 + e^2x^4\left(a+b\right.\right.$$

$$= d^2 \int \left(a+b\cosh^{-1}(cx)\right)^2 \, dx + (2de) \int x^2\left(a+b\cosh^{-1}(cx)\right)^2 \, dx +$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5\left(a+b\right.$$

$$= -\frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{-1+cx}}{}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\right.}{c}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1-}}{}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 -$$

Mathematica [A]
time = 0.32, size = 299, normalized size = 0.83

$$\frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4) - 30ab\sqrt{-1+cx}\,\sqrt{1+cx}\,(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4)) + 2b^2cx(360e^2+60c^2e(25d+ex^2)+c^4(3375d^2+250dex^2+27e^2x^4)) - 30b\left(-15ac^5x(15d^2+10dex^2+3e^2x^4)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))\right)\cosh^{-1}(cx) + 225b^2c^5x(15d^2+10dex^2+3e^2x^4)\cosh^{-1}(cx)^2}{3375c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2
+ 9*e^2*x^4)) + 2*b^2*c*x*(360*e^2 + 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^
2 + 250*d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3
*e^2*x^4) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^
2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcCosh[c*x] + 225*b^2*c^5*x*
(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x]^2)/(3375*c^5)
```

Maple [A]
time = 1.55, size = 367, normalized size = 1.02

$$\frac{\frac{a^2\left(\frac{1}{5}e^2x^5c^5+\frac{2}{3}x^3c^5de+x\,c^5d^2\right)}{c^4} + \frac{b^2\left(\frac{\left(25\operatorname{arccosh}(cx)^2+2\right)e^2x^5c^5}{125} - \frac{2x^4c^4\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)e^2}{25} + \frac{2\left(225\operatorname{arccosh}(cx)^2c^2d+50c^2d+12e\right)}{675}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(1/5*e^2*x^5*c^5+2/3*x^3*c^5*d*e+x*c^5*d^2)+1/c^4*b^2*(1/125*(
25*arccosh(c*x)^2+2)*e^2*x^5*c^5-2/25*x^4*c^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)*a
rccosh(c*x)*e^2+2/675*(225*arccosh(c*x)^2*c^2*d+50*c^2*d+12*e)*e*x^3*c^3-4/
225*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*(25*c^2*d+6*e)*x^2*c^2+1/225
*(225*arccosh(c*x)^2*c^4*d^2+450*c^4*d^2+200*c^2*d*e+48*e^2)*x*c-2/225*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*(225*c^4*d^2+100*c^2*d*e+24*e^2))+2*a*
b/c^4*(1/5*arccosh(c*x)*e^2*x^5*c^5+2/3*arccosh(c*x)*x^3*c^5*d*e+arccosh(c*
x)*x*c^5*d^2-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^
2+225*c^4*d^2+12*c^2*e^2*x^2+100*c^2*d*e+24*e^2)))
```

Maxima [A]
time = 0.28, size = 429, normalized size = 1.19

$$\frac{1}{5}b^2x^5\operatorname{arcosh}(cx)^2e^2+\frac{2}{3}b^2dx^3\operatorname{arcosh}(cx)^2e+\frac{1}{5}a^2x^5e^2+b^2d^2x\operatorname{arcosh}(cx)^2+\frac{2}{3}a^2dx^3e+2b^2d^2\left(x-\frac{\sqrt{c^2x^2-1}\operatorname{arcosh}(cx)}{c}\right)+a^2d^2x+\frac{4}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)abde-\frac{4}{27}\left(3c\left(\frac{\sqrt{c^2x^2-1}x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\operatorname{arcosh}(cx)-\frac{c^2x^3+6x}{c^2}\right)b^2de+\frac{2\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)abd^2}{c}+\frac{2}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)abe^2-\frac{2}{1125}\left(15\left(\frac{3\sqrt{c^2x^2-1}x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\operatorname{arcosh}(cx)-\frac{9c^4x^5+20c^2x^3+120x}{c^4}\right)b^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5*arccosh(c*x)^2*e^2 + 2/3*b^2*d*x^3*arccosh(c*x)^2*e + 1/5*a^2*x
^5*e^2 + b^2*d^2*x*arccosh(c*x)^2 + 2/3*a^2*d*x^3*e + 2*b^2*d^2*(x - sqrt(c
^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^2*x + 4/9*(3*x^3*arccosh(c*x) - c*(sqrt
(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d*e - 4/27*(3*c*(sqrt
(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) - (c^2*x^3 +
6*x)/c^2)*b^2*d*e + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^2/c + 2/
75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1
)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^
2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c*a
rccosh(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(309) = 618.
time = 0.38, size = 767, normalized size = 2.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(3375*(a^2 + 2*b^2)*c^5*d^2*x + 3*(9*(25*a^2 + 2*b^2)*c^5*x^5 + 40*b
^2*c^3*x^3 + 240*b^2*c*x)*cosh(1)^2 + 225*(3*b^2*c^5*x^5*cosh(1)^2 + 3*b^2*
c^5*x^5*sinh(1)^2 + 10*b^2*c^5*d*x^3*cosh(1) + 15*b^2*c^5*d^2*x + 2*(3*b^2*
c^5*x^5*cosh(1) + 5*b^2*c^5*d*x^3)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1))^2
+ 3*(9*(25*a^2 + 2*b^2)*c^5*x^5 + 40*b^2*c^3*x^3 + 240*b^2*c*x)*sinh(1)^2 +
 250*((9*a^2 + 2*b^2)*c^5*d*x^3 + 12*b^2*c^3*d*x)*cosh(1) + 30*(45*a*b*c^5*
```

```
x^5*cosh(1)^2 + 45*a*b*c^5*x^5*sinh(1)^2 + 150*a*b*c^5*d*x^3*cosh(1) + 225*
a*b*c^5*d^2*x + 30*(3*a*b*c^5*x^5*cosh(1) + 5*a*b*c^5*d*x^3)*sinh(1) - (225
*b^2*c^4*d^2 + 3*(3*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*cosh(1)^2 + 3*(3*b
^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sinh(1)^2 + 50*(b^2*c^4*d*x^2 + 2*b^2*c
^2*d)*cosh(1) + 2*(25*b^2*c^4*d*x^2 + 50*b^2*c^2*d + 3*(3*b^2*c^4*x^4 + 4*b
^2*c^2*x^2 + 8*b^2)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2
*x^2 - 1)) + 2*(125*(9*a^2 + 2*b^2)*c^5*d*x^3 + 1500*b^2*c^3*d*x + 3*(9*(25
*a^2 + 2*b^2)*c^5*x^5 + 40*b^2*c^3*x^3 + 240*b^2*c*x)*cosh(1))*sinh(1) - 30
*(225*a*b*c^4*d^2 + 3*(3*a*b*c^4*x^4 + 4*a*b*c^2*x^2 + 8*a*b)*cosh(1)^2 + 3
*(3*a*b*c^4*x^4 + 4*a*b*c^2*x^2 + 8*a*b)*sinh(1)^2 + 50*(a*b*c^4*d*x^2 + 2*
a*b*c^2*d)*cosh(1) + 2*(25*a*b*c^4*d*x^2 + 50*a*b*c^2*d + 3*(3*a*b*c^4*x^4
+ 4*a*b*c^2*x^2 + 8*a*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.58, size = 602, normalized size = 1.68

$$\begin{cases} a^2d^2x + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^5}{5} + 2abd^2x\operatorname{acosh}(cx) + \frac{4abdex^3\operatorname{acosh}(cx)}{3} + \frac{2abe^2x^5\operatorname{acosh}(cx)}{5} - \frac{2abd^2\sqrt{c^2x^2-1}}{c} - \frac{4abdex^2\sqrt{c^2x^2-1}}{9c} - \frac{2abe^2x^4\sqrt{c^2x^2-1}}{25c} - \frac{8abde\sqrt{c^2x^2-1}}{9c^3} - \frac{8abe^2x^2\sqrt{c^2x^2-1}}{75c^3} - \frac{16abe^2\sqrt{c^2x^2-1}}{75c^5} + b^2d^2x\operatorname{acosh}^2(cx) + 2b^2d^2x + \frac{2b^2dex^3\operatorname{acosh}^2(cx)}{3} + \frac{4b^2dex^3}{27} + \frac{b^2e^2x^5\operatorname{acosh}^2(cx)}{5} + \frac{2b^2e^2x^5}{125} - \frac{2b^2d^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{4b^2dex^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c} - \frac{2b^2e^2x^4\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{25c} + \frac{8b^2dex}{9c^2} + \frac{8b^2e^2x^3}{225c^2} - \frac{8b^2de\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c^3} - \frac{8b^2e^2x^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{75c^3} + \frac{16b^2e^2x}{75c^4} - \frac{16b^2e^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{75c^5} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)^2\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*acosh(c*x) + 4*a*b*d*e*x**3*acosh(c*x)/3 + 2*a*b*e**2*x**5*acosh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 - 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 8*a*b*d*e*sqrt(c**2*x**2 -
 1)/(9*c**3) - 8*a*b*e**2*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 - 1)/(75*c**5) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*acosh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*acos
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) + 8*b**2*d*e*x/(9*c**2) + 8*b**
2*e**2*x**3/(225*c**2) - 8*b**2*d*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3)
 - 8*b**2*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^2,x)
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^2, x)
```

3.527 $\int (d+ex^2) \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=168

$$2b^2dx+\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4be\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}\right.}{9c^3}$$

```
[Out] 2*b^2*d*x+4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arccosh(c*x))^2+1/3*e*x^3
*(a+b*arccosh(c*x))^2-2*b*d*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c-4/9*b*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/9*b*e*x^2*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]

time = 0.40, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5909, 5879, 5915, 8, 5883, 5939, 30}

$$-\frac{4be\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bex^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c}+\frac{4b^2ex}{9c^2}+2b^2dx+\frac{2}{27}b^2ex^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x]))/(9*c^3) - (2*b*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(9*c) + d*x*(a + b*ArcCosh[c*x])^2 + (e*x^3*(a + b*ArcC
osh[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right)^2 \, dx &= \int \left(d\left(a+b\cosh^{-1}(cx)\right)^2+ex^2\left(a+b\cosh^{-1}(cx)\right)^2\right) \, dx \\
&= d\int \left(a+b\cosh^{-1}(cx)\right)^2 \, dx+e\int x^2\left(a+b\cosh^{-1}(cx)\right)^2 \, dx \\
&= dx\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^2-(2bcd)\int \frac{x(a+}{\sqrt{-1}} \\
&= -\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{-1+cx}\,\sqrt{1}}{} \\
&= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4b}{} \\
&= 2b^2dx+\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{c}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 174, normalized size = 1.04

$$
\frac{9a^2c^3x(3d+ex^2)-6ab\sqrt{-1+cx}\,\sqrt{1+cx}\,(2e+c^2(9d+ex^2))+2b^2cx(6e+c^2(27d+ex^2))-6b\left(-3ac^3x(3d+ex^2)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(2e+c^2(9d+ex^2))\right)\cosh^{-1}(cx)+9b^2c^3x(3d+ex^2)\cosh^{-1}(cx)^2}{27c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*
(9*d + e*x^2)) + 2*b^2*c*x*(6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*
d + e*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*x^2)))*ArcC
osh[c*x] + 9*b^2*c^3*x*(3*d + e*x^2)*ArcCosh[c*x]^2)/(27*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \left(e\,x^2+d\right)\left(a+b\,\mathrm{arccosh}(cx)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.28, size = 222, normalized size = 1.32

$$
\frac{1}{3}\,b^2x^3\,\mathrm{arcosh}\left(cx\right)^2e+b^2dx\,\mathrm{arcosh}\left(cx\right)^2+\frac{1}{3}\,a^2x^3e+2\,b^2d\left(x-\frac{\sqrt{c^2x^2-1}\,\mathrm{arcosh}\left(cx\right)}{c}\right)+a^2dx+\frac{2}{9}\left(3\,x^3\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)abe-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\mathrm{arcosh}\left(cx\right)-\frac{c^2x^3+6\,x}{c^2}\right)b^2e+\frac{2\left(cx\,\mathrm{arcosh}\left(cx\right)-\sqrt{c^2x^2-1}\right)abd}{c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arccosh(c*x)^2*e + b^2*d*x*arccosh(c*x)^2 + 1/3*a^2*x^3*e + 2*b
^2*d*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d*x + 2/9*(3*x^3*arccosh(
c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*e - 2/2
7*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) -
 (c^2*x^3 + 6*x)/c^2)*b^2*e + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*
d/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(151) = 302$.
time = 0.37, size = 304, normalized size = 1.81

$$\frac{27\,(a^2+2\,b^2)c^3dx+9\,(b^2c^3x^3\cosh(1)+b^2c^3x^3\sinh(1)+3\,b^2c^3dx)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+((9\,a^2+2\,b^2)c^3x^3+12\,b^2cx)\cosh(1)+6\left(3\,abc^3x^3\cosh(1)+3\,abc^3x^3\sinh(1)+9\,abc^3dx-(9\,b^2c^2d+(b^2c^2x^2+2\,b^2)\cosh(1)+(b^2c^2x^2+2\,b^2)\sinh(1))\sqrt{c^2x^2-1}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+((9\,a^2+2\,b^2)c^3x^3+12\,b^2cx)\sinh(1)-6\,(9\,abc^2d+(abc^2x^2+2\,ab)\cosh(1)+(abc^2x^2+2\,ab)\sinh(1))\sqrt{c^2x^2-1}}{27\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(27*(a^2 + 2*b^2)*c^3*d*x + 9*(b^2*c^3*x^3*cosh(1) + b^2*c^3*x^3*sinh(
1) + 3*b^2*c^3*d*x)*log(c*x + sqrt(c^2*x^2 - 1))^2 + ((9*a^2 + 2*b^2)*c^3*x
^3 + 12*b^2*c*x)*cosh(1) + 6*(3*a*b*c^3*x^3*cosh(1) + 3*a*b*c^3*x^3*sinh(1)
 + 9*a*b*c^3*d*x - (9*b^2*c^2*d + (b^2*c^2*x^2 + 2*b^2)*cosh(1) + (b^2*c^2*
x^2 + 2*b^2)*sinh(1))*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) + ((9
*a^2 + 2*b^2)*c^3*x^3 + 12*b^2*c*x)*sinh(1) - 6*(9*a*b*c^2*d + (a*b*c^2*x^2
 + 2*a*b)*cosh(1) + (a*b*c^2*x^2 + 2*a*b)*sinh(1))*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.27, size = 286, normalized size = 1.70

$$\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{acosh}(cx)+\frac{2abex^3\operatorname{acosh}(cx)}{3}-\frac{2abd\sqrt{c^2x^2-1}}{c}-\frac{2abex^2\sqrt{c^2x^2-1}}{9c}-\frac{4abe\sqrt{c^2x^2-1}}{9c^3}+b^2dx\operatorname{acosh}^2(cx)+2b^2dx+\frac{b^2ex^3\operatorname{acosh}^2(cx)}{3}+\frac{2b^2ex^3}{27}-\frac{2b^2d\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c}-\frac{2b^2ex^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c}+\frac{4b^2ex}{9c^2}-\frac{4b^2e\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c^3} & \text{for}\ c\neq 0 \\ \left(a+\frac{i\pi b}{2}\right)^2\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*acosh(c*x) + 2*a*b*e*x**3*a
cosh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 -
 1)/(9*c) - 4*a*b*e*sqrt(c**2*x**2 - 1)/(9*c**3) + b**2*d*x*acosh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*acosh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x
)/(9*c) + 4*b**2*e*x/(9*c**2) - 4*b**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*
c**3), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{acosh}(c\,x))^2\,\left(e\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2), x)
```

3.528 $\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=51

$$2b^2x-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2$$

```
[Out] 2*b^2*x+x*(a+b*arccosh(c*x))^2-2*b*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c
```

Rubi [A]
time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5879, 5915, 8}

$$-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c + x*(a
+ b*ArcCosh[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx = x\left(a+b\cosh^{-1}(cx)\right)^2 - (2bc)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= -\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} + x\left(a+b\cosh^{-1}(cx)\right)^2 + \left(2b^2\right)\int$$
$$= 2b^2x - \frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} + x\left(a+b\cosh^{-1}(cx)\right)^2$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 1.65

$$\left(a^2+2b^2\right)x - \frac{2ab\sqrt{-1+cx}\,\sqrt{1+cx}}{c} + \frac{2b\left(acx - b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\cosh^{-1}(cx)}{c} + b^2x\cosh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + (2*b*(a*c*x - b*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])*ArcCosh[c*x])/c + b^2*x*ArcCosh[c*x]^2
```

Maple [A]
time = 1.73, size = 78, normalized size = 1.53

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arccosh}(cx)^2xc-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}+2cx\right)+2ab\left(cx\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx-\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arccosh}(cx)^2xc-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}+2cx\right)+2ab\left(cx\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx-\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arccosh(c*x)^2*x*c-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)+2*c*x)+2*a*b*(c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 72, normalized size = 1.41

$$b^2x\,\mathrm{arcosh}\,(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2-1}\,\mathrm{arcosh}\,(cx)}{c}\right) + a^2x + \frac{2\left(cx\,\mathrm{arcosh}\,(cx) - \sqrt{c^2x^2-1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*x
 + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.
time = 0.36, size = 96, normalized size = 1.88

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2-1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2-1}\,ab+2\left(abcx-\sqrt{c^2x^2-1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 - 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 - 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 - 1)*b^2)*log(c*x + sqrt(c^2*x^2 - 1
)))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.10, size = 88, normalized size = 1.73

$$\begin{cases} a^2x+2abx\operatorname{acosh}(cx)-\frac{2ab\sqrt{c^2x^2-1}}{c}+b^2x\operatorname{acosh}^2(cx)+2b^2x-\frac{2b^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} & \text{for } c\neq 0\\ x\left(a+\frac{i\pi b}{2}\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*acosh(c*x) - 2*a*b*sqrt(c**2*x**2 - 1)/c + b**2
*x*acosh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c, Ne(c
, 0)), (x*(a + I*pi*b/2)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.
time = 0.47, size = 111, normalized size = 2.18

$$2\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2-1}\,\log\left(cx+\sqrt{c^2x^2-1}\right)}{c^2}\right)\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2
 - 1))/c^2))*b^2 + a^2*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b \operatorname{acosh}(c x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2,x)
[Out] int((a + b*acosh(c*x))^2, x)
```

3.529 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=763

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\cdots$$

[Out] 1/2*(a+b*arccosh(c*x))^2*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccosh(c*x))^2*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arccosh(c*x))*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arccosh(c*x))*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)

Rubi [A]
time = 0.99, antiderivative size = 763, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5909, 5962, 5681, 2221, 2611, 2320, 6724}

$$-\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b^2\text{Li}_3\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b^2\text{Li}_3\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}+\frac{b^2\text{Li}_3\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}-\frac{b^2\text{Li}_3\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]

[Out] ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log

```
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcCos
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(\ldots}\right.}{2\sqrt{-\ldots}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{\ldots}{c\sqrt{-d}\ldots}\right.}{\ldots}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\ldots}{2\sqrt{\ldots}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\ldots}{2\sqrt{\ldots}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\ldots}{2\sqrt{\ldots}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\ldots}{2\sqrt{\ldots}}$$

Mathematica [A]
time = 0.40, size = 623, normalized size = 0.82

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])^2*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh
```

```
[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^A
rcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x
])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])] - 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(-
(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] - 2*b^2*PolyLog[3, (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + integrate(b^2*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))^2/(x^2*e + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sq
rt(c*x - 1))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d),x)
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{e\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2), x)
```

3.530 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Mathematica [A]
time = 9.15, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}(cx)\right)^2\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + sqrt(x^2*e + d)*x)*a^2 + integ
rate(sqrt(x^2*e + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(
x^2*e + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(x^2*e + d), x
)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{acosh}(cx))^2\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**2*sqrt(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\mathrm{acosh}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2), x)
```

3.531 $\int \frac{(a+b\cosh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^2}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + integrate(b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/sqrt(x^2*e + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1))/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/sqrt(x^2*e + d), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(1/2), x)
```

3.532 $\int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}} dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}} dx = \int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}} dx$$

Mathematica [A]
time = 12.75, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{(e\,x^2+d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(x^2*e + d)/(x
^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{(e\,x^2+d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(3/2), x)
```

3.533 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}},x\right)$$

[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 60.60, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]

[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^2}{\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(x^2*e + d)^(5/2) + 2*a*b*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(x^2*e + d)/(x
^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^2}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{(e\,x^2+d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(5/2), x)
```

3.534 $\int \frac{(d+ex^2)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$-\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}-\frac{de\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2bc^3}-\frac{e^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bc^5}-\frac{de\text{Chi}\left(\frac{3(a+\dots$$

```
[Out] d^2*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c+1/2*d*e*cosh(a/b)*Shi((a+b*arcc
osh(c*x))/b)/b/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^5+1/2*d*
e*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b/c^3+3/16*e^2*cosh(3*a/b)*Shi(3*
(a+b*arccosh(c*x))/b)/b/c^5+1/16*e^2*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b
)/b/c^5-d^2*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b/c-1/2*d*e*Chi((a+b*arccos
h(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b/c^
5-1/2*d*e*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b
*arccosh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Chi(5*(a+b*arccosh(c*x))/b)*si
nh(5*a/b)/b/c^5
```

Rubi [A]
time = 0.52, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5909, 5881, 3384, 3379, 3382, 5887, 5556}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^5}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{2bc^3}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^2*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) - (d*e*CoshIn
tegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(2*b*c^3) - (e^2*CoshIntegral[(a
+ b*ArcCosh[c*x])/b]*Sinh[a/b])/(8*b*c^5) - (d*e*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b]*Sinh[(3*a)/b])/(2*b*c^3) - (3*e^2*CoshIntegral[(3*(a + b*Arc
Cosh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^5) - (e^2*CoshIntegral[(5*(a + b*ArcC
osh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^5) + (d^2*Cosh[a/b]*SinhIntegral[(a +
b*ArcCosh[c*x])/b])/(b*c) + (d*e*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x]
)/b])/(2*b*c^3) + (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b
*c^5) + (d*e*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(2*b*c
^3) + (3*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16*b*
c^5) + (e^2*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/(16*b*c
^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d^2}{a+b\cosh^{-1}(cx)} + \frac{2dex^2}{a+b\cosh^{-1}(cx)} + \frac{e^2x^4}{a+b\cosh^{-1}(cx)}\right)\,dx$$

$$= d^2\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx + (2de)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx + e^2\int \frac{x^4}{a+b\cosh^{-1}(cx)}\,d$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{(2de)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x\right.}{c^3}$$

$$= \frac{(2de)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)\,dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{e^2\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)} + \frac{3\sinh}{16(a}\right.\right.}{}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{(de)\text{Subst}\left(\int \frac{\sin}{a}\right.}{}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{S}}{}$$

$$= -\frac{de\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2bc^3} - \frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^5} - \frac{d^2\text{Chi}\left(\frac{a+b}{}\right.}{}$$

Mathematica [A]

time = 0.39, size = 254, normalized size = 0.65

$$\frac{-2(8c^4d^2+4c^2de+e^2)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - e(8c^2d+3e)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - e^2\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{5a}{b}\right) + 16c^4d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 8c^2de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 2e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 8c^2de\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + e^2\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{16bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b
] - e*(8*c^2*d + 3*e)*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] -
e^2*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 16*c^4*d^2*Cosh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*d*e*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*
d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^2*Cosh[(5*a)/b]*SinhIntegral[5*(
a/b + ArcCosh[c*x])])/(16*b*c^5)
```

Maple [A]

time = 9.42, size = 380, normalized size = 0.98

method	result
derivativedivides	$-\frac{e^2 e^{-\frac{5a}{b}}\,\text{expIntegral}\left(1,-5\,\text{arccosh}(cx)-\frac{5a}{b}\right)}{32c^4b} + \frac{e^2 e^{\frac{5a}{b}}\,\text{expIntegral}\left(1,5\,\text{arccosh}(cx)+\frac{5a}{b}\right)}{32c^4b} + \frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)d^2}{2b} + \frac{e^{\frac{a}{b}}}{}$

default	$\frac{-\frac{e^{2} \mathrm{e}^{-\frac{5a}{b}} \operatorname{expIntegral}\left(1,-5 \operatorname{arccosh}(cx)-\frac{5a}{b}\right)}{32c^{4}b}+\frac{e^{2} \mathrm{e}^{\frac{5a}{b}} \operatorname{expIntegral}\left(1,5 \operatorname{arccosh}(cx)+\frac{5a}{b}\right)}{32c^{4}b}+\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right) d^{2}}{2b}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)+1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d*e+1/16/c^4/b*exp(a/b)*Ei(
1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d^2-1/4/c^2
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arccosh
(c*x)-a/b)*e^2+1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arccosh(c*x)-3*a/b)*d-3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a
/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*d*x^2*e + d^2)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^{2}\right)^{2}}{a + b \operatorname{acosh}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x)),x)
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x)), x)
```

3.535 $\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$-\frac{(4c^2d+e)\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bc^3}-\frac{e\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bc^3}+\frac{(4c^2d+e)\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}}{b}\right.}{4bc^3}$$

```
[Out] 1/4*(4*c^2*d+e)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^3+1/4*e*cosh(3*a/b)
*Shi(3*(a+b*arccosh(c*x))/b)/b/c^3-1/4*(4*c^2*d+e)*Chi((a+b*arccosh(c*x))/b
)*sinh(a/b)/b/c^3-1/4*e*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b/c^3
```

Rubi [A]
time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5909, 5881, 3384, 3379, 3382, 5887, 5556}

$$-\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4bc^3}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) - (e*CoshIntegr
al[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(4*b*c^3) - (e*CoshIntegral[(3*(a + b
*ArcCosh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^3) + (d*Cosh[a/b]*SinhIntegral[(a
+ b*ArcCosh[c*x])/b])/(b*c) + (e*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x]
)/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b]
)/(4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d}{a+b\cosh^{-1}(cx)} + \frac{ex^2}{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx$$

$$= -\frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \cos\right.}{c^3}$$

$$= \frac{e\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\right.}{bc}$$

$$= -\frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{e\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\right.}{}$$

$$= -\frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Su}}{}$$

$$= -\frac{e\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e\text{Chi}\left(\frac{3a}{b}+3\,\text{c}\right.}{}$$

Mathematica [A]

time = 0.17, size = 125, normalized size = 0.90

$$\frac{-\left(\left(4c^2d+e\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)\right) - e\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) + 4c^2d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d + e)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - e*CoshIntegr
al[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d*Cosh[a/b]*SinhIntegral[a
/b + ArcCosh[c*x]] + e*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + e*Cosh[
(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]])/(4*b*c^3)
```

Maple [A]

time = 7.38, size = 178, normalized size = 1.28

method	result
derivativedivides	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\text{expIntegral}\left(1,-3\,\text{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2b}+\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}}{c}$
default	$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\text{expIntegral}\left(1,-3\,\text{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2b}+\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8*e/c^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8*e/c^2/b*exp(3
*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x
)-a/b)*d-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x^2)/(a + b*acosh(c*x)), x)
```

3.536 $\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c-Chi((a+b*arccosh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5881, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) + (Cosh[a/b]*Sinh
Integral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, \right.}{bc}$$
$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.85

$$-\frac{\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-1),x]
[Out] -((CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]])/(b*c))
```

Maple [A]
time = 3.66, size = 56, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c*(1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x)),x)
[Out] Integral(1/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{acosh}(c\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x)),x)
[Out] int(1/(a + b*acosh(c*x)), x)
```

3.537 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\,\mathrm{arccosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((x^2*e + d)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*x^2*e + a*d + (b*x^2*e + b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)), x)
```

3.538 $\int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)} dx = \int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)} dx$$

Mathematica [A]
time = 2.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 \left(a+b\cosh^{-1}(cx)\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*
d^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^2), x)
```

3.539 $\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/(b*arccosh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(b*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{a + b\operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{acosh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x)),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x)), x)
```

3.540 $\int \frac{1}{\sqrt{d+ex^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2*e + d)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a*x^2*e + a*d + (b*x^2*e + b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}{(cx)}) \sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(1/2)), x)
```

3.541 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*
b*d*x^2*e + b*d^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right) \left(d + ex^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(3/2)), x)
```

3.542 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 2.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3
 + (b*x^6*e^3 + 3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right) \left(d + ex^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(5/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(5/2)), x)
```

3.543 $\int \frac{(d+ex^2)^2}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=510

$$-\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}$$

```
[Out] d^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/2*d*e*Chi((a+b*arccosh(c*x)
)/b)*cosh(a/b)/b^2/c^3+1/8*e^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c^5+
3/2*d*e*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3+9/16*e^2*Chi(3*(a+b
*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^5+5/16*e^2*Chi(5*(a+b*arccosh(c*x))/b)*
cosh(5*a/b)/b^2/c^5-d^2*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-1/2*d*e*S
hi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Shi((a+b*arccosh(c*x))/b
)*sinh(a/b)/b^2/c^5-3/2*d*e*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3
-9/16*e^2*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Shi(5*(a
+b*arccosh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c
/(a+b*arccosh(c*x))-2*d*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(
c*x))-e^2*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.61, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5909, 5880, 5953, 3384, 3379, 3382, 5885}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^5}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{de\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c^3}-\frac{de\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c^3}+\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^4*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshI
ntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (d*e*Cosh[a/b]*CoshIntegral[(a +
 b*ArcCosh[c*x])/b])/(2*b^2*c^3) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCo
sh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcC
osh[c*x]))/b])/(2*b^2*c^3) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*Ar
cCosh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b
*ArcCosh[c*x]))/b])/(16*b^2*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c*x])/b])/(b^2*c) - (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(2*b^2*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(8*b^2*
c^5) - (3*d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(2*b^
2*c^3) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(16
*b^2*c^5) - (5*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c*x]))/b])/
(16*b^2*c^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
```

```
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2}{(a+b\cosh^{-1}(cx))^2}\,dx &= \int \left(\frac{d^2}{(a+b\cosh^{-1}(cx))^2}+\frac{2dex^2}{(a+b\cosh^{-1}(cx))^2}+\frac{e^2x^4}{(a+b\cosh^{-1}(cx))^2}\right)dx \\
&= d^2\int \frac{1}{(a+b\cosh^{-1}(cx))^2}\,dx+(2de)\int \frac{x^2}{(a+b\cosh^{-1}(cx))^2}\,dx+e^2\int \frac{}{(a+b\ } \\
&= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{2dex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{e^2x^4\sqrt{-1+cx}\,\sqrt{1}}{bc\,(a+b\cosh^{-1}(c} \\
&= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{2dex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{e^2x^4\sqrt{-1+cx}\,\sqrt{1}}{bc\,(a+b\cosh^{-1}(c} \\
&= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{2dex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{e^2x^4\sqrt{-1+cx}\,\sqrt{1}}{bc\,(a+b\cosh^{-1}(c} \\
&= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{2dex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\,(a+b\cosh^{-1}(cx))}-\frac{e^2x^4\sqrt{-1+cx}\,\sqrt{1}}{bc\,(a+b\cosh^{-1}(c}
\end{aligned}$$

Mathematica [A]
time = 1.40, size = 663, normalized size = 1.30

$$-\frac{16bc^4d^2\sqrt{\frac{-1+cx}{1+cx}}+16bc^5d^2x\sqrt{\frac{-1+cx}{1+cx}}+32bc^4dex^2\sqrt{\frac{-1+cx}{1+cx}}+32bc^5dex^3\sqrt{\frac{-1+cx}{1+cx}}+16bc^4e^2x^4\sqrt{\frac{-1+cx}{1+cx}}+16bc^5e^2x^5\sqrt{\frac{-1+cx}{1+cx}}-2(8c^4d^2+4c^2de+e^2)(a+b\cosh^{-1}(cx))\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3e(8c^2d+3e)(a+b\cosh^{-1}(cx))\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5ae^2\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5be^2\cosh^{-1}(cx)\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+16ac^4d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+8ac^2de\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+2ae^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh\right.}{16b^2c^5\,(a+b\cosh^{-1}(cx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^4*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c^5*d^2*x*Sqrt[(-1 +
c*x)/(1 + c*x)] + 32*b*c^4*d*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 32*b*c^5*d*
e*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c^4*e^2*x^4*Sqrt[(-1 + c*x)/(1 + c*
x)] + 16*b*c^5*e^2*x^5*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*(8*c^4*d^2 + 4*c^2*d*
e + e^2)*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] -
3*e*(8*c^2*d + 3*e)*(a + b*ArcCosh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b
+ ArcCosh[c*x])] - 5*a*e^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c*x]
)] - 5*b*e^2*ArcCosh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c*x])
] + 16*a*c^4*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*a*c^2*d*e*S
inh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 2*a*e^2*Sinh[a/b]*SinhIntegral[
```

```
a/b + ArcCosh[c*x]] + 16*b*c^4*d^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 8*b*c^2*d*e*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + Arc
Cosh[c*x]] + 2*b*e^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]
] + 24*a*c^2*d*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 9*a*e
^2*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 24*b*c^2*d*e*ArcCos
h[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 9*b*e^2*ArcCosh
[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*e^2*Sinh[(5*
a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*e^2*ArcCosh[c*x]*Sinh[(5*a
)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])])/(b^2*c^5*(a + b*ArcCosh[c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(478) = 956$.
time = 10.28, size = 1102, normalized size = 2.16

method	result	size
derivativedivides	Expression too large to display	1102
default	Expression too large to display	1102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/32*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+12*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c^5*x^5-20*c^3*x^3+5*c*x)*e^2
/c^4/b/(a+b*arccosh(c*x))-5/32*e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5
*a/b)-1/32/b*e^2/c^4*(16*c^5*x^5-20*c^3*x^3+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+5*c*x-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/(a+b*arccosh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)
-5*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2
*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+1/4*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+
c*x)*d*e/c^2/b/(a+b*arccosh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arccosh(c*x
)+a/b)+1/16*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arccosh(c*x))
-1/16/c^4*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c
*x)-a/b)-1/4/c^2/b*d*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
-1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/16/c^4/b^2*e^2*exp(-a/b)*Ei(1
,-arccosh(c*x)-a/b)+1/4*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/
2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*x)*d*e/c^2/b/(a+b*arccosh(c*x))+3/32*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*
x)*e^2/c^4/b/(a+b*arccosh(c*x))-3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x
)+3*a/b)*d-9/32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e
/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(a+b*arccosh(c*x))*d-3/32/c^4*e^2/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
```

```
-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*d-9/32/c^4*e^2/b^2*e
xp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^7*e^2 + (2*c^3*d*e - c*e^2)*x^5 - c*d^2*x + (c^3*d^2 - 2*c*d*e)*x^3
 + (c^2*x^6*e^2 + (2*c^2*d*e - e^2)*x^4 + (c^2*d^2 - 2*d*e)*x^2 - d^2)*sqrt
(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2
*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((5*c^5*x^8*e^2 + 2*(3*c
^5*d*e - 5*c^3*e^2)*x^6 + (c^5*d^2 - 12*c^3*d*e + 5*c*e^2)*x^4 + (5*c^3*x^6
*e^2 + 3*(2*c^3*d*e - c*e^2)*x^4 + c*d^2 + (c^3*d^2 - 2*c*d*e)*x^2)*(c*x +
1)*(c*x - 1) + c*d^2 - 2*(c^3*d^2 - 3*c*d*e)*x^2 + (10*c^4*x^7*e^2 + (12*c^
4*d*e - 13*c^2*e^2)*x^5 + 2*(c^4*d^2 - 7*c^2*d*e + 2*e^2)*x^3 - (c^2*d^2 -
4*d*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*
c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*d*x^2*e + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*
x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + e x^{2}\right)^{2}}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x))^2, x)
```

3.544 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=257

$$-\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}+\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}$$

```
[Out] d*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/4*e*Chi((a+b*arccosh(c*x))/b)
*cosh(a/b)/b^2/c^3+3/4*e*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3-d*
Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-1/4*e*Shi((a+b*arccosh(c*x))/b)*s
inh(a/b)/b^2/c^3-3/4*e*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))-e*x^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.37, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5909, 5880, 5953, 3384, 3379, 3382, 5885}

$$\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshIn
tegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e*Cosh[a/b]*CoshIntegral[(a + b*
ArcCosh[c*x])/b])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*A
rcCosh[c*x]))/b])/(4*b^2*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*
x])/b])/(b^2*c) - (e*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(4*b^2
*c^3) - (3*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b])/(4*b^2
*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx \\
&= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{(cd)\int \frac{}{\sqrt{-1+cx}\,\sqrt{1+}}}{b} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\text{Subst}\Big(\int \frac{\cosh(x)}{a+bx}\,dx,x,}{bc} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\Big(\int}{} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\Big(\frac{a}{b}+\cosh}{b^2c}
\end{aligned}
$$

Mathematica [A]
time = 0.69, size = 338, normalized size = 1.32

$$
-\frac{4bc^2d\sqrt{\frac{-1+cx}{1+cx}}+4bc^3dx\sqrt{\frac{-1+cx}{1+cx}}+4bc^2ex^2\sqrt{\frac{-1+cx}{1+cx}}+4bc^3ex^3\sqrt{\frac{-1+cx}{1+cx}}-(4c^2d+e)\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3e\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4ac^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+ae\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4bc^2d\cosh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+be\cosh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3ae\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+3be\cosh^{-1}(cx)\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{4b^2c^3\left(a+b\cosh^{-1}(cx)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/4*(4*b*c^2*d*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*d*x*Sqrt[(-1 + c*x)/(1
 + c*x)] + 4*b*c^2*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*e*x^3*Sqrt[(-
1 + c*x)/(1 + c*x)] - (4*c^2*d + e)*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshInte
gral[a/b + ArcCosh[c*x]] - 3*e*(a + b*ArcCosh[c*x])*Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcCosh[c*x])] + 4*a*c^2*d*Sinh[a/b]*SinhIntegral[a/b + ArcCos
h[c*x]] + a*e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 4*b*c^2*d*ArcCos
h[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*e*ArcCosh[c*x]*Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*e*Sinh[(3*a)/b]*SinhIntegral[3*(
a/b + ArcCosh[c*x])] + 3*b*e*ArcCosh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b
 + ArcCosh[c*x])])/(b^2*c^3*(a + b*ArcCosh[c*x]))
```

Maple [A]
time = 7.84, size = 465, normalized size = 1.81

method	result

derivativedivides	$\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e}{8c^2b(a+b\,\text{arccosh}(cx))}-\frac{3e\,e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e(}{}$
default	$\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e}{8c^2b(a+b\,\text{arccosh}(cx))}-\frac{3e\,e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e/c^2/b/(a+b*arccosh(c*x))-3/8*e/c^2/b^2*exp(3*a/b)*Ei(1
,3*arccosh(c*x)+3*a/b)-1/8*e/c^2/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8*e/c^2/b^
2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+c*x)*d/b/(a+b*arccosh(c*x))+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e/c^2/b
/(a+b*arccosh(c*x))-1/2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/8/c^2/b^2*e
xp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a
+b*arccosh(c*x))*d-1/8/c^2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh
(c*x))*e-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d-1/8/c^2/b^2*exp(-a/b)*
Ei(1,-arccosh(c*x)-a/b)*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^5*e + (c^3*d - c*e)*x^3 - c*d*x + (c^2*x^4*e + (c^2*d - e)*x^2 - d)
*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^
2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*x^6*e + (c^
5*d - 6*c^3*e)*x^4 + (3*c^3*x^4*e + (c^3*d - c*e)*x^2 + c*d)*(c*x + 1)*(c*x
 - 1) - (2*c^3*d - 3*c*e)*x^2 + (6*c^4*x^5*e + (2*c^4*d - 7*c^2*e)*x^3 - (c
^2*d - 2*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*
(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x
^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt
(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral((x^2*e + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**2,x)
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{(a+b\operatorname{acosh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^2,x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^2, x)
```

3.545 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}$$

```
[Out] Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c-Shi((a+b*arccosh(c*x))/b)*sinh(a/
b)/b^2/c-(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.21, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5880, 5953, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) + (Cosh[a/b]*C
oshIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c) - (Sinh[a/b]*SinhIntegral[(a +
 b*ArcCosh[c*x])/b])/(b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}\,dx}{b}\\
&= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{bc}\\
&= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{bc}-\text{sin}\\
&= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{c}\right.}{b^2c}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 80, normalized size = 0.89

$$\frac{-\frac{b\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}{a+b\cosh^{-1}(cx)}+\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(a + b*ArcCosh[c*x])) + Cosh[a/
b]*CoshIntegral[a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]])/(b^2*c)
```

Maple [A]
time = 4.15, size = 125, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{-\sqrt{cx-1}\,\sqrt{cx+1}+cx}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{cx-1}\,\sqrt{cx+1}}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{ar}\right.}{2b^2}}{c}$
default	$\frac{\frac{-\sqrt{cx-1}\,\sqrt{cx+1}+cx}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{cx-1}\,\sqrt{cx+1}}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{ar}\right.}{2b^2}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)/b/(a+b*arccosh(c*x))-1/2/b^2*ex
p(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*
arccosh(c*x))-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^2 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate((c^4*x^4 - 2*c^2*x^2 + (c^2*x^2 + 1)*(c*x + 1)*(c*x - 1) + (2*
c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x +
1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2
*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2,x)
[Out] Integral((a + b*acosh(c*x))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^2,x)
[Out] int(1/(a + b*acosh(c*x))^2, x)
```

3.546 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^4*e
 - a*b*c*d + (a*b*c^3*d - a*b*c*e)*x^2 + (a*b*c^2*x^3*e + a*b*c^2*d*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) + (b^2*c^3*x^4*e - b^2*c*d + (b^2*c^3*d - b^2*c*e)*
x^2 + (b^2*c^2*x^3*e + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate((c^5*x^6*e - (c^5*d + 2*c^3*e)*x^
4 + (c^3*x^4*e - (c^3*d + 3*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d
+ c*e)*x^2 + (2*c^4*x^5*e - (2*c^4*d + 5*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) - c*d)/(a*b*c^5*x^8*e^2 + 2*(a*b*c^5*d*e - a*b*c^3*
e^2)*x^6 + a*b*c*d^2 + (a*b*c^5*d^2 - 4*a*b*c^3*d*e + a*b*c*e^2)*x^4 + (a*b
*c^3*x^6*e^2 + 2*a*b*c^3*d*x^4*e + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) - 2
*(a*b*c^3*d^2 - a*b*c*d*e)*x^2 + 2*(a*b*c^4*x^7*e^2 - a*b*c^2*d^2*x + (2*a*
b*c^4*d*e - a*b*c^2*e^2)*x^5 + (a*b*c^4*d^2 - 2*a*b*c^2*d*e)*x^3)*sqrt(c*x
+ 1)*sqrt(c*x - 1) + (b^2*c^5*x^8*e^2 + 2*(b^2*c^5*d*e - b^2*c^3*e^2)*x^6 +
 b^2*c*d^2 + (b^2*c^5*d^2 - 4*b^2*c^3*d*e + b^2*c*e^2)*x^4 + (b^2*c^3*x^6*e
^2 + 2*b^2*c^3*d*x^4*e + b^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^3*
d^2 - b^2*c*d*e)*x^2 + 2*(b^2*c^4*x^7*e^2 - b^2*c^2*d^2*x + (2*b^2*c^4*d*e
- b^2*c^2*e^2)*x^5 + (b^2*c^4*d^2 - 2*b^2*c^2*d*e)*x^3)*sqrt(c*x + 1)*sqrt(
c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arccosh(c*x)^2 + 2*(a*b
*x^2*e + a*b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2}\left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)), x)
```

3.547 $\int \frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{1}{(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex^2)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^6*e
^2 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*x^4 + (a*b*c^3*d^2 - 2*a*b*c*d
*e)*x^2 + (a*b*c^2*x^5*e^2 + 2*a*b*c^2*d*x^3*e + a*b*c^2*d^2*x)*sqrt(c*x +
1)*sqrt(c*x - 1) + (b^2*c^3*x^6*e^2 - b^2*c*d^2 + (2*b^2*c^3*d*e - b^2*c*e^
2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*d*e)*x^2 + (b^2*c^2*x^5*e^2 + 2*b^2*c^2*d*x
^3*e + b^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))) - integrate((3*c^5*x^6*e - (c^5*d + 6*c^3*e)*x^4 + (3*c^3*x
^4*e - (c^3*d + 5*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d + 3*c*e)*x
^2 + (6*c^4*x^5*e - (2*c^4*d + 11*c^2*e)*x^3 + (c^2*d + 4*e)*x)*sqrt(c*x +
1)*sqrt(c*x - 1) - c*d)/(a*b*c^5*x^10*e^3 + (3*a*b*c^5*d*e^2 - 2*a*b*c^3*e^
3)*x^8 + (3*a*b*c^5*d^2*e - 6*a*b*c^3*d*e^2 + a*b*c*e^3)*x^6 + a*b*c*d^3 +
(a*b*c^5*d^3 - 6*a*b*c^3*d^2*e + 3*a*b*c*d*e^2)*x^4 + (a*b*c^3*x^8*e^3 + 3*
a*b*c^3*d*x^6*e^2 + 3*a*b*c^3*d^2*x^4*e + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x -
 1) - (2*a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + 2*(a*b*c^4*x^9*e^3 - a*b*c^2*d^
3*x + (3*a*b*c^4*d*e^2 - a*b*c^2*e^3)*x^7 + 3*(a*b*c^4*d^2*e - a*b*c^2*d*e^
2)*x^5 + (a*b*c^4*d^3 - 3*a*b*c^2*d^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) +
 (b^2*c^5*x^10*e^3 + (3*b^2*c^5*d*e^2 - 2*b^2*c^3*e^3)*x^8 + (3*b^2*c^5*d^2
*e - 6*b^2*c^3*d*e^2 + b^2*c*e^3)*x^6 + b^2*c*d^3 + (b^2*c^5*d^3 - 6*b^2*c^
3*d^2*e + 3*b^2*c*d*e^2)*x^4 + (b^2*c^3*x^8*e^3 + 3*b^2*c^3*d*x^6*e^2 + 3*b
^2*c^3*d^2*x^4*e + b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) - (2*b^2*c^3*d^3 -
3*b^2*c*d^2*e)*x^2 + 2*(b^2*c^4*x^9*e^3 - b^2*c^2*d^3*x + (3*b^2*c^4*d*e^2
- b^2*c^2*e^3)*x^7 + 3*(b^2*c^4*d^2*e - b^2*c^2*d*e^2)*x^5 + (b^2*c^4*d^3 -
 3*b^2*c^2*d^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*
x^2*e + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)
*arccosh(c*x)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{acosh}\left(c\,x\right))^2\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^2), x)
```

3.548 $\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx = \int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 20.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)*sqrt(x^2*e + d
)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))) + integrate((2*c^5*x^6*e + (c^5*d - 4*c^3*e)*x^4 + (2*c^3*
x^4*e + c^3*d*x^2 + c*d)*(c*x + 1)*(c*x - 1) - 2*(c^3*d - c*e)*x^2 + (4*c^4
*x^5*e + 2*(c^4*d - 2*c^2*e)*x^3 - (c^2*d - e)*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + c*d)*sqrt(x^2*e + d)/(a*b*c^5*x^6*e + (a*b*c^5*d - 2*a*b*c^3*e)*x^4 +
a*b*c*d + (a*b*c^3*x^4*e + a*b*c^3*d*x^2)*(c*x + 1)*(c*x - 1) - (2*a*b*c^3*
d - a*b*c*e)*x^2 + 2*(a*b*c^4*x^5*e - a*b*c^2*d*x + (a*b*c^4*d - a*b*c^2*e)
*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6*e + (b^2*c^5*d - 2*b^2*c^3
*e)*x^4 + b^2*c*d + (b^2*c^3*x^4*e + b^2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) - (
2*b^2*c^3*d - b^2*c*e)*x^2 + 2*(b^2*c^4*x^5*e - b^2*c^2*d*x + (b^2*c^4*d -
b^2*c^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{(a + b\operatorname{acosh}{(cx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x))^2, x)
```

3.549 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 17.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*x^2
+ sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*sqrt(x^2*e + d)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x - a*b*c)*sqrt(x^2*e + d)) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (
(c^3*d + 2*c*e)*x^2 + c*d)*(c*x + 1)*(c*x - 1) + (2*(c^4*d + c^2*e)*x^3 - (
c^2*d + e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/((b^2*c^5*x^6*e + (b^2*c^5
*d - 2*b^2*c^3*e)*x^4 + b^2*c*d + (b^2*c^3*x^4*e + b^2*c^3*d*x^2)*(c*x + 1)
*(c*x - 1) - (2*b^2*c^3*d - b^2*c*e)*x^2 + 2*(b^2*c^4*x^5*e - b^2*c^2*d*x +
 (b^2*c^4*d - b^2*c^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*x^6*e + (a*b*c^5*d - 2*a*
b*c^3*e)*x^4 + a*b*c*d + (a*b*c^3*x^4*e + a*b*c^3*d*x^2)*(c*x + 1)*(c*x - 1
) - (2*a*b*c^3*d - a*b*c*e)*x^2 + 2*(a*b*c^4*x^5*e - a*b*c^2*d*x + (a*b*c^4
*d - a*b*c^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arccosh(c
*x)^2 + 2*(a*b*x^2*e + a*b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2} \sqrt{d + e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{e\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

3.550 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 165.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*x^4*
e - b^2*c*d + (b^2*c^3*d - b^2*c*e)*x^2 + (b^2*c^2*x^3*e + b^2*c^2*d*x)*sqr
t(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1)) + (a*b*c^3*x^4*e - a*b*c*d + (a*b*c^3*d - a*b*c*e)*x^2 + (a*b*c^2*x^3
*e + a*b*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)) - integrate
((2*c^5*x^6*e - (c^5*d + 4*c^3*e)*x^4 + (2*c^3*x^4*e - (c^3*d + 4*c*e)*x^2
- c*d)*(c*x + 1)*(c*x - 1) + 2*(c^3*d + c*e)*x^2 + (4*c^4*x^5*e - 2*(c^4*d
+ 4*c^2*e)*x^3 + (c^2*d + 3*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*
c^5*x^8*e^2 + 2*(b^2*c^5*d*e - b^2*c^3*e^2)*x^6 + b^2*c*d^2 + (b^2*c^5*d^2
- 4*b^2*c^3*d*e + b^2*c*e^2)*x^4 + (b^2*c^3*x^6*e^2 + 2*b^2*c^3*d*x^4*e + b
^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^3*d^2 - b^2*c*d*e)*x^2 + 2*(
b^2*c^4*x^7*e^2 - b^2*c^2*d^2*x + (2*b^2*c^4*d*e - b^2*c^2*e^2)*x^5 + (b^2*
c^4*d^2 - 2*b^2*c^2*d*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*x^8*e^2 + 2*(a*b*c^5*d*e
- a*b*c^3*e^2)*x^6 + a*b*c*d^2 + (a*b*c^5*d^2 - 4*a*b*c^3*d*e + a*b*c*e^2)*
x^4 + (a*b*c^3*x^6*e^2 + 2*a*b*c^3*d*x^4*e + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*
x - 1) - 2*(a*b*c^3*d^2 - a*b*c*d*e)*x^2 + 2*(a*b*c^4*x^7*e^2 - a*b*c^2*d^2
*x + (2*a*b*c^4*d*e - a*b*c^2*e^2)*x^5 + (a*b*c^4*d^2 - 2*a*b*c^2*d*e)*x^3)
*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*
e^2 + 2*b^2*d*x^2*e + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^
2*e + a*b*d^2)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

3.551 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*x^6*
e^2 - b^2*c*d^2 + (2*b^2*c^3*d*e - b^2*c*e^2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*
d*e)*x^2 + (b^2*c^2*x^5*e^2 + 2*b^2*c^2*d*x^3*e + b^2*c^2*d^2*x)*sqrt(c*x +
 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 (a*b*c^3*x^6*e^2 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*x^4 + (a*b*c^3*
d^2 - 2*a*b*c*d*e)*x^2 + (a*b*c^2*x^5*e^2 + 2*a*b*c^2*d*x^3*e + a*b*c^2*d^2
*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)) - integrate((4*c^5*x^6*e
- (c^5*d + 8*c^3*e)*x^4 + (4*c^3*x^4*e - (c^3*d + 6*c*e)*x^2 - c*d)*(c*x +
1)*(c*x - 1) + 2*(c^3*d + 2*c*e)*x^2 + (8*c^4*x^5*e - 2*(c^4*d + 7*c^2*e)*x
^3 + (c^2*d + 5*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*c^5*x^10*e^3
 + (3*b^2*c^5*d*e^2 - 2*b^2*c^3*e^3)*x^8 + (3*b^2*c^5*d^2*e - 6*b^2*c^3*d*e
^2 + b^2*c*e^3)*x^6 + b^2*c*d^3 + (b^2*c^5*d^3 - 6*b^2*c^3*d^2*e + 3*b^2*c*
d*e^2)*x^4 + (b^2*c^3*x^8*e^3 + 3*b^2*c^3*d*x^6*e^2 + 3*b^2*c^3*d^2*x^4*e +
 b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) - (2*b^2*c^3*d^3 - 3*b^2*c*d^2*e)*x^2
 + 2*(b^2*c^4*x^9*e^3 - b^2*c^2*d^3*x + (3*b^2*c^4*d*e^2 - b^2*c^2*e^3)*x^7
 + 3*(b^2*c^4*d^2*e - b^2*c^2*d*e^2)*x^5 + (b^2*c^4*d^3 - 3*b^2*c^2*d^2*e)*
x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2*e + d)*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)) + (a*b*c^5*x^10*e^3 + (3*a*b*c^5*d*e^2 - 2*a*b*c^3*e^3)*x^8 +
 (3*a*b*c^5*d^2*e - 6*a*b*c^3*d*e^2 + a*b*c*e^3)*x^6 + a*b*c*d^3 + (a*b*c^5
*d^3 - 6*a*b*c^3*d^2*e + 3*a*b*c*d*e^2)*x^4 + (a*b*c^3*x^8*e^3 + 3*a*b*c^3*
d*x^6*e^2 + 3*a*b*c^3*d^2*x^4*e + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) - (2
*a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + 2*(a*b*c^4*x^9*e^3 - a*b*c^2*d^3*x + (3
*a*b*c^4*d*e^2 - a*b*c^2*e^3)*x^7 + 3*(a*b*c^4*d^2*e - a*b*c^2*d*e^2)*x^5 +
 (a*b*c^4*d^3 - 3*a*b*c^2*d^2*e)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(x^2
*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)/(a^2*x^6*e^3 + 3*a^2*d*x^4*e^2 + 3*a^2*d^2*x^2*e +
 a^2*d^3 + (b^2*x^6*e^3 + 3*b^2*d*x^4*e^2 + 3*b^2*d^2*x^2*e + b^2*d^3)*arcc
osh(c*x)^2 + 2*(a*b*x^6*e^3 + 3*a*b*d*x^4*e^2 + 3*a*b*d^2*x^2*e + a*b*d^3)*
arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2} \left(d + ex^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

3.552 $\int (d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}\, dx$

Optimal. Leaf size=672

$$d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,d^2e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b}}{}\right.}{4c}$$

```
[Out] -1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2
)*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)-1/72*d*e*exp(3*a/b)*erf(3^(1/
2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*e^2
*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*P
i^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)
*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/192*e^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^
(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)-1/4*d^2*exp(a/b)*erf
((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a/b)*erf(
(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/32*e^2*exp(a/b)*er
f((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*erfi((a+b*
arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)-1/8*d*e*erfi((a+b*
arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32*e^2*erfi((a
+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+d^2*x*(a+b*ar
ccosh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arccosh(c*x))^(1/2)+1/5*e^2*x^5*(a+b*arc
cosh(c*x))^(1/2)
```

Rubi [A]
time = 1.59, antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5909, 5879, 5953, 3388, 2211, 2236, 2235, 5884, 3393}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^2e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{320c^5}-\frac{\sqrt{\pi}\sqrt{b}\,dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{\pi}\sqrt{b}\,dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,dee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{\pi}\sqrt{b}\,d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,d^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+d^2x\sqrt{a+b\cosh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcCosh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcCosh[c*x]])/5 - (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3) - (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5) - (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(64*c^5) - (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
```

```
*ArcCosh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) - (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}\, dx = \int \left(d^2\sqrt{a+b\cosh^{-1}(cx)} + 2dex^2\sqrt{a+b\cosh^{-1}(cx)} + e^2x^4\sqrt{a -$$

$$= d^2\int \sqrt{a+b\cosh^{-1}(cx)}\, dx + (2de)\int x^2\sqrt{a+b\cosh^{-1}(cx)}\, dx +$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+$$

Mathematica [**A**]
time = 4.39, size = 536, normalized size = 0.80

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcCosh[c*x]] + 8*b*c^4*d^2*Arc
Cosh[c*x]*Sqrt[a/b + ArcCosh[c*x]] - b*e*(4*c^2*d + e)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, a/b + ArcCosh[c
*x]] - 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])
```

```
^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcCosh[c*x]))/b] - E^((2*a)/b)*(25*Sqrt[3]*
b*e*(8*c^2*d + 3*e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])^2/
b^2)]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 450*E^((2*a)/b)*(8*a*c^4*d^
2*Sqrt[-((a + b*ArcCosh[c*x])/b)] + 8*b*c^4*d^2*ArcCosh[c*x]*Sqrt[-((a + b*
ArcCosh[c*x])/b)] + b*e*(4*c^2*d + e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a +
b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] + b*e*E^((6*
a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]*(
25*Sqrt[3]*(8*c^2*d + 3*e)*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b] + 9*Sqrt[
5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcCosh[c*x]))/b]))))/(7200*c^5*E^((5
*a)/b)*(a + b*ArcCosh[c*x])^(3/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\mathrm{arccosh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
[Out] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^2*sqrt(b*arccosh(c*x) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(c x\right)}\left(d+e x^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

3.553 $\int (d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=322

$$dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,ee^{a/b}\sqrt{\pi}\,\mathrm{E}}{}$$

```
[Out] -1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3
^(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*
b^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/4*d*exp(a/b)*erf((a+b*arccosh(c*x
))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arccosh(c*x))
^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arccosh(c*x))^(1/2)/b^
(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)-1/16*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arccosh(c*x))^(1/2)+1/3*e*x^3*
(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]

time = 0.83, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5909, 5879, 5953, 3388, 2211, 2236, 2235, 5884, 3393}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcCosh[c*x]] + (e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
```

```
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}\,dx &= \int \left(d\sqrt{a+b\cosh^{-1}(cx)}+ex^2\sqrt{a+b\cosh^{-1}(cx)}\right)dx \\
&= d\int \sqrt{a+b\cosh^{-1}(cx)}\,dx+e\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{1}{2}(bcd)\int \frac{}{\sqrt{-1-}} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{(bd)\text{Subst}\left(\int \frac{c}{\sqrt{a}}\right.}{} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{(bd)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\right.}{} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}}{} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}}{} \\
&= dx\sqrt{a+b\cosh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}}{}
\end{aligned}$$

Mathematica [A]

time = 1.88, size = 317, normalized size = 0.98

$$\frac{de^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}+\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+9e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right)\right)}{72c^3\sqrt{-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4
*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqr
t[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E
^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \sqrt{a+b\,\mathrm{arccosh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)*sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(cx\right)}\left(d+ex^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2), x)
```

3.554 $\int \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$x\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

```
[Out] -1/4*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*
erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*a
rccosh(c*x))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 5953, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,\sqrt{b}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\,\sqrt{b}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cosh^{-1}(cx)}\,dx &= x\sqrt{a+b\cosh^{-1}(cx)}-\frac{1}{2}(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}\,dx\\
&= x\sqrt{a+b\cosh^{-1}(cx)}-\frac{b\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{2c}\\
&= x\sqrt{a+b\cosh^{-1}(cx)}-\frac{b\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{4c}-\frac{b\text{Subst}\left(\int \right.}{}\\
&= x\sqrt{a+b\cosh^{-1}(cx)}-\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{2c}-\frac{\text{Subst}\left(\int \right.}{}\\
&= x\sqrt{a+b\cosh^{-1}(cx)}-\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{e}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 100, normalized size = 0.98

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]],x]
[Out] (Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqr
t[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b
*ArcCosh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arccosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2),x)
[Out] int((a+b*arccosh(c*x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,\text{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2), x)
```

3.555 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

Rubi [A]

time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx=\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A]

time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arccosh}\left(cx\right)}}{e\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(x^2*e + d), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\operatorname{acosh}(cx)}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + b \operatorname{acosh}(c x)}}{e x^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2), x)
```

3.556 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]
[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Mathematica [A]
time = 14.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arccosh}\,(cx)}}{(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(x^2*e + d)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{acosh}{\left(cx \right)}}}{\left(d + ex^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d)**2,x)
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2)^2,x)
[Out] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

3.557 $\int (d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=442

$$-\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{3c^3}-\frac{bex^2\sqrt{-1+c}}{}$$

```
[Out] d*x*(a+b*arccosh(c*x))^(3/2)+1/3*e*x^3*(a+b*arccosh(c*x))^(3/2)-1/288*b^(3/
2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/c^3/exp(3*a/b)-3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arccosh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)+3/32*b^(3/2)*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*
x))^(1/2)/c-1/3*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2)/c^
3-1/6*b*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2)/c
```

Rubi [A]
time = 1.12, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5909, 5879, 5915, 5881, 3389, 2211, 2236, 2235, 5884, 5939, 5887, 5556}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}\,b^{3/2}e e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}\,b^{3/2}d e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}d e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{be\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{3c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{3bd\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{bex^2\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) - (b*e
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*e*x^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + d*x*(a + b*A
rcCosh[c*x])^(3/2) + (e*x^3*(a + b*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*d*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*
E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)
*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8
*c*E^(a/b)) + (3*b^(3/2)*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx = \int \left(d\left(a+b\cosh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\right)\,dx$$

$$= d\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx + e\int x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bcd)\int$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+}}{}$$

Mathematica [A]
time = 2.33, size = 812, normalized size = 1.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])
/Sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a
 + b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcCosh[c*x]]*(9*E
^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] +
 Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] +
 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b
)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a +
 b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^
2)]) + (b*d*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*
x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((
2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]))/Sqrt[b]))/(8*c) + (Sqrt[b]*e*(9*(-12*Sqrt[b]*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[c*x]*Sqrt[a
 + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqr
t[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[
3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*
a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*
a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*ArcCosh[c*x
]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(288*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)*(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acosh}(cx))^{\frac{3}{2}} \left(d + ex^2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)*(d + e*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^(3/2)*(d + e*x^2), x)
```

3.558 $\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}\right.}{8c}$$

```
[Out] x*(a+b*arccosh(c*x))^(3/2)-3/8*b^(3/2)*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2
)/c
```

Rubi [A]
time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5879, 5915, 5881, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{4}(\ldots$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b\ldots}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b\ldots}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b\ldots}{\ldots}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{3b^{3}\ldots}{\ldots}$$

Mathematica [A]
time = 0.51, size = 269, normalized size = 1.92

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}+\frac{b\left(-12\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{a+b\cosh^{-1}(cx)}+8cx\cosh^{-1}(cx)\sqrt{a+b\cosh^{-1}(cx)}+\frac{(2a+3b)\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)}{\sqrt{b}}+\frac{(2a-3b)\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)}{\sqrt{b}}\right)}{8c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c
*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a
/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*c)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{acosh}(cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2),x)
[Out] int((a + b*acosh(c*x))^(3/2), x)
```

3.559 $\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2},x\right)$$

[Out] Unintegrable((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2}\,dx$$

Mathematica [A]
time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{d+ex^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]

[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^{\frac{3}{2}}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(x^2*e + d), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^{\frac{3}{2}}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)/(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2), x)
```

3.560 $\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2}, x\right)$$

[Out] Unintegrable((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2} dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2} dx = \int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2} dx$$

Mathematica [A]
time = 8.76, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^{3/2}}{(d+ex^2)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]

[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^{\frac{3}{2}}}{(e\,x^2+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(x^2*e + d)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)/(d + e*x**2)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

3.561 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$-\frac{d^2e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{dee^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}-\frac{e^2e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,c^5}$$

```
[Out] -1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)
*Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)-1/12*d*e*exp(3*a/b)*erf(3^(1/2)
*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e*er
fi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b
)/b^(1/2)-1/2*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c
/b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^
3/b^(1/2)-1/16*e^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/
c^5/b^(1/2)+1/2*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a
/b)/b^(1/2)+1/4*d*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp
(a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^5/
exp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)+1/32*e^2*erfi(3^(1/2)*(a+b*arccosh(c*x
))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5909, 5881, 3389, 2211, 2236, 2235, 5887, 5556}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}-\frac{\sqrt{\frac{\pi}{5}}e^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{5}}e^2e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}c^5}-\frac{\sqrt{\pi}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -1/2*(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(Sqrt[b]*
c) - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b
]*c^3) - (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*S
qrt[b]*c^5) - (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
*x]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) - (e^2*E^((5*a)/b)*Sqr
t[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) +
 (d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)
) + (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^
```

```
(a/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*
c^5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
```

```
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{2dex^2}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx+e^2\int \frac{}{\sqrt{a+b}}$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{bc}+\frac{(2de)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,d\right.}{c^3}$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(cx)\right)}{2bc}+\frac{d^2\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a\right.}{2bc}$$

$$= -\frac{d^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{bc}+\frac{d^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{\right.}{bc}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{2\sqrt{b}\,c}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{2\sqrt{b}\,c}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{dee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{4\sqrt{b}\,c^3}$$

Mathematica [**A**]

time = 0.79, size = 530, normalized size = 0.87

$$\frac{e^{-\frac{5a}{b}}\left(30(8c^4d^2+4c^2de+e^2)e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+3\sqrt{5}\,e^2\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(cx))}{b}\right)+40\sqrt{3}\,c^2dee^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+15\sqrt{3}\,e^2e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+120c^2dee^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+30e^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+40\sqrt{3}\,c^2dee^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+15\sqrt{3}\,e^2e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(cx))}{b}\right)+3\sqrt{5}\,e^2e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(cx))}{b}\right)\right)}{480c^5\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (30*(8*c^4*d^2 + 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, a/b + ArcCosh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcCosh[c*x])/b)]*
Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*Sq
rt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 15*
Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
 b*ArcCosh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/
b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 120*c^2*d*e*E^((4*a)/b)*Sqrt[-(
(a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 30*e^2*E^(
(4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/
b)] + 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3
*(a + b*ArcCosh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c
*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*e^2*E^((10*a)/b)*Sq
rt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b])/(480*c^5*E^(
(5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{arccosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{acosh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^2}{\sqrt{a+b\operatorname{acosh}(c\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x))^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x))^(1/2), x)
```

3.562 $\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{ee^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\ldots}}{\ldots}\right)}{8\sqrt{b}\,c^3}$$

```
[Out] -1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)-1/2*d*exp(a/b)*erf((a+b*arccosh(c*x))
^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1
/2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/
2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)+1/8*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5909, 5881, 3389, 2211, 2236, 2235, 5887, 5556}

$$-\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -1/2*(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(Sqrt[b]*c)
 - (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^
3) - (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/
(2*Sqrt[b]*c*E^(a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
*x]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d}{\sqrt{a+b\cosh^{-1}(cx)}}+\frac{ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx + e\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= -\frac{d\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x\right.}{c^3}$$

$$= -\frac{d\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{d\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a\right.}{2bc}$$

$$= -\frac{d\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{d\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{bc}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{2\sqrt{b}\,c}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right.}{2\sqrt{b}\,c}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}$$

Mathematica [A]
time = 0.45, size = 213, normalized size = 0.74

$$\frac{e^{-\frac{3a}{b}}\left(3(4c^2d+e)\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\,e\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+3(4c^2d+e)\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)+\sqrt{3}\,ee^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\right)}{24c^3\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (3*(4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b
*ArcCosh[c*x]))/b] + 3*(4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x]
```

```
)/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*e*E^((6*a)/b)*Sqrt[a/
b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b])/(24*c^3*E^((3*a)/
b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{a+b\,\mathrm{acosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*acosh(c*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(b*arccosh(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^(1/2),x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^(1/2), x)
```

3.563 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*
erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5881, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -1/2*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(Sqrt[b]*c) +
 (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+\ldots\right.}{2bc}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\ldots}\right.}{bc}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.14

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(2*c*E^(
a/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(c*x))^(1/2), x)
```

3.564 $\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx=\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\sqrt{a+b\,\mathrm{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)*sqrt(b*arccosh(c*x) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}{\left(cx \right)}} \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*acosh(c*x))*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)), x)
```

3.565 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx = \int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2 \sqrt{a+b\,\mathrm{arccosh}\,(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*sqrt(b*arccosh(c*x) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \left(d + e x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*acosh(c*x))*(d + e*x**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

3.566 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=358

$$-\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{de^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{ee^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\right.}{}$$

```
[Out] d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e*e
xp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erfi((
a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)+1/4*e*erfi((a+
b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)+1/4*e*exp(3*a/
b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c
^3+1/4*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^
(3/2)/c^3/exp(3*a/b)-2*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
^(1/2)-2*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5909, 5880, 5953, 3388, 2211, 2236, 2235, 5885}

$$\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2d\sqrt{cx-1}\,\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (2*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (d*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (e*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(
a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*
E^(a/b)) + (e*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
```

```
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx &= \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}+\frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\right)dx \\
&= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx+e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx \\
&= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(2cd)\int \frac{}{\sqrt{-1+cx}\,\sqrt{}}}{} \\
&= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(2d)\text{Subst}\left(\int \frac{\cosh(x}{\sqrt{a+}}\right.}{} \\
&= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{d\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\right.}{bc} \\
&= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(2d)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\right.}{} \\
&= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{2ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+}}{}\right.}{b^{3/2}c}
\end{aligned}$$

Mathematica [A]
time = 1.36, size = 268, normalized size = 0.75

$$\frac{e^{-\frac{3a}{b}}\left(-\left(\left(4c^2d+e\right)e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sqrt{3}\,e\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+\left(4c^2d+e\right)e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)-e^{\frac{3a}{b}}\left(2\left(4c^2d+e\right)\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)+\sqrt{3}\,ee^{\frac{3a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+2e\sinh\left(3\cosh^{-1}(cx)\right)\right)\right)}{4bc^3\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-((4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]]) + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c*x]))/b] + (4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - E^((3*a)/b)*(2*(4*c^2*d + e)*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sqrt[3]*e*E^((3*a)/b)*Sqrt[a/b + ArcC
osh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 2*e*Sinh[3*ArcCosh[c*x]]
))/(4*b*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^{2}}{\left(a + b\,\mathrm{acosh}\left(cx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^(3/2),x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^(3/2), x)
```

3.567 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

```
[Out] exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+b
*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5880, 5953, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} + \frac{2}{}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{er}}{}$$

Mathematica [A]
time = 0.22, size = 132, normalized size = 1.10

$$\frac{e^{-\frac{a}{b}}\left(-2e^{a/b}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx) - e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right) + \sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b
)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(b*c*E^(a/b)*Sqrt[a + b*ArcCosh[c
*x]])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^(3/2),x)
[Out] int(1/(a + b*acosh(c*x))^(3/2), x)
```

3.568 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}} \, dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}} \, dx = \int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}} \, dx$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)\left(a+b\,\mathrm{arccosh}\,(cx)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2*e + d)*(b*arccosh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))^{\frac{3}{2}}(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
[Out] Integral(1/((a + b*acosh(c*x))**(3/2)*(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)), x)
```

3.569 $\int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\cosh^{-1}(cx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2*e + d)^2*(b*arccosh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```